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RESEARCH ARTICLE

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Control charts to monitor rates and proportions

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Abstract
In this paper, we call attention for monitoring proportions and rates when they are not results of Bernoulli experiments. To deal with this problem, usually control charts based on Beta distributions are built as this distribution is a well-known one. However, in practice, there are other distributions to be considered as Simplex and Unit Gamma distributions. The impact of the speed to signal a shift in the average proportion in terms of out-of-control average run length is measured when the control limits determined under a Beta distribution is equivocally used to monitor individual rates from Simplex or Unit Gamma distributions or vice versa.

KEYWORDS
average run length, Beta distribution, Simplex distribution, Unit Gamma distribution

1 | INTRODUCTION

Control charts have been the most used tool to monitor the stability of process parameters (like mean, variance, or nonconforming fraction). It is desirable that the goods/products/items are being manufactured to meet the client's requirements and in absence of special/assignable causes. Most recently, their applications have been extended to other areas than manufacturing. For example, they have been used to verify the stability of a service quality or the price of stock or the surveillance of a disease to declare if the epidemic level is reached or not, and so.

In many practical situations, there is interest in monitoring rates/proportions of components in a product. When they are results from Bernoulli experiments, the *np* and *p* charts may be applied to monitor the stability of nonconforming proportion.[1] Good review papers about attribute charts can be found in the literature as these written by Szarka and Woodall[2] and Woodall.[3] Some recent contributions on *np* or *p* charts varying estimator, sampling, or decision scheme can be cited.[4-7] For a single quality characteristic normally distributed, many contributors have proposed “pure” attribute charts based on the results of classifications of the values of the quality characteristic by a gauge to monitor process parameters as mean[8-12] or variance.[13,14] Its extensions to monitor bivariate process parameters can be found in the literature: for monitoring a vector of mean[15] and covariance-variance matrix.[16] Other authors have combined attribute (as results of gauge classification) and variable charts[17-20] to improve the performance in signaling out-of-control state. Attribute charts have also been used to monitor parameters of Weibull and[21,22] exponential[23] distributions.

However, there are situations that rates or proportions are not results from Bernoulli experiments although assume values in the range (0, 1). For example, the monitoring of the proportion of the drug components (of a medicine) in pharmaceutical industries is essential to assure its effectiveness. Or the proportion of components in a casting product or the monitoring of the unemployment rate to drive the resources in public politics. Thus, the former mentioned control charts cannot be applied for this purpose.

The well-known probability distribution that deals with this type of random variable is Beta distribution, and a Shewhart-type control chart considering this baseline distribution was developed by SantAnna and ten

Caten.[24] Recently, Bayer et al[25] extended considering beta regression control chart to monitor proportions and rates. However, other probability functions modelling rates/proportions in the range (0, 1) are available in the literature like Simplex[26] or Unit Gamma[27] distributions.

The aim of this paper is to measure the impact of the speed to signal shifts in the average proportion measured in terms of usual performance metrics, as out-of-control average run length (ARL_1), out-of-control median run length (MRL_1), and out-of-control standard deviation run length ($SDRL_1$) when control limits determined under a Beta distribution are inappropriately used to monitor rates/proportions from other distributions than Beta like Simplex or Unit Gamma distributions or vice versa.

This paper is organized as follows: this introductory section. In section 2, a brief review of the main probability distributions for modelling proportions or rate in the unit interval is presented. The Shewhart-type control charts to monitor proportions are the subject of section 3. The performance of the control charts (for Beta, Simplex and Unit Gamma) are determined for a set of shifts. Additionally, the impact in the speed to signal changes in the average proportion when the control limits obtained under a baseline distribution are misused for others distributions. These results are discussed in section 4. Applications to a real data set are presented in section 5, and conclusion remarks are outlined in Section 6.

2 | SOME DISTRIBUTIONS FOR MODELLING RATES AND PROPORTIONS

In what follows, we shall briefly present three different distributions that can be used to model data that assume values in the unit interval, namely, the Beta, Simplex, and Unit Gamma distributions.

2.1 | Beta distribution

A random variable Y follows a Beta distribution with shape parameters $\delta > 0$ and $\gamma > 0$, denoted by $Y \sim \mathrm{B}(\delta, \gamma)$, if its cumulative distribution function is given by

$$F(y|\delta,\gamma) = I_y(\delta,\gamma), \quad 0 < y < 1, \tag{1}$$

where $I_x(\delta,\gamma) = \dfrac{B_x(\delta,\gamma)}{B(\delta,\gamma)}$ is the incomplete beta function ratio, $B_x(\delta,\gamma) = \int_0^x \omega^{\delta-1}(1-\omega)^{\gamma-1}\mathrm{d}\omega$ is the incomplete function, $B(\delta,\gamma) = \dfrac{\Gamma(\delta)\Gamma(\gamma)}{\Gamma(\delta+\gamma)}$ is the beta function, and $\Gamma(\delta) = \int_0^\infty \omega^{\delta-1}\mathrm{e}^{-\omega}\mathrm{d}\omega$.

The probability density function (pdf) associated with 91) is

$$f(y|\delta,\gamma) = \frac{y^{\delta-1}(1-y)^{\gamma-1}}{B(\delta,\gamma)}, \quad 0 < y < 1. \tag{2}$$

The mean and variance associated with (2) are given by

$$\mathrm{E}(Y) =: \mu = \frac{\delta}{\delta+\gamma} \tag{3}$$

and

$$\mathrm{Var}(Y) = \frac{\delta\gamma}{(\delta+\gamma)^2(\delta+\gamma+1)}, \tag{4}$$

respectively. Let, in (2)

$$\delta = \mu\phi \quad \text{and} \quad \gamma = (1-\mu)\phi. \tag{5}$$

From (5), the Beta density in (2) can be written as

$$f(y|\mu,\phi) = \frac{y^{\mu\phi-1}(1-y)^{(1-\mu)\phi-1}}{B(\mu\phi,(1-\mu)\phi)}, \quad 0 < y < 1. \tag{6}$$

Thus, under this parameterization, it follows from Equations 3 and 4 that

$$\mathrm{E}(Y) = \mu \quad \text{and} \quad \mathrm{Var}(Y) = \frac{\mu(1-\mu)}{\phi+1}.$$

2.2 | Simplex distribution

A random variable Y follows a Simplex distribution,[26] denoted by $Y \sim \mathrm{S}(\mu,\sigma^2)$ if its pdf is given by

$$f(y|\mu,\sigma^2) = \{2\pi\sigma^2[y(1-y)]^3\}^{-\frac{1}{2}} \exp\left[-\frac{1}{2\sigma^2}d(y;\mu)\right], \quad 0 < y < 1, \tag{7}$$

where

$$d(y;\mu) = \frac{(y-\mu)^2}{y(1-y)\mu^2(1-\mu)^2}$$

is the deviance function, $0 < \mu < 1$, and $\sigma^2 > 0$. The Simplex distribution has a unimode if $\sigma \leq 4/\sqrt{3}$; otherwise, it yields multimodes.

