Modelling and Optimization

INF170

#14:Vehicle Routing Problem

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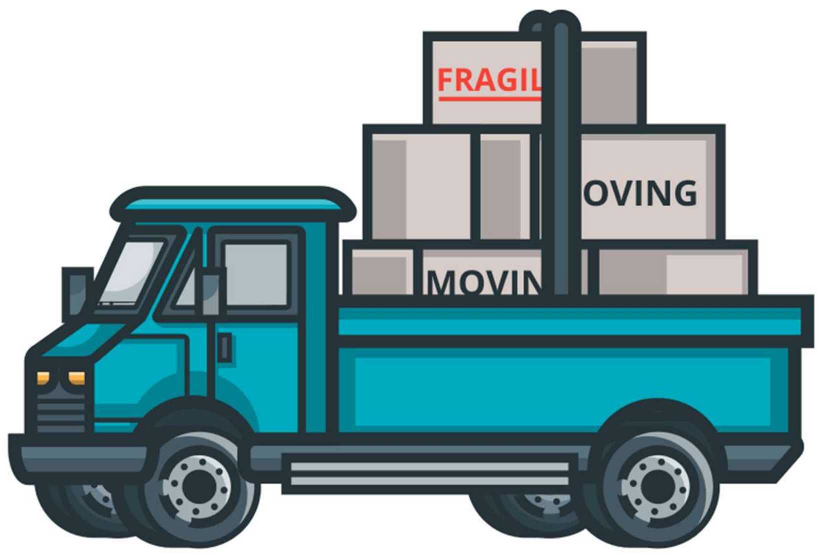
Agenda

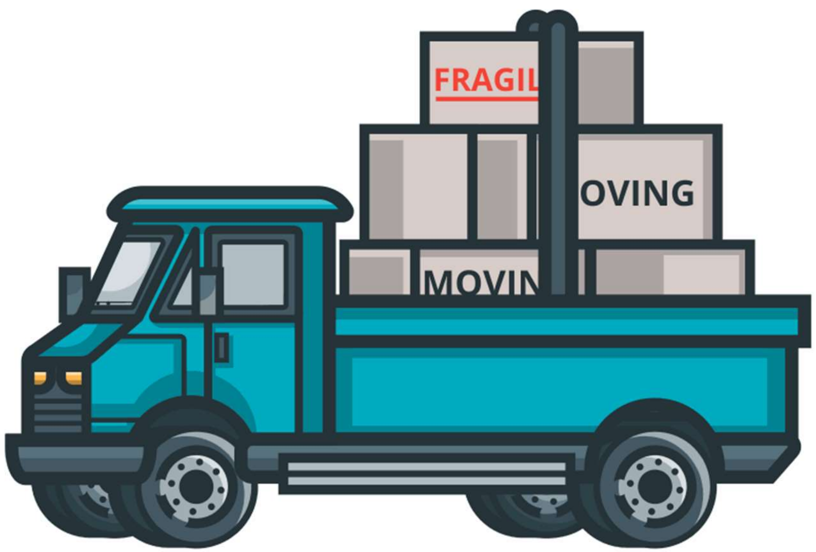
- Vehicle Routing Problems
- VRP Models
- Heuristics for VRP
  - Nearest Neighbors
  - Savings heuristics (e.g. Clarke and Wright)
  - Cluster-first route-second (e.g. Sweep algorithm)

- Given: A set of transportation requests and a fleet of vehicles.



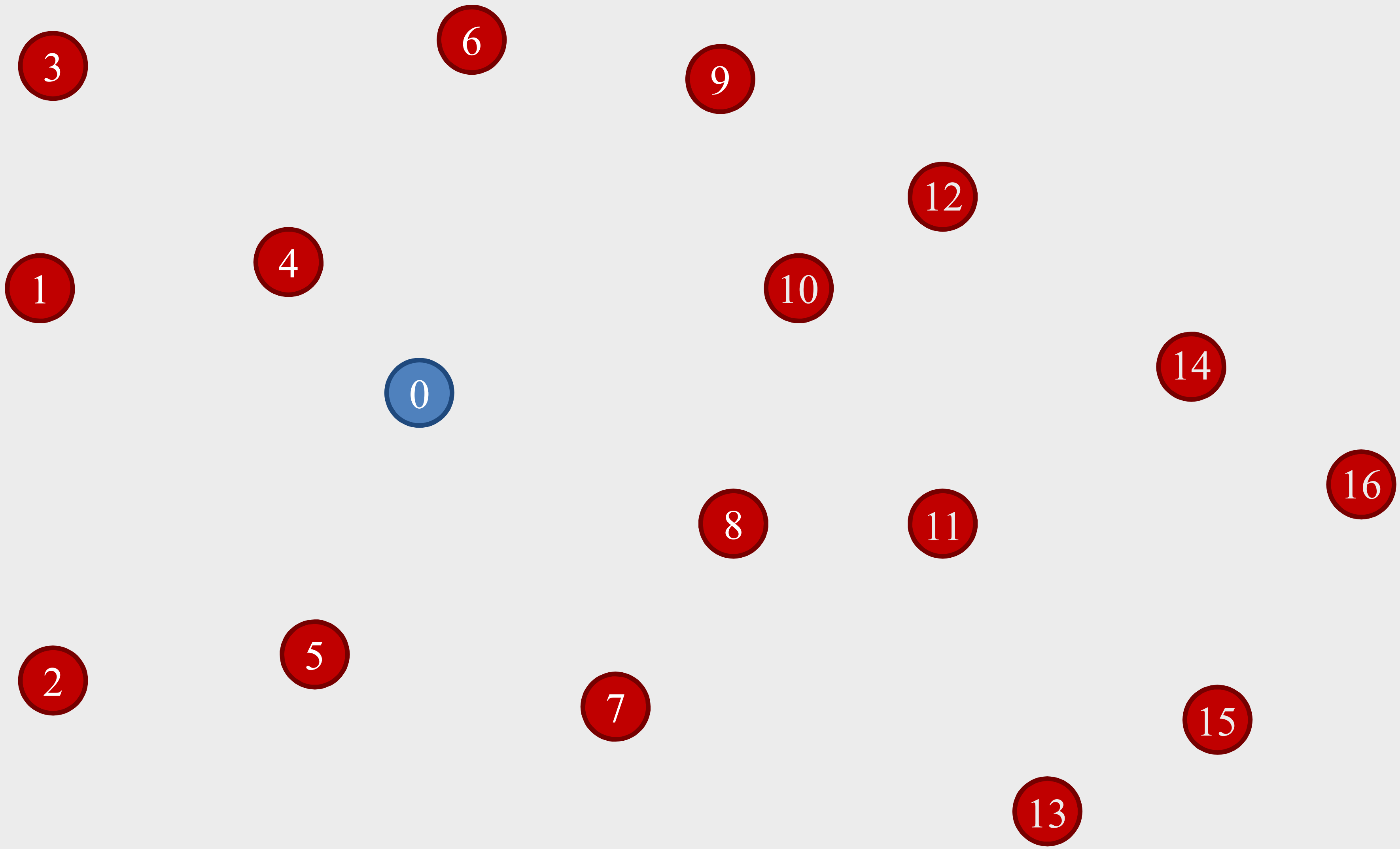
- Task: Find a plan to perform (all) transportation requests with the given vehicle fleet at minimum cost, in particular, decide which vehicle handles which request in which sequence so that all vehicle tours can be feasibly executed.

Vehicle Routing Problems

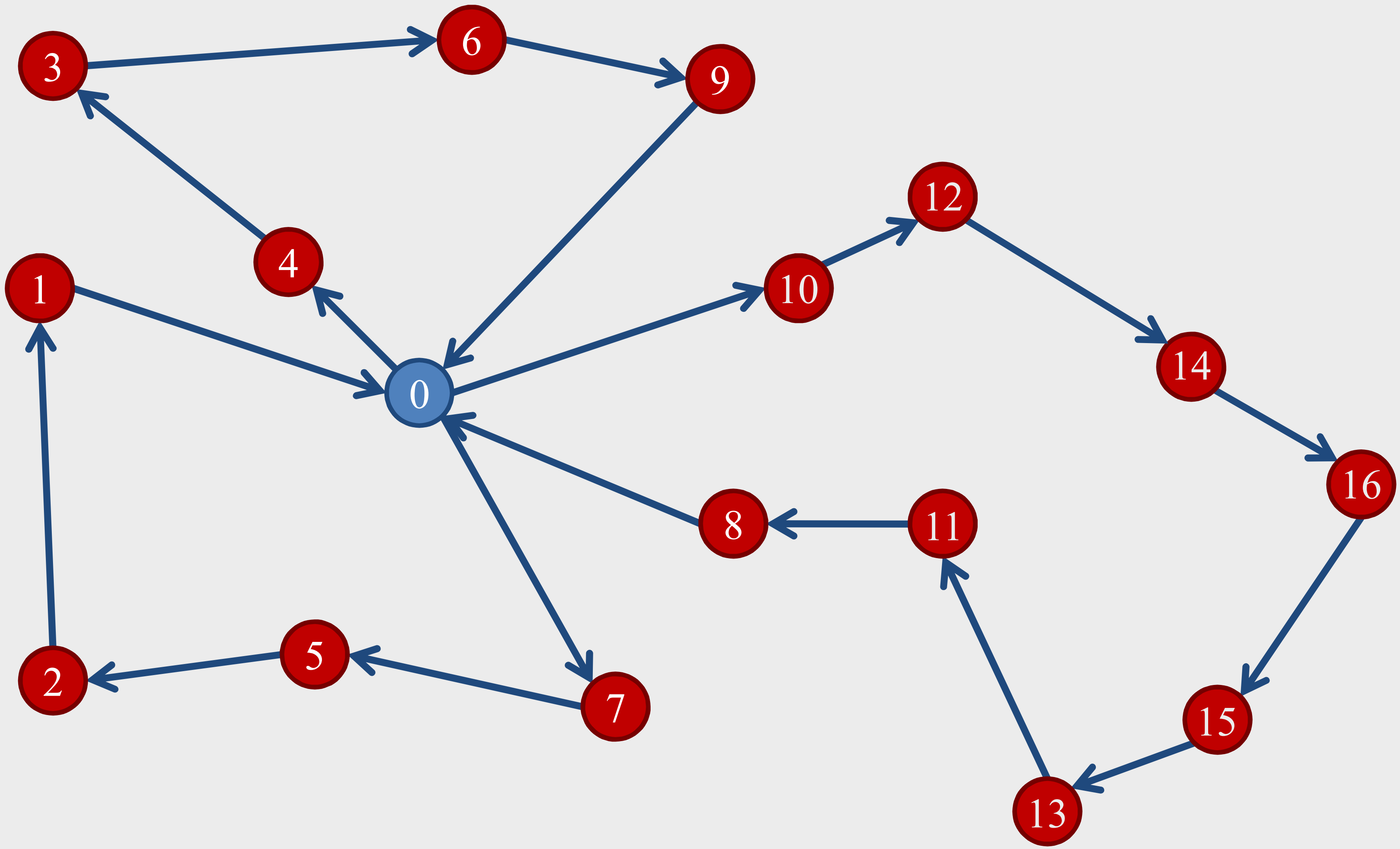


- First introduced by Dantzig and Ramser, 1959 as "truck dispatching problem"
- Called VRP by Christofides, 1976
- Since then thousands of papers !
- Practically relevant
- Very difficult to solve (NP-hard)
- Testbed for all new methods

