LendBook
a Lending Limit Order Book

Breaking Lab *

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Abstract

A lending limit order book is a non-custodial and permissionless lending protocol that enables users to borrow assets from limit orders collateralized by their own limit orders. This new financial primitive offers users multiple benefits: low liquidation penalty, high loan-to-value, high leverage, stop loss orders with guaranteed stop price, interest-bearing limit orders and leveraged limit orders. The protocol is immune to the risk of bad debt and can operate without the need for off-chain governance.

Current issues with lending protocols

Lending protocols provide users with the ability to lend and borrow cryptoassets in a decentralized, permissionless, and trustless manner. However, their growth has been impeded by a common birth flaw: the risk of incurring bad debt. A bad debt appears when the price of the deposited collateral rapidly falls and its value no longer covers the outstanding loan, putting the protocol at risk of insolvency

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and bank run. The loss can be exacerbated by the lack of sufficient liquidity in the pools in which the collateral assets are exchanged to cover borrowed assets during liquidation events.[1]

To mitigate this existential risk, lending protocols impose various constraints on the borrowers side, such as high collateral-to-debt ratios and high liquidation costs. They also limit lending markets to high-quality assets. These constraints impair users' experience by making borrowing more expensive and riskier and by restricting the range of borrowable assets. Despite many innovative features introduced in the sector since its emergence in 2018, significant improvement of users experience is still awaiting a foolproof solution for the risk of bad debt.

LendBook is a new lending primitive which eliminates the insolvency risk, can achieve full decentralization by getting rid of off-chain risk management and brings along the way a host of new benefits for lenders and borrowers. Section 1 explains what is a lending limit order book. Section 2 presents the details of its functioning. Section 3 describes the interest rate mode. Section 4 compile the list of all benefits brought by LendBook.

1   What is a lending limit order book

A lending limit order book (LLOB) is a non-custodial and permissionless order book that enables users to place limit orders and borrow the assets from. Borrowing positions are collateralized by sell orders. Figure 1 presents the double queue organization of a typical central limit order book. On the left-hand side, lenders place buy orders which traders find profitable to take if the price decreases below their limit prices. On the right-hand side, they place sell orders which are filled if the price increases above their limit prices. Rectangles' height indicate the amount of assets lenders have committed in their limit orders.

Fig.   2 shows how a lending primitive is attached to a limit order book.

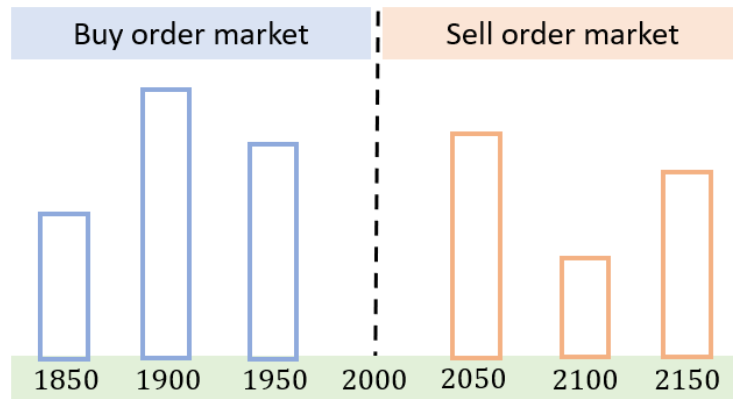[1]See RiskDAO for examples of bad debt events.

Figure 1: A graphical representation of a central limit order book (market price is 2000).

Lenders allow traders to borrow their assets deposited in buy orders in exchange of an interest rate. The rectangles with a blue background represent the total assets borrowed from the orders at the corresponding limit price.
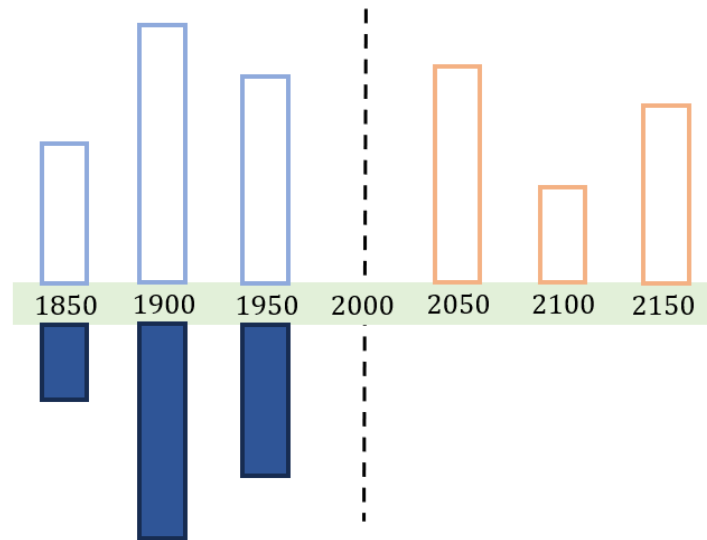

Figure 2: A central limit order book with lending functionalities.