The mean and variance of Y are given by

$$\mathrm{E}(Y) = \mu$$

and

$$\mathrm{Var}(Y) = \frac{1}{\sqrt{2\sigma^2}} \exp\left\{\frac{1}{2\sigma^2\mu^2(1-\mu)^2}\right\} \Gamma\left(\frac{1}{2}, \frac{1}{2\sigma^2\mu^2(1-\mu)^2}\right),$$

respectively, where $\Gamma(a,b)$ is the incomplete gamma function defined by $\Gamma(a,b) = \int_b^\infty t^{a-1}\mathrm{e}^{-t}\mathrm{d}t$.

2.3 | Unit gamma distribution

The last distribution we shall present is the Unit Gamma model proposed by Grassia.[27] The random variable Y follows a Unit Gamma distribution with parameters θ and τ, denoted by $Y \sim \mathrm{uGA}(\theta, \tau)$, if its pdf is given by

$$f(y|\theta,\tau) = \frac{\theta^\tau}{\Gamma(\tau)} y^{\theta-1}\left[\log\left(\frac{1}{y}\right)\right]^{\tau-1}, \quad 0 < y < 1,$$

where $\theta > 0$ and $\tau > 0$. The mean and variance are given by

$$\mathrm{E}(Y) = \mu = \left[\frac{\theta}{\theta+1}\right]^\tau$$

and

$$\mathrm{Var}(Y) = \left[\frac{\theta}{\theta+2}\right]^\tau - \left[\frac{\theta}{\theta+1}\right]^{2\tau},$$

respectively.

Mousa et al[28] considered a new parameterization for the Unit Gamma distribution. Let $\theta = \dfrac{\mu^{1/\tau}}{(1-\mu^{1/\tau})}$, from this new parameterization, the Unit Gamma density can be written as

$$f(y|\mu,\tau) = \frac{\left[\frac{\mu^{1/\tau}}{1-\mu^{1/\tau}}\right]^\tau}{\Gamma(\tau)} y^{\frac{\mu^{1/\tau}}{1-\mu^{1/\tau}}-1}\left[\log\left(\frac{1}{y}\right)\right]^{\tau-1}, \quad 0 < y < 1, \tag{8}$$

where $0 < \mu < 1$ and $\tau > 0$. Thus, the mean and variance are given by

$$\mathrm{E}(Y) = \mu \quad \text{and} \quad \mathrm{Var}(Y) = \mu\left[\frac{1}{(2-\mu^{1/\tau})^\tau} - \mu\right].$$

3 | CONTROL CHARTS TO MONITOR INDIVIDUAL OBSERVATIONS OF RATES AND PROPORTIONS

Usually, to monitor nonconforming proportions in the production process, the p and np control charts have been employed for such purposes. In this case, the proportions, most of them, are results of Bernoulli experiments. Due to its discreteness nature (of the binomial distribution) and depending on the sample size, the symmetric control limits (used in Shewhart-type control chart) determined by the approximated normal distribution may not reach the desired values of ARL_0 like 370. Thus, procedures have been proposed by many researchers to improve the performance of np or p charts.[29-36]

When the monitored proportions are not results of Bernoulli experiments, control charts can also be built using distributions with the random variable defined in the range (0, 1) like Beta or Simplex or Unit Gamma distributions.

Fixed the type I error α, the lower and upper probability control limits LCL_B and UCL_B for Beta chart are determined such that

$$P(Y < LCL_B|\mu_{0B},\phi_0) = P(Y > UCL_B|\mu_{0B},\phi_0) = \alpha/2$$
$$\int_0^{LCL_B} f(y|\mu_{0B},\phi_0) = \int_{UCL_B}^1 f(y|\mu_{0B},\phi_0) = \alpha/2,$$

where μ_{0B} and ϕ_0, respectively, are the in-control average proportion and dispersion parameter of Beta chart and $f(y|\mu_{0B},\phi_0)$ expressed in (6). The probability control limits LCL_S and UCL_S for Simplex chart are similarly calculated as

$$P(Y < LCL_S|\mu_{0S},\sigma_0^2) = P(Y > UCL_S|\mu_{0S},\sigma_0^2) = \alpha/2$$
$$\int_0^{LCL_S} f(y|\mu_{0S},\sigma_0^2) = \int_{UCL_S}^1 f(y|\mu_{0S},\sigma_0^2) = \alpha/2,$$

with μ_{0S} and σ_0^2, respectively, the in-control average proportion and precision parameter of Simplex chart, $f(y|\mu_{0S},\sigma_0^2)$ in (7), and finally, the probability control limits LCL_G and UCL_G for Unit Gamma chart are obtained as

$$P(Y < LCL_G|\mu_{0G},\tau_0) = P(Y > UCL_G|\mu_{0G},\tau_0) = \alpha/2$$
$$\int_0^{LCL_G} f(y|\mu_{0G},\tau_0) = \int_{UCL_G}^1 f(y|\mu_{0G},\tau_0) = \alpha/2,$$

with μ_{0G},τ_0, respectively, the in-control average proportion and precision parameter of Unit Gamma control chart and $f(y|\mu_{0G},\tau_0)$ in (8).

The power $(1 - \beta)$ of these control charts can be calculated as

$$1 - \beta = P(Y < LCL_i|\Theta_1) + P(Y > UCL_i|\Theta_1),$$

$i = B, S, G$, with $\Theta_{1i} = \{(\mu_{1B},\varphi_1),(\mu_{1S},\sigma_1^2),(\mu_{1G},\tau_1)\}$, respectively, the out-of-control parameters of Beta, Simplex, and Unit Gamma control charts. Usually, the out-of-control average proportion μ_{1i} is expressed in terms of in-control average proportion as $\mu_{1i} = \mu_{0i} + k\sqrt{\mathrm{Var}(Y)}$ or $\mu_{1i} = \mu_{0i} + \Delta_i$ or $\mu_{1i} = \mu_{0i}\Delta_i$ depending on the context of interest. In this paper, we consider shifts only on the average proportion and no shifts on the dispersion parameters.

In case of unilateral control charts, the control limits UCL_i or LCL_i are similarly calculated with $P(Y < LCL_i|\Theta_0) = \alpha$ or $P(Y > UCL_i|\Theta_0) = \alpha$, $i = B, S, G$ with $\Theta_0 = \{(\mu_{0B},\varphi_0),(\mu_{0S},\sigma_0^2),(\mu_{0G},\tau_0)\}$, respectively, the in-control parameters of Beta, Simplex, and Unit Gamma control charts. And the power $(1-\beta)$ is obtained as $1-\beta = P(Y < LCL_i|\Theta_1)$ or $1 - \beta = P(Y > UCL_i|\Theta_1)$, $i = B, S, G$ with $\Theta_1 = \{(\mu_{1B},\varphi_0),(\mu_{1S},\sigma_0^2),(\mu_{1G},\tau_0)\}$, respectively, the out-of-control parameters of Beta, Simplex, and Unit Gamma control charts.

