Measurement Fusion for Track Initialization of Objects Ejected by a Carrier

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Abstract

This work deals with objects ejected simultaneously by a moving carrier. The objects leave the carrier like "star fireworks", (an expanding cluster) with common-magnitude lateral velocities (same speed) orthogonal on the carrier's velocity vector and a common longitudinal ejection velocity (along the carrier's velocity vector). These objects are unguided, so they can be tracked with conventional algorithms for ballistic targets [3, 2], so the most accurate initialization of their tracks is of interest. The approach developed in this work is to take advantage of the commonality of the ejection speeds among the objects of interest, both the lateral as well as the longitudinal one, to estimate their velocities. The two parameters that need to be estimated to initialize the ejected objects' tracks are the dispersion (lateral) speed and longitudinal ejection speed. This will be accomplished by fusing the measurements from the ejected objects once they are resolved via a Maximum Likelihood (ML) approach. This requires the Likelihood Function (LF) of a parameter of interest (in the present case the two speeds) based on the available radar position measurements, i.e. the probability density function (pdf) of these measurements conditioned on the parameter. Since both parameters of interest are speeds (velocity magnitudes), there is no well-defined pdf of the measurements conditioned on these speeds. Consequently, we introduce, for each parameter, the new notion'of "parameter-observable function of the measurements", which (i) has a well-defined pdf conditioned on the parameter and (ii) provides all the information about the parameter, i.e. it is a sufficient statistic. The accuracy of estimation presented here for the two speeds is shown to be significantly better than the conventional two-point differencing approach [1] applied for each ejected object separately. A second method of estimating these speeds is using the radar's Doppler measurements. A comparison between the two methods — using position measurements and Doppler — is carried out. The combined use of both is also explored.

1 Introduction

2 Problem Formulation

A carrier is moving at a high altitude and is at time t_o at location

$$\mathbf{x}_c = [x_c \;\; y_c \;\; z_c]^\top [[xc]] \tag{1}$$

in the Cartesian East-North-Up (ENU) coordinate system centered at the radar In spherical coordinates, also centered at the radar, the location is

$$\boldsymbol{\xi}_c = [r_c \;\; a_c \;\; e_c]^\top [[carrsph]] \tag{2}$$

where the azimuth a is clockwise from N and the elevation e is w.r.t. the EN (horizontal) plane.

Its velocity vector $\mathbf{v}_c = v_c \mathbf{1}_{vc}$ at time t_o, where the carrier velocity unit vector is

$$\mathbf{1}_{vc} = [\sin\alpha_c \cos\epsilon_c \;\;\; \cos\alpha_c \cos\epsilon_c \;\;\; \sin\epsilon_c]^\top [[1vc]] \tag{3}$$

where we denote the pointing azimuth and elevation angles of $\mathbf{v}_c$ as α_c and ϵ_c in the ENU coordinate system centered at the radar tracking it, respectively. This vector will be needed for the generation of the ejected objects' dispersion velocities, which will be orthogonal to it.

This carrier is tracked by the radar with revisit intervals T. At time t_o the carrier opens to eject n objects. These objects have a common longitudinal (to the carrier) velocity $\mathbf{v}_L$ in addition to the carrier velocity $\mathbf{v}_c$, and lateral (dispersion) velocities $\mathbf{v}_{\delta i}$, $i = 1, \ldots, n$ (orthogonal to $\mathbf{v}_c$) with the same magnitude (speed) v_δ and the angles between them are $2\pi/n$. Their lateral motion is in the style of “star fireworks". While the goal is to eventually track separately the ejected objects, it is obvious that initially they will be unresolved from the carrier and among each other — they will appear as an extended object. As the ejected objects move forward from the carrier and from each other, they will become eventually resolved.

The goal of the proposed approach is to take advantage, after the ejected objects become resolved, of the same-magnitude lateral ejection velocities (i.e. same speed) and the common longitudinal ejection velocity of the ejected objects in developing an algorithm that can be run in real time and will yield accurate and consistent estimates of the speeds of the ejected objects to initialize their tracks.

3 Generation of the “star fireworks" velocities

While the ejected objects form an expanding circle if viewed with a line of sight (LOS) aligned with the carrier's velocity vector, the radar LOS is not aligned with this vector. Consequently, they will appear to the radar as if on an ellipse with unequal angles between them. Their velocities will be generated symmetrically in a horizontal circle centered at the radar and then translated and rotated to be orthogonal to the carrier velocity $\mathbf{v}_c$ as follows.