The protocol ensures that any position borrowing from a pool of orders can be liquidated when part or all assets of the pool are taken. The alignment of the two events significantly streamlines the settlement process for all parties.

Users trade asset pairs in which only one of the two assets can be borrowed and the other serve as collateral. In other words, assets which serve as collateral cannot be borrowed and borrowable assets cannot serve as collateral.

To concentrate liquidity, lenders are permitted to deposit assets within a restricted set of limit prices. A pool of limit orders groups all assets deposited at the same limit price allowing users to deposit and borrow from the pools, rather than from individual orders.

Example

Let's illustrate how a lending operation works in the simplest case of two actors and two orders. Market price is 2000. Alice posts a buy order of 3 ETH at price 1900 USDC. To do so, she deposits $3 \times 1900 = 5700$ USDC in the protocol's USDC pool. Bob is willing to borrow 3800 USDC from Alice's buy order. He places a sell order of 2 ETH at 2200 (or any price greater than market price) and deposits 2 ETH in the ETH pool. With 2 ETH of collateral, he can then borrow $2 \times 1900 = 3800$ USDC from Alice. The financial flows are summarized in Fig. 3.
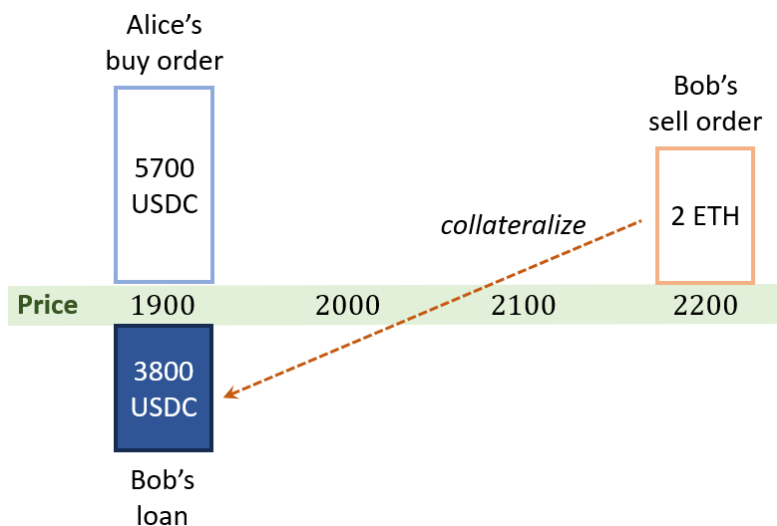

Figure 3: Bob borrows 3800 USDC from Alice, collateralized by his sell order at 2200.

If the price decreases to 1900, a taker swaps Alice's remaining 1900 USDC for 1 ETH, which triggers the closing of Bob's position. Bob keeps the borrowed USDC and Alice is given 2 ETH taken from Bob's collateral (see Fig. 4).
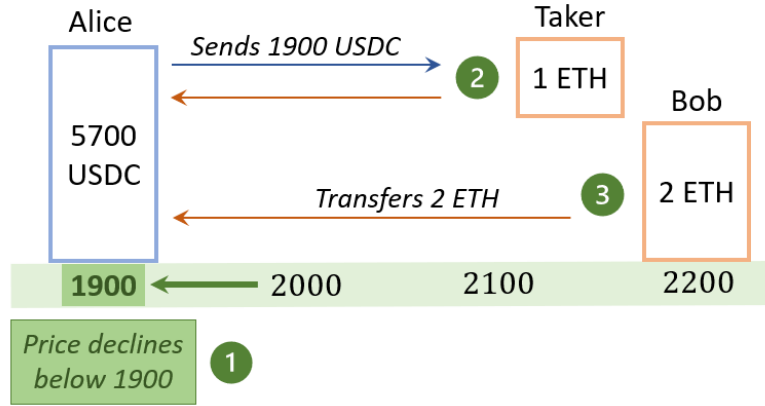

Figure 4: If the price decreases to 1900 (1), Alice's remaining assets are taken (2). Bob's collateral is transferred to Alice to complete the filling (3).

Nothing changes for Alice compared to a non-borrowed buy order. Importantly, Bob's assets are not market sold but directly transferred to Alice who is paid back with the collateral.

Before reviewing the benefits of implementing a lending order book, let's dive into the details of its functioning.