Vehicle Routing Problem (VRP)



Vehicle Routing Problem (VRP)



VRP – Applications

Context	Network	Vehicles	Locations	Tasks
Distribution logistics	Road network	Trucks, lorries	Customers like stores	Deliveries
Urban waste management	Streets of a city	Trucks with compactors	Streets with their 2 sides	Collect waste
Laundry service	Hospital corridors	Carts w/o engines	Bedrooms with patients	Change linen
Inspection of power lines	Electric network	Technicians on foot	Electric cables, poles	Visual checks
Laser plotter	A plot	Laser beam	Items of the plot	Plot an item
Manufacturing industry	Electronic board	Robotic arm	Slots for components	Insert a component
Spying activities	Earth surface	Satellites	Airbases, troops	Take pictures

Vehicle Routing Problems

- Vehicle Routing problems concern the distribution of goods between depots and final users

General Formulation

Input: Vehicles, depot, road network, costs and customers requirements.

Task: Find a collection of routes, each performed by a single vehicle, and starting and ending at the depot, such that:

- requirement of customers are fulfilled,
- operational constraints are satisfied and
- the global transportation cost is minimized.

VRP – Fomulation

- Let $G = (V, A)$ be a graph where $V = \{1 \ldots n\}$ is a set of vertices representing cities and A the links between the vertices

- Every links (i, j) $i \neq j$ is associated a non-negative distance matrix $C = (c_{ij})$

$$x_{ij} = \begin{cases} 1 & \text{if link from city } i \text{ to city } j \text{ is included in the } VRP \text{ solution} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, i,j \in V$$

The Dantzig, Fulkerson & Johnson (DFJ) model

$$x_{ij} = \begin{cases} 1 & \text{if link from city } i \text{ to city } j \text{ is included in the TSP tour} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, i,j \in N$$

$$min\, z = \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

s.t.

$$\sum_{j \in N} x_{ij} = 1 \quad \forall\, i \in N$$

$$\sum_{i \in N} x_{ij} = 1 \quad \forall\, j \in N$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij} = |S| - 1 \quad \forall\, S \subset N, 2 \leq |S| \leq |N| - 1$$

$$x_{ij} \in \{0,1\} \quad \forall\, i,j \in N$$

VRP – Fomulation

$$min\, z = \sum_{i \in V} \sum_{\substack{j \in V \\ j \neq i}} c_{ij} x_{ij}$$

s.t.

$$\sum_{\substack{j \in V \\ j \neq i}} x_{ij} = 1 \quad \forall\, i \in V$$

$$\sum_{\substack{i \in V \\ i \neq j}} x_{ij} = 1 \quad \forall\, j \in V$$

$$\sum_{i \in S} \sum_{\substack{j \in S \\ j \neq i}} x_{ij} = |S| - v(S) \quad \forall\, S \subset V \backslash \{1\}, |S| \geq 2$$

$$x_{ij} \in \{0,1\} \qquad \forall\ \{i,j\} \in A,\ i \neq j$$

The Miller, Tucker & Zemlin (MTZ) model

$$x_{ij} = \begin{cases} 1 & \text{if link from city } i \text{ to city } j \text{ is included in the TSP tour} \\ 0 & \text{otherwise} \end{cases} \quad \forall\, i,j \in N$$

u_i = number of cities visited at city i.

$$min\, z = \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

s.t.

$$\sum_{j \in N} x_{ij} = 1 \quad \forall\, i \in N$$

$$\sum_{i \in N} x_{ij} = 1 \quad \forall\, j \in N$$

$$u_i + 1 \leq u_j + |N|\,(1 - x_{ij})\,, \quad \text{for } i\,, j \geq 2 \in N\,, i \neq j$$

$$x_{ij} \in \{0,1\} \ \forall\, i,j \in N\,, \quad u_i \geq 0 \quad \forall i \in N$$

CVRP – Fomulation

- We use two nodes to represent the same single depot and impose that all routes must start on 0 and return to $n + 1$
- fleet with K vehicles available in a single depot
- Each vehicle has a maximum capacity Q
- Each node has a demand d_i, such that $d_i > 0$ for each i and $d_0 = d_{n+1} = 0$
- y_j is a continuous decision variable corresponding to the cumulated demand on the route that visits node $j \in N$ up to this visit.

CVRP – FOMULATION

$$min\, z = \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} c_{ij} x_{ij}$$

s.t.

$$\sum_{\substack{j=1 \\ j \neq i}}^{n+1} x_{ij} = 1 \quad \forall\, i = 1, \dots, n$$

$$\sum_{\substack{i=0 \\ i \neq h}}^{n} x_{ih} - \sum_{\substack{j=1 \\ j \neq h}}^{n+1} x_{hj} = 0 \qquad \forall\, h = 1, \dots, n$$

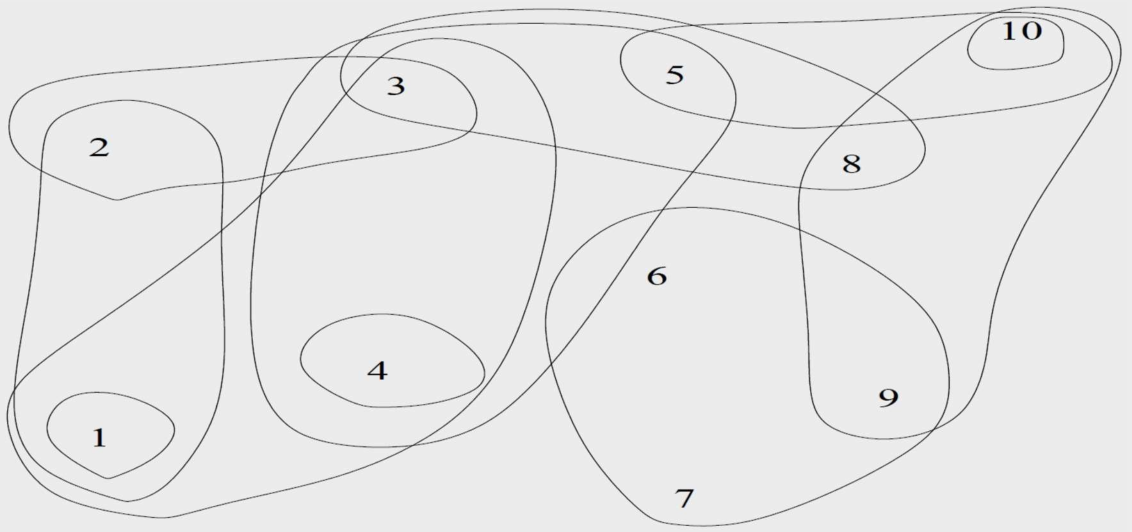
$$\sum_{j=1}^{n} x_{0j} \leq K$$

$$y_j \geq y_i + d_j x_{ij} - Q\left(1 - x_{ij}\right) \quad \forall\, i, j = 0, \dots, n+1$$

$$d_i \leq y_i \leq Q \quad \forall\, i = 0, \dots, n+1$$

$$x_{ij} \in \{0,1\} \qquad \forall\, i, j = 0, \dots, n+1$$

Set Partitioning Problem



1	1	1	0	0	1	0	0	0	0	0	0		0	=	1
2	0	1	1	0	0	0	0	0	0	0	0		0	=	1
3	0	0	1	0	1	1	1	0	0	0	0		0	=	1
4	0	0	0	1	1	0	0	0	0	0	0		0	=	1
5	0	0	0	0	0	1	1	1	0	0	0		0	=	1
6	0	0	0	0	0	0	0	0	0	1	0		0	=	1
7	0	0	0	0	0	0	0	0	0	1	0	...	0	=	1
8	0	0	0	0	0	0	1	0	1	0	0		0	=	1
9	0	0	0	0	0	0	0	0	1	1	0		0	=	1
10	0	0	0	0	0	0	0	1	1	0	1		0	=	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮
m	0	0	0	0	0	0	0	0	0	0	0		1	=	1
	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}		c_n		

VRP – Set Partitioning Fomulation

- Currently, the most efficient exact methods for solving VRP variants are based on SP formulations.
- The variables in these formulations correspond to feasible routes of the problem.
- Let $\mathscr{R}$ be the set of routes that satisfy the problem requirements.
- For example, in the CVRP, a route in $\mathscr{R}$ must start and finish at the depot, visit at most once a customer, respect the vehicle capacity and guarantee that if the route arrives to a customer than it has to leave this customer.
- Let λ_r be the binary decision variable that is equal to 1 if and only if the route $r \in \mathscr{R}$ is selected.

