A Pickup and Delivery problem with multiple time windows in supply chain industry

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abstract

This is a test Shaw (1997). Hopefully it works..

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Chapter 1

Introduction

The automobile industry is one of the most significant industries in Europe today which is facing high cost-reduction pressure from fierce international competition and difficult conditions.4flow AG (2018) The supply chain process in automobile industry requires alot of flexibility in their planning and modelling of transportation which often takes the form some sort of vehicle transportation problem. In planning the most efficient way of organizing your transportation network the classical VRPTW from is likely the first problem that comes to mind.

Parragh et al. (2008) The Vehicle Routing Problem with Time Window Constraints (VRPTW) is defined as the problem of minimizing costs when a fleet of homogeneus vehicles has to destribute goods from a depot to a set of customers satisfying time windows and capacity constraints Cordeau and Groupe d'études et de recherche en analyse des décisions (Montréal(2000). Another classical problem is the PDPTW, Pickup and Delivery Problem with Time Windows, with multiple vehicles is another problem that has been widely researched and solved for example in Nanry and Barnes (2000).

Cordeau and Groupe d'études et de recherche en analyse des décisions (Montréal(2000) Presents a VRPTW Problem as an extension of the Capacitated Vehicle routing problem, with hard and soft timewindows, where soft can be broken agains a cost and hard time-windows cannot. They present a multi-commodity network flow formulation of the problem and use approximation methods to derive upper bounds and use Lagrangean relaxation and column generation to derive lower bounds. To produce integer solutions they use cutting and branching strategies and finally they also present special cases and extensions. Solomon (1987) formalized an algorithm for general PDP Lu and Dessouky (2006) insertion based heuristic Kalantari et al. (1985) An algorithm for the travelling salesman problem with pickup and delivery of customers Dumas et al. (1991) pickup and delivery problem with time windows Fagerholt et al. (2010) speed optimization and reduction of fuel emissions Berbeglia et al. (2010) dynamic and static pickup and delivery problems survey and classification scheme Berbeglia et al. (2007) static Zhou (2013) Inventory management and inbound logistics optimization Savelsbergh and Sol (1995) General pickup and delivery problem. Dror et al. (1989) Properties and solution framework

Christiansen and Fagerholt (2002) present a ship scheduling PDP problem of bulk cargoes with multiple time windows. They also handle specific shipping industry challenges regarding ship idle time, transport risks due to weather and unpredictable service time at ports. They use a set partitioning approach to solve

the problem and found that you could increase the robustness of the schedules on account of costs. Christiansen et al. (2004) Ship routing and scheduling Hemmati et al. (2014) Consider a class of cargo ship routing and scheduling problems from tramp and industrial shipping industry. They provide solutions to a wide range of benchmark instances both optimal for smaller instances, using a commercial mixed-integer programming solver. And they present adaptice large neighbourhood search heuristics to larger instances. They also provide the benchmark instances and an instance generator. Hemmati et al. (2016) A iterative two-phase hybrid metaheuristic for multi-product short sea inventory routing problem Ferreira et al. (2018) variable neighbourhood search for vehicle routing problem with multiple time windows Desrosiers et al. (1995) Time constrained routing and scheduling

In this paper we will propose a mathematical formulation to a multivehicle PDPTW problem, with certain adjustments to satisfy the needs of an automobile manufacturing company. To these type of companies the inbound production side of the supply chain is more in focus and require ajdustments on the classical PDPTW. In these type of industries companies often facing challenges related to the production side of the problem, which is often referred to as looking at the problem from an inbound perspective. Inbound oriented perspective means that you have less focus on the the supplier side of the of the production, and on the means of transportation as long as what is being transported arrives on time for production. This changes our problem away from the classical problems mentioned above. Instead of having a customer oriented view, we must instead shift our view to focus on getting a set of orders from a set of suppliers to a set of factories where the orders are needed for production.

Being more inbound oriented also leads to having more focus on the problems that sometimes occur within the factory and the delivery part of the problem. Some manufacturers might have alot of traffic on their factories, and want to limit that, saying that a vehicle can only visit a certain amount of docks for each visit. This might lead to more ineffective solutions and more use of vehicles in some cases however, for the inbound oriented producer it is a nessecary limitation. To our knowledge dock constraints has not been researched before in PDPTW problems.

Another result from the inbound thinker is that instead of using your own fleet of vehicles, you hire a logistics carrier to do your transports. The carries might have different cost structures and payment methods which you have no influence over. Leading you to need a model that takes that into account. We focus here on dealing with a carrier that offers a varying cost per kilometer. Meaning that the cost parameter of the standard PDP will change depending on how far a vehicle is travelling. Using a carrier also changes the problem to that you dont care so much where the vehicle is travelling from to its first pickup. The vehicle rather starts at its first pickup point because the cost of the car manufacturer starts only from the time of the first pickup. Before pickup and after delivery the carrier has the costs. To our knowledge very little research has been made into using a logistics carrier as vehicle fleet.

In car manufacturing business you might have production periods that go over several days with strict opening times on delivery. This leads to the problem of multiple time windows. The manufacturer might only want the parts delivered in the morning on one of three days but have a break in the middle of each time window (lunch). There might also be advantageous to move the delivery of one order to the

next day to bundle orders together and we want our model to be able to handle such problems.

