CS320 Concepts of programming languages
Part II, Quiz 2
12/06/2023

Name:
BU Id:
Section A1 – B1

Question 1 (1 point)
The operational semantics describes the behavior of a program at a level of abstraction that is independent from a particular machine model.
(a) True
(b) False

Question 2 (1 point)
The operational semantics is usually described using a formal grammar.
(a) True
(b) False

Consider the following language with booleans, and integers L_0:

$$\textit{constants} \quad \langle const \rangle ::= boolean \mid int \mid \texttt{error}$$
$$\textit{expressions} \quad \langle expr \rangle ::= \langle const \rangle \mid \textbf{add}\ (\langle expr \rangle, \langle expr \rangle) \mid \textbf{eq}\ (\langle expr \rangle, \langle expr \rangle)$$

Consider the following rules defining the operational semantics of L_0. In this operational semantics a configuration is just an expression, which we will denote using the meta-variables $x, y, \ldots$. We use the notation $x \Rightarrow y$ to say that from the configuration/expression x we can get in one step to the configuration/expression y. Similarly, we use the notation $x \Rightarrow^n y$ to say that from the configuration/expression x we can get, in n steps, to the configuration/expression y. Here $\mathbb{Z}$ is the set of all integers, and $\mathbb{B}$ is the set of all booleans. $+$, and $=$ are the usual mathematical notion of integer addition, and equality.

$$\frac{}{x \Rightarrow^0 x}\ \textsc{Multi-Base} \qquad \frac{x \Rightarrow^n y \qquad y \Rightarrow z}{x \Rightarrow^{n+1} z}\ \textsc{Multi-Ind} \qquad \frac{x \Rightarrow x'}{\textbf{add}\ (x, y) \Rightarrow\ \textbf{add}\ (x', y)}\ \textsc{Add-Left}$$

$$\frac{x \in \mathbb{Z} \qquad y \Rightarrow y'}{\textbf{add}\ (x, y) \Rightarrow\ \textbf{add}\ (x, y')}\ \textsc{Add-Right} \qquad \frac{x \in \mathbb{Z} \qquad y \in \mathbb{Z}}{\textbf{add}\ (x, y) \Rightarrow (x + y)}\ \textsc{Add-Success}$$

$$\frac{x \in \mathbb{B} \cup \{\texttt{error}\}}{\textbf{add}\ (x, y) \Rightarrow \texttt{error}}\ \textsc{Add-Left-Error} \qquad \frac{x \in \mathbb{Z} \qquad y \in \mathbb{B} \cup \{\texttt{error}\}}{\textbf{add}\ (x, y) \Rightarrow \texttt{error}}\ \textsc{Add-Right-Error}$$

$$\frac{x \Rightarrow x'}{\textbf{eq}\ (x, y) \Rightarrow\ \textbf{eq}\ (x', y)}\ \textsc{Eq-Left} \qquad \frac{x \in \mathbb{Z} \qquad y \Rightarrow y'}{\textbf{eq}\ (x, y) \Rightarrow\ \textbf{eq}\ (x, y')}\ \textsc{Eq-Right}$$

$$\frac{x \in \mathbb{Z}}{\textbf{eq}\ (x, x) \Rightarrow\ \textbf{true}}\ \textsc{Eq-True} \qquad \frac{x, y \in \mathbb{Z} \qquad x \neq y}{\textbf{eq}\ (x, y) \Rightarrow\ \textbf{false}}\ \textsc{Eq-False}$$

$$\frac{x \in \mathbb{B} \cup \{\texttt{error}\}}{\textbf{eq}\ (x, y) \Rightarrow \texttt{error}}\ \textsc{Eq-Left-Error} \qquad \frac{x \in \mathbb{Z} \qquad y \in \mathbb{B} \cup \{\texttt{error}\}}{\textbf{eq}\ (x, y) \Rightarrow \texttt{error}}\ \textsc{Eq-Right-Error}$$

Question 3 (2 points)
Consider the operational semantics defined above, which of the following judgments cannot be proved with it.

1. `Add(Add(2,3),Add(4,5))` $\Rightarrow^3$ `14`
2. `Add(Add(2,3),Add(4,5))` $\Rightarrow$ `Add(5,Add(4,5))`
3. `Add(Add(2,3),Add(4,5))` $\Rightarrow$ `Add(Add(2,3),9)`
4. `Add(Add(2,3),Add(4,5))` $\Rightarrow$ `Add(5,9)`

Question 4 (2 points)
Consider the operational semantics defined above, which of the following judgments cannot be proved with it.

1. `Add(Eq(2,3),1)` $\Rightarrow^2$ `error`
2. `Eq(Add(2,3),2)` $\Rightarrow^2$ `error`
3. `Add(error,1)` $\Rightarrow^1$ `error`
4. `Add(1,error)` $\Rightarrow$ `error`

Question 5 (3 points)
Prove the following judgement by drawing its derivation tree:

`Add(Add(2,3),1)` $\Rightarrow_2$ `6`

Question 6 (3 points)

Prove the following judgement by drawing its derivation tree:

`add(eq(2,3),eq(3,3))` $\Rightarrow_2$ `error`