2 Loan-to-value

The maximum LTV ratio defines the maximum amount of assets that can be borrowed for a given quantity of collateral. Suppose a user deposits C units of collateral asset in a sell order. To borrow D units of quote assets from a buy order at limit price L, his collateral must be enough to be traded against his debt if the limit order is filled: $D/L \leq C$. D/L is the minimum collateral required if the debt D is liquidated. In practice, his borrowing capacity is capped by the

liquidation loan-to-value (LLTV < 1):

$$\frac{D}{L} \leq \text{LLTV}.C \qquad (1)$$

The LLTV is the LTV ratio above which positions are eligible to liquidation. It is computed by using the collateral assessed at limit price (see Subsection 5.2).

Example (continued): With 1 ETH deposited in a sell order, Bob can borrow from buy orders at 1900, with LLTV = 0.98, up to 0.98 × 1900 = 1962 USDC.

The condition easily extends to users depositing in several sell orders and borrowing from several buy orders. Borrowing capacity of a user depositing collateral C in pools l and borrowing quote assets D in pools j is:

$$\sum_j \frac{D_j}{L_j} \leq \text{LLTV} \sum_l C_l$$

Example: Bob wants to borrow 1800 USDC at limit price 1800 and 2200 USDC at limit price 1600. To do so, he needs to deposit at least, with LLTV = 0.98, (2200/1600 + 1800/1800)/0.98 = 2.42 ETH in one or several sell orders.

3 Interest rate model

3.1 Pool's excess liquidity

Not all assets deposited in a pool of buy orders can be borrowed, as sufficient incentives must be preserved for traders to take the non-borrowed part of the assets when the price hits the limit price. Pool i's excess liquidity (EL) is defined by

$$\text{EL}_i = \phi D_i - B_i$$

where D_i are total deposits in the pool and B_i borrowed assets from the pool. The parameter $\phi \in (0, 1)$ defines the proportion of the deposits available for borrowers (e.g. 90%). Pool i's EL must always be positive. Borrowing or withdrawing from

the pool reduces pool's EL and makes less assets available to borrow or withdraw for other users.

The interest rate is the price of the liquidity present in each pool and guarantees the equilibrium between demand and supply.

3.2   Interest rate curve

Interest rate algorithmically increases with pool's utilization rate, which is the ratio of the total borrowed assets to available supplied assets:

$$U_i = \frac{B_i}{\phi D_i}$$

so that either lenders will be able to exit the pool and users borrow from the pool or interest rate will be very high (possibly up to 100%). The utilization rate has a maximum value of 100%. There exists as many utilization rate and interest rate than active pools. The higher the pool's utilization rate, the higher the interest rate. The borrowers' interest rate curve is split in two parts around an optimal utilization rate U^*. Before U^* the slope is small, after it begins rising sharply:

$$U_i \leq U^* : R_i = \alpha + \beta \frac{U_i}{U^*}$$

$$U_i > U^* : R_i = \alpha + \beta + \gamma \frac{U_i - U^*}{1 - U^*}$$

with α, β and $\gamma > \beta$ three positive parameters. R_i is paid by users borrowing from pool i. Lenders earn an interest rate scaled down by the pool's utilization rate:

$$R_i^l = \frac{B_i}{D_i} R_i$$

Deposits in sell orders cannot be borrowed and do not earn an interest rate.

3.3 Interest-based liquidation

As time goes on, the borrowed amount increases due to the compound interest rate. A collateralized position can become under-collateralized and subject to liquidation. This type of liquidation is distinct from liquidations triggered by the filling of limit orders.

For a user to avoid liquidation, the sum of his debts, interest rate included, divided by the pools' limit prices L_j from which the debt originates must be lower than the sum of all collateral assets deposited in sell orders scaled down by the LLTV < 1:

$$\sum_j \frac{\mathrm{D}_j}{L_j} \leq \mathrm{LLTV} \sum_l \mathrm{C}_l$$

This is a safety mechanism to protect lenders from borrowers' insolvency due to growing interest rate. Interest rate-based liquidations are at the initiative of external actors who receive in exchange a reward up to 3% of the liquidated assets.

Example (continued): Price is 2000. A liquidator repays Bob's debt equal to 2071 USDC. He receives in exchange 1.03 × 2071/2000 = 1.067 ETH. Bob is left with 1900 USDC and 1.1 - 1.067 = 0.033 ETH.

4   Pools' parameters and asset tiers

Pools' key parameters depend on the volatility of the listed asset pairs. Three asset tiers exist: pegged assets, volatile assets and long-tail assets. See Table 1 for examples.

4.1   Multiplicative step

The first key parameter is the multiplicative step used to compute limit prices at which users can deposit and borrow assets. The more volatile the assets the

larger the step. For instance, a 10% step is used for volatile top-tier assets (see Table 1). The aim is to restrict the number of borrowable pools to 3 or 4.