Consider a circle of unit radius at the origin of the ENU coordinate system with n points on it at locations

$$
\begin{aligned}
v_{xi} &= \sin i2\pi/n \\
v_{yi} &= \cos i2\pi/n \\
v_{zi} &= 0 \\
& \quad i = 1, \ldots, n
\end{aligned}
\tag{4}
$$

i.e. with the axis of this circle represented by the unit vector

$$
\mathbf{1}_a = \mathbf{1}_U = [0 \;\; 0 \;\; 1]^\top [[1a]] \tag{5}
$$

pointing Up in the ENU coordinates. The radius 1 will be the initial object dispersion speed and the corresponding vectors

$$
\mathbf{1}_{\mathbf{v}i} = [v_{xi} \; v_{yi} \; v_{zi}]^\top [[1vi]] \tag{6}
$$

will be translated to originate at $\mathbf{x}_c$ (the position where the ejection takes place) and rotated so they are all orthogonal to $\mathbf{v}_c$, i.e. the axis unit vector $\mathbf{1}_a$ will become aligned with the carrier velocity unit vector $\mathbf{1}_c$.

The ENU coordinates will be denoted as xyz and this coordinate system will be rotated in two stages to align the axis of the circle $\mathbf{1}_a = \mathbf{1}_z$ with $\mathbf{1}_c$. In order to obtain the dispersion velocities orthogonal to $\mathbf{v}_c$, the first rotation of xyz will be around $\mathbf{1}_z = \mathbf{1}_U$ from y to x to obtain $x'y'z'$ such that $\mathbf{1}_{y'}$ will have the carrier's velocity azimuth α_c in ENU. The rotation matrix for this is

$$
A_1 = \begin{bmatrix} \cos\alpha_c & \sin\alpha_c & 0 \\ -\sin\alpha_c & \cos\alpha_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}
$$

The second rotation will be around $\mathbf{1}_{x'}$ from z' to y' to obtain $x''y''z''$ such that $\mathbf{1}_{z''}$ has the elevation angle ϵ of $\mathbf{v}_c$. This rotation (by $\pi/2 - \epsilon$) is given by the matrix

$$
A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\epsilon_c & \cos\epsilon_c \\ 0 & -\cos\epsilon_c & \sin\epsilon_c \end{bmatrix} \tag{8}
$$

With the above rotations of $\mathbf{1}_a = \mathbf{1}_U = \mathbf{1}_z$ yielding $\mathbf{1}_{z''} = \mathbf{1}_c$, the dispersion velocities unity vectors (6) will become

$$
\mathbf{1}_{\mathbf{v}\delta i} = A_2 A_1 \mathbf{1}_{\mathbf{v}i} \triangleq A \mathbf{1}_{\mathbf{v}i} [[1vdeltai]] \tag{9}
$$

where

$$
A = \begin{bmatrix} \cos\alpha_c & \sin\alpha_c \sin\epsilon_c & \sin\alpha_c \cos\epsilon_c \\ -\sin\alpha_c & \cos\alpha_c \sin\epsilon_c & \cos\alpha_c \cos\epsilon_c \\ 0 & -\cos\epsilon_c & \sin\epsilon_c \end{bmatrix} [[rotA]] \tag{10}
$$

Note that the last column of A is the same as $\mathbf{1}_{vc}$ in (3) and the columns of A are mutually orthogonal.

Thus, since each $\mathbf{v}_{\delta i}$ is a linear combination $\mathbf{1}_{x''}$ and $\mathbf{1}_{y''}$, its unit vector has the property

$$\mathbf{1}_{\mathbf{v}\delta i} \perp \mathbf{v}_c[[1dispi]] \tag{11}$$

The dispersion velocities are (for known magnitude v_δ)

$$\mathbf{v}_{\delta i} = v_\delta \mathbf{1}_{\mathbf{v}\delta i}[[vdeltai]] \tag{12}$$

which completes the generation of the dispersion velocities orthogonal to $\mathbf{v}_c$.

Each object ejected from the carrier has initial position $\mathbf{x}_c$ and their initial velocities are

$$\mathbf{v}_i(t_o) = \mathbf{v}_c + \mathbf{v}_{\delta i} + \mathbf{v}_L[[vi]] \tag{13}$$

where v_δ is the dispersion (lateral) ejection speed and

$$\mathbf{v}_L = v_L \mathbf{1}_{vc} \tag{14}$$

is the longitudinal ejection velocity with speed v_L.