In Favaretto et al. (2007) they propose a time window model to handle multiple time windows and multiple visits. We are using a modified version of this model to handle the multiple time windows in this paper to handle the time window constraints. However as they focus on satisfying a set of customers instead of production companies our model differs greatly when it comes to the rest of the constraints.

To make a model that is as realistic as possible it is important to integrate each of the above aspects and take them all into account. One aspect of the model could greatly influence the decision of another so it is important to build a model that satisfies all the aspects above. Research often forcuses on solving single issues, like the multiple time windows, to handle this specific problem for a certain industry. However handeling all of the above mentioned aspects have to our knowledge never been researched before and makes it therefore an important problem to solve.

The goal of this paper is to present a realistic mathematical model that can be further developed into a realistic model solving pickup and delivery problems on a daily/weekly basis for manufacturing companies. The goal is also to have a mathematical basis to be used in a solver such as AMPL to solve a small instance, and finally to make a heuristic model that can solve the problem efficiently and be used by companies in similar sort of industries.

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Chapter 2

Model Description

Like mentioned above to take the perspective of a vehicle manufacturing company, we see that we have a certain set of factories where the products are produced, each with a set of demanded parts/orders for production. The parts, or orders (typically named transport orders but we refer to orders here), can be delivered from a set of suppliers. At a given point in time (could be delivery for one day or a week) we consider a planning problem with a demand for orders that should be satisfied with the given suppliers using vehicles provided by different logistic carriers.

v	-	vehicle
i	-	node
f	-	factory
p	-	timewindow
s	-	stop location
α	-	distance interval in cost structure
β	-	weight interval in cost structure

Table 2.1: Indices

The vehicles have different capacities(kg and volume), cost structures, incompatabilities and start at the first pickup location at the first pickup time (ie. costs to get from start to first pickup are not relevant to the manufacturer). The cost structure depend on the total distance a vehicle is driving, the maximum weight transported by that vehicle, and a fix cost. There is also a cost for stopping at a node.

When a delivery is assigned to a vehicle, the vehicle must load the delivery from the supplier, and deliver the delivery at the factory dock. The docks (each reperesented by a different node) in each factory can differ according to which order is being delivered and there is a limit to how many docks each vehicle can unload at per visit to the factory.

Each delivery/pickup can have several time windows, lapsing sometimes over several days. If a car arrives before a time window it has to wait.

The mathematical formulation of the problem will now follow. The problem can be viewed as a graph $G(E, N)$ where $N = \{0, ..., 2n\}$ are the verticies and n is the number of orders in the problem, and $E = \{(i, j) : i, j \in N, i \neq j\}$ represent the edges in the graph.

	Sets	
N	-	nodes $\{1, .., 2n\}$ where n is number of orders
V	-	vehicles
E	-	edges
E_v	-	edges visitable by vehicle v
N_v	-	nodes visitable by vehicle v
N^P	-	pickup nodes
N^D	-	delivery Nodes
F	-	factories
N_f	-	delivery nodes for factory f
A_v	-	index of elements in the cost where α goes from $(1, .., \gamma_v)$ and β from $(1, .., \mu_v)$
P_i	-	set of time windows at node i, $\{1, .., \pi_i\}$
T_i	-	set of time parameters $[\underline{T_{ip}}, \overline{T_{ip}}]$ at node i where $p \in P_i$
S	-	set of stops indicating a pickup/delivery location
L_s	-	Sets of nodes sharing a stop location $s \in S$

The set of vehicles used is denoted by V and weight capacity of each vehicle $v \in V$ is denoted by K_v^{kg} and volume capacity is denoted K_v^{vol}. The set of Edges that each vehicle can traverse is represented by E_v. Since n is the number of orders in the problem, then if i is a specific pickup-node then $i+n$ corresponds to the delivery node for the same order. The set of pickup Nodes (supplier docks) we denote using N^P and each delivery node (Factory dock) is denoted by N^D. All nodes are therefore equivalent to $N = N^P \cup N^D$. Each Factory, $f \in F$, also has a set of Nodes belonging to the same factory wich we denote N_f. Since all factories are delivery nodes these sets only include delivery nodes. Each vehicle has a set of Nodes it can travel to represented by N_v. This set also includes an origin node, $o(v)$ and a destination node $d(v)$ which is a fictive start and ending point unique to each vehicle v. The distance and costs from here to the first pickup is zero. The factory docking limit is denoted by H_f.