Illustration with a step of 10%: Market price is 2000. Alice may deposit assets in buy orders at limit price 1900, $1900/1.1 = 1727$ or $1900/1.1^2 = 1579$, and so on, but not at limit prices in between.

Table 1: Asset tiers and multiplicative step

Asset tier	Multiplicative step	**Examples**
Pegged assets	1%	sDAI/USDC, wstETH/ETH
Top-tier volatile assets	10%	WBTC/ETH, USDC/ETH, UNI/ETH
Long-tail assets	15%	RPL/ETH, RUNE/ETH, RBN/ETH

4.2 Liquidation fees

The more volatile the assets, the higher the risk taken by lenders in case of price decrease, and consequently the higher the liquidation fees (see Table 2).

Table 2: Asset tiers and liquidation fees

Asset tier	Liquidation fees
Pegged assets	0.5%
Volatile assets	2%
Long-tail assets*	5%

The main goals of liquidation fees are to compensate lenders for receiving the collateral and give borrowers incentives to repay their loan before liquidation.

4.3 Liquidation Loan-to-value

The more volatile the assets the lower the LLTV (see Table 3).

Table 3: Asset tier and liquidation LTV

Asset tier	LLTV
Pegged assets	99.5%
Volatile assets	98%
Long-tail assets	95%

5 Benefits of using LendBook

The benefits of a lending order book are multiple: low liquidation penalty, high leverage, stop loss orders with guaranteed stop price, programmability of leverage strategies, automated market making and minimized governance. Let's review them one by one.

5.1 Low liquidation costs

The prevalent liquidation mechanism in lending protocols allows a liquidator to repay a fraction of the borrower's debt and acquire its collateral at a discount. In LendBook, when the price crosses a limit price, the closing of a borrowing position does not rely on the active monitoring of liquidators but on that of takers. Traders, by taking the non-borrowed part of the assets, initiate the internal transfer from the borrowers to the lenders. Since lenders agree to receive the collateral as a payment, the protocol does not need to incentivize bots to liquidate unhealthy positions on time.

Borrowers pay liquidation fee rates between 0.5 and 5%, depending on asset volatility (see Table 2). The rate are on average lower than in other lending protocols, which are 5% and more.

Furthermore, lenders, not bots, earn liquidation fees in LendBook. This adds an extra source of income for lenders on top of lending rates.

5.2 High Loan-To-Value and leverage

It is standard to calculate the max LTV by dividing the maximum borrowing capacity D_{max} by the current market value of the collateral:

$$\text{max LTV} = \frac{D_{max}}{pC}$$

Replacing D_{max} by the maximum borrowable amount, given collateral C and limit price L, from (1) gives LendBook's maximum LTV:

$$\text{max LTV} = \text{LLTV}\frac{L}{p} \tag{2}$$

The maximum LTV depends on two factors: the value of the LLTV and the distance of the pool's limit price L to current price p. The closer p to the limit price, the closer the max LTV to LLTV.

Example (continued): With 1 ETH deposited in a sell order, Bob can borrow from buy orders at 1900 up to LLTV × 1900 USDC. His maximum LTV is: LLTV 1900/p. For LLTV = 0.96 and p = 2000, his max LTV is 0.92.

This is different from other lending protocols in which a borrower could in theory borrow up to the LLTV. In LendBook this is only the case when the market price is arbitrarily close to the limit price. Depending on where the market price lies relative to the nearest limit price, the max LTV fluctuates between two bounds (see Table 4).

Table 4: Asset tier and max LTV

Asset tier	minimum max LTV	maximum max LTV
Pegged assets	98.5%	99.5%
Volatile assets	88%	98%
Long-tail assets	80%	95%

As in other protocols, users can borrow quote tokens and swap them to amplify their position in base tokens. High LTV directly translates into high lever-

age. Abstracting from gas and swap costs, the maximum leverage factor is:

$$\frac{1}{1 - \max \text{LTV}}$$

Maximum leverage in LendBook ranges from high to extremely high (see Table 5).

Table 5: Asset tier and max leverage

Asset tier	minimum max leverage	maximum max leverage
Pegged assets	67×	200×
Volatile assets	8×	50×
Long-tail assets	5×	20×

5.3 Stop loss and take profit orders

5.3.1 Stop loss

A stop-loss order allows traders to close long positions by selling the assets or a short position by buying the assets. In LendBook, users open stop-loss orders by borrowing assets from limit orders. The stop price in case of price decrease (or increase) is the limit price of the buy (sell) order from which they borrow.

Example: Bob deposits 2 ETH in a sell order and borrows from Alice's buy order at 1900: 0.98 × 2 × 1900 = 3724 USDC. if the price decreases to 1900, Alice's buy order is taken. Bob keeps the 3724 USDC and gives up his 2 ETH. This is as if he benefits from a stop loss (sell ETH when its price decreases) at the guaranteed price of 1900. His stop price is Alice's limit price.