In the production processes, the performance of the control chart is usually measured in terms of the number of samples (the run length) until a signal is observed and the most used metric is the average run length (ARL). In the current context, the observations y_i are independent as at every sampling interval, a single proportion is observed, and a decision is taken on the production process (stops or continues). Thus, the run length follows a geometric distribution with parameter p. When the process is in-control, $p = \alpha$ and the process is out-of-control, $p = (1 - \beta)$. As the run length is high asymmetrically distributed, other metrics than $ARL = 1/p$ as the standard deviation of the run length, $\left(SDRL = \sqrt{(1-p)/p^2} \right)$ or the median run length, ($MRL = \ln(0.5)/\ln(1-p)$) are also considered. When the process is in-control, large values of $ARL_0 = 1/\alpha$ is desirable, but in a case of out-of-control low values of $ARL_1 = 1/(1 - \beta)$ is preferable.

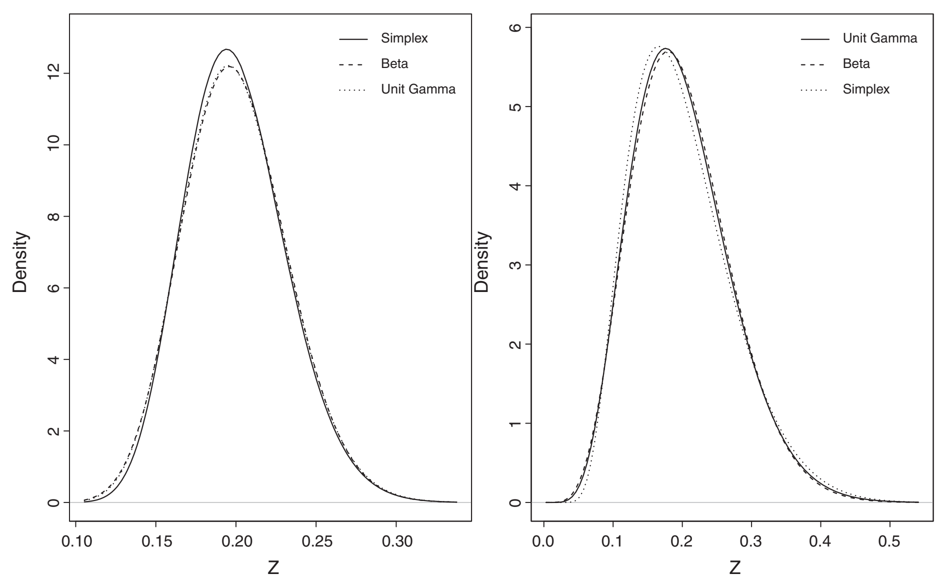
4 | PERFORMANCE OF CONTROL CHARTS

In this section, the performance (measured in terms of $ARL_1, MRL_1, SDRL_1$) of the three control charts to detect different sizes of shifts are obtained. In addition, the impact in the speed to detect the shifts in the average proportion if a set of control limits determined under distribution "A" is wrongly used to monitor proportions under distribution "B" or vice versa. All results presented in this section are obtained considering as in-control average proportion $\mu_{0B} = \mu_{0S} = \mu_{0G} = 0.2$ and out-of-control $\mu_{1B} = \mu_{1S} = \mu_{1G} = 0.2 \pm \varepsilon, \varepsilon = \{0.02, 0.04, 0.06, 0.08\}$. Four levels of dispersion/precision are chosen for each distribution such that they present similar probability distribution function. The dispersion/precision parameters are shown in Table 1, where the cases 1 (−) and 4 (+) denote, respectively, the cases with lower and higher variances. Probability density functions of cases 2 and 4 are shown in Figure 1. The other cases also have similar probability density functions and are not shown here.

Table 2 presents the performance of the control charts (in terms of ARL, $SDRL$, and MRL) when the average proportion shifts in two directions (increase and decrease) using error type I $\alpha = 0.0027$ to yield $ARL_0 = 370$. The results are organized in three blocks of nine columns. The first block (columns 3-11) is related to Beta control chart, the second to Simplex control chart, and the last to Unit Gamma control chart. At every block, the first three columns are the performance metrics when the correct control limits are used (Beta/Simplex/UGamma-true), and other six columns are the metrics when incorrect control limits are employed. For example, columns 3 to 5 are the true performance of Beta chart (that is, the Beta control limits are correctly used). Columns 6 to 8 and columns 9 to 11 are, respectively, the performance metrics employing

TABLE 1 Dispersion parameters

Case	Beta ϕ	Simplex σ^2	Unit Gamma τ
1 (−)	290	0.37	1.55
2	148	0.5	96
3	80	0.71	51
4(+)	31	1.2	20



(A) Case 2 (B) Case 4

FIGURE 1 Beta, Simplex, and Unit Gamma probability density function (pdf) of cases 2 and 4