Parameters

n	-	amount of orders
K_v^{kg}	-	weight capacity of vehicle $v \in V$
K_v^{vol}	-	volume capacity of vehicle $v \in V$
$o(v)$	-	starting node of vehicle v
$d(v)$	-	ending node of vehicle v
Q_i^{kg}	-	weight of order at node $i \in N$
Q_i^{vol}	-	volume of order at node $i \in N$
H_f	-	docking limit at factory $f \in F$
T_{ijv}	-	travel time for vehicle $v \in V$ over edge $(i,j) \in E_v$
π_i	-	amount of time windows at node $i \in N$
$\overline{T_{ip}}$	-	upper bound time of time window $p \in P_i$ at node $i \in N$
$\underline{T_{ip}}$	-	lower bound time of time window $p \in P_i$ at node $i \in N$
γ_v	-	amount of distance intervals for vehicle v
μ_v	-	amount of weight intervals for vehicle v
$C_{v\alpha\beta}^{km}$	-	cost per distance unit (km) in cost matrix element $(\alpha,\beta) \in A_v$ for vehicle v
$C_{v\alpha\beta}^{kg}$	-	cost per weight unit (kg) in cost matrix element $(\alpha,\beta) \in A_v$ for vehicle v
$C_{v\alpha\beta}^{fix}$	-	fixed cost in index $(\alpha,\beta) \in A_v$ for vehicle v
C_i^{stop}	-	costs of making a stop at node i
C_i	-	cost of not transporting order $i \in N^P$
D_{ij}	-	distance between node $i \in N$ and $j \in N$
B_α	-	distance for interval α in cost matrix A_v column index
Z_β	-	weight for interval β in cost matrix A_v row index

Each delivery node has a variable h_i indicating how many docks have been visited including the node i.

Each pickup node has a weight Q_i^{kg} and a volume Q_i^{vol} parameter indicating the weight and volume of the order at that node. Each node has a set T_i of time windows represented by $[\underline{T_{ip}}, \overline{T_{ip}}] \in [0,T]$ where $p \in P_i = \{0,1,...,\pi_i\}$ and all nodes should be picked up and delivered within given timewindows. Each node has a current time based on when its being served, denoted by t_i and where $i \in N$. The distance from node i to node j is denoted by D_{ij} and the time for each vehicle v to travel between them is represented by T_{ijv}.

Each time a vehicle v makes a stop and a node i there will be a stop cost represented by C_i^{stop}. The costs of vehicle v depends on the total distance of that vehicle and the maximum weight transported. Each possible interval of weight and distance is represented by an index pair (α,β), where α is the distance interval index ranging $(1..\gamma_v$ and β is the weight interval ranging from $(1..\mu_v)$. Together these pairs make a matrix we refer to in this paper as a cost matrix. Each type of cost has a matrix, including distance, weight, and fix costs and the cost in a certain interval (α,β) is represented by $C_{v\alpha\beta}^{cost-type}$. The total distance travelled by vehicle v will be denoted by the variables $d_{v\alpha\beta}$ for each $(\alpha,\beta) \in A_v$, where only one variable per vehicle will have the value equal to the total distance of that vehicle. The maximum weight transported by a vehicle is represented by $l_{v\alpha\beta}$ and also only one of these variables per vehicle will have a the corresponding value. Which $d_{v\alpha\beta}$ and $l_{v\alpha\beta}$ has a value will be determined by the binary variable $b_{v\alpha\beta}$ and the distance insterval parameter B_α and the weight interval parameter Z_β.

Variables

x_{ijv}	-	binary indicating travel from node $i \in N$ to $j \in N$ of vehicle $v \in V$
y_i	-	binary indicating that an order $i \in N^P$ is not picked up
l_{iv}^{kg}	-	weight of vehicle v after visiting node i
l_{iv}^{vol}	-	volume of vehicle v after visiting node i
h_i	-	docking times in factory after visiting node $i \in N_f$
t_i	-	time after visiting node $i \in N$
u_{ip}	-	binary indicating usage of time window $p \in P_i$ at node i
$d_{v\alpha\beta}$	-	total distance travelled of vehicle $v \in V$ if it fits in interval $(\alpha, \beta) \in A_v$
$b_{v\alpha\beta}$	-	binary indicating interval $(\alpha, \beta) \in A_v$ for vehicle $v \in V$
$l_{v\alpha\beta}$	-	the highest weight trasported by vehicle $v \in V$ for interval $(\alpha, \beta) \in A_v$

l_{iv}^{kg} is the weight and l_{iv}^{vol} is the volume on the vehicle v leaving node i. x_{ijv} is a binary vairable indicating if vehicle v is travelling between i and j node. The cost of not transporting an order will be represented by C_i for each node i, with a corresponding binary variable y_i, indicating that an order is not picked up.

$$\min \sum_{v \in V} \sum_{(\alpha,\beta) \in A_v} (C_{v\alpha\beta}^{km} d_{v\alpha\beta} + C_{v\alpha\beta}^{kg} l_{v\alpha\beta} + C_{v\alpha\beta}^{fix} b_{v\alpha\beta}) + \sum_{v \in V} \sum_{s \in S} \sum_{\substack{i \in L_s \\ j \in N_v \notin L_s}} C_i^{stop} x_{ijv} + \sum_{i \in N^P} C_i y_i \tag{2.1}$$

subject to:

$$\sum_{v \in V} \sum_{j \in N_v} x_{ijv} + y_i = 1, \qquad i \in N^P \tag{2.2}$$

$$\sum_{j \in N_v} x_{ijv} - \sum_{j \in N_v} x_{jiv} = 0, \qquad v \in V, i \in N_v \notin \{o(v), d(v)\} \tag{2.3}$$

$$\sum_{j \in N_v} x_{o(v)jv} = 1, \qquad v \in V \tag{2.4}$$

$$\sum_{j \in N_v} x_{jd(v)v} = 1, \qquad v \in V \tag{2.5}$$

$$\sum_{j \in N_v} x_{ijv} - \sum_{j \in N_v} x_{(i+n)jv} = 0, \qquad v \in V, i \in N_v^P \tag{2.6}$$