In traditional or crypto finance, once the stop price is met, the stop loss order becomes a market order and is executed at the next available price. The obtained price can be significantly less favorable than the specified price when

markets move fast. Here the stop price is guaranteed by the filling of the sell order at the limit price.

5.3.2   Take profit

In addition, by posting their collateral in the order book, borrowers can program in advance the price at which they exit their strategy, which is the limit price of their collateral order.

Example: By depositing ETH in a sell order at 2200, Bob benefits from a take-profit at the same price. If the market price increases to 2200, Bob's sell order is taken first. His 2 ETH are exchanged against 4400 USDC from which 3724 are used to pay back his borrowing position (cf. Rule R3). He keeps the 3724 USDC borrowed from Alice and earns a profit of 676 USDC.

Setting in advance an exit price is an integral part of risk management in case of leveraged position.

Example: Bob borrows 3724 USDC from Alice, exchanges the amount for 1.86 ETH at market price 2000 and deposits the amount in his sell order at 2200. He then borrows 0.98 × 1.86 × 1900 = 3445, sells for 1.72 ETH and deposits the proceeds in his sell order. If the price hits 2200, his sell order is filled and his borrowing position is closed out. The protocol pays back Bob's debt of 3724 + 3445 = 7169 USDC with the 2200 × (2 + 1.86 + 1.72) ETH = 12276 USDC of his sell order. Bob's leveraged profit is 12276 - 7169 - 2 × 2200 = 707 USDC.

Fig. 5 shows the price interval over which Bob makes a profit or a loss and at which prices his position is closed out in both directions.

The combination of an order book with a lending protocol unlocks a rich set of strategies that borrowers can fine-tune and program in advance. They can spread their position over several collateral orders and pools with different limit prices. This way, their borrowing can be progressively reduced as the price reaches well-specified thresholds.
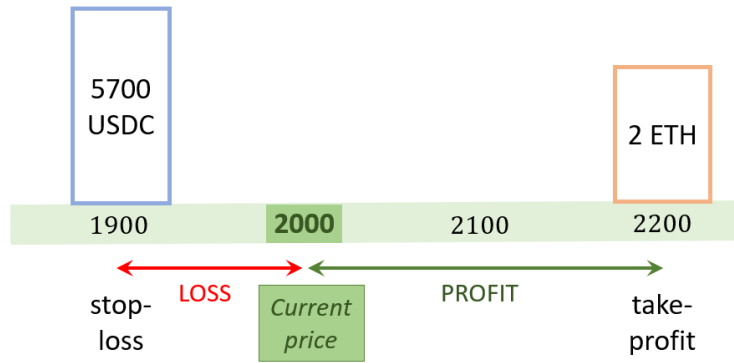
Figure 5: When Bob borrows at 1900 and places a collateral sell order at 2200, his leverage is closed out for a profit if his sell order is taken first and for a loss if the buy order is taken first.
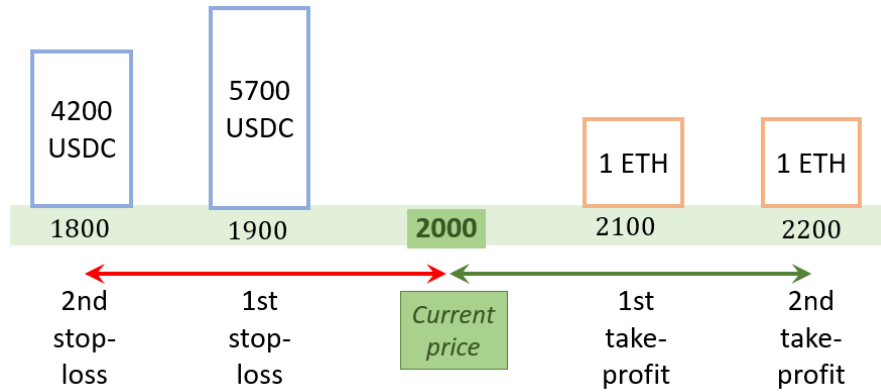

Figure 6: Bob borrows USDC from a pool at 1900 and another one at 1800, collateralized by a sell order at 2100 and another one at 2200. This allows Bob to gradually deleverage his positions at a loss or for a profit.

In Fig. 6, Bob is gradually liquidated at prices 1900 and 1800. If the price increases, he progressively takes profit at 2100 and 2200.

