INF319: A Variable Cost Pickup and Delivery Problem with Multiple Time Windows

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Agenda

- Automobile Manufacturing Industry
- Pickup and Delivery problems
- The Inbound Manufacturer
  - Suppliers and Manufacturers
  - Carrier Vehicle Fleet
  - Multiple Time Windows
- The Solution: A VCPDPMTW Model
- Mathematical Model
- References

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Automobile Manufacturing Industry

- One of the most significant industries in Europe
- Highly competitive global market
- Increasing cost pressure
- High flexibility in processes and structures
- Supply chain concept and active management of outbound and inbound flow essential
- Highly complex models with many variables
- Optimization and pickup and delivery models key tools

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Pickup and delivery problems

- Well researched problem with a long history.
- Solomon (1987) described an algorithm to the general pickup and delivery problem.
- Vehicle Routing with Stochastic demands Dror et al. (1989)
- Multi-vehicle PDPTW, Desrosiers et al. (1995)
- NP-hard problems -> heuristics, Savelsberg and Sol (1995), and Lu et al. (2004)
- Parragh et al. (2006) - comprehensive survey on pickup and delivery problems and classified PDP in sub-categories.
- Maritime versions of the Problem: Hybrid Cargo generating and routing heuristic in Christiansen et al. (2002), Speed optimization to reduce fuel emissions Christiansen et al. (2004), Hemmati et al. (2014), Hemmati et al. (2016)
- Zhou (2013) proposed a model to reduce the inbound transportation costs for a food processing company.
- PDP with multiple time windows was proposed by, Favaretto et al. (2007) and Ferreira et al. (2018)

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The Inbound manufacturer

- An inbound perspective -> need for comprehensive model
- Suppliers and Factories
  - Docking limitation
- Carrier vehicle fleet
  - Before and after irrelevance
  - Carriers cost structure
- Multiple time windows
  - Weekly planning priod
  - Inbound oriented thinking

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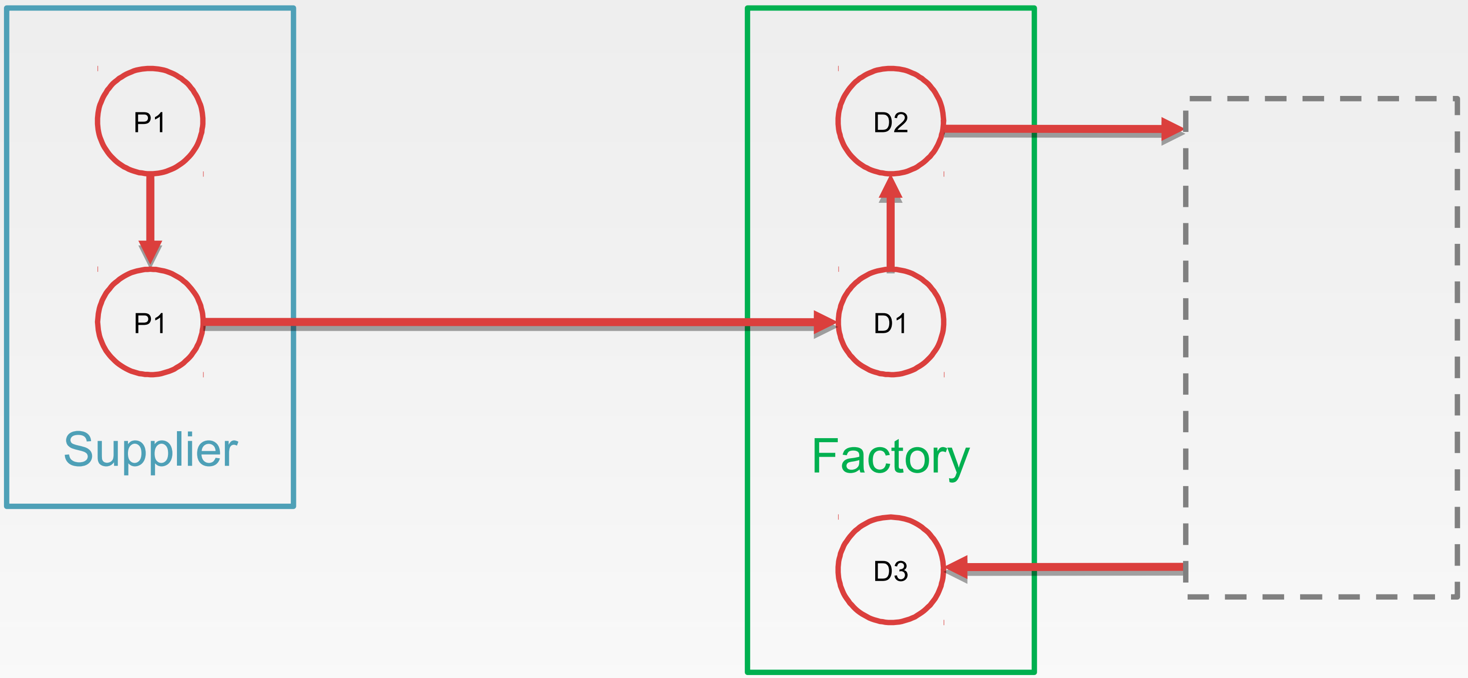
Suppliers and factories



Factory: No more than
2 stops per visit!

