Modelling and Optimization

INF170

#3:Maximizing Profits and Diet Model

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Fall Semester
2018

Agenda

- Maximizing Profits (AMPL Book, Chapter 1.)
  - Graphical Method
- Diet Model (AMPL Book, Chapter 2.)

Maximizing Profits

Maximizing Profits

A steel company must decide how to allocate next week's time on a rolling mill. The mill takes unfinished slabs of steel as input, and can produce either of two semi-finished products, which we will call bands and coils.

The mill's two products come off the rolling line at different rates:

Tons per hour:	Bands	200
	Coils	140

and they also have different profitabilities:

Profit per ton:	Bands	$25
	Coils	$30

Weekly production amounts:

Maximum tons:	Bands	6,000
	Coils	4,000

If 40 hours of production time are available this week, how many tons of bands and how many tons of coils should be produced to bring in the greatest total profit?

Maximizing Profits

Variables:

X_B : the number of tons of bands to be produced

X_C : the number of tons of coils to be produced

Objective function: Maximization

Total profit = (profit per ton of bands) × X_B

+ (profit per ton of coils) × X_C

Maximize $25\ X_B + 30\ X_C$

Maximizing Profits

Subject to (Constraints):

- Production limits

$$0 \leq X_B \leq 6000$$

$$0 \leq X_C \leq 4000$$

- 40 hours of production time are available this week

The total hours to produce all these tons is then given by

(hours to make a ton of bands) × X_B

+ (hours to make a ton of coils) × X_C

$$(1/200)\, X_B + (1/140)\, X_C \leq 40.$$

Maximizing Profits

Maximize $25\ X_B + 30\ X_C$

Subject to :