VRP – Set Partitioning Fomulation

$$min\, z = \sum_{r \in \mathcal{R}} c_r \lambda_r$$

s.t.

$$\sum_{r \in \mathcal{R}} a_{ri} \lambda_r = 1 \quad \forall\, i \in N$$

$$\sum_{r \in \mathcal{R}} \lambda_r \leq K$$

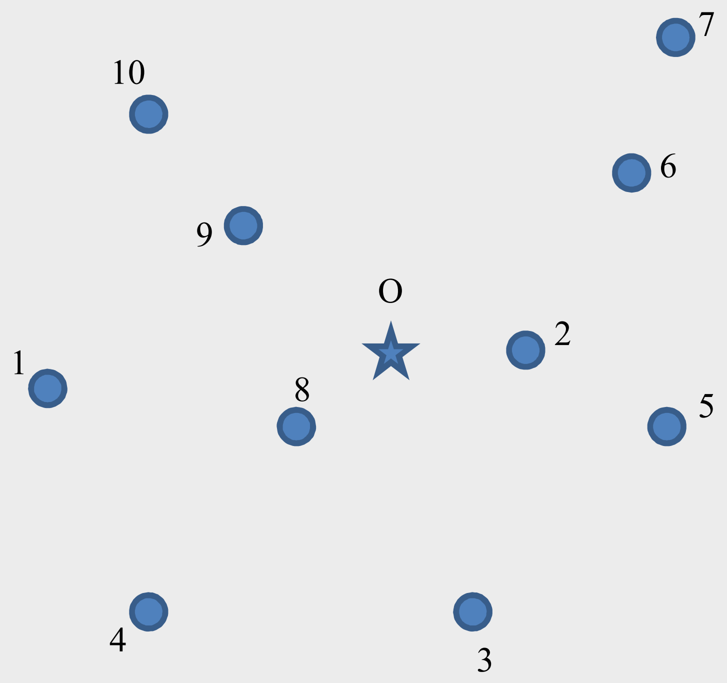
$$\lambda_r \in \{0,1\} \qquad r \in \mathscr{R}$$

CAPACITED VEHICLE ROUTING PROBLEM (CVRP)

Dis.	1	2	3	4	5	6	7	8	9	10	O
1	0	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	10	0	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	10.8	7.1	0	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	6.3	10.6	7	0	12.1	15.6	19.4	5.8	11.2	14	8.6
5	13	3.6	6.4	12.1	0	7.1	11	8	10.8	14.2	6.3
6	13.4	5.4	12.4	15.6	7.1	0	4.1	9.9	8.1	10.2	7.1
7	16.4	9.5	16.5	19.4	11	4.1	0	13.6	10.3	11.2	10.8
8	5.1	5.4	6.4	5.8	8	9.9	13.6	0	6.1	9.5	2.8
9	6.4	7.2	12.1	11.2	10.8	8.1	10.3	6.1	0	3.6	5
10	8.2	10.6	15.7	14	14.2	10.2	11.2	9.5	3.6	0	8.6
O	7.1	3	7.3	8.6	6.3	7.1	10.8	2.8	5	8.6	0

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10



VRP – Set Partitioning Fomulation

- How many potential routes are there?

$$\approx \sum_{k=1}^{n} \binom{n}{k} \left(\frac{1}{2}\right) (k+1-1)! = \left(\frac{1}{2}\right) \sum_{k=1}^{n} \frac{n!}{(n-k)!}$$

- In our example with 10 nodes plus one depot ~ 5 millions
- Not all routes are feasible!
- In practice, routes are generated in a separate "sub-problem" and fed into the MILP "master problem"
- Very active research area – solving very large scale optimization problems

Capacited Vehicle Routing Problem (CVRP)

Dis.	1	2	3	4	5	6	7	8	9	10	O
1	0	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	10	0	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	10.8	7.1	0	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	6.3	10.6	7	0	12.1	15.6	19.4	5.8	11.2	14	8.6
5	13	3.6	6.4	12.1	0	7.1	11	8	10.8	14.2	6.3
6	13.4	5.4	12.4	15.6	7.1	0	4.1	9.9	8.1	10.2	7.1
7	16.4	9.5	16.5	19.4	11	4.1	0	13.6	10.3	11.2	10.8
8	5.1	5.4	6.4	5.8	8	9.9	13.6	0	6.1	9.5	2.8
9	6.4	7.2	12.1	11.2	10.8	8.1	10.3	6.1	0	3.6	5
10	8.2	10.6	15.7	14	14.2	10.2	11.2	9.5	3.6	0	8.6
O	7.1	3	7.3	8.6	6.3	7.1	10.8	2.8	5	8.6	0

D	2	1	3	1	2	7	2	3	4	3

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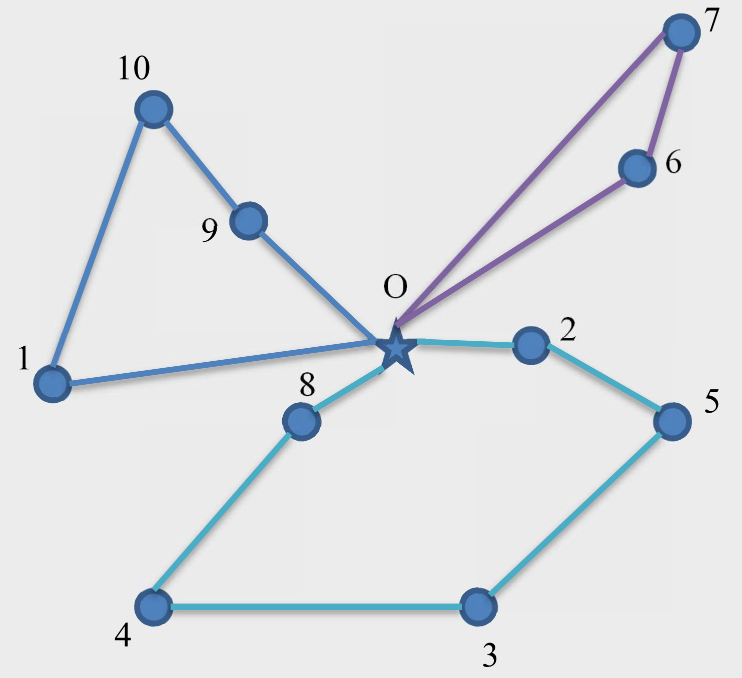


Tours: O-8-4-3-5-2-O
O-9-10-1-O
O-6-7-O

Length: 74.5

CVRP – Nearest Neighbor Heuristic

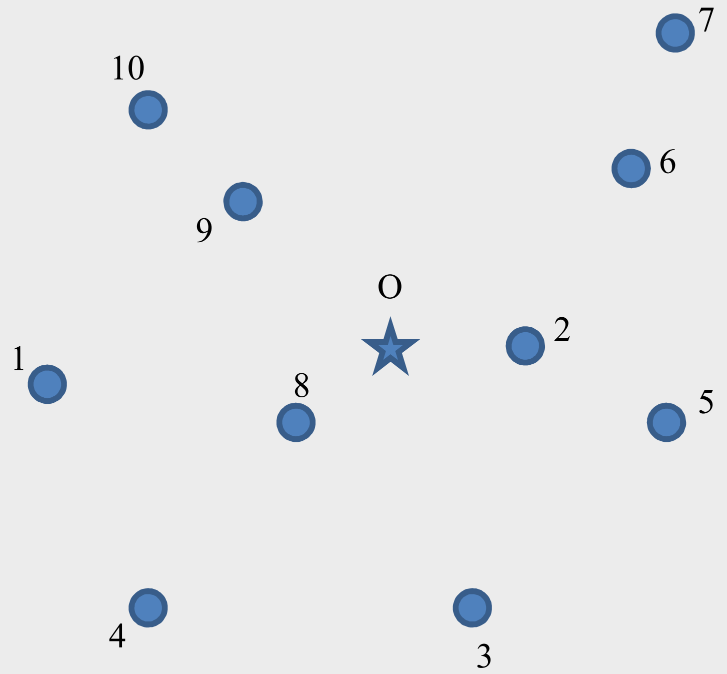
1. Start with a node at the beginning of the tour (depot node)
2. Find the node closest to the last node and add to the current tour. If the closest node is already added to one of the tours or if by adding the node we violate the capacity then select next closest node.
3. If because of the capacity you can not pick any other node come back from the current node to the depot and start a new tour (go to step 1)
4. Go to step 2 until all nodes have been added
5. Come back to the depot at the end!

CVRP – Nearest Neighbor Heuristic

Dis.	1	2	3	4	5	6	7	8	9	10	O
1	0	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	10	0	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	10.8	7.1	0	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	6.3	10.6	7	0	12.1	15.6	19.4	5.8	11.2	14	8.6
5	13	3.6	6.4	12.1	0	7.1	11	8	10.8	14.2	6.3
6	13.4	5.4	12.4	15.6	7.1	0	4.1	9.9	8.1	10.2	7.1
7	16.4	9.5	16.5	19.4	11	4.1	0	13.6	10.3	11.2	10.8
8	5.1	5.4	6.4	5.8	8	9.9	13.6	0	6.1	9.5	2.8
9	6.4	7.2	12.1	11.2	10.8	8.1	10.3	6.1	0	3.6	5
10	8.2	10.6	15.7	14	14.2	10.2	11.2	9.5	3.6	0	8.6
O	7.1	3	7.3	8.6	6.3	7.1	10.8	2.8	5	8.6	0

Demand	2	1	3	1	2	7	2	3	4	3	-

Capacity of vehicles	10

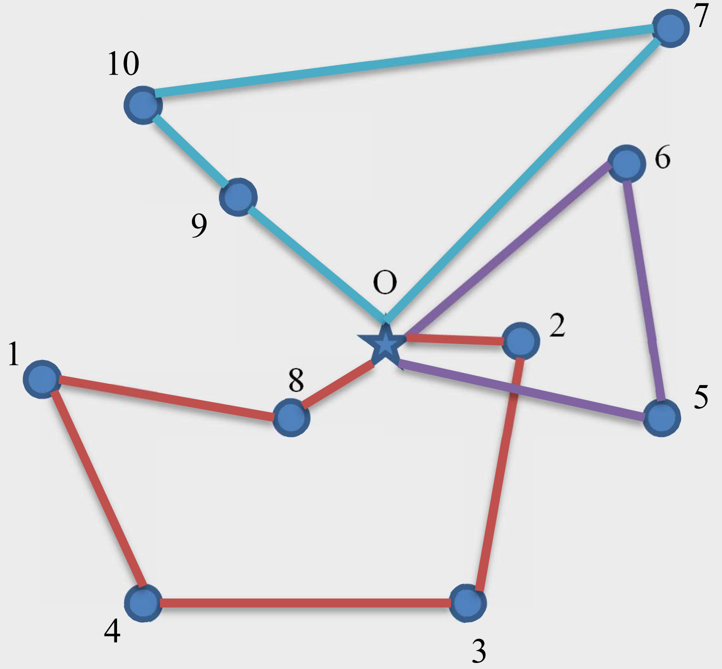


CVRP – Nearest Neighbor Heuristic

Dis.	1	2	3	4	5	6	7	8	9	10	O
1	0	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	10	0	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	10.8	7.1	0	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	6.3	10.6	7	0	12.1	15.6	19.4	5.8	11.2	14	8.6
5	13	3.6	6.4	12.1	0	7.1	11	8	10.8	14.2	6.3
6	13.4	5.4	12.4	15.6	7.1	0	4.1	9.9	8.1	10.2	7.1
7	16.4	9.5	16.5	19.4	11	4.1	0	13.6	10.3	11.2	10.8
8	5.1	5.4	6.4	5.8	8	9.9	13.6	0	6.1	9.5	2.8
9	6.4	7.2	12.1	11.2	10.8	8.1	10.3	6.1	0	3.6	5
10	8.2	10.6	15.7	14	14.2	10.2	11.2	9.5	3.6	0	8.6
O	7.1	3	7.3	8.6	6.3	7.1	10.8	2.8	5	8.6	0

