

# Pickup & Delivery Problem with Multiple Time Windows

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# 1 Introduction

TODO..

## 2 Problem Formulation

Our problem is typically for a vehicle manufacturing company that has a certain set of factories where the vehicles are produced, each with a set of demanded parts for production. The parts, or orders (typically named transport orders but we refer to orders here), can be delivered from a set of suppliers. At a given point in time (could be delivery for one day or a week) we consider a planning problem with a demand for orders that should be satisfied with the given suppliers using vehicles provided by different logistics companies.

The vehicles have different capacities, cost structures, incompatibilities and start at the first pickup location at the first pickup time (ie. costs to get from start to first pickup are not considered neither is time). The cost structure can be multidimensional taking into account total distance and volume in different formats (variable costs, fix costs etc).

When a delivery is assigned to a vehicle, the vehicle must load the delivery from the supplier, and deliver the delivery at the factory dock. The docks in each factory can differ according to which order is being delivered and there is a limit to how many docks each vehicle can visit at each factory.

Each delivery/pickup can have several time windows, lapsing sometimes over several days. If a car arrives before a time window it has to wait.

The mathematical formulation of the problem will now follow where the set of vehicles used is denoted by  $V$  and capacity of each vehicle  $v \in V$  is denoted by  $K_v$ . We let  $n$  be the number of orders in the problem, so if  $i$  is a specific pickup-node then  $i + n$  corresponds to the delivery node for the same order. Here it follows that for each vehicle we have a set  $O_v$  of orders that vehicle  $v \in V$  can transport based on the incompatibilities. The set of pickup Nodes (supplier docks) we denote using  $N^P$  and each delivery node (Factory dock) is denoted by  $N^D$ . All nodes are therefore represented by  $N = N^P \cup N^D$ . Each Factory,  $f \in F$ , also has a set of Nodes belonging to the same factory which we denote  $N_f$ . The delivery nodes that belong to one factory can then be represented as  $N_f^D = N_f \cap N^D$ . Each vehicle has a set of Nodes it can travel to (corresponding to orders  $O_v$ ) represented by  $N_v$ . This set also includes an origin node,  $o(v)$  and a destination node  $d(v)$  corresponding to the first pickup and the last delivery of vehicle  $v$ . The set of Edges that each vehicle can traverse is represented by  $E_v$ . The factory

docking limit is denoted by  $D_f$ . Each vehicle has a given amount of docks per factory it wants to deliver to represented by  $d_f^v$ .

Each node has a set of  $h_i$  time windows represented by  $[T_{ip}, \overline{T_{ip}}] \in [0, T]$  where  $p \in \{0, 1, \dots, h_i\}$  and all nodes should be picked up and delivered after time 0 and before time T. Each node has a current time based on when its being served, denoted by  $t_i$  and a current load, denoted by  $l_i$ , where  $i \in 2n$  and  $v \in V$ . The distance from node  $i$  to node  $j$  is denoted by  $d_{ij}$ . The cost of vehicle  $v$  driving from node  $i$  to  $j$  is denoted by  $C_{ij}^{vq(v)w(v)}$  and the corresponding time between the nodes  $T_{ij}^v$ .  $l_i^v$  is the load of vehicle  $v$  leaving node  $i$  and  $x_{ij}^v$  is a binary variable indicating if vehicle  $v$  is travelling between  $i$  and  $j$  node. The cost of not transporting an order will be the same for each order (should be set relatively high to avoid dummy transports) so we denote this with  $C^N$  and each order,  $i$ , not transported is represented by a binary variable  $y_i$ .

$$\min \sum_{v \in V} \sum_{s \in S} C_{vs} d_{vs} + \sum_{i \in N^P} C_i y_i \quad (1)$$

subject to:

$$\sum_{v \in V} \sum_{j \in N_v} x_{ijv} + y_i \geq 1, \quad i \in N^P \quad (2)$$

$$\sum_{j \in N_v} x_{ijv} - \sum_{j \in N_v} x_{jiv} = 0, \quad v \in V, i \in N_v \setminus \{o(v), d(v)\} \quad (3)$$

$$\sum_{j \in N_v} x_{o(v)jv} = 1, \quad v \in V \quad (4)$$

$$\sum_{j \in N_v} x_{jd(v)v} = 1, \quad v \in V \quad (5)$$

$$\sum_{j \in N_v} x_{ijv} - \sum_{j \in N_v} x_{(i+n)jv} = 0, \quad v \in V, i \in N_v^P \quad (6)$$