$$(1/200)\ X_B + (1/140)\ X_C \leq 40$$

$$0 \leq X_B \leq 6000$$

$$0 \leq X_C \leq 4000$$

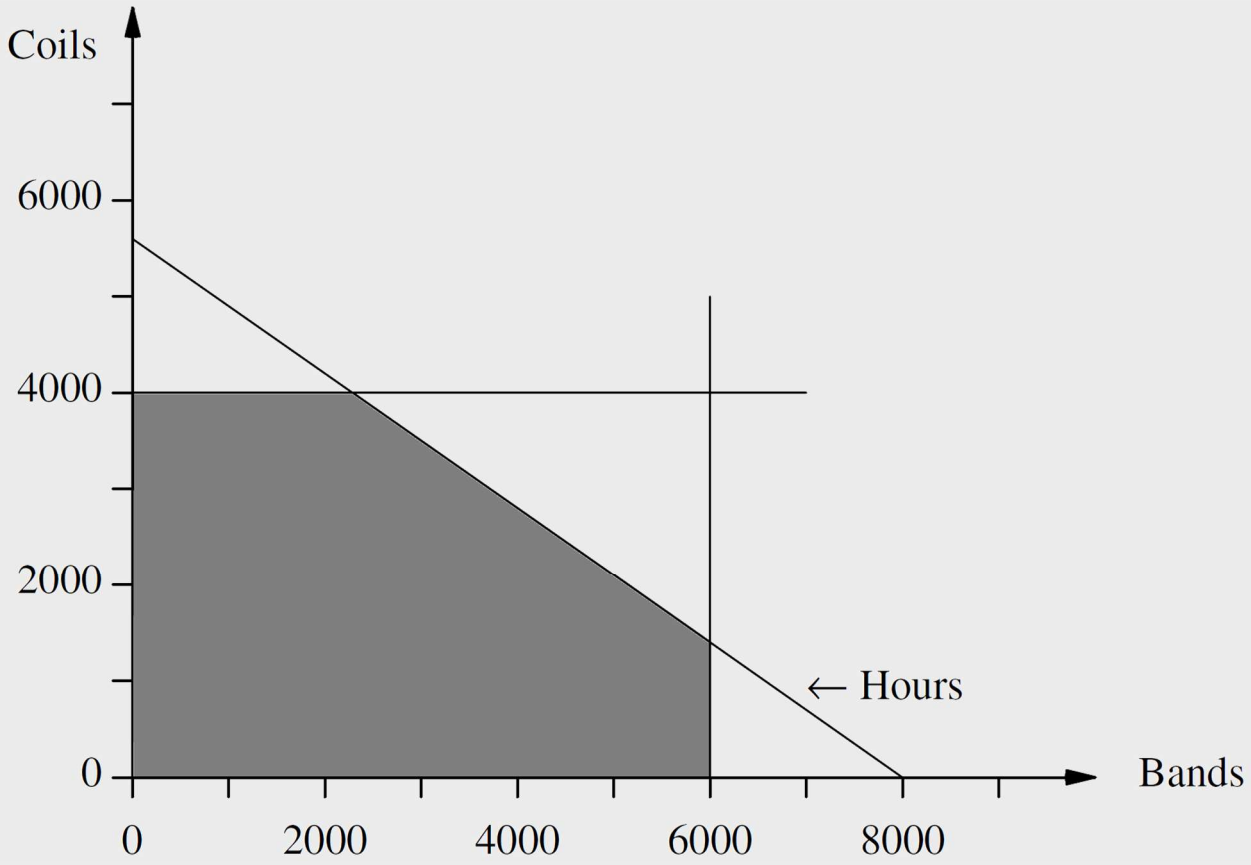
Maximizing Profits

Solution 1:

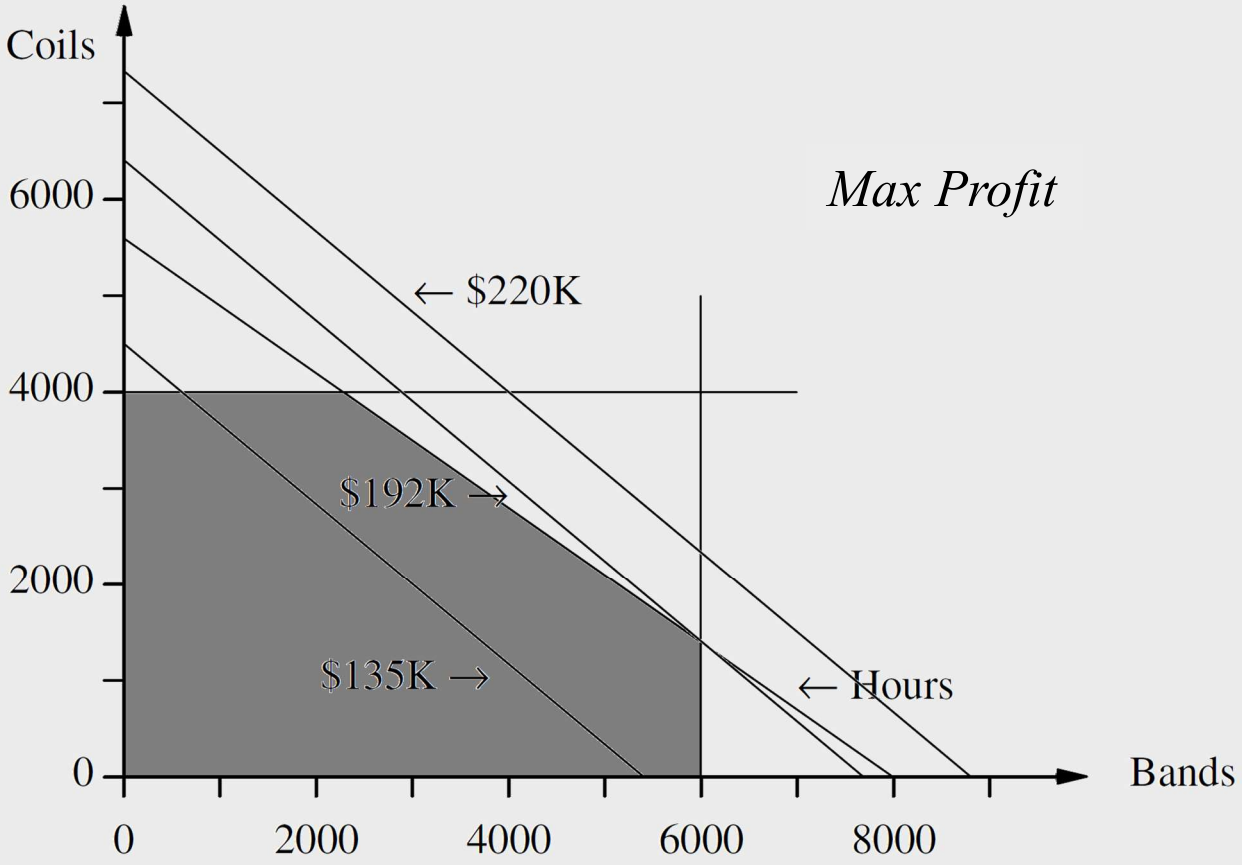
Profit per hour:	Bands	$5,000
	Coils	$4,200

- Produce as many bands as the production limit will allow — 6,000 tons, which takes 30 hours.
- The remaining 10 hours to make coils — 1,400 tons in all
- For a total profit of $192,000.