Demand	2	1	3	1	2	7	2	3	4	3	-

Capacity of vehicles	10

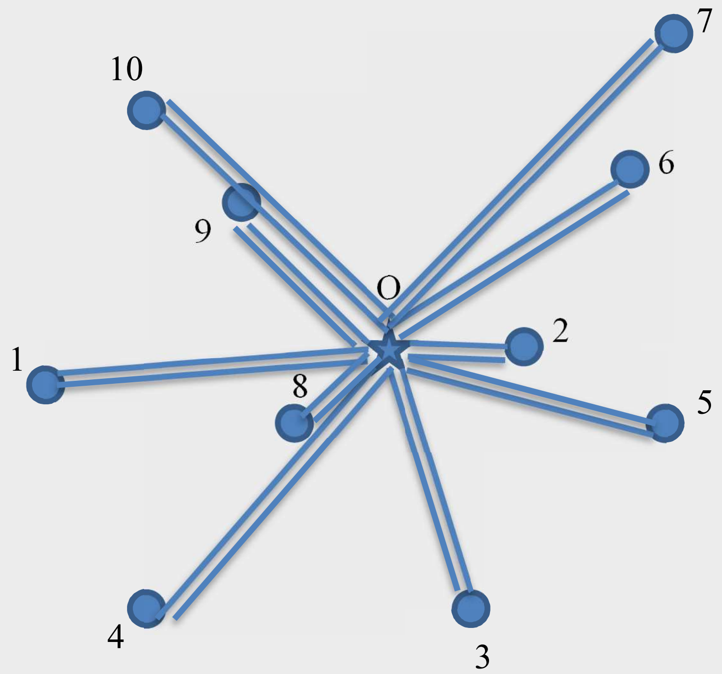


Tours: O-8-1-4-3-2- O
O-9-10-7-O
O-5-6-O

Length: 82.4

CVRP – Clark and Wright Savings Heuristic

- The basis for *C&W* algorithm is savings concept
- Assume *n* vehicles are available where *n* is the number of nodes.
- Each vehicle travels from the depot directly to a node and returns to the depot.



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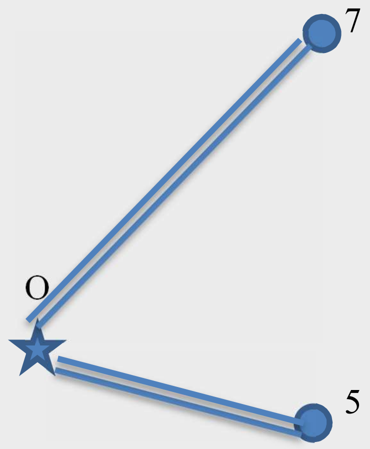

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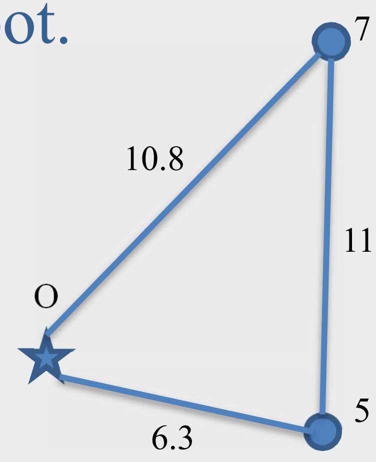
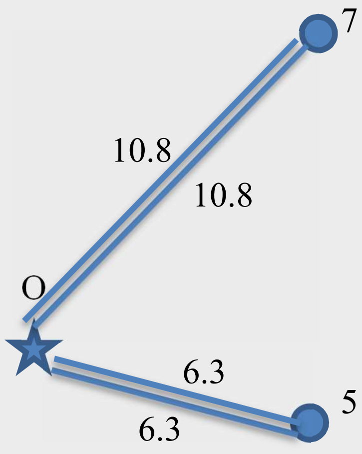
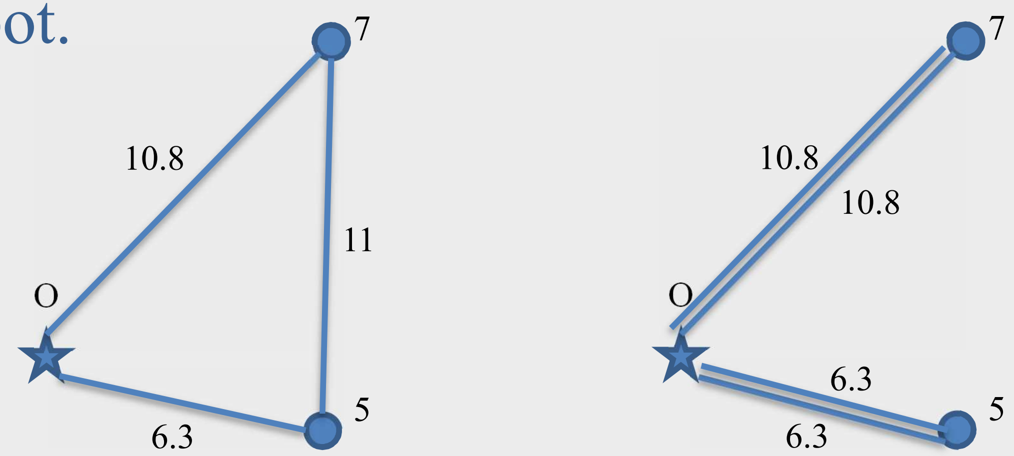




Length: 10.8 + 6.3 + 11 = 28.1

Length: 2*10.8 + 2*6.3 = 34.2

- The basis for *C&W* algorithm is savings concept
- Assume *n* vehicles are available where *n* is the number of nodes.
- Each vehicle travels from the depot directly to a node and returns to the depot.



Saving: (2*10.8 + 2*6.3) – (10.8 + 6.3 + 11) = 10.8 + 6.3 – 11 = 6.1

CVRP – Clark and Wright Savings Heuristic

1. Calculate savings $s_{i,j} = c_{O,i} + c_{O,j} - c_{i,j}$ for every pair (i, j) of demand nodes.

2. Rank and process the savings $s_{i,j}$ in descending order of magnitude.

3. For the savings $s_{i,j}$ under consideration, include arc (i, j) in a route only if:

   - No route or vehicle constraints will be violated by adding it in a route and
   - Nodes i and j are first or last nodes to/from the origin in their current route.

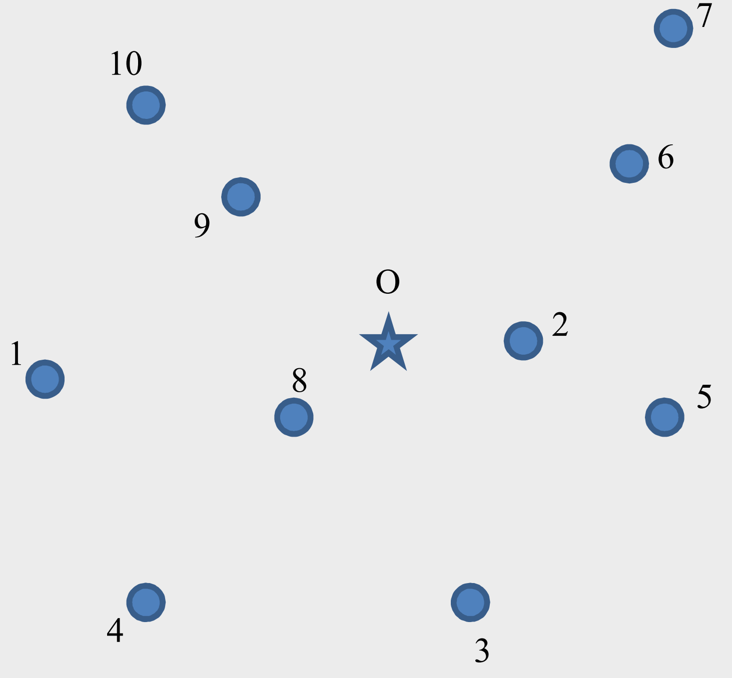
4. If the savings list has not been exhausted, return to Step 3, processing the next entry in the list; otherwise, stop.