TABLE 2 Comparing the performance of the control charts-bidirectional

		Beta - True			Simplex			UGamma			Simplex - True			Beta			UGamma			UGamma - True			Beta			Simplex		
	μ	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
Case 1:	0.12	1.26	0.57	0.44	**1.21**	**0.5**	**0.4**	**1.39**	**0.74**	**0.55**	1.09	0.31	0.28	**1.13**	**0.38**	**0.32**	**1.25**	**0.56**	**0.43**	1.4	0.75	0.55	**1.28**	**0.6**	**0.46**	**1.23**	**0.53**	**0.41**
$\phi = 290$	0.14	2.34	1.77	1.24	**2.1**	**1.52**	**1.07**	**2.99**	**2.44**	**1.7**	2.15	1.57	1.11	**2.5**	**1.94**	**1.36**	**3.56**	**3.02**	**2.1**	2.85	2.3	1.6	**2.28**	**1.71**	**1.2**	**2.07**	**1.49**	**1.05**
$\sigma = 0.37$	0.16	8.04	7.52	5.22	6.6	6.08	4.22	12.49	11.98	8.31	9.34	8.83	6.12	12.32	11.81	8.19	22.95	22.44	15.56	10	9.49	6.58	6.8	6.28	4.36	5.7	5.18	3.59
$\tau = 155$	0.18	54.6	54.1	37.5	40.35	39.85	27.62	105.26	104.76	72.61	70.24	69.74	48.34	103.42	102.92	71.34	242.45	241.95	167.71	63.21	62.71	43.47	35.72	35.22	24.41	27.37	26.87	18.62
	0.2	370.37	369.87	256.37	**333.95**	**333.45**	**231.13**	1028.5	1028	712.56	370.46	369.96	256.44	**371.34**	**370.84**	**257.05**	1080.23	1079.73	748.41	370.3	369.8	256.33	155.73	155.23	107.6	146.39	145.89	101.12
	0.22	69.71	69.21	47.97	91.56	91.06	63.12	178.02	177.52	123.05	55.04	54.54	37.8	43.89	43.39	30.07	95.21	94.71	65.65	90.28	89.78	62.23	41.69	41.19	28.55	52.04	51.54	35.72
	0.24	12.26	11.75	8.15	15.05	14.54	10.08	24.7	24.19	16.77	10.52	10.01	6.94	**8.97**	**8.46**	**5.86**	15.43	14.92	10.34	17.82	17.31	12	9.85	9.34	6.47	11.72	11.21	7.77
	0.26	3.71	3.17	2.21	**4.26**	**3.73**	**2.59**	6.01	5.49	3.81	3.71	3.17	2.21	**3.34**	**2.8**	**1.95**	**4.81**	**4.28**	**2.97**	5.36	4.83	3.36	**3.51**	**2.97**	**2.07**	**3.97**	**3.43**	**2.39**
	0.28	1.78	1.18	0.84	**1.93**	**1.34**	**0.95**	**2.38**	**1.81**	**1.27**	1.98	1.39	0.99	**1.85**	**1.25**	**0.89**	**2.34**	**1.77**	**1.24**	2.36	1.79	1.26	**1.81**	**1.21**	**0.86**	**1.95**	**1.36**	**0.96**
	LCL	0.135			0.138			0.131			0.138			0.135			0.131			0.131			0.135			0.138		
	UCL	0.275			0.278			0.285			0.278			0.275			0.285			0.285			0.275			0.278		
Case 2	0.12	2.36	1.79	1.26	**1.88**	**1.29**	**0.91**	**2.24**	**1.67**	**1.17**	1.87	1.28	0.91	**2.73**	**2.17**	**1.52**	**2.5**	**1.94**	**1.36**	2.23	1.66	1.16	**2.35**	**1.78**	**1.25**	**1.86**	**1.26**	**0.9**
$\phi = 148$	0.14	5.72	5.2	3.61	**3.9**	**3.36**	**2.34**	**5.26**	**4.73**	**3.29**	5.72	5.2	3.61	11.14	10.63	7.37	9.6	9.09	6.3	5.36	4.83	3.36	**5.85**	**5.33**	**3.7**	**3.93**	**3.39**	**2.36**
$\sigma = 0.5$	0.16	20.11	19.6	13.59	11.52	11.01	7.63	17.84	17.33	12.02	24.83	24.32	16.86	61.46	60.96	42.25	50.41	49.91	34.59	18.84	18.33	12.71	21.36	20.85	14.46	11.92	11.41	7.91
$\tau = 96$	0.18	100.51	100.01	69.32	48.27	47.77	33.11	86.24	85.74	59.43	128.23	127.73	88.54	380.55	380.05	263.43	302.39	301.89	209.25	95.63	95.13	65.94	112.22	111.72	77.44	52	51.5	35.7
	0.2	370.37	369.87	256.37	211.09	210.59	145.97	**359.18**	**358.68**	**248.62**	370.48	369.98	256.45	562.52	562.02	389.56	593.77	593.27	411.22	370.37	369.87	256.37	**373.12**	**372.62**	**258.28**	223.02	222.52	154.24
	0.22	129.21	128.71	89.21	**128.71**	**128.21**	**88.87**	148.13	147.63	102.33	90	89.5	62.04	**88.79**	**88.29**	**61.2**	100.42	99.92	69.26	138.59	138.09	95.72	121.2	120.7	83.66	121.75	121.25	84.04
	0.24	32.24	31.74	22	**33.09**	**32.59**	**22.59**	**36.31**	**35.81**	**24.82**	21.68	21.17	14.68	**21.22**	**20.71**	**14.36**	**23.35**	**22.84**	**15.84**	35.74	35.24	24.42	**31.79**	**31.29**	**21.69**	**32.64**	**32.14**	**22.28**
	0.26	10.61	10.1	7	**10.83**	**10.32**	**7.15**	**11.6**	**11.09**	**7.69**	7.94	7.42	5.15	**7.81**	**7.29**	**5.06**	**8.4**	**7.88**	**5.47**	11.8	11.29	7.83	**10.73**	**10.22**	**7.09**	**10.97**	**10.46**	**7.25**
	0.28	4.53	4	2.78	**4.62**	**4.09**	**2.84**	**4.89**	**4.36**	**3.03**	3.94	3.4	2.37	**3.89**	**3.35**	**2.33**	**4.11**	**3.58**	**2.49**	4.97	4.44	3.09	**4.62**	**4.09**	**2.84**	**4.7**	**4.17**	**2.9**
	LCL	0.1132			0.1205			0.1147			0.1205			0.1132			0.1147			0.1147			0.1132			0.1205		
	UCL	0.308			0.3095			0.3107			0.3095			0.308			0.3107			0.3107			0.308			0.3095		
Case 3	0.12	5.11	4.58	3.18	**3.58**	**3.04**	**2.12**	**4.75**	**4.22**	**2.93**	5.94	5.42	3.76	12.47	11.96	8.29	10.66	10.15	7.04	4.85	4.32	3	5.24	4.71	3.27	3.61	3.07	2.14
$\phi = 80$	0.14	12.99	12.48	8.65	7.97	7.45	5.17	**11.77**	**11.26**	**7.81**	19.22	18.71	12.97	49.3	48.8	33.82	40.55	40.05	27.76	12.31	11.8	8.18	13.67	13.16	9.12	8.16	7.64	5.3
$\sigma = 0.71$	0.16	40.85	40.35	27.97	21.76	21.25	14.73	36.03	35.53	24.63	64.92	64.42	44.65	195.18	194.68	134.94	155.76	155.26	107.62	38.78	38.28	26.53	44.3	43.8	30.36	22.76	22.25	15.43
$\tau = 51$	0.18	150.86	150.36	104.22	234.83	234.33	162.42	131.6	131.1	90.87	213.7	213.2	147.78	646.57	646.07	447.82	538.9	538.4	373.19	144.77	144.27	100	165.72	165.22	114.52	76.72	76.22	52.83
	0.2	370.37	369.87	256.37	256.62	256.12	177.53	**384.12**	**383.62**	**265.9**	370.35	369.85	256.36	413.27	412.77	286.11	497.08	496.58	344.2	370.37	369.87	256.37	**342.53**	**342.03**	**237.08**	240.04	239.54	166.04
	0.22	195.11	194.61	134.89	97.36	96.86	67.14	251.03	250.53	173.65	135.09	134.59	93.29	100.36	99.86	69.22	125.53	125.03	86.66	213.14	212.64	147.39	166.24	165.74	114.88	222.6	222.1	153.95
	0.24	67.47	66.97	46.42	97.12	96.62	66.97	86.65	86.15	59.71	42.76	42.26	29.29	32.52	32.02	22.19	**39.15**	**38.65**	**26.79**	78.28	77.78	53.91	61.24	60.74	42.1	87.66	87.16	60.41
	0.26	26.33	25.83	17.9	36.49	35.99	24.94	32.76	32.26	22.36	17.56	17.05	11.82	14.01	13.5	9.36	**16.3**	**15.79**	**10.95**	31.52	31.02	21.5	25.5	24.99	17.33	**35**	**34.5**	**23.91**
	0.28	11.93	11.42	7.92	15.76	15.25	10.57	14.37	13.86	9.61	8.93	8.42	5.84	**7.42**	**6.9**	**4.79**	**8.4**	**7.88**	**5.47**	14.37	13.86	9.61	**11.96**	**11.45**	**7.94**	**15.76**	**15.25**	**10.57**
	LCL	0.0884			0.0968			0.0899			0.0968			0.0884			0.0899			0.0899			0.0884			0.0968		
	UCL	0.3506			0.3586			0.356			0.3586			0.3506			0.356			0.356			0.3506			0.3586		
Case 4	0.12	15.68	15.17	10.52	7.45	6.93	4.81	12.69	12.18	8.44	35.02	34.52	23.93	335.75	335.25	232.38	167.38	166.88	115.67	14.82	14.31	9.92	18.96	18.45	12.79	8.07	7.55	5.24
$\phi = 31$	0.14	36.3	35.8	24.81	14.63	14.12	9.79	28.09	27.59	19.12	80.59	80.09	55.51	936.45	935.95	648.75	443.01	442.51	306.72	34.21	33.71	23.36	45.95	45.45	31.5	16.33	15.82	10.97
$\sigma = 1.2$	0.16	90.21	89.71	62.18	31.11	30.61	21.22	67.19	66.69	46.23	173.86	173.36	120.16	2043.34	2042.84	1415.99	1037.9	1037.4	719.07	85.15	84.65	58.67	118.1	117.6	81.51	35.9	35.4	24.54
$\tau = 20$	0.18	220.61	220.11	152.57	70.59	70.09	48.58	166.1	165.6	114.78	332.35	331.85	230.02	1264.89	1264.39	876.41	1250.58	1250.08	866.49	210.67	210.17	145.68	274.11	273.61	189.65	84.11	83.61	57.95
	0.2	370.37	369.87	256.37	160.93	160.43	111.2	**341.86**	**341.36**	**236.61**	370	369.5	256.12	**354.02**	**353.52**	**245.04**	471.04	470.54	326.15	370.37	369.87	256.37	**360.68**	**360.18**	**249.66**	189.6	189.1	131.07
	0.22	289.82	289.32	200.54	**282.74**	**282.24**	**195.63**	361.97	361.47	250.55	191.01	190.51	132.05	120.56	120.06	83.22	159.17	158.67	109.98	316.76	316.26	219.21	244.07	243.57	168.83	291.62	291.12	201.79
	0.24	155.75	155.25	107.61	259.18	258.68	179.3	213.74	213.24	147.81	84.09	83.59	57.94	52.37	51.87	35.95	66.59	66.09	45.81	184.2	183.7	127.33	135.41	134.91	93.51	234.96	234.46	162.52
	0.26	81.39	80.89	56.07	151.15	150.65	104.42	111.39	110.89	76.86	41.59	41.09	28.48	27.26	26.76	18.55	33.52	33.02	22.89	102.06	101.56	70.4	75.56	75.06	52.03	138.69	138.19	95.79
	0.28	44.63	44.13	30.59	81.19	80.69	55.93	59.75	59.25	41.07	23.42	22.91	15.88	16.19	15.68	10.87	19.37	18.86	13.08	58.31	57.81	40.07	43.87	43.37	30.06	78.9	78.4	54.34
	LCL	0.0449			0.0594			0.0485			0.0594			0.0449			0.0485			0.0485			0.0449			0.0594		
	UCL	0.4517			0.4742			0.4628			0.4742			0.4517			0.4628			0.4628			0.4517			0.4742		