$$l_{iv}^{kg} + Q_j^{kg} - l_{jv}^{kg} \leq K_v^{kg}(1 - x_{ijv}), \qquad v \in V, j \in N_v^P, (i,j) \in E_v \tag{2.7}$$

$$l_{iv}^{kg} - Q_j^{kg} - l_{(j+n)v}^{kg} \leq K_v^{kg}(1 - x_{i(j+n)v}), \qquad v \in V, j \in N_v^P, (i, n+j) \in E_v \tag{2.8}$$

$$0 \leq l_{iv}^{kg} \leq K_v^{kg}, \qquad v \in V, i \in N_v^P \tag{2.9}$$

$$l_{iv}^{vol} + Q_j^{vol} - l_{jv}^{vol} \leq K_v^{vol}(1 - x_{ijv}), \qquad v \in V, j \in N_v^P, (i,j) \in E_v \tag{2.10}$$

$$l_{iv}^{vol} - Q_j^{vol} - l_{(j+n)v}^{vol} \leq K_v^{vol}(1 - x_{i(j+n)v}), \qquad v \in V, j \in N_v^P, (i, n+j) \in E_v \tag{2.11}$$

$$0 \leq l_{iv}^{vol} \leq K_v^{vol}, \qquad v \in V, i \in N_v^P \tag{2.12}$$

$$h_i + 1 - h_j \leq (H_f + 1)(1 - x_{ijv}), \qquad v \in V, f \in F, i \in N_f, j \in N_f, j \neq i \tag{2.13}$$

$$h_j \leq H_f, \qquad v \in V, f \in F, j \in N_f, \tag{2.14}$$

$$h_j \geq \sum_{\substack{i \in N_v \\ i \notin N_f}} (x_{ijv}) \qquad v \in V, j \in N_f \tag{2.15}$$

$$\sum_{p \in P_i} u_{ip} = 1, \qquad i \in N \tag{2.16}$$

$$\sum_{p \in P_i} u_{ip} \underline{T_{ip}} \leq t_i, \qquad i \in N \tag{2.17}$$

$$\sum_{p \in P_i} u_{ip} \overline{T_{ip}} \geq t_i, \qquad i \in N \tag{2.18}$$

$$t_i + T_{ijv} - t_j \leq (\overline{T_{i\pi_i}} + T_{ijv})(1 - x_{ijv}), \qquad v \in V, (i,j) \in E_v \tag{2.19}$$

$$t_i + T_{i(i+n)v} - t_{(i+n)} \leq 0, \qquad v \in V, i \in N_v^P \tag{2.20}$$

$$\sum_{(\alpha,\beta) \in A_v} d_{v\alpha\beta} = \sum_{(i,j) \in E_v} x_{ijv} D_{ij}, \qquad v \in V \tag{2.21}$$

$$\sum_{(\alpha,\beta) \in A_v} l_{v\alpha\beta} \geq l_{iv}^{kg} \qquad v \in V, i \in N_v \tag{2.22}$$

$$B_{(\alpha-1)} b_{v\alpha\beta} \leq d_{v\alpha\beta} \leq B_\alpha b_{v\alpha\beta}, \qquad v \in V, (\alpha,\beta) \in A_v \tag{2.23}$$

$$Z_{(\beta-1)} b_{v\alpha\beta} \leq l_{v\alpha\beta} \leq Z_\beta b_{v\alpha\beta}, \qquad v \in V, (\alpha,\beta) \in A_v \tag{2.24}$$

$$\sum_{(\alpha,\beta) \in A_v} b_{v\alpha\beta} \leq \sum_{j \in N_v} x_{o(v)jv}, \qquad v \in V \tag{2.25}$$

$$h_i, t_i \geq 0, \qquad i \in N \tag{2.26}$$

$$u_{ip} \in \{0,1\}, \qquad i \in N, p \in P_i \tag{2.27}$$

$$b_{v\alpha\beta} \in \{0,1\}, \qquad v \in V, (\alpha,\beta) \in A_v \tag{2.28}$$

$$d_{v\alpha\beta}, l_{v\alpha\beta} \geq 0 \qquad v \in V, (\alpha,\beta) \in A_v \tag{2.29}$$

$$y_i \in \{0,1\}, \qquad i \in N^P \tag{2.30}$$

$$x_{ijv} \in \{0,1\}, \qquad v \in V, (i,j) \in E_v \tag{2.31}$$

The objective function (**??**) sums up to the cost of all vehicles given corresponding costs from their cost matrix. Costs could be variable per distance, weight, fixed and/or related to stops made. Loads not transported will be penilized with costs and the aim is to minimize the sum of all these costs. Constraint 2.2 secures that a load is picked up once and only by one vehicle or not picked up at all. **??** makes certain that if a load at a node j is travelled to it is also left. This is not the case for

origin and final destination node. Then **??** makes sure that origin is only left once by each vehicle, and **??** ensures that desination is only arrived to once. Finally **??** is there to say if a load is picked up it must also be delivered.