The position of the ejected object i at resolution time t_ρ is modeled using CV (constant velocity) motion plus gravity

$$\mathbf{x}_i(t_\rho) = \mathbf{x}_c(t_o) + \mathbf{v}_i(t_o)(t_\rho - t_o) + [0 \;\; 0 \;\; -1]^\top g \frac{(t_\rho - t_o)^2}{2}[[motioni]] \tag{15}$$

where g is the gravity acceleration. The position of the carrier at t_ρ is

$$\mathbf{x}_c(t_\rho) = \mathbf{x}_c(t_o) + v_{ce}\mathbf{1}_{vc}(t_\rho - t_o) + [0 \;\; 0 \;\; -1]^\top g \frac{(t_\rho - t_o)^2}{2}[[motionc]] \tag{16}$$

where v_{ce} is the carrier speed after ejection, given in (43).

The estimation of the dispersion (lateral) ejection speed v_δ as well as of the longitudinal ejection speed v_L using first position measurements, then Doppler measurements are discussed in the next section.

4 Modeling the Observables at Ejection Time and Estimation of the Ejection Speeds

It can be assumed that, since ejection requires opening the shell of the carrier, the radar will detect a sudden increase in the RCS (radar cross section) of the carrier. However, since the RCS fluctuates, a more reliable procedure for detecting the ejection is using the radar Doppler measurements, as discussed later in this section. The time when the carrier opening is detected is denoted as t_o.

Following the opening of the carrier, the objects are ejected in a symmetric manner (star fireworks style) around the axis of the carrier. This is modeled in Sec. 3 with dispersion velocities orthogonal to the carrier velocity. The ejected objects also have a common forward ejection velocity.

The carrier will slow down somewhat due the forward velocities of the ejected objects because of the momentum conservation. Ideally (with perfect resolution), the radar will see an elliptical shape of the dispersing objects (assuming the radar-to-carrier LOS is not aligned with the carrier velocity vector). However, since the radar has limited resolution, it will see initially an unresolved extended target of elliptical shape. Within a short time the ejected objects separate from each other. Denote the time at which the objects become resolved at t_ρ. Then one has a measurement for each as the initial estimate of their locations. To obtain the initial estimates of their velocities the procedure described in the sequel is proposed.

Assuming the dispersion velocities have the same magnitudes, the following “customized" ML approach can be used to take advantage of this:

The radar obtains at t_ρ the resolved range-azimuth-elevation position measurements of the ejected objects (in spherical coordinates centered at the radar)

$$\mathbf{u}_i(t_\rho) = [u_{ir} \;\; u_{i\alpha} \;\; u_{i\epsilon}]^\top = \mathbf{h}[\mathbf{x}_i(t_\rho)] + \mathbf{w}_i(t_\rho) \qquad i = 1, \ldots, n[[msphi]] \tag{17}$$

where $\mathbf{h}$ is the Cartesian-to-spherical conversion [2]

$$\mathbf{h}(x, y, z) = \left[\sqrt{x^2 + y^2 + z^2} \quad \tan^{-1} x/y \quad \tan^{-1} z/\sqrt{x^2 + y^2}\right]^\top [[h]] \tag{18}$$

and $\mathbf{w}_i$ the range-az-el measurement noise with independent components and independent across the objects, and

$$w_{ir} \sim \mathcal{N}(0, \sigma_r^2)[[wir]] \tag{19}$$

$$w_{i\alpha} \sim \mathcal{N}(0, \sigma_a^2)[[wia]] \tag{20}$$

$$w_{i\epsilon} \sim \mathcal{N}(0, \sigma_e^2)[[wie]] \tag{21}$$

The measurements are converted into Cartesian

$$\mathbf{z}_i(t_\rho) = \mathbf{h}^{-1}[\mathbf{u}_i(t_\rho)][[sph2cart]] \tag{22}$$

where

$$\mathbf{h}^{-1}(r, a, e) = [r \sin e \cos a \;\; r \sin e \sin a \;\; r \cos e]^\top [[h1]] \tag{23}$$