Solution 2: Graphically!



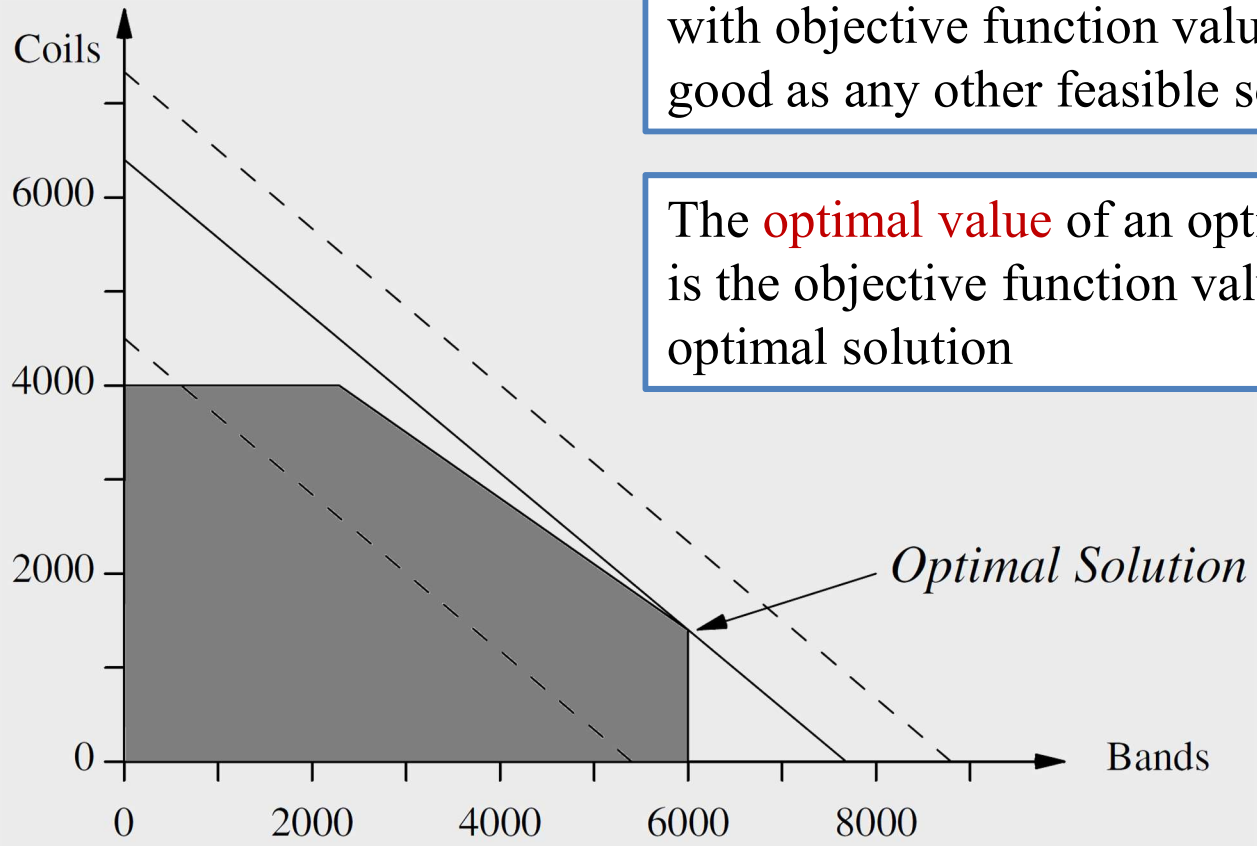
Solution 2: Graphically!



Solution 2: Graphically!