Regarding weight, constraint **??** ensures that if a load at node j is picked up, the weight before plus the weight of the load at node j minus the weight leaving the node is equal to zero. If not the summation cannot exceed the capacity of the vehicle. Also **??** makes certain that at delivery of node j the weight before delivery minus the weight of the load at j minus the weight after, is equal to zero, ie. the weight of the vehicle after is lighter than before by exactly the weight of the load at j. The final weight constraint **??** says that weight leaving node i always has to be between 0 and the capacity of the vehicle.

The next constraints **??**-**??**, ensures the same as for the weight but for volume, that each load is increased by the volume of the order, that the volume is decreased once delivered and that the volume at any node is between 0 and the volume capacity of that vehicle.

It follows from constrain 2.13 that within a certain factory, if you travel between two nodes, the amount of stops you have made should always be iterated by one. If you are not travelling between them, the summation cannot be bigger than one plus the limitation corresponding to that factory. The next constraint on factories 2.14, ensures that any node visited in a factory cannot exceed the docking limit. Then 2.15 makes sure that when a vehicle enters a factory from outside, the docking amount gets an initial value of 1, and if one is not travelling from i to j the value has to be greater than or equal to 0.

Constraint 2.16 ensures only one timewindow is used per node, and 2.17 says that the time a node i is visited has to exceed a lower timewindow limit. 2.18 ensures the same upper bound timewindow is used. Then constraint 2.19 ensures that the travel from one node to the next is appropriately increased by the travel time between them and for any other two nodes the values cannot exceed a large constant. Finally 2.20 ensures that the time a vehicle visits the delivery of an order must always be after the pickup of that order.

From 2.21 we have that for each vehicle, the sum of the total distance variables has to be equal to the total travel distance of that vehicle. Then regarding weight 2.22 ensures that the sum of the max weight of a vehicle v is greater than or equal to the weight at all nodes visited by that vehicle. Constraint 2.23 ensures that for each vehicle v the total distance variable can only exist in the appropriate distance interval. The same is the case in 2.24 for maximum weight in the appropriate weight interval. Finally 2.25 says that if a vehicle is not leaving its origin node, there cannot be a cost interval binary for that vehicle, which in turn ensures that we dont calculate the fixed costs of said vehicle.

Constraints 2.26 to 2.31 are ensuring positive and binary variables where appropriate.

Chapter 3

Solution Method

Describe Local search algorithms and refer some papers. These algorithms can have difficulties moving from one solution space to another, even when this is cosidered in the metaheuristics.

A way to deal with this could be to, refer to some paper that tries to implement some form of heuristic to search larger neighbourhoods.

3.1 Local Neighbourhood Search

3.2 Simmulated Annealing

Chapter 4

An Adaptive Large Neighbourhood Search Aproach

This chapter explains how we have applied ALNS to the PDPMTW. The model in this paper differs to models from papers like Ropke and Pisinger (2006) in the following way:

1. We use only one removal and one insertion heuristic in each search iteration in what we call an operator. We describe each of these operators and their heuristics in 4.2.

4.1 Initial Solution

Many different algorithms have been made for finding an initial solution. In our paper we have chosen to simply start with an initial solution where no orders are assigned to any vehicle which is very efficient and will not lead us to getting This is a test Shaw (1997). Hopefully it works..

4.2 Heuristics

This section presents the operators or heuristics used by our ALNS algorithm. The first three operators 4.2.1, 4.2.2, 4.2.3, we call shuffeling operators. They try to quickly schuffle a given solution around to find new solutions in the same neighbourhood. The last four operators, 4.2.4, 4.2.5, 4.2.6, 4.2.7, are using removal and reinsertion heuristics. These are used partly to move from one neighbourhood to another, and partly for local search depending on the amount of solution elements being reinserted.

Removal and reinsertion heuristics are well reserached tools used in solving PDP. We have implemented and adapted the most succesful heuristics and created some new ones to help solve our specific problem.

4.2.1 Swap first fit

This operator simply swaps the pickup and delivery of two orders first time it fits. This operator is very efficient and jumps randomly around a neighbourhoods solution space.

4.2.2 3-exchange

The 3-exchange operator selects a random vehicle with at least two assigned orders, and performs an exchange of 3 assigned positions until a feasible new combination is found. This can be illustrated as follows:
A vehicles schedule before exchange:
[1 **1** 2 **2** **3** 3]
A 3-exchange performed on highlighted positions (two, four and five) on this schedule would result in the following:
[1 **3** 2 **1** **2** 3] TODO: make an illustration here with arrows
This heuristic is very fast as the exchanges are fast operations and the feasibility check of a vehicles schedule is a very effective operation. Like the swap heuristic from the previous section this operator jumps randomly around a neighbourhoods solution space.

4.2.3 2-opt

The 2-opt operator used in this paper is based on the 2-opt-L operator from ref: Carcasse2006. It selects a random vehicle with more than 2 orders (it breaks out after not finding any fitting vehicle after o tires, where o is the amount of orders in the problem). For the selected vehicle it divides up the route of the vehicle in 3 parts. All orders up until the index i of the vehicle route are inserted normally. Orders from the index j+1 until the end of the route are also inserted normally. then finally orders from the index i+1 until index j are inserted in inverse order. If the new route has a smaller cost than the original route the route the route is remembered as the new best route. When all reverses have been performed on the current route, the best route is selected as the new route. This operation is continued until no improvement can be made ie. the best possible schedule for the selected vehicle has been found.