CVRP – Clark and Wright Savings Heuristic

Dis.	1	2	3	4	5	6	7	8	9	10	O
1	0	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	10	0	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	10.8	7.1	0	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	6.3	10.6	7	0	12.1	15.6	19.4	5.8	11.2	14	8.6
5	13	3.6	6.4	12.1	0	7.1	11	8	10.8	14.2	6.3
6	13.4	5.4	12.4	15.6	7.1	0	4.1	9.9	8.1	10.2	7.1
7	16.4	9.5	16.5	19.4	11	4.1	0	13.6	10.3	11.2	10.8
8	5.1	5.4	6.4	5.8	8	9.9	13.6	0	6.1	9.5	2.8
9	6.4	7.2	12.1	11.2	10.8	8.1	10.3	6.1	0	3.6	5
10	8.2	10.6	15.7	14	14.2	10.2	11.2	9.5	3.6	0	8.6
O	7.1	3	7.3	8.6	6.3	7.1	10.8	2.8	5	8.6	0

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

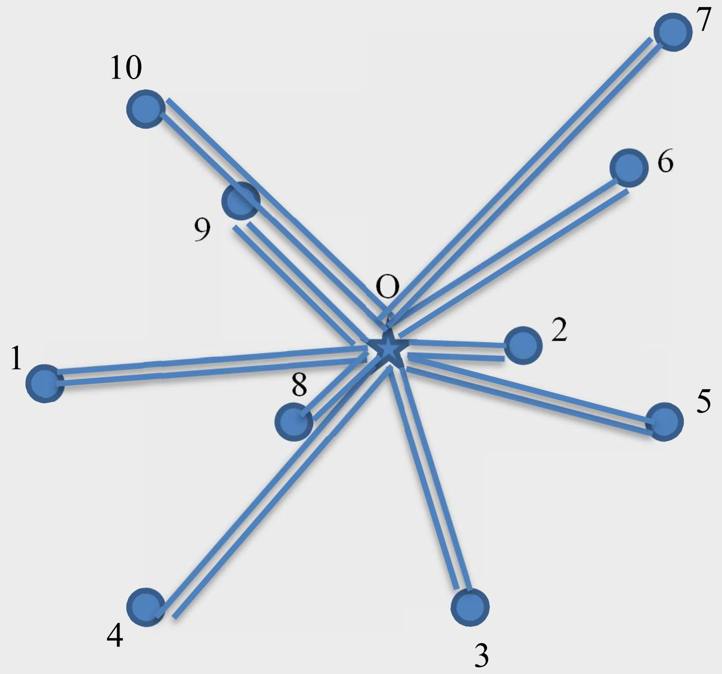


CVRP – Clark and Wright Savings Heuristic

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10



Length: 133.2

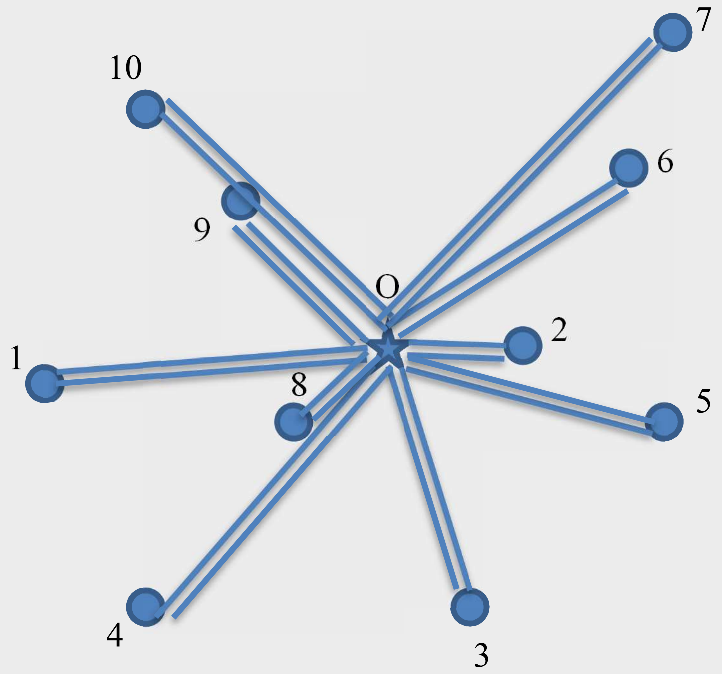
CVRP – Clark and Wright Savings Heuristic

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

(i-j)	S_{ij}
(7-6)	13.8
(10-9)	10
(4-1)	9.3
(4-3)	8.9
(10-7)	8.2
(10-1)	7.5
(5-3)	7.2
(6-5)	6.3
(7-5)	6.1
(5-2)	5.7
(9-1)	5.7
(8-4)	5.6
(9-7)	5.5
(10-6)	5.5
(8-1)	4.8
(6-2)	4.7
(7-2)	4.3
(9-6)	4
(8-3)	3.7
(3-1)	3.5
(3-2)	3.2
(10-4)	3.2
(5-4)	2.8
(9-4)	2.4
(6-3)	2
(10-8)	1.9
(9-8)	1.7
(7-3)	1.6
(7-1)	1.5
(8-5)	1.2
(4-2)	1
(10-2)	1



Length: 133.2

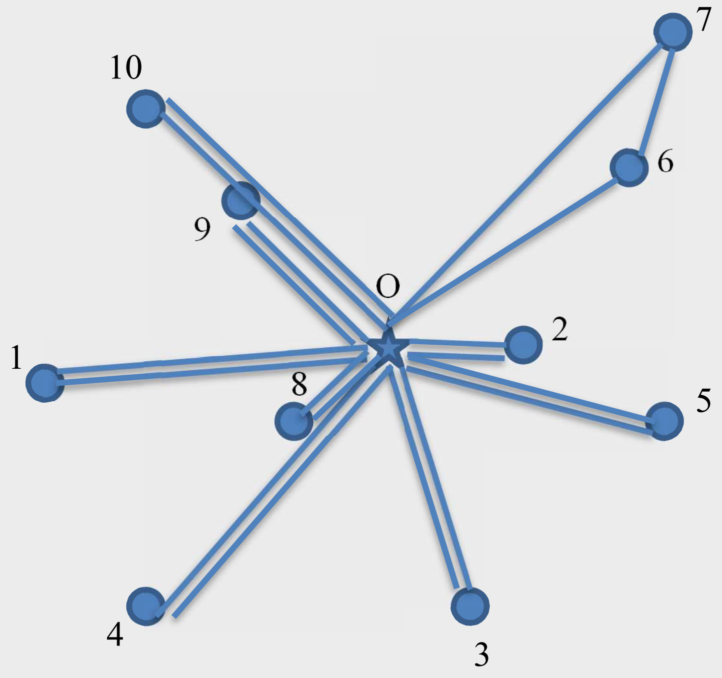
CVRP – Clark and Wright Savings Heuristic

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

(i-j)	S_{ij}
(7-6)	13.8
(10-9)	10
(4-1)	9.3
(4-3)	8.9
(10-7)	8.2
(10-1)	7.5
(5-3)	7.2
(6-5)	6.3
(7-5)	6.1
(5-2)	5.7
(9-1)	5.7
(8-4)	5.6
(9-7)	5.5
(10-6)	5.5
(8-1)	4.8
(6-2)	4.7
(7-2)	4.3
(9-6)	4
(8-3)	3.7
(3-1)	3.5
(3-2)	3.2
(10-4)	3.2
(5-4)	2.8
(9-4)	2.4
(6-3)	2
(10-8)	1.9
(9-8)	1.7
(7-3)	1.6
(7-1)	1.5
(8-5)	1.2
(4-2)	1
(10-2)	1



Length: 119.4

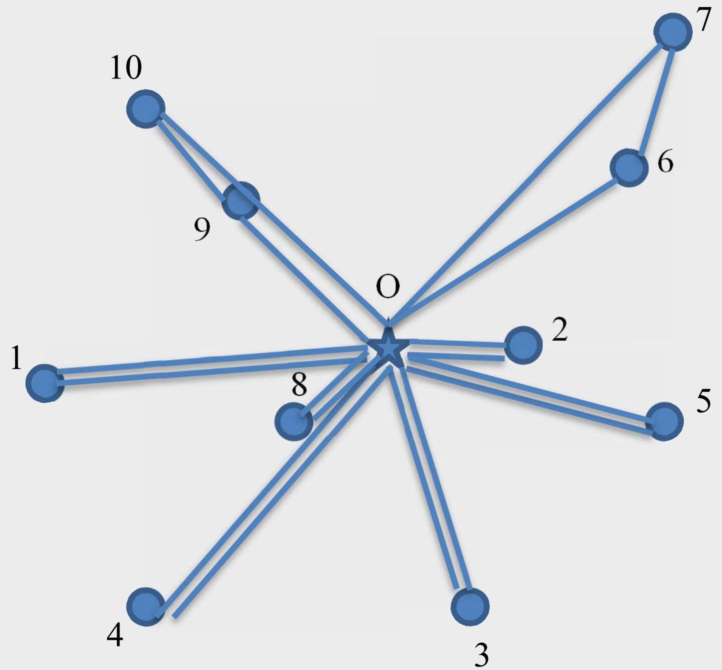
CVRP – Clark and Wright Savings Heuristic

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

(i-j)	S_{ij}
~~(7-6)~~	~~13.8~~
(10-9)	10
(4-1)	9.3
(4-3)	8.9
(10-7)	8.2
(10-1)	7.5
(5-3)	7.2
(6-5)	6.3
(7-5)	6.1
(5-2)	5.7
(9-1)	5.7
(8-4)	5.6
(9-7)	5.5
(10-6)	5.5
(8-1)	4.8
(6-2)	4.7
(7-2)	4.3
(9-6)	4
(8-3)	3.7
(3-1)	3.5
(3-2)	3.2
(10-4)	3.2
(5-4)	2.8
(9-4)	2.4
(6-3)	2
(10-8)	1.9
(9-8)	1.7
(7-3)	1.6
(7-1)	1.5
(8-5)	1.2
(4-2)	1
(10-2)	1



Length: 109.4

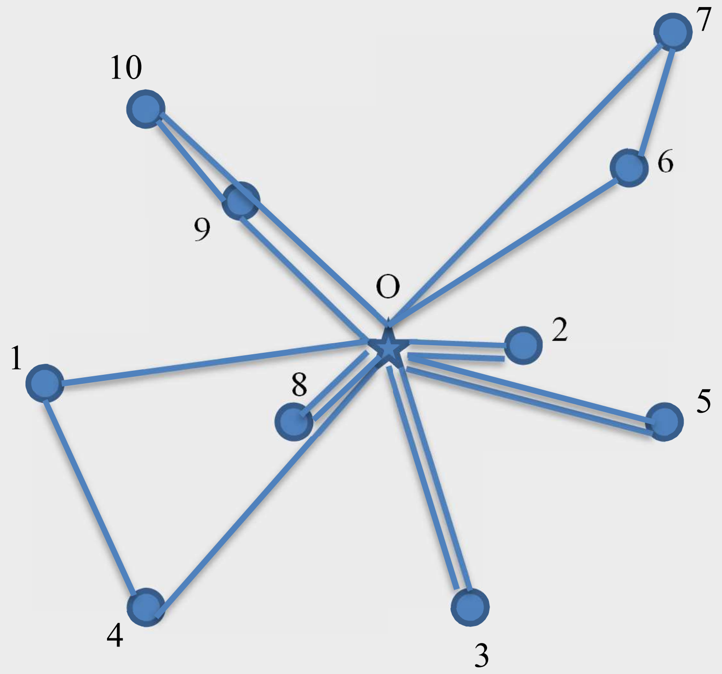
CVRP – Clark and Wright Savings Heuristic