These will be in an elliptical shape since they are seen by radar at an angle. For the purpose of fusing them to estimate the common speed of the ejected objects, they will be centered at

$$\mathbf{z}_0(t_\rho) = (1/n) \sum_{i=1}^{n} \mathbf{z}_i(t_\rho)[[center]] \tag{24}$$

and rotated with the inverse of A given in (10) to be in an ideally circular shape. These rotated and centered measurements will be

$$\mathbf{y}_i = A^{-1}[\mathbf{z}_i(t_\rho) - \mathbf{z}_0(t_\rho)] \qquad i = 1, \ldots, n[[rotctdri]] \tag{25}$$

Denote the set of the above measurements as

$$Y = \{\mathbf{y}_i\}_{i=1}^{n}[[rotctrd]] \tag{26}$$

4.1 Estimation of the dispersion speed with the parameter-observable function of the position measurements

The Likelihood Function (LF) of the dispersion speed v_δ based on Y requires the pdf of Y conditioned on v_δ, which, however, does not exist since v_δ is not sufficient for this. Consequently, we define the parameter-observable function $f_\delta(Y)$ for v_δ as the function of Y that (i) has a well defined pdf conditioned on v_δ and (ii) provides all the information about v_δ.[1] The ML formulation for estimation of the dispersion speed v_δ is then based on this dispersion-observable $f_\delta(Y)$, conditioned on v_δ. This function $f_\delta(Y)$ consists of the distances

$$f_\delta(Y) = \{d_i\}_{i=1}^{n/2} \tag{27}$$

where

$$d_i = \|\mathbf{y}_i - \mathbf{y}_{i+n/2}\| = 2(t_\rho - t_o)v_\delta + w_i \qquad i = 1, \dots, n/2[[dist]] \tag{28}$$

and $\mathbf{y}_{i+n/2}$ is the measurement on the opposite location to $\mathbf{y}_i$ on the circle and the position noise corresponding to the difference in (28) is

$$w_i \sim \mathcal{N}(0, 2\sigma_w^2) \tag{29}$$

with the 3d position measurement noise MSE (mean square error) for a single measurement

$$\sigma_w^2 = \mathrm{tr}(R) \tag{30}$$

where R is the (position) measurement noise covariance matrix in Cartesian, obtained via the standard transformation for small angular errors or the unbiased transformation [1].

Thus, the following LF is used

$$\begin{aligned}\Lambda[v_\delta; f_\delta(Y)] &= \prod_{i=1}^{n/2} p[\|\mathbf{y}_i - \mathbf{y}_{i+n/2}\| \,|v_\delta] \\ &= (2\pi\sqrt{2}\sigma_w)^{-n/4} \exp\left\{-\frac{[\|\mathbf{y}_i - \mathbf{y}_{i+n/2}\| - 2(t_\rho - t_o)v_\delta]^2}{2(2\sigma_w^2)}\right\}[[lfvd]] \end{aligned} \tag{31}$$

The ML estimate (MLE) of v_δ from (31) — the fusion equation of the measurements at t_δ for the dispersion speed — is

$$\hat{v}_\delta = \frac{1}{n/2}\frac{\sum_{i=1}^{n/2}\|\mathbf{y}_i - \mathbf{y}_{i+n/2}\|}{2(t_\rho - t_o)} \tag{32}$$

with variance

$$\sigma_{v_\delta}^2 = \frac{1}{n}\frac{2\sigma_w^2}{t_\rho - t_o}[[sigmavdisp]] \tag{33}$$

The above is n times more accurate than the variance of the velocity of each ejected object obtained from the conventional two-point differencing, which is [1]

$$\sigma_{v2}^2 = 2\sigma_w^2/(t_\rho - t_o)[[sigmav2pt]] \tag{34}$$

[1]The classical definition of the sufficient statistic $g_\theta(Y)$ for a parameter θ is that the pdf the measurements Y conditioned on θ can be written as $p(Y|\theta) = f_1[g_\theta(Y), \theta]f_2(Y)$ and, while $f_1[g_\theta(Y), \theta] = p[g_\theta(Y)|\theta]$ is the LF of θ based on the θ-observable function of the measurements $g_\theta(Y)$, in the present case one does not have $p(Y|\theta)$.