$$l_{iv} + Q_j - l_{jv} \leq K_v(1 - x_{ijv}), \quad v \in V, j \in N_v^P, (i, j) \in E_v \quad (7)$$

$$l_{iv} - Q_j - l_{jv} \leq K_v(1 - x_{i(j+n)v}), \quad v \in V, j \in N_v^P, (i, n+j) \in E_v \quad (8)$$

$$0 \leq l_{iv} \leq K_v, \quad v \in V, i \in N_v^P \quad (9)$$

$$h_j - 1 - h_i \leq H_f(1 - x_{ijv}), \quad v \in V, (i, j) \in N_f^D, f \in F \quad (10)$$

$$h_i \leq H_f x_{ijv}, \quad v \in V, j \in E_v \setminus N_f^D, i \in N_f^D, f \in F \quad (11)$$

$$h_i = x_{ijv}, \quad v \in V, i \in N_f^D, j \in E_v \setminus N_f^D, f \in F \quad (12)$$

$$0 \leq h_i, \quad i \in N \quad (13)$$

TODO: possible that 13 is redundant? Should be covered by 12 right?

$$\sum_{p=1}^{h_i} u_{ip} = 1, \quad i \in N \quad (14)$$

$$\sum_{p=1}^{h_i} u_{ip} \underline{T}_{ip} \leq t_i, \quad i \in N \quad (15)$$

$$\sum_{p=1}^{h_i} u_{ip} \overline{T}_{ip} \geq t_i, \quad i \in N \quad (16)$$

$$t_i + T_{ijv} + W_j - t_j \leq M(1 - x_{ijv}), \quad v \in V, (i, j) \in E_v \quad (17)$$

$$t_i + T_{ijv} + W_j - t_j \geq M(x_{ijv} - 1), \quad v \in V, (i, j) \in E_v \quad (18)$$

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TODO: Still working on integrating the weight here. WIP

$$\sum_{s \in S} d_{vs} = \sum_{v \in V} \sum_{(i,j) \in E_v} x_{ijv} D_{ijv}, \quad v \in V \quad (19)$$

$$B_{(s-1)} w_{vs} \leq d_{vs} \leq B_s w_{vs} \quad (20)$$

$$\sum_{s \in S} w_{sv} = 1, \quad v \in V \quad (21)$$

$$y_i \in \{0, 1\} \quad i \in N^P \quad (22)$$

$$x_{ijv} \in \{0, 1\} \quad v \in V, (i, j) \in A_v \quad (23)$$

The objective function (1) sums up to the cost of the spot cars and the aim is to minimize these costs.

List of Variables and parameters:

$v \in V$  - Vehicles

$n$  - number of orders

$s \in S$  - size of cost structure regarding distance

$i$  - orders

$K_v$  - Vehicle Capacity

$l_{iv}$  - Load at node  $i$  with vehicle  $v$

$O_v$  - Set of orders  $i$  that vehicle  $v$  can transport

$N^P$  - pickup nodes

$N^D$  - delivery Nodes

$N_f^D$  - Delivery nodes for factory  $f$

$E_v$  - Edges/Arcs the vehicle  $v$  can traverse

$f \in F$  - Factory

$H_f$  - Factory docking limit

$h_i$  - factory visits at node  $i$

$T_{ia}$  - Node  $i$  has a sets of  $a$  time windows in tuples  $[\underline{T}_{ia}, \overline{T}_{ia}]$  where  $a \in A$

$T_{ijv}$  - Travel time from  $i$  to  $j$  using  $v$

$u_{ip}$  - binary variable for timewindow  $p$  at node  $i$

$M$  - Arbitrary large constant

$W_i$  - Waiting time at node  $i$

$Q_j$  - weight of order at node  $j$

$x_{ijv}$  - Binary variable indicating travel from  $i$  to  $j$  by vehicle  $v$

$y_i$  - binary indicating order not picked up

$d_{vs}$  - total distance for vehicle  $v$

$C_{vs}$  - Cost per km for vehicle  $v$  in cost structure  $s$

$w_{vs}$  - binary indicating which interval vehicle  $v$  is in cost structure  $s$

$B_s$  - distance determening interval  $s$  in cost structure

### 3 Solution in AMPL

TODO..

### 4 References

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