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

(i-j)	S_{ij}
~~(7-6)~~	~~13.8~~
~~(10-9)~~	~~10~~
(4-1)	9.3
(4-3)	8.9
(10-7)	8.2
(10-1)	7.5
(5-3)	7.2
(6-5)	6.3
(7-5)	6.1
(5-2)	5.7
(9-1)	5.7
(8-4)	5.6
(9-7)	5.5
(10-6)	5.5
(8-1)	4.8
(6-2)	4.7
(7-2)	4.3
(9-6)	4
(8-3)	3.7
(3-1)	3.5
(3-2)	3.2
(10-4)	3.2
(5-4)	2.8
(9-4)	2.4
(6-3)	2
(10-8)	1.9
(9-8)	1.7
(7-3)	1.6
(7-1)	1.5
(8-5)	1.2
(4-2)	1
(10-2)	1



Length: 100.1

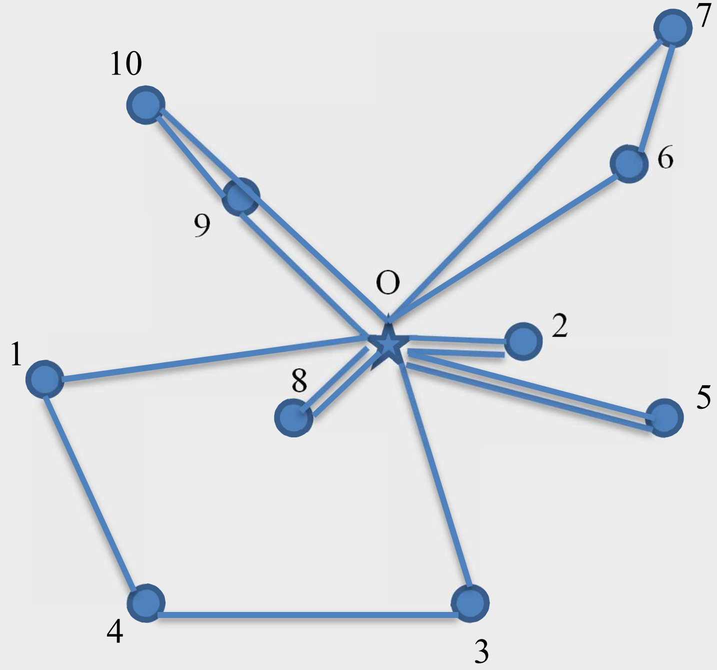
CVRP – Clark and Wright Savings Heuristic

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

(i-j)	S_{ij}
~~(7-6)~~	~~13.8~~
~~(10-9)~~	~~10~~
~~(4-1)~~	~~9.3~~
(4-3)	8.9
(10-7)	8.2
(10-1)	7.5
(5-3)	7.2
(6-5)	6.3
(7-5)	6.1
(5-2)	5.7
(9-1)	5.7
(8-4)	5.6
(9-7)	5.5
(10-6)	5.5
(8-1)	4.8
(6-2)	4.7
(7-2)	4.3
(9-6)	4
(8-3)	3.7
(3-1)	3.5
(3-2)	3.2
(10-4)	3.2
(5-4)	2.8
(9-4)	2.4
(6-3)	2
(10-8)	1.9
(9-8)	1.7
(7-3)	1.6
(7-1)	1.5
(8-5)	1.2
(4-2)	1
(10-2)	1



Length: 91.2

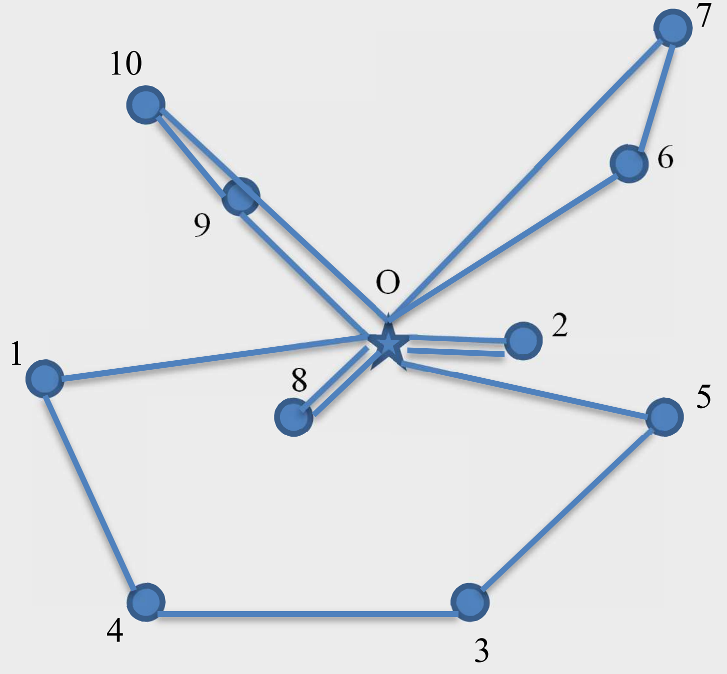
CVRP – Clark and Wright Savings Heuristic

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

(i-j)	S_{ij}
~~(7-6)~~	~~13.8~~
~~(10-9)~~	~~10~~
~~(4-1)~~	~~9.3~~
~~(4-3)~~	~~8.9~~
(10-7)	8.2
(10-1)	7.5
(5-3)	7.2
(6-5)	6.3
(7-5)	6.1
(5-2)	5.7
(9-1)	5.7
(8-4)	5.6
(9-7)	5.5
(10-6)	5.5
(8-1)	4.8
(6-2)	4.7
(7-2)	4.3
(9-6)	4
(8-3)	3.7
(3-1)	3.5
(3-2)	3.2
(10-4)	3.2
(5-4)	2.8
(9-4)	2.4
(6-3)	2
(10-8)	1.9
(9-8)	1.7
(7-3)	1.6
(7-1)	1.5
(8-5)	1.2
(4-2)	1
(10-2)	1



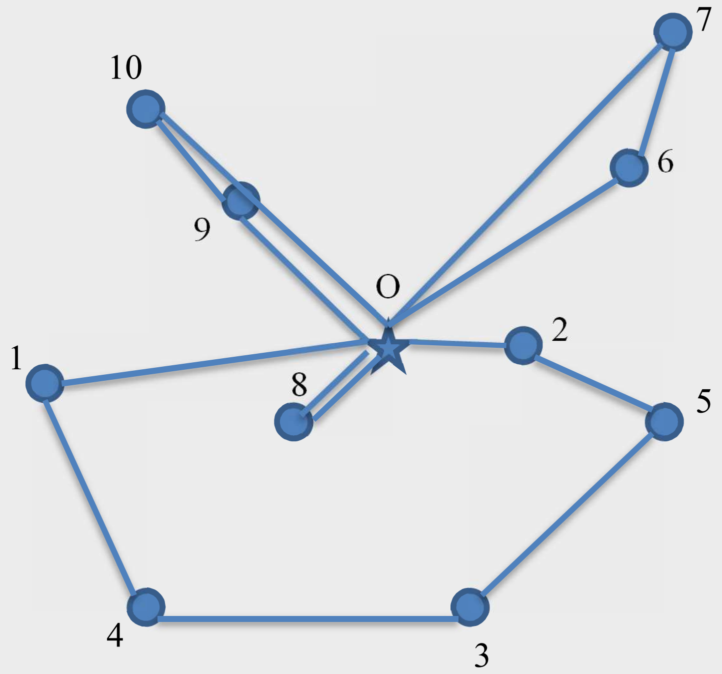
Length: 84

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

(i-j)	S_{ij}
~~(7-6)~~	~~13.8~~
~~(10-9)~~	~~10~~
~~(4-1)~~	~~9.3~~
~~(4-3)~~	~~8.9~~
(10-7)	8.2
(10-1)	7.5
~~(5-3)~~	~~7.2~~
(6-5)	6.3
(7-5)	6.1
(5-2)	5.7
(9-1)	5.7
(8-4)	5.6
(9-7)	5.5
(10-6)	5.5
(8-1)	4.8
(6-2)	4.7
(7-2)	4.3
(9-6)	4
(8-3)	3.7
(3-1)	3.5
(3-2)	3.2
(10-4)	3.2
(5-4)	2.8
(9-4)	2.4
(6-3)	2
(10-8)	1.9
(9-8)	1.7
(7-3)	1.6
(7-1)	1.5
(8-5)	1.2
(4-2)	1
(10-2)	1



Length: 78.3

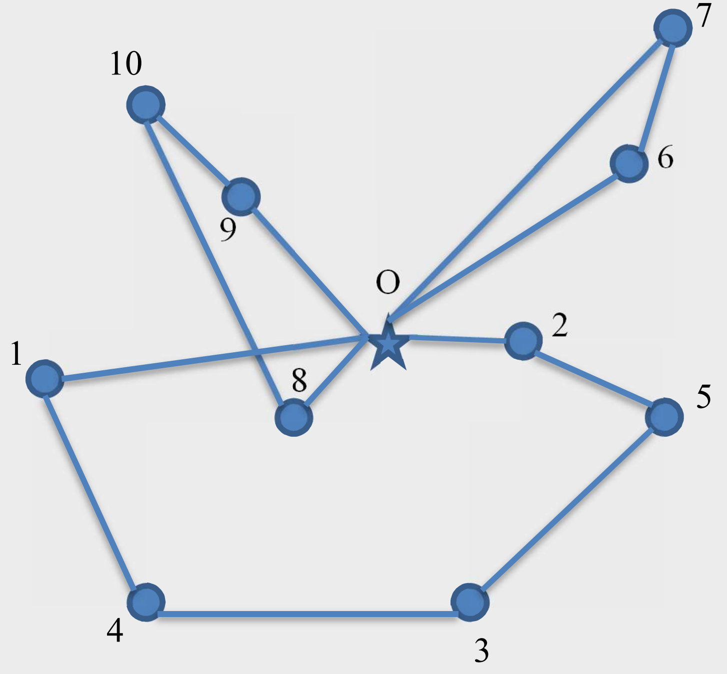
CVRP – Clark and Wright Savings Heuristic