4.2.4 Remove random and insert first fit

Removes randomly between 2 orders and 10 percent of the amount of orders and reinserts them randomly in their first possible position. This operator is used to jump from one neighbourhood to another and is trying to search for possible solutions regardless of the cost they produce.

4.2.5 Remove non-clustered and insert clustered

This heuristic tries to remove orders that are bundled together but belong to different clusters. It then tries to bundle orders together in the best possible way. The orders pickup and delivery locations are all assorted into clusters based on the distance between them. We use a hirarchial single linkage clustering algorithm to cluster locations together into k clusters. (TODO: make algorithm table for herarchial single linkage algorithm) Then we compare each set of k cluster to eachother using the siluette coefficient, and keep the best one. (TODO: make algorithm table for siluette coefficient)

To choose which order to remove we have ranked the orders based on how many clusters are shared within a vehicles schedule. If an orders pickup and delivery shares

no cluster with any other pickup or delivery nodes the rank is 0. If the order shares the same cluster with both pickup and delivery node as all other pickup and delivery nodes within a vehicle it is given a rank 1. This leads us to quite easily differenciate between a well clustered order and a not well clustered one. The orders are sorted based on their rank and we choose the order to pickup that has the lowest rank, using some randomization, based on the value p. We choose the order on the position k^p in the rank where k is a random number. We have chosen $p = 4$ in this paper.

To reinsert the orders we have chosen to try and maximize the cluster rank we introduced above. This means we find the vehicle where the rank is the highest and insert the order in the best possible position in the chosen vehicle.

4.2.6 Remove Worst and Insert Greedy

Removing orders in the most costful positions and reinserting it in its cheapest (greedy) position seems to be a reasonable way of moving towards a better solution. We therefore propose a heuristic that removes the orders with the highest cost C_i. The C_i is calclated as the increase in a vehicles schedule cost with the chosen order. We remove orders again based on the same randomness factor explained above. We do this by first ranking the orders based on cost and choose the order in the k^p position.

The reinsertion is using a greedy algorithm where it simply checks to find the best possible possition for the order and inserts it there.

4.2.7 Remove Similar and Regret Insert

The removal heuristic used in this operator is based on Shaw (1997) with slight modifications based on our PDPMTWF problem. It removes orders that share specific similar qualities, as the basic idea is that by replacing these orders by eachother we find new, hopefully better, solutions. Another good reason to remove similar orders is that it could be advantagous to reinsert these orders togther on the same vehicle since they share alot of the same properties in regards to distance, time etc. We define a relatedness factor r_{ij} which represents how much the order i is related to the order j. The lower the value of r_{ij} the more the two orders i and j are related. The relatedness of two orders we base here on the following properties: a distance property, a weight property, an overlapping timewindow property, a property indicating if the same vehicles can be used to serve each request, and finally if the orders belong to the same factory.

The relatedness factor is given by the following equation:

$$r_{ij} = \psi(D_{ij}+D_{(i+n)(j+n)})+\omega|Q_i-Q_j|+\phi(1-\frac{|V_i \cap V_j|}{max(|V_i|,|V_j|)})+\tau G_{ij}+\chi(U_{ij}+U_{(i+n)(j+n)}) \tag{4.1}$$

D_{ij}, Q_i, are defined in the problem formulation section and these values have been normalised to result in a value between [0..1]. V_i is the set of vehicles that can serve order i. The parameter G_{ij} is 1 if i belong to another factory than j and 0 if

they belong to the same factory. U_{ij} is the timewindows at the pickup and delivery location, equal to 0 if the two orders have identical time windows and 1 if not. It corresponds to the sum of overlapping time windows divided by the overlapping span of the two time window sets. It can be formulated as follows

$$U_{ij} = 1 - \frac{\sum_{\substack{p \in \pi_i \\ o \in \pi_j \\ \underline{T_{ip}} \leq \overline{T_{jo}} \\ \underline{T_{jo}} \leq \overline{T_{ip}}}} (\min(\overline{T_{ip}}, \overline{T_{jo}}) - \max(\underline{T_{ip}}, \underline{T_{jo}}))}{\max\left(\max_{p \in \pi_i} \overline{T_{ip}}, \max_{o \in \pi_j} \overline{T_{jo}}\right) - \min\left(\min_{p \in \pi_i} \underline{T_{ip}}, \min_{o \in \pi_j} \underline{T_{jo}}\right) - \sum_{\substack{p \in \pi_i \\ o \in \pi : q_j \\ \underline{T_{ip}} \geq \overline{T_{j(o-1)}} \\ \underline{T_{jo}} \geq \overline{T_{i(p-1)}}}} (\min(\underline{T_{ip}}, \underline{T_{jo}}) - \max(\overline{T_{i(p-1)}}, \overline{T_{j(o-} \tag{4.2}$$

Here $\overline{T_{ip}}$ and $\underline{T_{ip}}$ are the upper and lower time windows defined in the problem formulation. Thus the relatedness measure is given a value $0 \leq r_{ij} \leq 2\psi + \omega + \phi + \tau + \chi$. We have chosen the following values in this paper $\psi = 0.7$, $\omega = 1.0$, $\phi = 0.8$, $\tau = 0.3$, $\chi = 0.3$.

