Pickup & Delivery Problem with Mulitiple Time Windows

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TODO..

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Our problem is typically for a vehicle manufacturing company that has a certain set of factories where the vehicles are produced, each with a set of demanded parts for production. The parts, or orders (typically named transport orders but we refer to orders here), can be delivered from a set of suppliers. At a given point in time (could be delivery for one day or a week) we consider a planning problem with a demand for orders that should be satisfied with the given suppliers using vehicles provided by different logistics companies.

The vehicles have different capacities, cost structures, incompatabilities and start at the first pickup location at the first pickup time (ie. costs to get from start to first pickup are not considered neither is time). The cost structure can be multidimentional taking into account total distance and volume in different formats (variable costs, fix costs etc).

When a delivery is assigned to a vehicle, the vehicle must load the delivery from the supplier, and deliver the delivery at the factory dock. The docks in each factory can differ according to which order is being delivered and there is a limit to how many docks each vehicle can visit at each factory.

Each delivery/pickup can have several time windows, lapsing sometimes over several days. If a car arrives before a time window it has to wait.

The mathematical formulation of the problem will now follow where the set of vehicles used is denoted by V and capacity of each vehicle $v \in V$ is denoted by K_v. We let n be the number of orders in the problem, so if i is a specific pickup-node then $i + n$ corresponds to the delivery node for the same order. Here it follows that for each vehicle we have a set O_v of orders that vehicle $v \in V$ can transport based on the incompatabilities. The set of pickup Nodes (supplier docks) we denote using N^P and each delivery node (Factory dock) is denoted by N^D. Each Factory, $f \in F$, also has a set of Nodes belonging to the same factory wich we denote N_f. The delivery nodes that belong to one factory can then be represented as $N_f^P = N_f \subset N^D$. Each vehicle has a set of Nodes it can travel to (corresponding to orders O_v) represented by N_v. This set also includes an origin node, $o(v)$ and a destination node $d(v)$ corresponding to the first pickup and the last delivery of vehicle v. The set of Edges that each vehicle can traverse is represented by E_v. The factory docking limit is denoted by D_f. Each vehicle has a given

amount of docks per factory it wants to deliver to represented by d_f^v.

Each node has a set of h time windows represented by T_i^h which contains tuples of $[\underline{T_{ik}}, \overline{T_{ik}}]$ where $k \in \{0, 1, ..., h\}$. each vehicle has a current time based on which node its currently at, denoted by t_i^v and a current load, denoted by l_i^v, where $i \in n$ and $v \in V$. The distance from node i to node j is denoted by d_{ij}. The cost of vehicle v driving from node i to j is denoted by $C_{ij}^{vq(v)w(v)}$ and the corresponding time between the nodes T_{ij}^v. l_i^v is the load of vehicle v leaving node i and x_{ij}^v is a binary vairable indicating if vehicle v is travelling between i and j node. The cost of not transporting an order will be the same for each order (should be set relatively high to avoid dummy transports) so we denote this with C^N and each order, i, not transported is represented by a binary variable y_i.

$$min \sum_{v \epsilon V} \sum_{(i,j) \epsilon E_v} C_{ij}^v x_{ij}^v + \sum_{i \in N^P} C_i y_i \tag{1}$$

subject to:

$$\sum_{v \epsilon V} \sum_{j \epsilon N_v} x_{ij}^v + y_i >= 1, \qquad i \in N^P \tag{2}$$

$$\sum_{j \epsilon N_v} x_{ij}^v - \sum_{j \epsilon N_v} x_{ji}^v = 0, \qquad v \in V, i \in N_v \backslash \{o(v), d(v)\} \tag{3}$$

$$\sum_{j \epsilon N_v} x_{o(v)j}^v = 1, \qquad v \in V \tag{4}$$

$$\sum_{j \epsilon N_v} x_{jd(v)}^v = 1, \qquad v \in V \tag{5}$$

$$\sum_{j \epsilon N_v} x_{ij}^v - \sum_{j \epsilon N_v} x_{(i+n)j}^v = 0, \qquad v \in V, i \in N_v^P \tag{6}$$

$$0 \leq l_i^v \leq K_v, \qquad v \in V, i \in N_v^P \tag{7}$$

$$0 \leq d_{if}^v \leq D_f, \qquad v \in V, i \in N_v^D, f \in F \tag{8}$$

,

$$t_{iv} + T_{ij}^v - t_j^v \leq (\overline{T_{ik}} + T_{ij}^v)(1 - x_{ij}^v), \qquad v \in V, (i,j) \in A_v \tag{9}$$

$$t_i^v + T_{i(i+n)}^v - t_{(i+n)}^v \leq 0, \qquad v \in V, i \in N_v^P \tag{10}$$

,

$$q^v = \sum_{v \in V} \sum_{(i,j) \in E_v} x_{ij}^v q_{ij}^v, \qquad v \in V \tag{11}$$

$$0 \leq q_v \leq Q_1 w_1 \tag{12}$$

$$\underline{T_{ik}} \leq t_i^v \leq \overline{T_{ik}}, \qquad v \in V, i \in N_v, \tag{13}$$

$$y_i \in \{0,1\} \quad i \in N_v^P \tag{14}$$

$$x_{ij}^v \in \{0,1\} \quad v \in V, (i,j) \in A_v \tag{15}$$

The objective function (1) sums up to the cost of the spot cars and the aim is to minimize these costs.

List of Variables:
$v \in V$ - Vehicles
n - number of orders
i - orders
K_v - Vehicle Capacity
$w(v)$ - Max load for Vehicle v
$q(v)$ - Max travel distance for Vehicle v
O_v - Set of orders i that vehicle v can transport
N^P - pickup nodes
N^D - delivery Nodes
E^v - Edges/Arcs the vehicle v can traverse $f \in F$ - Factory
D_f - Factory docking limit
d_{if}^v - docking amount at node i per vehicle v and factory f
C_{ij}^v - Cost of transport from i to j with vehicle v
T_i^h - Node i has a sets of h time windows in tuples $[\underline{T_{ik}}, \overline{T_{ik}}]$ where $k \in \{0, 1, ..., h\}$
T_{ij}^v - Travel time from i to j using v E_v - Edges that vehicle v can traverse
l_i^v - Load at node i with vehicle v
x_{ij}^v - binary variable indicating travel from i to j with v
y_i - binary indicating order not picked up
C^N - cost of not transporting an order

3 Solution in AMPL

TODO..

4 References

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