4.2 Estimation of the objects' longitudinal ejection speed with the parameter-observable function of the position measurements

Denote the estimated position of the carrier at the time it opens, t_o, to eject the objects as $\hat{\mathbf{x}}_c(t_o)$, with covariance $P_p(t_o)$. The estimate of the center of the circle of the dispersed objects at t_ρ is given in (24) and has variance

$$\sigma_0^2 = (1/n)\sigma_w^2 \tag{35}$$

Similarly to (31), the LF of the longitudinal ejection speed v_L based on the parameter-observable function of the position measurements

$$f_L(Y) = \|\mathbf{z}_0(t_\rho) - \hat{\mathbf{x}}_c(t_o)\| - (t_\rho - t_o)\hat{v}_c \tag{36}$$

which comprises (24) and the (also needed) carrier position and speed estimates $\hat{\mathbf{x}}_c(t_o)$, $\hat{v}_c$, respectively, is written as

$$\begin{aligned}
\Lambda[v_L; f_L(Y)] &= \Lambda[v_L; \mathbf{z}_0(t_\rho), \hat{\mathbf{x}}_c(t_o)] = p[f_L(Y)|v_L] \\
&= \left\{2\pi[\sigma_w^2/n + \sigma_p^2 + (t_\rho - t_o)^2\sigma_{v_c}^2]\right\}^{-1/2} \\
&\times \exp\left\{-\frac{[\|\mathbf{z}_0(t_\rho) - \hat{\mathbf{x}}_c(t_o)\| - (t_\rho - t_o)(\hat{v}_c + v_L)]^2}{2(\sigma_w^2/n + \sigma_p^2 + (t_\rho - t_o)^2\sigma_{v_c}^2)}\right\} [[lfvL]]
\end{aligned} \tag{37}$$

where the carrier position MSE is

$$\sigma_p^2 = \mathrm{tr}(P_{\mathbf{x}_c}) = \mathrm{tr}[\mathrm{cov}(\mathbf{x}_c)] \tag{38}$$

with $P_{\mathbf{x}_c}$ the covariance matrix of the carrier's position from its track and the MSE of the carrier speed is

$$\sigma_{v_c}^2 = \mathrm{tr}(P_{\mathbf{v}_c}) = \mathrm{tr}[\mathrm{cov}(\mathbf{v}_c)] \tag{39}$$

with $P_{\mathbf{v}_c}$ the covariance matrix of the carrier's velocity from its track.

The MLE of the longitudinal ejection speed v_L, which fuses the position measurments, is

$$\hat{v}_L = \frac{\|\mathbf{z}_0(t_\rho) - \hat{\mathbf{x}}_p(t_o)\|}{t_\rho - t_o} - \hat{v}_c \tag{40}$$

with variance

$$\sigma_{\hat{v}_L}^2 = \frac{\sigma_w^2/n + \sigma_p^2}{(t_\rho - t_o)^2} + \sigma_{v_c}^2 [[sigmalongit]] \tag{41}$$

4.3 Estimation of the objects' ejection time and longitudinal ejection speed using Doppler measurements

With the longitudinal ejection speed v_L, the carrier speed (before ejection) v_c and the ejected mass m_e out of the total mass of the carrier before ejection m_c, the conservation of momentum at ejection is written as

$$m_e v_L + (m_c - m_e)v_{ce} = m_c v_c \tag{42}$$

which yields the carrier's speed after ejection, v_{ce} as

$$v_{ce} = \frac{m_c v_c - m_e v_L}{m_c - m_e} = \frac{v_c - (m_e/m_c) v_L}{1 - m_e/m_c} [[vce]] \tag{43}$$

The estimated longitudinal speed (51) should be used in the above for the carrier speed after ejection to obtain the estimate of the ejection time in the sequel.

Denote the unit vector of the Line of Sight (LOS) of the radar to the target as

$$\mathbf{1}_{LOS} = \mathbf{x}_c / \|\mathbf{x}_c\| \tag{44}$$

The Doppler measurement of the carrier just before the opening/ejection time t_o is (indicating now the time)

$$z_{Dc}(t_o^-) = v_c \phi_{cLOS} + w_D(t_o^-) [[zDct-]] \tag{45}$$

where

$$\phi_{cLOS} \triangleq \mathbf{1}_c^\top \mathbf{1}_{LOS} \tag{46}$$

is the projection factor of $\mathbf{1}_c$ on $\mathbf{1}_{LOS}$ to yield the range rate, and the Doppler measurement noise (to be assumed white) is

$$w_D \sim \mathcal{N}(0, \sigma_D^2) \tag{47}$$

Right after the ejection (denoted as t_o^+) the Doppler measurement of the carrier is

$$z_{Dc}(t_o^+) = v_{ce} \phi_{cLOS} + w_D(t_o^+) [[zDct+]] \tag{48}$$

where v_{ce} can be obtained from (43) using the estimate $\hat{v}_{LD}$ from (51) below.