equivocally control limits of Simplex and Unit Gamma charts in Beta chart. The second block (columns 12-20) is related of employing equivocally control limits of Beta and Unit Gamma charts in Simplex charts (the true metrics are under the group of the columns named Simplex-true), and finally, the third block (columns 21-29) is related to employing equivocally control limits of Beta and Simplex charts in Unit Gamma charts (the true metrics are under the group of the columns named UGamma-true).

According to Table 2, the use of equivocal control limits produce a great variety of impacts: earlier/latter false alarms; in some cases, no false alarms (like cases 2 to 4, the Beta control limits wrongly used in Simplex chart yield $ARL_0 > 1000$). Small shifts may not be detected for case 4 of Table 2 ($ARL_1 > ARL_0 = 370$).

Tables 3 to 4 are built to describe the results related to the shifts of the average proportion in a single direction, respectively, considering only increases or decreases. The organization of the results follows the same pattern of Table 2.

Most of results show coherence. For example, the use of Beta control limits in Simplex chart provides earlier false and true alarms. Thus, it is expected that the true and false signals are postponed when Simplex control limits are used in Beta charts.

Bold values in Tables 2 to 4 are the values of the metrics obtained with equivocal control limits but indeed similar to those using correct control limits. Few scenarios with no impact in all metrics (in-control and out-of-control) are observed: Simplex control limits in Beta chart and vice versa (see Table 3—case 2) and Unit Gamma control limits in Beta chart and vice versa for moderate/large shifts (see Table 2—case 2). And also, scenario with better performance as using Beta control limits in Simplex chart as in-control metrics are similar and out-of-control metrics lower than the truly ones (see Table 2—case 1 for moderate/large increases).

Certainly, the variance size plays an important role as no impact in larger shifts for the lower variance cases (cases 1 and 2) is observed (the out-of-control metrics are similar).

About the control limits of the different charts, numerically, they are very similar. Less impact in true alarms is observed when the difference of the correct control limits and equivocal ones is approximately $< |0.002|$ like cases 1 and 2 in Tables 2 to 4. Even when the equivocal in-control metrics are similar to the "true" metrics, it does not assure equal performance for the whole set of shift (similar only for medium/larger shifts). The tail (left and right) densities of the three distributions are quite different; this explains the different performance observed in the four cases. Thus, it is essential to have knowledge which distribution follows the proportions and rates to design the control charts correctly. When this information is unknown (assuming

TABLE 3 Comparing the performance of the control charts: Increases in average proportion