5.4 Absence of bad debt

A major implication of borrowing assets from limit orders is the dramatic simplification and high safety of the liquidation process. Borrowing positions cannot go

under-collateralized even in case of strong and rapid price action, gas fee spike, or blockchain congestion/downtime. While the risk of rapid price variation persists, it is borne by the maker of the limit order, creating an opportunity cost for them. Although this cost is inherent in all limit order books, in LendBook, lenders are compensated through an interest rate and liquidation fees.

The fact that lending pools stay well collateralized under any market conditions brings many benefits at the UX and governance levels. In particular, there is no need for supervision by third party, funding a safety module, liquidation costs, or borrowing restrictions such as supply caps, borrowing caps and low loan-to-value.

5.5 Governance minimization

The governance activity of lending protocols has significantly grown and become more complex over time. Managing the risks of pools has been progressively delegated to experts, whose mission is to monitor the asset markets and update pools' risk parameters. This involves assessing multiple risk factors, such as assets' on-chain liquidity, their price volatility and market capitalization.

Governance-driven risk management hinders protocols from scaling horizontally. The more assets are listed, the greater the number of risk parameters that need to be monitored and updated in real-time. The DAO, with its limited scope for attention and complex decision process, becomes a bottleneck in expanding to more chains and assets.

Moreover, despite committing considerable resources and expertise to risk management, protocols' solvency is still at risk of a lack of due diligence or governance failure. Lending protocols have also implemented and funded sizeable financial buffers to absorb shortfall events and protect lenders from bad debt. Those safety measures mitigate solvency risk at the expense of token holders.

In contrast, the functioning of a LLOB is fully algorithmic and automated. As pools' solvency does not rely on team's interventions or governance by a DAO,

full decentralization becomes a credible objective which LendBook will actively pursue. The protocol will ultimately be governance free with non-upgradeable smart contracts and parameters set at the time of contract deployment.

No governance process will be needed to whitelist approved tokens. Markets will be created permissionlessly by calling a factory contract. The number of assets that could be listed is only limited by the existence of a reliable price feed in the V1.[2] The V2 will expand to long-tail assets by getting rid of price oracles.

6 Use cases

LendBook has many use cases, most of them specifically allowed by the order book base layer.

6.1 Yield amplification

LendBook is exceptionally well-suited for yield amplification in markets with pegged assets in which one is a yield-bearing version of the other.

Let us illustrate the efficiency of the protocol with an example taken in the liquid staking derivative markets. The Uniswap V3 market wstETH/ETH endowed with 40m liquidity, is represented in Fig. 7. In this market, capital is spread between prices 1.1502 and 1.1508 (observed on Dec. 26th 2023).[3]

Assuming a similar liquidity distribution on LendBook, the order book representation is displayed in Fig. 8 where limit prices are spaced by the factor 1.001 (the spacing for pegged assets in Table 1). A strategy amplifying the yield consists in depositing wstETH on the sell order side and borrowing ETH on the

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[2]Aave V3 currently lists 20 or so assets on Ethereum. The additional list of admissible tokens with a Chainlink feed includes SHIB, GRT, SAND, APE, CVX, ANKR, SUSHI, RDNT, BADGER and PERP.
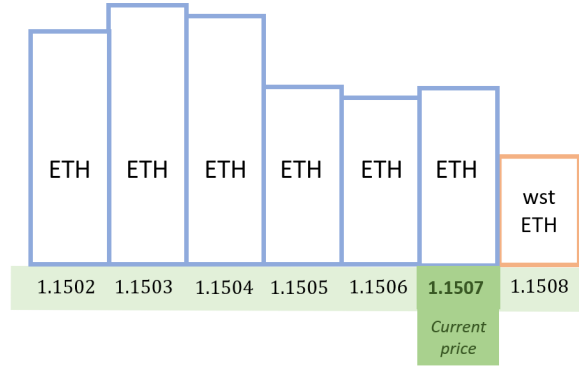
[3]See Uniswap's pool address.

Figure 7: Capital distribution in the Uniswap wstETH/ETH market (observed on Dec. 26th 2023).
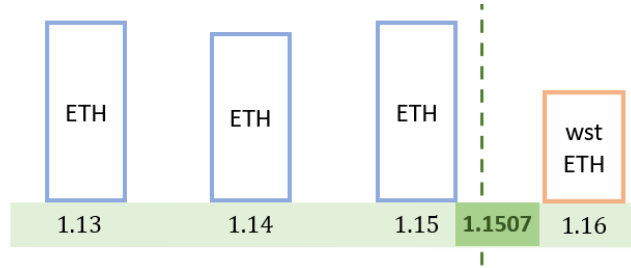

Figure 8: Capital distribution in the wstETH/ETH LendBook market, similar to the Uniswap liquidity pool, with a step factor 1.01.

buy order side. Given a market price of 1.1507, borrowing ETH at the limit price of 1.15 would give users a max LTV of $0.995 \times 1.15/1.1507 = 99.44\%$ (see Eq. 2). Exchanging on Uniswap the ETH for wstETH at a 0.01% fee rate would provide traders with an adjusted LTV of 99.43% (assuming no price impact). The maximum leverage factor is:

$$\frac{1}{1 - \max \text{LTV}} = \frac{1}{1 - 0.9943} = 175\times$$

In practice, the leverage factor will be less due to possible price impact and gas price. But even for a leverage factor of 100, given a wstETH APR of 3.3% and a borrow APY of 2.5%, leveraged APR would be $100 \times 0.8\% = 80\%$.