The time of the change from the measurement (45) to (48) can be used to provide the estimate of t_o.

The ejection velocities are dominated by the longitudinal ejection speed v_L, which is, typically, an order of magnitude larger than the lateral (dispersion) speed. While the ejected objects ($i = 1, \ldots, n$) are still unresolved at t_o^+, they will yield a single new Doppler measurement denoted as

$$z_D(t_o^+) = (v_c + v_L) \phi_{cLOS} + w_D(t_o^+) [[zD]] \tag{49}$$

where the Doppler measurement noise is

$$w_D \sim \mathcal{N}(0, \sigma_{w_D}^2) [[wD]] \tag{50}$$

Eq. (49) can be used to estimate v_L. This yields the Doppler-based estimate of v_L as

$$\hat{v}_{LD} = \frac{z_D(t_o^+)}{\phi_{cLOS}} - \hat{v}_c [[vhatLD]] \tag{51}$$

with variance

$$\sigma_{\hat{v}_{LD}}^2 = \frac{\sigma_{w_D}^2}{\phi_{cLOS}^2} + \sigma_{v_c}^2 [[sigmalongitD]] \tag{52}$$

4.4 Estimation of the objects' dispersion speed using Doppler measurements

The projection factor of the dispersion velocity of object i on the LOS, needed for the range rate of this object, is

$$\phi_{\delta iLOS} = \mathbf{1}_{\mathbf{v}\delta i}^{\top}\mathbf{1}_{LOS}[[projiLOS]] \tag{53}$$

where $\mathbf{1}_{\mathbf{v}\delta i}$ is given in (9). The Doppler measurement of object i, z_{Di}, consists of

1. its dispersion velocity with projection factor (53)
2. its (total) longitudinal velocity projected on the LOS
3. measurement noise w_{Di}, as in (50).

Note that item 2 above is common for all i. Thus, if we carry out differencing of "opposite" measurements as in (28), the longitudinal velocity cancels and

$$z_{Di} - z_{D,i+n/2} = 2v_\delta \phi_{\delta iLOS} + w_{Di} + w_{D,i+n/2} \qquad i = 1, \ldots, n/2[[Dvi]] \tag{54}$$

since for opposite measurements

$$\phi_{\delta iLOS} = -\phi_{\delta,i+n/2,LOS} \tag{55}$$

Then the fusion based estimate of the dispersion speed using the Doppler measurements (54) is

$$\hat{v}_{\delta D} = \frac{1}{n/2}\sum_{i=1}^{n/2}\frac{(z_{Di} - z_{D,i+n/2})/2}{\phi_{\delta iLOS}} = \frac{1}{n}\sum_{i=1}^{n/2}\frac{z_{Di} - z_{D,i+n/2}}{\phi_{\delta iLOS}} \tag{56}$$

with variance

$$\sigma_{v\delta D}^2 = 1/n^2 \sum_{i=1}^{n/2}\frac{2\sigma_D^2}{\phi_{\delta iLOS}^2}[[sigmavdispD]] \tag{57}$$

5 Simulation Parameters

Radar location: at origin of the ENU coordinate system; all vectors are in this ENU

Radar revisit rate: $T = 20$Hz

Radar range measurement s.d.: 5m

Radar az and el measurement s.d.: 0.15mrad

Radar Doppler measurement s.d.: $\sigma_D = 1$m/s

Azimuth is in the horizontal plane (EN) clockwise from N

Elevation is w.r.t. the horizontal plane toward the U axis

Number of ejected objects: $n = 12$ (assumed known at this point)