The insertion part of this operator tries to improve on the insert greedy by calculating a regret value, c_i^*, which represents how much is lost by inserting this order in its second best position.

4.2.8 Wild operator

4.3 Choosing an operator

4.4 Adaptive weight adjustment

4.5 Acceptance criteria

4.6 Radical jump away from neighbourhood

ALNS algorithms are known to be good at searching locally aswell as globally. However they do sometimes get stuck in one neighbourhood, and it is important that our algorithm is able to adapt in these situations. If we for 500 iterations find no improvement in the solution we are assuming that we are stuck. In this case we need to do something to get unstuck. We have designed our algorithm so that in the case when no improvement in found, we run 100 iterations apart from the normal iterations, where we accept any new solution found regardless of the objective value. In these iterations we use what we call a wild operator. The wild operator is basically a collection of the most unspecific targeting operators from 4.2. We use the operators from section 4.2.4, 4.2.2 and 4.2.1 as these operators are not trying to improve and select targeted solutions but rather returns the first feasible solution they find. We also increase the amount of elements the operators are working on to effectively move further then the normal search iterations.

Chapter 5

Computational Results

This chapter will present the results from the following experiments:

- Wild escape algorithm evaluation
- Heuristic evaluation and selection
- Final composition of SCALNS evaluation

Before going into the experimental results we will describe the experimental setup. This icludes the techical side aswell as the setup of the analytical experiments themselves. After that we will explain the generation ofthe instances used in the evaluation sections.

5.1 Experimental Setup

In this section we describe the technical setup of the experiments aswell as the Analytical setup of our eperiments.

5.1.1 Technical Setup

The computational experiments in this paper is run on two different computers. The more demanding experiments are run on a 64-bit Windows 10 computer with a 3.4 Ghz i7-6700 quad core processor and 16GB RAM. We will shorten the name of this computer to "Windows computer". The less demanding experiments are run on a 64-bit Ubuntu 18.04 computer with a 1.8 Ghz quad core i7-8550u processor and 16GB RAM. We call this computer for short "Ubuntu computer". As a general rule through the rest of this paper; if not specified, the results were run on the Ubuntu computer.

The instance generator described in the next section was implemented in Java (version number). The mathematical model from section 2 is setup in AMPL (version number), using the Gurobi solver. All AMPL experiments were run on the Windows computer. The ALNS heuristics are implemented in Java version 10.0.4+13. These experiments were run partly on the Ubuntu and partly on the Windows computer. The statistical experiments are performed in Matlab R2019a version 9.6.0.1174912 on the Ubuntu computer.

5.1.2 Analytics Setup

In section 3 we described seven operators, aswell as a wild operator, some invented in this paper and others based on known ALNS heuristics. For our testing we generated five instance sets of each five instances, totally 25 instance. While testing the algorithm from section 4.6 on page 19, we used one instance set. All tests were run 10 times and results are given as an average and best objective value over the 10 runs aswell as an average running time over the runs. To analyse the performance of each of the heuristics, 5 reasonably sized instances was solved 10 times using each of the $2^7 = 128$ combinations of operators. These $2^7 = 128$ runs were done on the Windows computer.

To determine which of the heuristics influence the result we have performed several statistical tests, including ANOVA (III) and multiple linear regression analysis, and pairwise t-tests. For all statistical tests in this paper we have used a 95% confidence interval. After the analysis of the heuristics, a final composition was chosen for further testing. We then compared the performance of the final composition with solutions found by the mathematical model in AMPL. We also present figures which show the performance of our selected heuristics in a hphazardly selected single run.

For these tests we compare the results toward the best known solution from AMPL and ..... TODO. We use a 95% confidence level for all statistical experiments in this paper.

5.2 Instances

The instance generator was created based on real data from an anonumous costomer of 4flow. Following, we will describe how we designed the generator and how we generated the instances used in the analytical part of the paper.

5.2.1 Generate Instances based on real data

The 4flow data gives information about the number of orders $|N|$, locations $|L|$, factories $|F|$. The amount of vehicles $|V|$ are kept large compared to the amount of orders to make sure there are enough but not too many vehicles. A solution that uses all but one vehicle is the ideal here and we found $|N|/2 \leq |V| \leq 2/3|N|$ to be fitting. $|N|, |V|, |L|, |F|$ are given as input to the generator. In addition the data from 4flow gave us information about the size of a vehicle and its compatabilities. Some vehicles might have cooling possibilities amd some speical equipment required for transport of special goods or equipment required at the pickup or delivery location etc.). Other information aquired by the data was travel distances, cost structure etc.

To keep the instances feasible but still as realistic as possbile it makes sense to limit the data to different possibilities. Our data was generated with the following properties:

- Orders are assigned to pickup and delivery locations randomly. Orders assigned to the same location are given the same stop L_s
- Each delivery location is assigned to a factory at random N_f.