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

(i-j)	S_{ij}
~~(7-6)~~	~~13.8~~
~~(10-9)~~	~~10~~
~~(4-1)~~	~~9.3~~
~~(4-3)~~	~~8.9~~
(10-7)	8.2
(10-1)	7.5
~~(5-3)~~	~~7.2~~
(6-5)	6.3
(7-5)	6.1
~~(5-2)~~	~~5.7~~
(9-1)	5.7
(8-4)	5.6
(9-7)	5.5
(10-6)	5.5
(8-1)	4.8
(6-2)	4.7
(7-2)	4.3
(9-6)	4
(8-3)	3.7
(3-1)	3.5
(3-2)	3.2
(10-4)	3.2
(5-4)	2.8
(9-4)	2.4
(6-3)	2
(10-8)	1.9
(9-8)	1.7
(7-3)	1.6
(7-1)	1.5
(8-5)	1.2
(4-2)	1
(10-2)	1



Length: 76.4

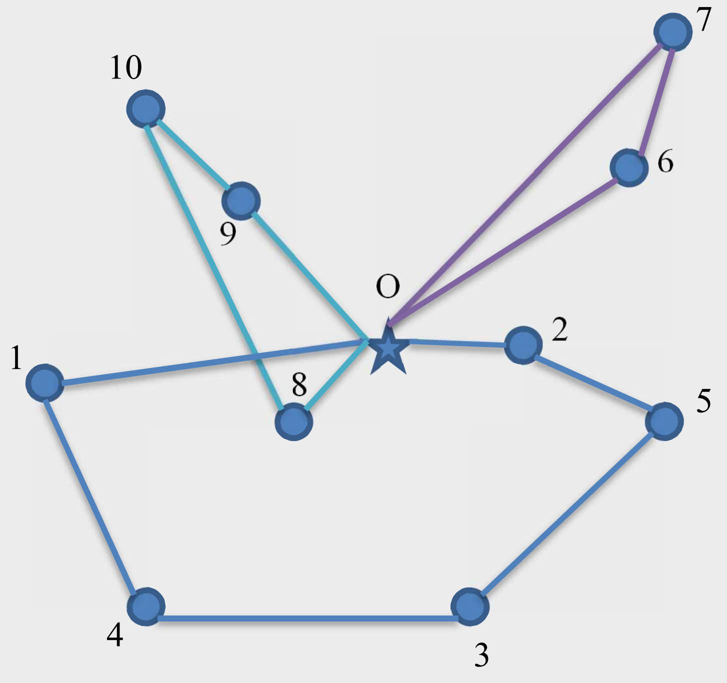
CVRP – Clark and Wright Savings Heuristic

Dis/ Sav	1	2	3	4	5	6	7	8	9	10	O
1	-	10	10.8	6.3	13	13.4	16.4	5.1	6.4	8.2	7.1
2	0.1	-	7.1	10.6	3.6	5.4	9.5	5.4	7.2	10.6	3
3	3.5	3.2	-	7	6.4	12.4	16.5	6.4	12.1	15.7	7.3
4	9.3	1	8.9	-	12.1	15.6	19.4	5.8	11.2	14	8.6
5	0.4	5.7	7.2	2.8	-	7.1	11	8	10.8	14.2	6.3
6	0.7	4.7	2	0.1	6.3	-	4.1	9.9	8.1	10.2	7.1
7	1.5	4.3	1.6	0	6.1	13.8	-	13.6	10.3	11.2	10.8
8	4.8	0.4	3.7	5.6	1.2	0	0	-	6.1	9.5	2.8
9	5.7	0.8	0.2	2.4	0.5	4	5.5	1.7	-	3.6	5
10	7.5	1	0.2	3.2	0.7	5.5	8.2	1.9	10	-	8.6

D	2	1	3	1	2	7	2	3	4	3

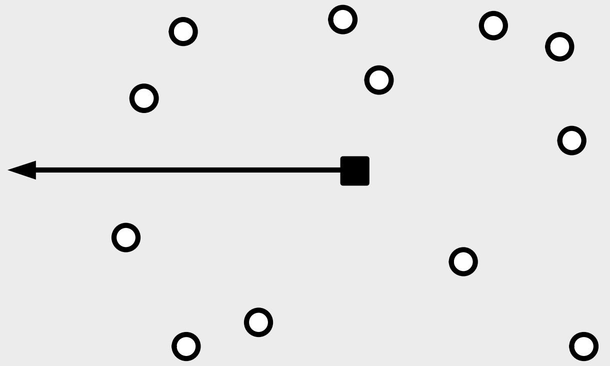
Capacity of vehicles	10

(i-j)	S_{ij}
~~(7-6)~~	~~13.8~~
~~(10-9)~~	~~10~~
~~(4-1)~~	~~9.3~~
~~(4-3)~~	~~8.9~~
(10-7)	8.2
(10-1)	7.5
~~(5-3)~~	~~7.2~~
(6-5)	6.3
(7-5)	6.1
~~(5-2)~~	~~5.7~~
(9-1)	5.7
(8-4)	5.6
(9-7)	5.5
(10-6)	5.5
(8-1)	4.8
(6-2)	4.7
(7-2)	4.3
(9-6)	4
(8-3)	3.7
(3-1)	3.5
(3-2)	3.2
(10-4)	3.2
(5-4)	2.8
(9-4)	2.4
(6-3)	2
~~(10-8)~~	~~1.9~~
(9-8)	1.7
(7-3)	1.6
(7-1)	1.5
(8-5)	1.2
(4-2)	1
(10-2)	1