6.2 Long-tail assets

Oracleless lending will be implemented in the V2 with the aim of addressing the market of long-tail assets for which no reliable price oracles are available. This section explains how LendBook will implement oracleless lending.

In V1, a price oracle is used to prevent users from taking at a loss buy orders which assets are borrowed.

Example: Market price is 2000. Alice deposits 4000 USDC in a buy order at 1900. Bob deposits 2 ETH in a sell order and borrows 3800 USDC from Alice's buy order. Alice's limit order cannot be filled as long as the price feed indicates a market price above 1900.

This restriction safeguards against premature liquidations by external parties. While filling orders at an unfair price is costly, the cost could be mitigated by filling a small quantity, or even negated if the taker is the lender. This could result to an undue profit at the expense of borrowers.

Example (continued): In absence of restriction, Alice could fill the remaining assets in the pool of buy order at market price 2000, which would liquidate Bob's position. Bob is left with 3800 USDC in exchange of 2 ETH worth 4000 at current price. Alice's profit is (2 + 200/1900) × 2000 - 4000 = 210 USDC.

It is possible to guarantee that limit orders are executed at a profit without relying on an oracle with some extra code. Consider a trader who is willing to take the available amount Y from a buy order which limit price is L_b. The filling is now preceded by the taker placing during a short time window the amount Y/L_b in a sell order at the same limit price L_b. If the sell order is unprofitable to take (meaning $(p - L_b)Y/L_b < 0$), the order offering Y at price L_b is profitable and the liquidation is allowed.

In the converse case $((p - L_b)Y/L_b > 0)$, the sell order will be filled at the expense of the maker, before liquidation takes place. The mechanism, which necessitates one honest taker, infers the relevant range of the market price and imposes a penalty on users trying to liquidate well collateralized loans.

Example (variant): To fill the remaining assets in the buy order at market price 2000, the taker has first to post a sell order with the amount 200/1900 = 0.11 ETH at limit price 1900. If the market price is lower than 1900, the sell order cannot be filled at a profit (0.11 × (p - 1900) < 0) and the taker is authorized to fill the buy order. Conversely, if the market price is higher than 1900, the sell order is filled and the maker loses her stake.

6.3   Leveraged limit orders

Two types of leverage exist in finance, leveraged longs, where users borrow quote assets and buy more base assets, and leveraged shorts where users borrow base assets and buy more quote assets. Borrowers expect the price of the base asset to increase in the first strategy and to decrease in the second one.

LendBook introduces a third type of leverage: leveraged limit orders, for which borrowers expect the price to decrease below L and then to reverse its course. Placing buy orders at price L is the relevant strategy to bet on this type of price dynamics. Users are now offered the possibility to leverage the strategy. They can do it in two ways.

The method consists in depositing collateral in sell orders, borrowing quote assets in buy orders and replacing the assets in a buy order with a higher limit price. The strategy allows to gradually raise the leverage as the price moves.

Example: In Fig. 9, Bob places a sell order of 2 ETH at 2200, borrows 3500 USDC from Alice at 1800 and replaces the amount in a buy order at 1900. If the price decreases to 1900, Bob's USDC are taken in exchange of 3500/1900 = 1.84 ETH. Bob is now leveraged long ETH. If the price reverts and increases above 1900, Bob profits.
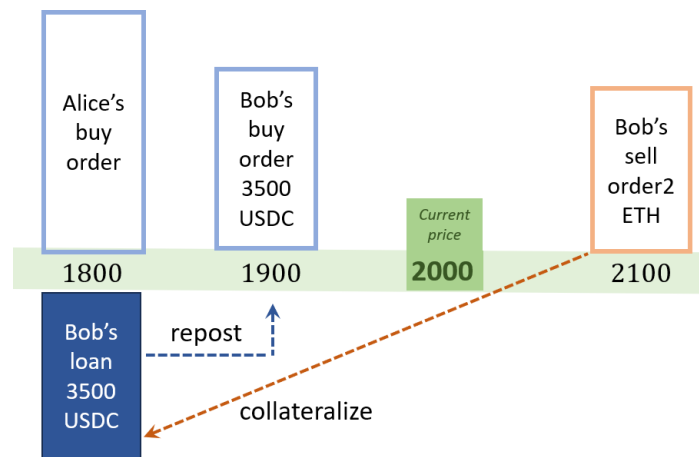
Figure 9: Bob replaces his borrowed USDC on the order book to leverage long his position if the price declines to 1900.