		Beta - True			Simplex			UGamma			Simplex - True			Beta			UGamma			UGamma - True			Beta			Simplex		
	μ	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
Case 1	0.2	370.37	369.87	256.37	493.34	492.84	341.61	1022.45	1021.95	708.36	370.26	369.76	256.3	284.73	284.23	197.01	720.38	719.88	498.98	370.37	369.87	256.37	162.91	162.41	112.57	205.54	205.04	142.12
$\phi = 290$	0.22	40.87	40.37	27.98	50.99	50.49	35	90.08	89.58	62.09	33.82	33.32	23.09	28.12	27.62	19.14	54.26	53.76	37.26	51.73	51.23	35.51	26.88	26.38	18.28	32.3	31.8	22.04
$\sigma = 0.37$	0.24	8.3	7.78	5.4	**9.74**	**9.23**	**6.4**	14.78	14.27	9.89	7.49	6.97	4.84	**6.6**	**6.08**	**4.22**	10.38	9.87	6.84	11.54	11.03	7.65	7.05	6.53	4.53	8.09	7.57	5.25
$\tau = 155$	0.26	2.87	2.32	1.62	**3.19**	**2.64**	**1.84**	**4.21**	**3.68**	**2.56**	2.97	2.42	1.69	**2.73**	**2.17**	**1.52**	**3.68**	**3.14**	**2.19**	3.93	>3.39	2.36	**2.8**	**2.24**	**1.57**	**3.07**	**2.52**	**1.76**
	0.28	1.77	1.17	0.83	1.64	1.02	0.74	1.92	1.33	0.94	1.72	1.11	0.8	1.64	1.02	0.74	1.97	1.38	0.98	1.94	1.35	0.96	1.58	0.96	0.69	1.67	1.06	0.76
	CL	0.27			0.272			0.278			0.272			0.27			0.278			0.278			0.27			0.272		
Case 2	0.2	370.37	369.87	256.37	**375.77**	**375.27**	**260.12**	423.83	423.33	293.43	370.36	369.86	256.37	**365.21**	**364.71**	**252.8**	416.09	415.59	288.06	370.37	369.87	256.37	325.69	325.19	225.4	330.22	329.72	228.54
$\phi = 148$	0.22	73.31	72.81	50.47	**74.2**	**73.7**	**51.08**	82.09	81.59	56.55	54.18	53.68	37.21	**53.61**	**53.11**	**36.81**	**59.18**	**58.68**	**40.67**	77.73	77.23	53.53	**69.67**	**69.17**	**47.94**	**70.49**	**69.99**	**48.51**
$\sigma = 0.5$	0.24	19.98	19.47	13.5	**20.18**	**19.67**	**13.64**	**21.9**	**21.39**	**14.83**	14.54	14.03	9.73	**14.42**	**13.91**	**9.64**	**15.54**	**15.03**	**10.42**	21.74	21.23	14.72	**19.86**	**19.35**	**13.42**	**20.05**	**19.54**	**13.55**
$\tau = 96$	0.26	7.27	6.75	4.68	**7.32**	**6.8**	**4.72**	**7.81**	**7.29**	**5.06**	5.87	5.35	3.71	**5.84**	**5.32**	**3.69**	**6.18**	**5.66**	**3.93**	7.91	7.39	5.13	**7.36**	**6.84**	**4.75**	**7.41**	**6.89**	**4.78**
	0.28	3.42	2.88	2	**3.44**	**2.9**	**2.02**	**3.61**	**3.07**	**2.14**	3.15	2.6	1.81	**3.14**	**2.59**	**1.81**	**3.27**	**2.72**	**1.9**	3.66	3.12	2.17	**3.46**	**2.92**	**2.03**	**3.48**	**2.94**	**2.05**
	CL	0.299			0.3			0.301			0.3			0.299			0.301			0.301			0.299			0.3		
Case 3	0.2	370.37	369.87	256.37	552.07	551.57	382.32	480.69	480.19	332.84	370.37	369.87	256.37	258.92	258.42	179.12	327.13	326.63	226.4	370.37	369.87	256.37	291.1	290.6	201.43	420.82	420.32	291.34
$\phi = 80$	0.22	110.7	110.2	76.38	157.64	157.14	108.92	139.42	138.92	96.29	82.06	81.56	56.53	61.69	61.19	42.41	74.32	73.82	51.17	110.48	109.98	76.23	96.19	95.69	66.33	134.07	133.57	92.58
$\sigma = 0.71$	0.24	39.49	38.99	27.02	53.78	53.28	36.93	48.3	47.8	33.13	27.46	26.96	18.69	21.82	21.31	14.78	25.95	25.45	17.64	44.76	44.26	30.68	36.91	36.41	25.24	49.59	49.09	34.03
$\tau = 51$	0.26	16.57	16.06	11.14	21.6	21.09	14.62	**19.69**	**19.18**	**13.3**	12.2	11.69	8.1	**10.13**	**9.62**	**6.67**	**11.44**	**10.93**	**7.58**	19.22	18.71	12.97	**16.24**	**15.73**	**10.91**	**21.03**	**20.52**	**14.23**
	0.28	8.07	7.55	5.24	16.08	15.57	10.8	**9.33**	**8.82**	**6.11**	6.63	6.11	4.24	**5.7**	**5.18**	**3.59**	**6.29**	**5.77**	**4**	9.37	8.86	6.14	**8.11**	**7.59**	**5.27**	**10.12**	**9.61**	**6.66**
	CL	0.338			0.346			0.343			0.346			0.338			0.343			0.343			0.338			0.346		
Case 4	0.2	370.37	369.87	256.37	758.21	757.71	525.2	509.91	509.41	353.1	370.38	369.88	256.38	198.9	198.4	137.52	261.35	260.85	180.81	370.37	369.87	256.37	278.35	277.85	192.59	531.47	530.97	368.04
$\phi = 31$	0.22	173.39	172.89	119.84	338.09	337.59	234	232.65	232.15	160.91	123.04	122.54	84.94	73.39	72.89	50.52	92.08	91.58	63.48	187.54	187.04	129.65	143.04	142.54	98.8	264.38	263.88	182.91
$\sigma = 1.2$	0.24	87.26	86.76	60.14	162.14	161.64	112.04	114.58	114.08	79.07	53.15	52.65	36.49	34.36	33.86	23.47	41.62	41.12	28.5	100.5	100	69.31	77.81	77.31	53.59	139.13	138.63	96.09
$\tau = 20$	0.26	46.94	46.44	32.19	83.17	82.67	57.3	60.34	59.84	41.48	27.6	27.1	18.78	19.01	18.5	12.83	22.39	21.88	15.17	56.66	56.16	38.93	44.54	44.04	30.52	77	76.5	53.03
	0.28	26.87	26.37	18.28	45.42	44.92	31.13	33.82	33.32	23.09	16.37	15.86	11	11.86	11.35	7.87	**13.66**	**13.15**	**9.12**	33.46	32.96	22.84	26.72	26.22	18.17	44.63	44.13	30.59
	CL	0.4313			0.4524			0.4408			0.4524			0.4313			0.4408			0.4408			0.4313			0.4524		