Ejection at: $t_o = 0$

Resolution time: $t_\rho = 1$s

Lateral ejection (dispersion) speed: $v_\delta = 20$m/s

Longitudinal ejection speed: $v_L = 200\text{m/s}$

Ejection speed: $v_e = \sqrt{v_\delta^2 + v_L^2} = 201 m/s \approx v_L$

Carrier position at ejection: $\mathbf{x}_c = [40\ 40\ 40]^\top\text{km}$

Carrier velocity at ejection: $\mathbf{v}_c = [-3.2\ \ 0\ \ -3]^\top\text{km/s}$

Carrier velocity az $\alpha_c = -90°$, el $\epsilon_c = -43°$

Slant range from radar to carrier at ejection: $40\sqrt{3} \approx 70\text{km}$

Radar range resolution cell: 10m (the minimum distance in range for two targets to be resolved)

Radar angular resolution cell: 20mrad (at 70km the minimum distance in cross-range in at lea+t one of the angular coordinates for two targets to be resolved is 70km x 0.15mrad=20m)

Position measurement RMSE: $\sigma_w = 10\text{m}$

Carrier position RMSE (from track): $\sigma_p = 5\text{m}$

Carrier velocity RMSE (from track): $\sigma_v = 5\text{m/s}$

Mass of ejected objects relative to total carrier mass before ejection: $m_e/m_c = 0.1$

6 Results

6.1 The true positions of the ejected objects

The following calculations (option A) should be performed:

1. Object dispersion velocities' unit vectors around the radar (6)
2. Object dispersion velocities' unit vectors around the carrier (9)
3. Object dispersion velocities around the carrier with known v_δ (12)
4. Object position at resolution time with known v_δ and v_L (15)
5. Carrier position at object resolution time (16)
6. Spherical coordinate measurements for each object (17) with

   A. zero noises for now

   B. noises according to (19)–(21).
7. Conversion to Cartesian of the spherical measurements (22[[sph2cart]]
8. Center of object measurements (24)
9. Rotate the centered measurements into (ideally) circular shape (26[[rotctrdi]]

The first results I would like to see

	Dispersion speed	Longitudinal speed
From position measurements	value from (33)	value from (57)
From Doppler measurements	value from (41)	value from (52)

Table 1: Standard deviations of speed estimates (m/s).

[[tsdspeeds]]

(R1) A 2d plot of the az, el (with scale in mrad) of the true positions of the ejected objects at $t_{20} = 20T$s, i.e. 1s after ejection (the circular dispersion will appear as elliptical) — item 6 above; Also the az-el for the center — item 8 above, converted to spherical.

(R2) Table of the true ranges from the radar of the 12 objects u_{ir}, $i = 1, \ldots, n$ at t_{20} their center nad the carrier range converted from (16); v_{ce} with true value from (43).

(R3) A 3d plot of the positions of the carrier (which moves with a slightly reduced speed (v_{ce} with true value from (43)) following the ejection) and the ejected objects — to get the overall picture at t_{20} (i.e. 1s after ejection). The ejected objects move with the dispersion speed v_δ and longitudinal speed $v_L + v_c$.

These should give an idea for the scenario considered and the resolution of the objects.

The estimation approach (after we see the results of the 3 items above) should be to find the best fitting circle (which appears as an ellipse for the radar) centered at the predicted location of the carrier, where the fitting should be the squared distance between the dispersed objects' measurements (initially without noises) and the circle, normalized by the cross-range s.d.

The carrier motion should be assumed CV in x, y while the z motion is CV+gravity ($\ddot{z} = -g$).

Table 1 shows the values of the dispersion and longitudinal speed standard deviations (s.d.) from the two procedures introduced in the work. Both are significantly better than the overall speed s.d. from the baseline 2-point differencing (34), which is ...m/s.

Noisy simulations:

(i) Add noises in r-az-el to the ejected object positions at t_{20} and overlay their positions to the plot 1.

(ii) Determination of the “opposite" objects and the number of objects in the ejection pattern.

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JPDAS multi-target tracking algorithm for cluster bombs tracking

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Results of simulations for a radar tracking problem that show proposed JPDAS has better tracking performance than JPDAF is presented. Abstract PDAF is a method of updating targets state estimation by using posteriori probability that measurements are originated from existing target in multi-target tracking. In this paper, we propose a multi-target tracking algorithm for falling cluster bombs separated from a mother bomb based on JPDAS method which is obtained by applying fixed-interval smoothing technique to JPDAF. The performance of JPDAF and JPDAS multi-target tracking algorithm is compared by observing the average of the difference between targets' state estimations obtained from 100 independent executions of two algorithms and targets' true states. Based on this, results of simulations for a radar tracking problem that show proposed JPDAS has better tracking performance than JPDAF is presented.

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