7 Conclusion

Lending protocols are an essential building block for blockchains' applications. They have grown to represent tens of billions of dollars in value. However, the vast majority of this value is held in smart contracts which management is still partially centralized. A complete decentralization process has failed so far, due to a persistent risk of insolvency, which management creates points of centralization.

LendBook's immunity to insolvency risk marks a significant advancement in the defi space. There is no concept of bad debt that might need to be absorbed by a DAO treasury / insurance fund or socialized across lenders. The radically innovative design unlocks many new features like high LTV and leverage, borrowing programmability and interest-bearing limit orders. This also makes possible the protocol to operate in a fully decentralized way.

Appendix: Continuous compounding

Interest income accrues every second using the block timestamp. R_0 is the initial interest rate at date 0. n_1 seconds later, the interest rate changes to R_1, n_2 seconds later to R_2 and so forth.

Borrowing interest rate accumulator (BIRA)

A BIRA is the continuously compounded interest rate since genesis date 0 and present T is $e^{\mathrm{TWIR}_{0 \to T}} - 1$ where $\mathrm{TWIR}_{0 \to T}$ is the time-weighted sum of interest rates since origin:

$$\mathrm{TWIR}_{0 \to T} = n_1 \frac{R_0}{N} + (n_2 - n_1)\frac{R_1}{N} + ... + (n_T - n_{T-1})\frac{R_{T-1}}{N}$$

with R_t/N the sub-period annualized interest rate valid between t and $t+1$ and N the number of seconds in a year. There are as many BRA as buy order pools.

Suppose a user borrows assets between dates t and T (present). Borrower's interest rate is $e^{\mathrm{TWIR}_{t \to T}} - 1$ where

$$\mathrm{TWIR}_{t \to T} = \mathrm{TWIR}_{0 \to T} - \mathrm{TWIR}_{0 \to t}$$

$$= (n_{t+1} - n_t)\frac{R_t}{N} + ... + (n_T - n_{T-1})\frac{R_{T-1}}{N}$$

Applying a third-order Taylor expansion:

$$e^{\mathrm{TWIR}_{t \to T}} - 1 \approx \mathrm{TWIR}_{t \to T} + \frac{\mathrm{TWIR}_{t \to T}^2}{2} + \frac{\mathrm{TWIR}_{t \to T}^3}{6}$$

Lending interest rate accumulator (LIRA)

The interest rate earned by individual depositors can be tracked by applying current pool's interest rate to deposit size normalized by the pool's utilization rate.

While TWIR_T applies to borrowed assets, TUWIR_T, the date T time- and UR-weighted sum of interest rates (or LIRA) applies to deposits:

$$\text{TUWIR}_T = n_1 \frac{R_0}{N} U_0 + (n_2 - n_1) \frac{R_1}{N} U_1 + ... + (n_T - n_{T-1}) \frac{R_{T-1}}{N} U_{T-1}$$

as 1 dollar of borrowed assets is equal to 1 dollar of borrowable deposits multiplied by the pool's utilization rate B/D. There are as many LIRA as buy order pools.

Suppose a lender deposits borrowable assets between dates $t \geq 0$ and T (present). Lender's interest rate is $e^{\text{dTUWIR}_{t \to T}} - 1$ with $\text{dTUWIR}_{t \to T} = \text{TUWIR}_T - \text{TUWIR}_t$. The third-order Taylor expansion is:

$$e^{\text{dTUWIR}_{t \to T}} - 1 \approx \text{dTUWIR}_{t \to T} + \frac{\text{dTUWIR}_{t \to T}^2}{2} + \frac{\text{dTUWIR}_{t \to T}^3}{6}$$

APY

APY represents the effective annual interest rate over a one-year period. It considers the effect of compounding, which means that interest is calculated not only on the initial principal but also on the accumulated interest from previous periods.

The borrowing APY starting at date t is: $e^{R_t^b} - 1$, with R_t^b the instant borrow rate multiplied by the number of seconds in a year. Applying a third-order Taylor expansion:

$$e^{R_t^b} - 1 \approx R_t^b + \frac{(R_t^b)^2}{2} + \frac{(R_t^b)^3}{6}$$

The lending APY is the borrowing APY scaled down by the utilization rate U:

$$e^{R_t^l} - 1 \approx U.R_t^b + \frac{(U.R_t^b)^2}{2} + \frac{(U.R_t^b)^3}{6}$$

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