TABLE 4 Performance of the control charts: Decreases in the average proportion

		Beta - True		Simplex			UGamma			Simplex - True			Beta			UGamma			UGamma - True			Beta			Simplex		
	μ	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
Case 1	0.2	370.37	369.87	256.37	267.92	267.42	185.36	809.99	809.49	561.1	370.37	369.87	256.37	537.4	536.9	372.15	1337.7	1337.2	926.88	370.37	369.87	256.37	187.05	186.55	129.31	141.07	140.57	97.44
$\phi = 290$	0.18	33.05	32.55	22.56	26.17	25.67	17.79	58.53	58.03	40.22	40.68	40.18	27.85	54.52	54.02	37.44	112.99	112.49	77.97	38.03	37.53	26.01	23.1	22.59	15.66	18.85	18.34	12.72
$\sigma = 0.37$	0.16	5.79	5.27	3.66	4.99	4.46	3.1	8.41	7.89	5.48	6.39	5.87	4.07	7.82	7.3	5.07	13.09	12.58	8.72	7.06	6.54	4.54	5.09	4.56	3.17	4.46	3.93	2.73
$\tau = 155$	0.14	1.96	1.37	0.97	**1.82**	**1.22**	**0.87**	**2.4**	**1.83**	**1.29**	1.78	1.18	0.84	**1.96**	**1.37**	**0.97**	**2.58**	**2.02**	**1.41**	2.33	1.76	1.24	**1.94**	**1.35**	**0.96**	**1.81**	**1.21**	**0.86**
	0.12	1.79	1.19	0.85	**1.15**	**0.42**	**0.34**	**1.27**	**0.59**	**0.45**	1.05	0.23	0.23	**1.07**	**0.27**	**0.25**	**1.14**	**0.4**	**0.33**	1.29	0.61	0.46	**1.2**	**0.49**	**0.39**	**1.17**	**0.45**	**0.36**
	CL	0.1396			0.1417			0.1359			0.1417			0.1396			0.1359			0.1359			0.1396			0.1417		
Case 2	0.2	370.37	369.87	256.37	172.24	171.74	119.04	316.59	316.09	219.1	370.3	369.8	256.33	1026.21	1025.71	710.97	830.47	829.97	575.29	370.37	369.87	256.37	437.45	436.95	302.87	194.59	194.09	134.53
$\phi = 148$	0.18	58.89	58.39	40.47	32.67	32.17	22.3	**52.02**	**51.52**	**35.71**	71.56	71.05	49.25	171.05	170.55	118.22	142.57	142.07	98.48	56.43	55.93	38.77	64.3	63.8	44.22	34.21	33.71	23.36
$\sigma = 0.5$	0.16	13.28	12.77	8.85	8.52	8	5.55	**12.11**	**11.6**	**8.04**	15.47	14.96	10.37	31.17	30.67	21.26	26.98	26.48	18.35	12.56	12.05	8.35	13.83	13.32	9.24	8.71	8.19	5.68
$\tau = 96$	0.14	4.29	3.76	2.61	**3.19**	**2.64**	**1.84**	**4.03**	**3.49**	**2.43**	4.1	3.57	2.48	6.74	6.22	4.32	6.05	5.53	3.84	4.07	3.53	2.46	**4.43**	**3.9**	**2.71**	**3.19**	**2.64**	**1.84**
	0.12	1.99	1.4	0.99	**1.67**	**1.06**	**0.76**	**1.91**	**1.32**	**0.93**	1.57	0.95	0.68	**2.04**	**1.46**	**1.03**	**1.92**	**1.33**	**0.94**	1.9	1.31	0.93	**1.97**	**1.38**	**0.98**	**1.66**	**1.05**	**0.75**
	CL	0.1185			0.125			0.1198			0.125			0.1185			0.1198			0.1198			0.1185			0.125		
Case 3	0.2	370.37	369.87	256.37	177.81	177.31	122.9	322.4	321.9	223.12	370.37	369.87	256.37	1039.92	1039.42	720.47	852.11	851.61	590.29	370.37	369.87	256.37	429.49	428.99	297.35	196.75	196.25	136.03
$\phi = 80$	0.18	88.45	87.95	60.96	48.04	47.54	32.95	78.78	78.28	54.26	117.72	117.22	81.25	299.06	298.56	206.95	249.67	249.17	172.71	85.61	85.11	58.99	96.8	96.3	66.75	50.79	50.29	34.86
$\sigma = 0.71$	0.16	25.54	25.04	17.35	15.65	15.14	10.5	**23.26**	**22.75**	**15.77**	37.51	37.01	25.65	85.2	84.7	58.71	72.62	72.12	49.99	24.43	23.92	16.58	**26.96**	**26.46**	**18.34**	16.11	15.6	10.82
$\tau = 51$	0.14	9	8.49	5.88	6.19	5.67	3.93	**8.38**	**7.86**	**5.45**	12.15	11.64	8.07	24.1	23.59	16.36	21.13	20.62	14.3	8.6	8.08	5.61	**9.28**	**8.77**	**6.08**	6.27	5.75	3.99
	0.12	3.91	3.37	2.35	**3**	**2.45**	**1.71**	**3.72**	**3.18**	**2.21**	4.2	3.67	2.55	7.09	6.57	4.56	6.38	5.86	4.07	3.75	3.21	2.23	**3.95**	**3.41**	**2.37**	**3**	**2.45**	**1.71**
	CL	0.0946			0.102			0.0959			0.102			0.0946			0.0959			0.0959			0.0946			0.102		
Case 4	0.2	370.37	369.87	256.37	129.76	129.26	89.6	270.61	270.11	187.23	370.41	369.91	256.4	3142.94	3142.44	2178.17	1661.38	1660.88	1151.23	370.37	369.87	256.37	529.4	528.9	366.61	149.46	148.96	103.25
$\phi = 31$	0.18	136.69	136.19	94.4	51.27	50.77	35.19	103.79	103.29	71.59	189.71	189.21	131.15	1418.63	1418.13	982.97	777.81	777.31	538.79	132.04	131.54	91.18	180.5	180	124.77	59.9	59.4	41.17
$\sigma = 1.2$	0.16	54.28	53.78	37.28	22.38	21.87	15.16	42.82	42.32	29.33	95.1	94.6	65.57	625.65	625.15	433.32	355.86	355.36	246.32	51.59	51.09	35.41	67.54	67.04	46.47	26.21	25.71	17.82
$\tau = 20$	0.14	23.31	22.8	15.81	11.5	10.99	7.62	19.09	18.58	12.88	45.98	45.48	31.52	264.02	263.52	182.66	156.06	155.56	107.83	22.05	21.54	14.93	27.67	27.17	18.83	12.5	11.99	8.31
	0.12	10.88	10.37	7.19	6.13	5.61	3.89	**9.25**	**8.74**	**6.06**	21.07	20.56	14.26	103.75	103.25	71.57	64.01	63.51	44.02	10.3	9.79	6.79	**12.41**	**11.9**	**8.25**	6.5	5.98	4.15
	CL	0.0514			0.0643			0.0546			0.0643			0.0514			0.0546			0.0546			0.0514			0.0643		

that the data are collected in-control state at Phase I), the following steps may help the practitioners when they are faced with such problem:

1. List the candidate distributions.
2. Obtain the maximum likelihood estimates of the parameters of the candidate distributions.
3. Calculate Akaike information criterion (AIC) and Bayesian information criterion (BIC) or alternatively get the p values of some well-known adherence tests—Kolmogorov-Smirnov, Anderson-Darling, Cramér von Mises, etc—for each candidate distribution.
4. Choose the distribution associated with the lowest value of AIC, BIC, or the largest p value.
5. Check the appropriateness of the choice by comparing the histogram of the data set and the probability density function of the best candidate distribution. If the choice is adequate, then find the control limits and apply the control chart, otherwise, go back to step 1.

5 | APPLICATION TO A REAL DATA SET

In this section, we illustrate our methodology by using a real data set. We consider a data set from SantAnna and ten Caten[24] consisting of a data set of the study of contaminated peanut by toxic substances in 34 batches of 120 pounds. The variable monitored is the proportion of non-contaminated peanuts. The data are 0.971, 0.979, 0.982, 0.971, 0.957, 0.961, 0.956, 0.972, 0.889, 0.961, 0.982, 0.975, 0.942, 0.932, 0.908, 0.970, 0.985, 0.933, 0.858, 0.987, 0.958, 0.909, 0.859, 0.863, 0.811, 0.877, 0.798, 0.855, 0.788, 0.821, 0.830, 0.718, 0.642, and 0.658. Obviously, due to the genesis of the Beta, Simplex and Unit gamma distributions, rates and proportions are by excellence ideally modelled by these distributions. Thus, the use of the Beta, Simplex, and Unit Gamma distributions for fitting this data set is well justified. In order to estimate the parameters of these models, we adopt the maximum likelihood method and all the computations were done using the R software. We will use the first 20 observations as the Phase I sample and the remaining 14 values as the Phase II observations, to demonstrate the application of the proposed control charts. The empirical mean and standard deviation for the first 20 observations are, respectively, 0.9536 and 0.0345.