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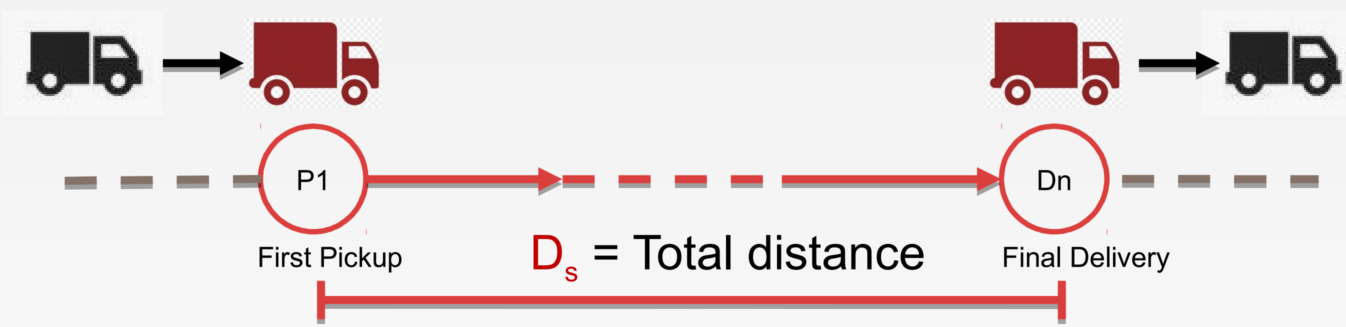
Carrier vehicle fleet

- Costs only relevant from pickup to delivery
- Carrier determines how costs are calculated
  - often cost per km depending on interval



Distance interval	**D_1 < 100**	**D_2 < 200**	**D_3 > 200**
C_s = cost per km	C_1 = 20	C_2 = 30	C_3 = 40

Total cost = $\Sigma (D_s * C_s)$

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Multiple Time Windows

- Multiple time windows
- Pickup/delivery must be within one of the possible time windows
- Inbound perspective

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Solution: A VCPDPMTW Model

- Our model: A Variable Cost Pickup and Delivery Problem with Multiple Time Windows (VCPDPMTW)
- Compehensive model
- Not previously touched to my knowledge
- Goal is to solve the demands of the inbound oriented manufacturer

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Mathematical model

Sets

N – nodes $\{1, .., 2n\}$
V – vehicles
E – edges
E_v – edges visitable by vehicle v
N_v – nodes visitable by vehicle v
N^P – pickup nodes
N^D – delivery Nodes
F – factories
N_f – delivery nodes for factory f
S – cost structure
P_i – time windows at node i, $\{1, .., \pi_i\}$
T_i – timewindows at node i $[\underline{T_{ip}}, \overline{T_{ip}}]$ where $p \in P_i$

Parameters

n – amount of orders
K_v – weight limit of vehicle $v \in V$
Q_i – weight of order at node $i \in N$
H_f – docking limit at factory $f \in F$
T_{ijv} – travel time from node $i \in N$ to $j \in N$ for vehicle $v \in V$
π_i – amount of time windows at node $i \in N$
$\overline{T_{ip}}$ – upper bound time of time window $p \in P_i$ at node $i \in N$
$\underline{T_{ip}}$ – lower bound time of time window $p \in P_i$ at node $i \in N$
C_{vs} – cost per distance unit in structure interval $s \in S$ for vehicle v
C_i – cost of not transporting order at node $i \in N^P$
D_{ij} – distance between node $i \in N$ and $j \in N$
B_s – distance for interval $s \in S$ in cost structure

Variables

x_{ijv} – binary indicating travel from node $i \in N$ to $j \in N$ of vehicle $v \in V$
y_i – binary indicating no pickup of order at node $i \in N^P$
l_{iv} – load of vehicle v at node i
$o(v)$ – origin node of vehicle v
$d(v)$ – destination node of vehicle v
h_i – docking times in factory at node $i \in N_f$
t_i – time of visit at node $i \in N$
w_i – waiting time at node i
u_{ip} – binary indicating usage of time window $p \in P_i$ at node i
d_{vs} – total distance travelled of vehicle $v \in V$ if it fits in interval $s \in S$
b_{vs} – binary indicating correct interval $s \in S$ for vehicle $v \in V$