An optimal solution is a feasible solution with objective function value at least as good as any other feasible solution

The optimal value of an optimization model is the objective function value of any optimal solution



Maximizing Profits

Solution 3:

```
var XB;

var XC;

maximize Profit: 25 * XB + 30 * XC;

subject to Time: (1/200) * XB + (1/140) * XC <= 40;

subject to B_limit: 0 <= XB <= 6000;

subject to C_limit: 0 <= XC <= 4000;
solve;
display XB, XC;
```

Maximizing Profits

How about adding another product, steel plate!

	rate	profit	limits
Bands	200	25	6000
Coils	140	30	4000
Plate	160	29	3500

Objective: 196400

$X_B = 6000$ $X_C = 0$ $X_P = 1600$

Maximizing Profits

What if we have a commitment on producing each product!

	rate	profit	commit	limits
Bands	200	25	1000	6000
Coils	140	30	500	4000
Plate	160	29	750	3500

Objective: 194828.5714

$X_B = 6000$ $X_C = 500$ $X_P = 1028.57$

Maximizing Profits

Two stage!

	rate:reheat	rate:roll	profit	commit	limits
bands	200	200	25	1000	6000
coils	200	140	30	500	4000
plate	200	160	29	750	3500
t_avail	35	40			

```
subject to Time_reheat:     ... + (1/200) * XC + ... <= 35;

subject to Time_roll:       ... + (1/140) * XC + ... <= 40;
```

Objective: 190071.4286

$X_B = 3357.14$ $X_C = 500$ $X_P = 3142.86$

Diet Model

Diet Model

Find a mix of foods that satisfies requirements on the amounts of various vitamins!

Prices per package

BEEF	beef	$3.19
CHK	chicken	$2.59
FISH	fish	$2.29
HAM	ham	$2.89
MCH	mac & cheese	$1.89
MTL	meat loaf	$1.99
SPG	spaghetti	$1.99
TUR	turkey	$2.49

Percentages, per package, of the minimum daily requirements for vitamins

	A	C	B1	B2
BEEF	60%	20%	10%	15%
CHK	8	0	20	20
FISH	8	10	15	10
HAM	40	40	35	10
MCH	15	35	15	15
MTL	70	30	15	15
SPG	25	50	25	15
TUR	60	20	15	10

The problem is to find the cheapest combination of packages that will meet a week's requirements — that is, at least 700% of the daily requirement for each vitamin!

Diet Model

Variables:

X_{BEEF} : the number of packages of beef to be purchased

X_{CHK} : the number of packages of chicken to be purchased

…

Objective function: Minimization of the Total Cost of the Diet

$$\text{Minimize} \quad 3.19\, X_{BEEF} + 2.59\, X_{CHK} + 2.29\, X_{FISH} + 2.89\, X_{HAM} +$$
$$1.89\, X_{MCH} + 1.99\, X_{MTL} + 1.99\, X_{SPG} + 2.49\, X_{TUR}$$

Diet Model

Subject to (Constraints):

- total percentage of vitamin "A" daily requirement met

$$60\ X_{BEEF} + 8\ X_{CHK} + 8\ X_{FISH} + 40\ X_{HAM} +$$

$$15\ X_{MCH} + 70\ X_{MTL} + 25\ X_{SPG} + 60\ X_{TUR} \geq 700$$

- total percentage of vitamin "C" daily requirement met

…

Diet Model

```
var Xbeef >= 0; var Xchk >= 0; var Xfish >= 0;
var Xham  >= 0; var Xmch >= 0; var Xmtl >= 0;
var Xspg  >= 0; var Xtur >= 0;
minimize cost:
        3.19*Xbeef + 2.59*Xchk + 2.29*Xfish + 2.89*Xham +
        1.89*Xmch + 1.99*Xmtl + 1.99*Xspg + 2.49*Xtur;
subject to A:
        60*Xbeef + 8*Xchk + 8*Xfish + 40*Xham +
        15*Xmch + 70*Xmtl + 25*Xspg + 60*Xtur >= 700;
subject to C:
        20*Xbeef + 0*Xchk + 10*Xfish + 40*Xham +
        35*Xmch + 30*Xmtl + 50*Xspg + 20*Xtur >= 700;
subject to B1:
        10*Xbeef + 20*Xchk + 15*Xfish + 35*Xham +
        15*Xmch + 15*Xmtl + 25*Xspg + 15*Xtur >= 700;
subject to B2:
        15*Xbeef + 20*Xchk + 10*Xfish + 10*Xham +
        15*Xmch + 15*Xmtl + 15*Xspg + 10*Xtur >= 700;
solve;
display Xbeef,Xchk,Xfish,Xham,Xmch,Xmtl,Xspg,Xtur;
```

```
Xmch = 46.6667
Objective: 88.2
```