The maximum likelihood estimates of the parameters (with corresponding standard errors in parentheses), AIC and BIC for the Beta, Simplex, and Unit Gamma models for the first 20 observations are listed in Table 5. Since the values of the AIC and BIC are smaller for the Simplex distribution compared with those values of the other models, the Simplex distribution seems to be a very competitive model for these data.

Plots of the pdf of the Beta, Simplex and Unit Gamma fitted models to these data are displayed in Figure 2. They indicate that the Simplex distribution is superior to the other distributions in terms of model fitting. Additionally, it is evident that the Beta and Unit gamma distributions presents the same fit to the current data.

TABLE 5 Estimates of the parameters (standard errors in parentheses) and goodness-of-fit statistics for the proportion of noncontaminated peanut

Model		ML Estimates	AIC	BIC
Beta	$\hat{\mu}$	0.9533(0.0066)	−85.455	−83.464
	$\hat{\phi}$	40.442(15.948)		
Simplex	$\hat{\mu}$	0.9534(0.0072)	−88.653	−86.662
	$\hat{\sigma}^2$	3.5711(0.5653)		
Unit Gamma	$\hat{\mu}$	0.9534(0.0072)	−85.455	−83.463
	$\hat{\tau}$	2.2798(0.6715)		

Abbreviation: AIC, Akaike information criterion.

In Figure 3, we present the control chart for the complete set of the available data (Phase I: 1-20; Phase II: 21-34) using error type I $\alpha = 0.0027$ to yield $\text{ARL}_0 = 370$. The control chart in Figure 3 does not trigger an alarm during Phase I, so we obtain no contradiction against the models. Note that the Beta and Unit gamma distributions present the same LCL and UCL to the current data.

Let us now proceed to Phase II analysis. Here, the Beta and Unit Gamma control chart triggers for the first time a signal at sample 25, whereas the Simplex control chart counterpart gives a signal at sample 32. Much earlier false alarms ($ARL_0 = 139$; $ARL_0 = 166.85$) is the consequence in case of the Beta or Unit Gamma control limits used equivocally in Simplex chart (see Table 6). Now, consider that the manager suspects that the quality of peanuts has deteriorated from $\mu_0 = 0.95$ to $\mu_1 = 0.8$. For such shift, no impact is observed in the performance metrics with all ARL_1 around three (see Table 6).

In a nutshell, our analysis clearly showed that the three models may be very useful in practice for describing proportion data.

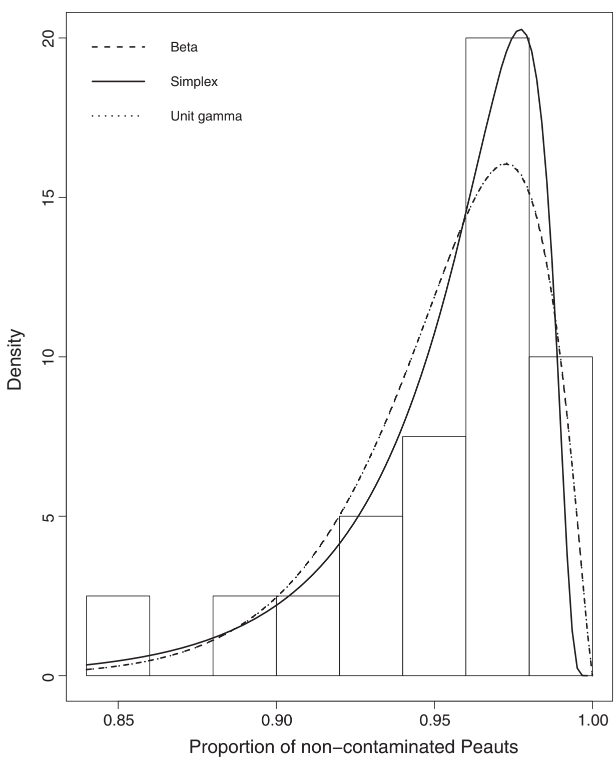


FIGURE 2 Estimated probability density function (pdf) for the Beta, Simplex, and Unit Gamma models for the proportion of noncontaminated peanut

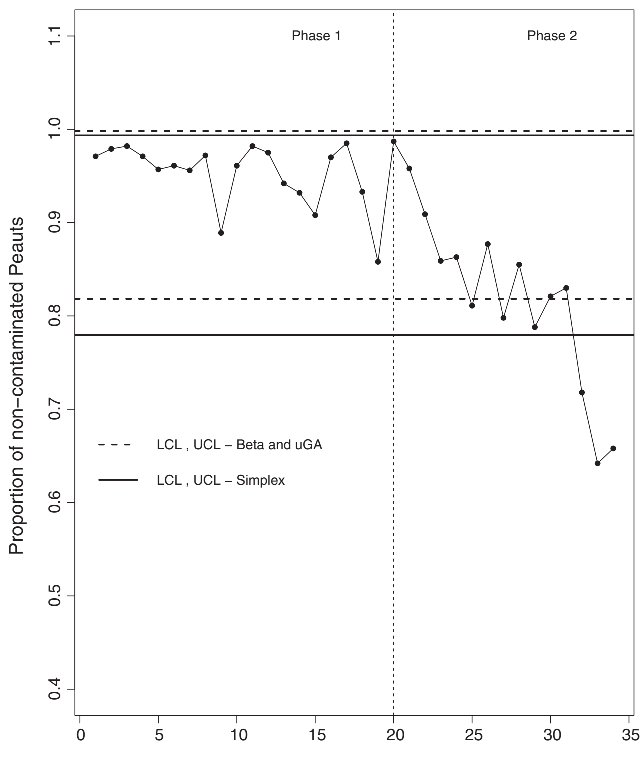


FIGURE 3 Control chart (in terms of ARL_1) for the proportion of noncontaminated peanut (Phase I: 1-20; Phase II: 21-34)

TABLE 6 Comparing the performance for the proportion of noncontaminated peanut

	Simplex - True			Beta			UGamma		
μ	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
0.95	370.61	370.11	256.54	139.10	138.60	96.07	166.85	166.35	115.31
0.8	3.27	2.72	1.89	2.54	1.98	1.38	2.62	2.05	1.44
LCL	0.758			0.812			0.806		
UCL	0.993			0.997			0.998		

6 | CONCLUDING REMARKS

In this paper, we call attention for monitoring proportions and rates when they are not results of Bernoulli experiments. In practice, control charts based on Beta distributions are usually built as this distribution is a well-known one. However, there are other distributions to be considered as for example, Simplex and Unit Gamma distributions. The use of equivocal control limits provokes a great variety of impacts like anticipation/postponement of false alarms or even no false alarm and also no detection mainly in case of small shifts. The variance magnitude and shift sizes play an important role as lower impacts are observed for larger shifts and small variances.

Thus, to monitor proportions or rates (not from Bernoulli experiments and unknown baseline distribution), it is essential to identify which distribution better fits the data set among many candidate distributions and then design the control chart to avoid the use of equivocal control limits. Additionally, some guidelines are suggested for the practitioners when they are faced with this problem.

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