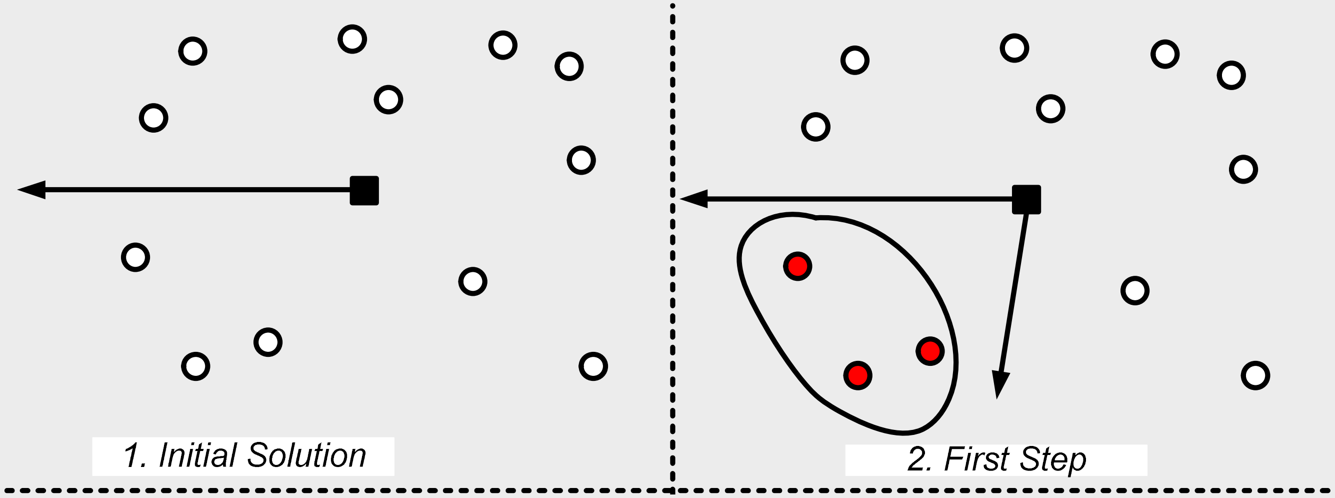
Length: 76.4

VRP – Sweep Algorithm

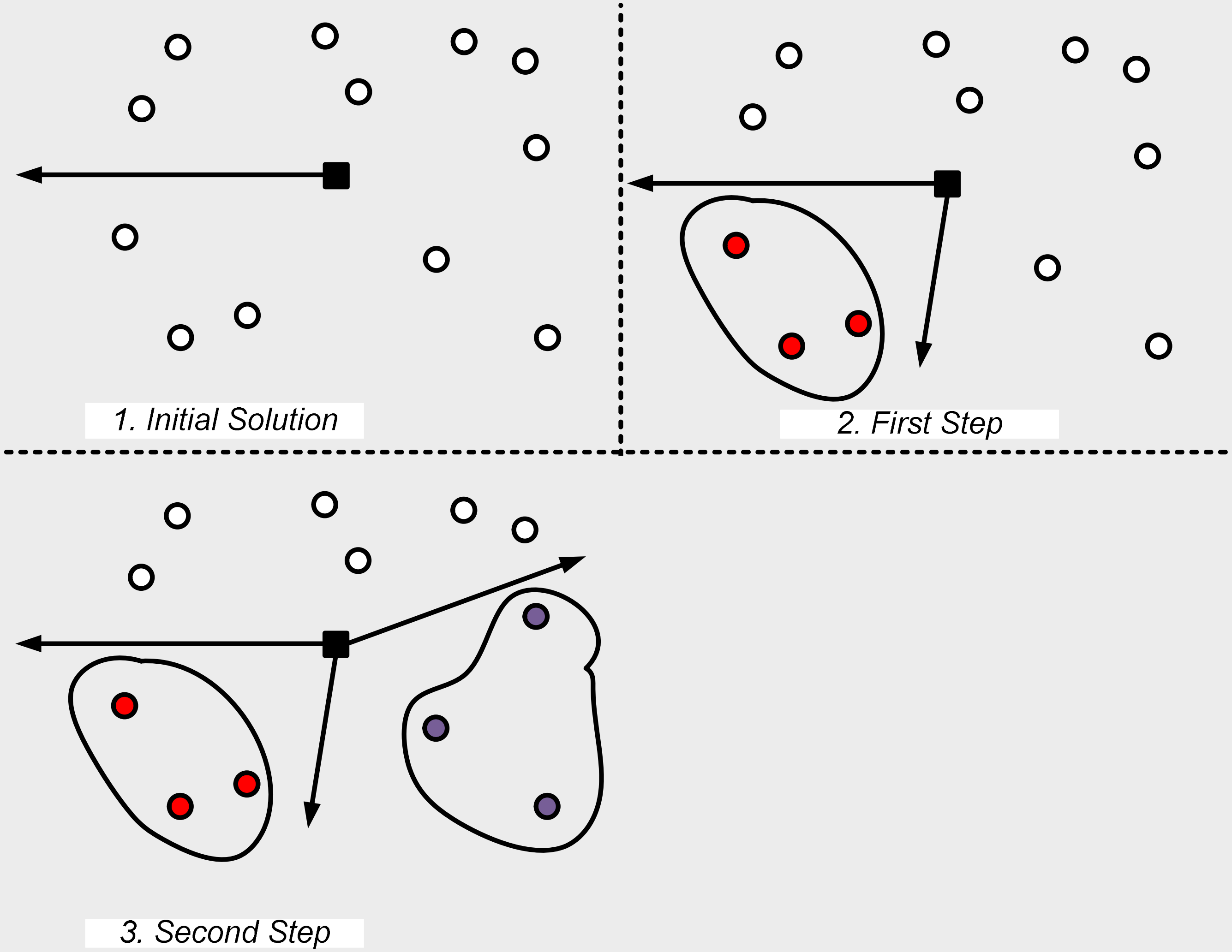


1. Initial Solution

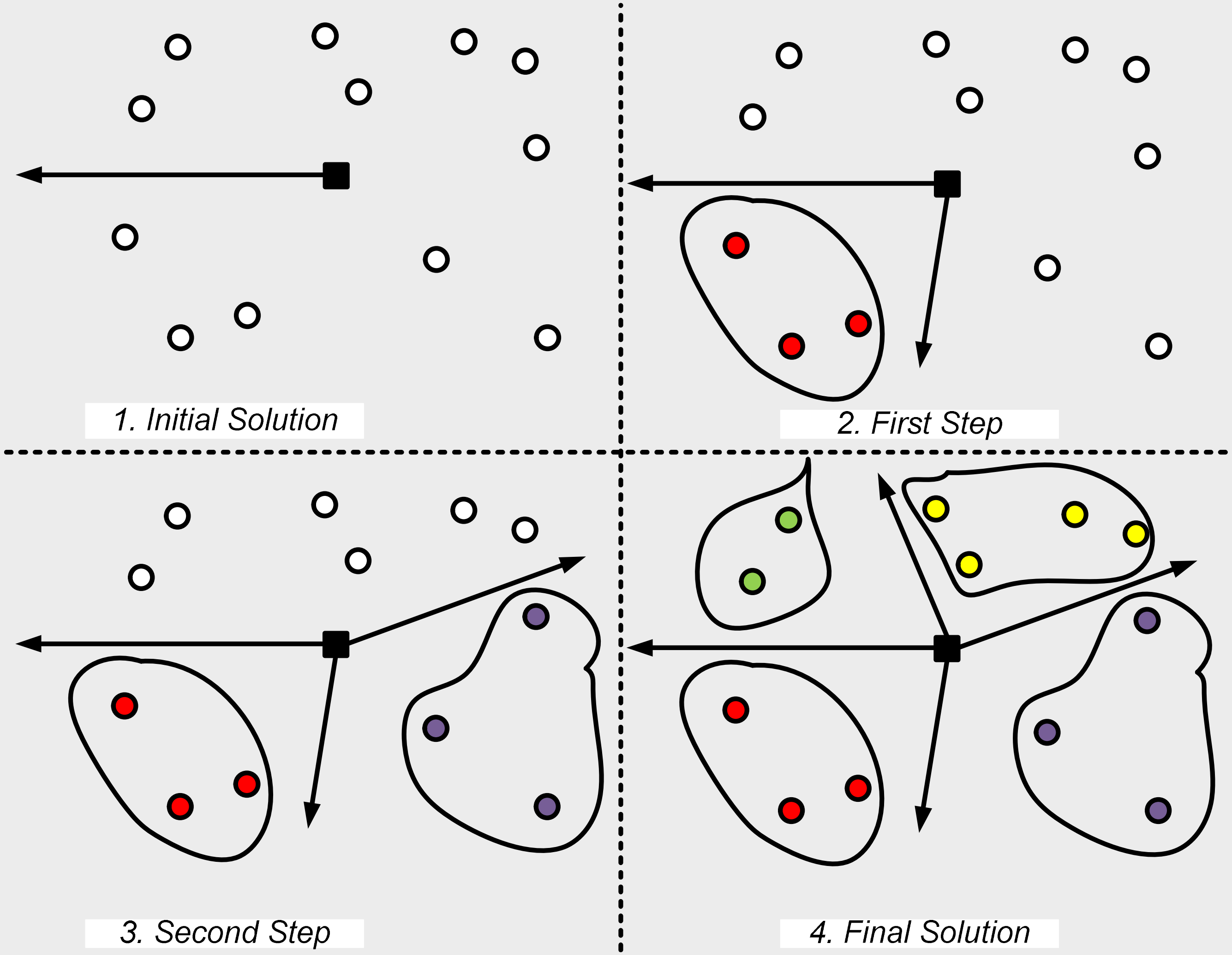
VRP – Sweep Algorithm



VRP – Sweep Algorithm



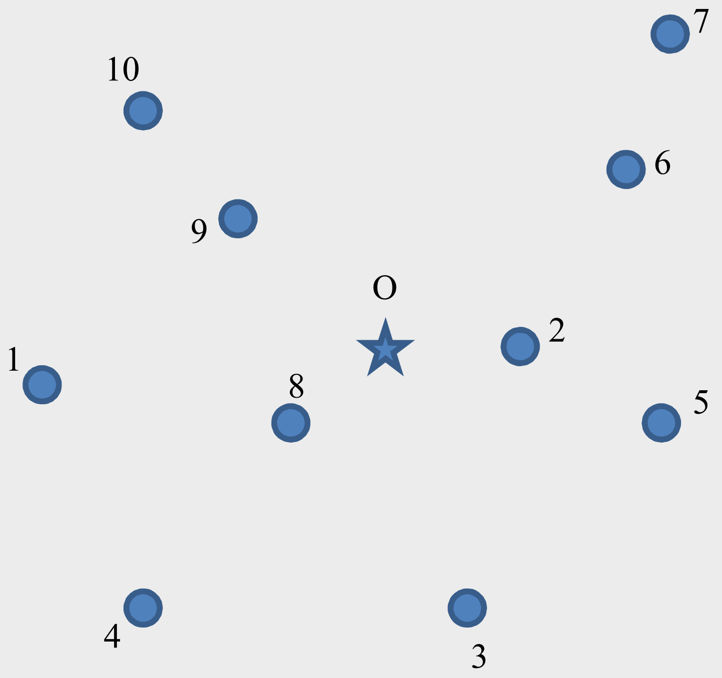
VRP – Sweep Algorithm



VRP – Sweep Algorithm

Nodes	1	2	3	4	5	6	7	8	9	10
Demand	2	1	3	1	2	7	2	3	4	3

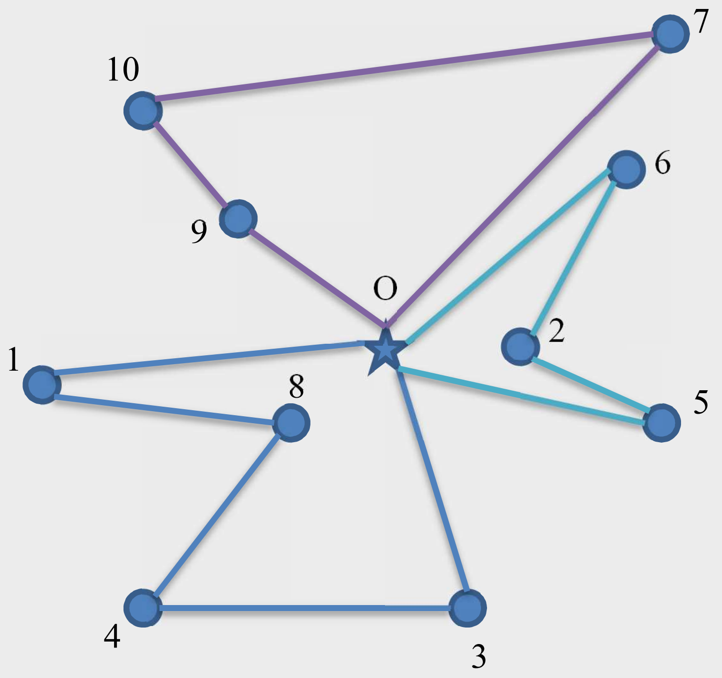
Capacity of vehicles	10



VRP – Sweep Algorithm

Nodes	1	2	3	4	5	6	7	8	9	10
Demand	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10

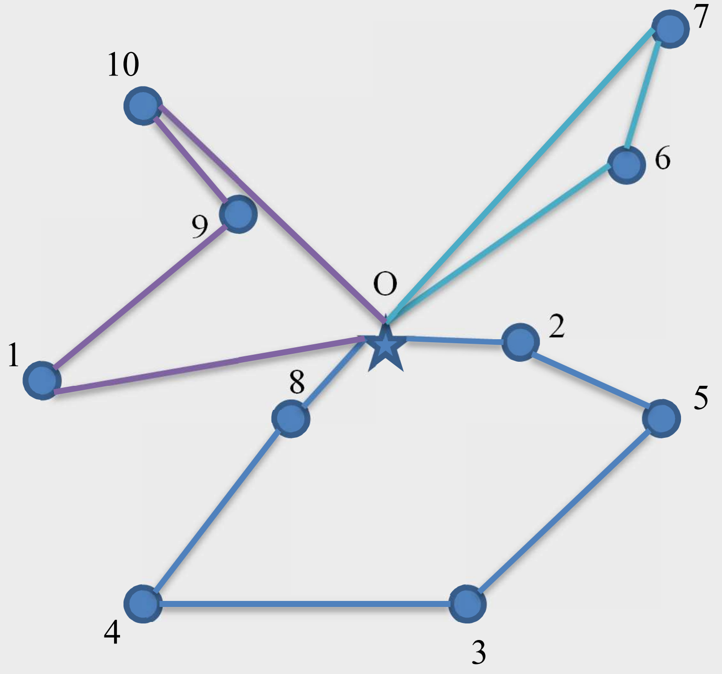


Length: 85.3

VRP – Sweep Algorithm

Nodes	1	2	3	4	5	6	7	8	9	10
Demand	2	1	3	1	2	7	2	3	4	3

Capacity of vehicles	10



Length: 77.4

IN CLASS ASSIGNMENT (TEST) #2:

- ✓ Closed-book
- ✓ Based on slides
- ✓ Modeling + AMPL

LECTURE #15:

PRODUCTION PLANNING AND SCHEDULING PROBLEMS