Diet Model

```
var Xbeef >= 0; var Xchk >= 0; var Xfish >= 0;
var Xham  >= 0; var Xmch >= 0; var Xmtl >= 0;
var Xspg  >= 0; var Xtur >= 0;
minimize cost:
        3.19*Xbeef + 2.59*Xchk + 2.29*Xfish + 2.89*Xham +
        1.89*Xmch + 1.99*Xmtl + 1.99*Xspg + 2.49*Xtur;
subject to A:
        60*Xbeef + 8*Xchk + 8*Xfish + 40*Xham +
        15*Xmch + 70*Xmtl + 25*Xspg + 60*Xtur = 700;
subject to C:
        20*Xbeef + 0*Xchk + 10*Xfish + 40*Xham +
        35*Xmch + 30*Xmtl + 50*Xspg + 20*Xtur = 700;
subject to B1:
        10*Xbeef + 20*Xchk + 15*Xfish + 35*Xham +
        15*Xmch + 15*Xmtl + 25*Xspg + 15*Xtur = 700;
subject to B2:
        15*Xbeef + 20*Xchk + 10*Xfish + 10*Xham +
        15*Xmch + 15*Xmtl + 15*Xspg + 10*Xtur = 700;
solve;
display Xbeef,Xchk,Xfish,Xham,Xmch,Xmtl,Xspg,Xtur;
```

```
Xchk = 19.5
Xmch = 16.3
Xmtl = 4.3
Objective: 89.99
```

Diet Model

	cost	min	max
BEEF	3.19	0	100
CHK	2.59	0	100
FISH	2.29	0	100
HAM	2.89	0	100
MCH	1.89	0	100
MTL	1.99	0	100
SPG	1.99	0	100
TUR	2.49	0	100

	A	C	B1	B2
BEEF	60%	20%	10%	15%
CHK	8	0	20	20
FISH	8	10	15	10
HAM	40	40	35	10
MCH	15	35	15	15
MTL	70	30	15	15
SPG	25	50	25	15
TUR	60	20	15	10

	min	max
A	700	10000
C	700	10000
B1	700	10000
B2	700	10000

```
Xmch = 46.6667
Objective: 88.2
```

	Diet
A	700
B1	700
B2	700
C	1633.33

Diet Model

	cost	min	max
BEEF	3.19	2	100
CHK	2.59	2	100
FISH	2.29	2	100
HAM	2.89	2	100
MCH	1.89	2	100
MTL	1.99	2	100
SPG	1.99	2	100
TUR	2.49	2	100

	A	C	B1	B2
BEEF	60%	20%	10%	15%
CHK	8	0	20	20
FISH	8	10	15	10
HAM	40	40	35	10
MCH	15	35	15	15
MTL	70	30	15	15
SPG	25	50	25	15
TUR	60	20	15	10

	min	max
A	700	10000
C	700	10000
B1	700	10000
B2	700	10000

```
Xbeef = 2
Xchk  = 2
Xfish = 2
Xham  = 2
Xmch  = 34
Xmtl  = 2
Xspg  = 2
Xtur  = 2
Objective: 99.12
```

	Diet
A	1052
B1	780
B2	700
C	1530

Diet Model

	cost	min	max	A	C	B1	B2	NA	CAL
BEEF	3.19	2	10	60	20	10	15	938	295
CHK	2.59	2	10	8	0	20	20	2180	770
FISH	2.29	2	10	8	10	15	10	945	440
HAM	2.89	2	10	40	40	35	10	278	430
MCH	1.89	2	10	15	35	15	15	1182	315
MTL	1.99	2	10	70	30	15	15	896	400
SPG	1.99	2	10	25	50	25	15	1329	370
TUR	2.49	2	10	60	20	15	10	1397	450

	min	max
A	700	20000
C	700	20000
B1	700	20000
B2	**700**	20000
NA	0	**40000**
CAL	16000	24000