Mathematical model

- Objective function:

$$min \sum_{v \epsilon V} \sum_{s \epsilon S} C_{vs} d_{vs} + \sum_{i \in N^P} C_i y_i \qquad (1)$$

Mathematical model

- Travel constraints:

subject to:

$$\sum_{v \epsilon V} \sum_{j \epsilon N_v} x_{ijv} + y_i = 1, \qquad i \in N^P \tag{2}$$

$$\sum_{j \epsilon N_v} x_{ijv} - \sum_{j \epsilon N_v} x_{jiv} = 0, \qquad v \in V, i \in N_v \notin \{o(v), d(v)\} \tag{3}$$

$$\sum_{j \epsilon N_v} x_{o(v)jv} = 1, \qquad v \in V \tag{4}$$

$$\sum_{j \epsilon N_v} x_{jd(v)v} = 1, \qquad v \in V \tag{5}$$

$$\sum_{j \epsilon N_v} x_{ijv} - \sum_{j \epsilon N_v} x_{(i+n)jv} = 0, \qquad v \in V, i \in N_v^P \tag{6}$$

Mathematical model

- Weight constraints:

$$l_{iv} + Q_j - l_{jv} \leq K_v(1 - x_{ijv}), \qquad v \in V, j \in N_v^P, (i,j) \in E_v \tag{7}$$

$$l_{iv} - Q_j - l_{jv} \leq K_v(1 - x_{i(j+n)v}), \qquad v \in V, j \in N_v^P, (i, n+j) \in E_v \tag{8}$$

$$0 \leq l_{iv} \leq K_v, \qquad v \in V, i \in N_v^P \tag{9}$$

Mathematical model

- Weight constraints:

$$l_{iv} + Q_j - l_{jv} \leq K_v(1 - x_{ijv}), \qquad v \in V, j \in N_v^P, (i,j) \in E_v \tag{7}$$

$$l_{iv} - Q_j - l_{jv} \leq K_v(1 - x_{i(j+n)v}), \qquad v \in V, j \in N_v^P, (i, n+j) \in E_v \tag{8}$$

$$0 \leq l_{iv} \leq K_v, \qquad v \in V, i \in N_v^P \tag{9}$$

Mathematical model

- Docking constraints:

$$h_i + 1 - h_j \leq H_f(1 - x_{ijv}), \qquad v \in V, (i,j) \in E_v, i,j \in N_f, f \in F \tag{10}$$

$$h_j \leq H_f \sum_{i \epsilon N_v} (x_{ijv}), \qquad v \in V, j \in N_f, f \in F \tag{11}$$

$$h_j = \sum_{i \epsilon N_v} (x_{ijv}), \qquad v \in V, (i,j) \in E_v, i \notin N_f, j \in N_f \tag{12}$$

Mathematical model

- Multiple time window constraints:

$$\sum_{p \in P_i} u_{ip} = 1, \qquad i \in N \tag{13}$$

$$\sum_{p \in P_i} u_{ip} \underline{T_{ip}} \leq t_i, \qquad i \in N \tag{14}$$

$$\sum_{p \in P_i} u_{ip} \overline{T_{ip}} \geq t_i, \qquad i \in N \tag{15}$$

$$\begin{aligned} t_i + T_{ijv} + w_j - t_j \leq (\overline{T_{\pi_i i}} + \\ T_{ijv} + w_j)(1 - x_{ijv}), \end{aligned} \qquad v \in V, (i,j) \in E_v \tag{16}$$

Mathematical model

- Cost structure constraints:

$$\sum_{s \in S} d_{vs} = \sum_{(i,j) \in E_v} x_{ijv} D_{ij}, \qquad v \in V \tag{17}$$

$$B_{(s-1)} b_{vs} \leq d_{vs} \leq B_s b_{vs}, \qquad v \in V, s \in S \tag{18}$$

$$\sum_{s \in S} b_{vs} = 1, \qquad v \in V \tag{19}$$

- Reminder! Objective function:

$$min \sum_{v \epsilon V} \sum_{s \epsilon S} C_{vs} d_{vs} + \sum_{i \in N^P} C_i y_i \tag{1}$$

Mathematical model

- Final variable constraints:

$$h_i \geq 0, \qquad i \in N \tag{20}$$

$$u_{ip} \in \{0,1\}, \qquad i \in N, p \in P_i \tag{21}$$

$$b_{vs} \in \{0,1\}, \qquad v \in V, s \in S \tag{22}$$

$$y_i \in \{0,1\}, \qquad i \in N^P \tag{23}$$

$$x_{ijv} \in \{0,1\}, \qquad v \in V, (i,j) \in E_v \tag{24}$$

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