	Diet
A	1993.09
B1	841.091
B2	**601.091**
C	1272.55
NA	40000
CAL	17222.9

Diet Model

	cost	min	max	A	C	B1	B2	NA	CAL
BEEF	3.19	2	10	60	20	10	15	938	295
CHK	2.59	2	10	8	0	20	20	2180	770
FISH	2.29	2	10	8	10	15	10	945	440
HAM	2.89	2	10	40	40	35	10	278	430
MCH	1.89	2	10	15	35	15	15	1182	315
MTL	1.99	2	10	70	30	15	15	896	400
SPG	1.99	2	10	25	50	25	15	1329	370
TUR	2.49	2	10	60	20	15	10	1397	450

	min	max
A	700	20000
C	700	20000
B1	700	20000
B2	700	20000
NA	0	**50000**
CAL	16000	24000

```
Xbeef = 5.36061
Xchk  = 2
Xfish = 2
Xham  = 10
Xmch  = 10
Xmtl  = 10
Xspg  = 9.30605
Xtur  = 2
Objective: 118.0594032
```

	Diet
A	1956.29
B1	1036.26
B2	700
C	1682.51
NA	50000
CAL	19794.6

Diet Model

	cost	min	max	A	C	B1	B2	NA	CAL
BEEF	3.19	2	10	60	20	10	15	938	295
CHK	2.59	2	10	8	0	20	20	2180	770
FISH	2.29	2	10	8	10	15	10	945	440
HAM	2.89	2	10	40	40	35	10	278	430
MCH	1.89	2	10	15	35	15	15	1182	315
MTL	1.99	2	10	70	30	15	15	896	400
SPG	1.99	2	10	25	50	25	15	1329	370
TUR	2.49	2	10	60	20	15	10	1397	450

	min	max
A	700	20000
C	700	20000
B1	700	20000
B2	700	20000
NA	0	50000
CAL	16000	24000

```
Xbeef = 5
Xchk  = 2
Xfish = 2
Xham  = 10
Xmch  = 10
Xmtl  = 10
Xspg  = 9
Xtur  = 2
```

	Diet
A	1927
B1	1025
B2	**690**
C	1660
NA	49255
CAL	19575

Diet Model

	cost	min	max	A	C	B1	B2	NA	CAL
BEEF	3.19	2	10	60	20	10	15	938	295
CHK	2.59	2	10	8	0	20	20	2180	770
FISH	2.29	2	10	8	10	15	10	945	440
HAM	2.89	2	10	40	40	35	10	278	430
MCH	1.89	2	10	15	35	15	15	1182	315
MTL	1.99	2	10	70	30	15	15	896	400
SPG	1.99	2	10	25	50	25	15	1329	370
TUR	2.49	2	10	60	20	15	10	1397	450

	min	max
A	700	20000
C	700	20000
B1	700	20000
B2	700	20000
NA	0	50000
CAL	16000	24000

```
Xbeef = 6
Xchk  = 2
Xfish = 2
Xham  = 10
Xmch  = 10
Xmtl  = 10
Xspg  = 10
Xtur  = 2
```

	Diet
A	2012
B1	1060
B2	720
C	1730
NA	**51522**
CAL	20240

Diet Model

	cost	min	max	A	C	B1	B2	NA	CAL
BEEF	3.19	2	10	60	20	10	15	938	295
CHK	2.59	2	10	8	0	20	20	2180	770
FISH	2.29	2	10	8	10	15	10	945	440
HAM	2.89	2	10	40	40	35	10	278	430
MCH	1.89	2	10	15	35	15	15	1182	315
MTL	1.99	2	10	70	30	15	15	896	400
SPG	1.99	2	10	25	50	25	15	1329	370
TUR	2.49	2	10	60	20	15	10	1397	450

	min	max
A	700	20000
C	700	20000
B1	700	20000
B2	700	20000
NA	0	50000
CAL	16000	24000

```
Xbeef = 9
Xchk  = 2
Xfish = 2
Xham  = 8
Xmch  = 10
Xmtl  = 10
Xspg  = 7
Xtur  = 2
Objective: 119.3
```

	Diet
A	2037
B1	945
B2	700
C	1560
NA	49793
CAL	19155

ASSIGNMENT #1:

AMPL BOOK
CHAPTER 1. EXERCISES(1-6)

LECTURE #4:

GENERAL LP