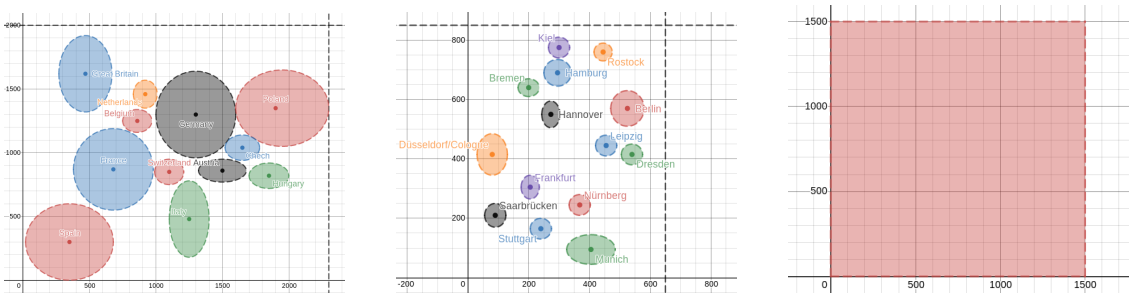
- Each location aswell as each order is assigned a special property with 5% probability. This will decide which vehicle can pickup which order, N_v^P and N_v^D
- We let the vehicles types be split up in 3 different vehicle types, small, medium, large, each with extected capacitied and capabilities.
  - Large vehicle: slower but compatible with all locations and orders, with $Q_v^{kg} = 24k$ and $Q_v^{vol} = 102$
  - Medium vehicle: medium fast and compatible with all locations but not orders with special compatability, with $Q_v^{kg} = 18k$ and $Q_v^{vol} = 71$
  - Small vehicle: fastest but not compatible with special locations and special orders, with $Q_v^{kg} = 12k$ and $Q_v^{vol} = 55$
- The distances d_{ij} were calculated using pytagoras on the randomly generated points described in the next section
- The travel time T_{ijv} were scaled with 60% of the travel distance, added with a random variation of $+/-10\%$ of the travel distance, multiplied by the speed of the vehicle (slow: $* = 105\%$, medium: $* = 102.5\%$).
- The cost matrices $C_{v\alpha\beta}^{km}$, $C_{v\alpha\beta}^{kg}$, $C_{v\alpha\beta}^{fix}$ were based on a real 4flow cost matrix, scaled to the size of the instance and to the size of the vehicle.
- The cost of no transport C_i was set to a minimum lower bound scaled based on the weight/volume/distance of the order.
- The stop costs C_{vi}^{stop} were calculated realtive to the size of the vehicle and the cost data from 4flow.
- Time windows $[\underline{T_{ip}}, \overline{T_{ip}}]$ were generated randomly based on typical factory opening hours. 1-2 timewindows per day, and 3-7 days per week saceld based on the instance size.

Random locations based on real georaphical data

Most of 4flow's customers are based either in Germany or in Europe. To make the instance generator as realistic as possible we have decided to split the instances into 3 geographical types; European, German and uniform geographically distributed locations. We made 2 maps based on real scale approximations of geographical data from National Geographics, in km. To simplify we have sticked to geographical points with an eliptic uniformly distributed area surrounding the point to represent a country or a city. figure 5.1 on the next page illustrates the areas of possible locations used in the generator. Larger elipses are more likely to be selected by the generator than smaller elipses.

For the selected elipse a point was selected within the elipse at random with a uniform distribution. For figure 5.1c on the facing page points were generated at random within the limits shown. From our 5 instance sets, two were generated using figure 5.1a on the next page, two with figure 5.1b on the facing page and one with the uniform distribution from figure 5.1c on the next page. If a point belong in the same factory as a previously generated point, that point was generated within a reasonable radius of three kilometer.

Figure 5.1: Area of random point generation



(a) Europe **(b)** Germany **(c)** Uniform

Shaded areas indicate a possible location generation. Larger areas are more likely to get picked

5.2.2 Generated Instances

For testing our algorithm in this paper, 5 instance sets of each 5 instances of varying sizes were generated. We have numbered the sets as follows:

- Set 1 and set 2 are generated based on the European map from figure 5.1a
- Set 3 and set 4 are generated on the german map from figure 5.1b
- Set 5 is generated on the uniform distribution from figure 5.1c

Each instance set contain representative instance sizes of our problem based on data from 4flow, $4, 12, 35, 80, 150$ orders using respectively $3, 7, 20, 4580$ vehicles and containing $7, 9, 22, 45, 85$ locations. The sizes of the instances and the instance set number are shown for each result presented in the following sections.

5.3 Initial Results

In this section we will present the data from our experiments. This data lead us to the final composition of our model. We have done this in 3 parts:

- Evaluation of wild escape algorithm
- Initial evaluation of heuristics
- Further evaluation of heuristics

In the last part of this section we will present the final composition of our model. The results from the final composition are presented in the next section.

5.3.1 Evaluation of the wild algorithm

To help our algorithm escape from a neighbourhood position we designed a wild algorithm described in section section 4.6 on page 19. To evaluate the implementaion of this operator we have decided to evaluate the result of running the complete ALNS with all heuristics included with three different modifications. First we run the algorithm without any escape algorithm. Secondly we ran the algorithm with

Table 5.1: ALNS with all heuristics using 3 different reset algorithms

			Initial	Average Objective			Best Objective			Running time (sec)		
Ord	Veh	Loc	Objective	No esc	Rnd rst	Wild esc	No esc	Rnd rst	Wild esc	No Esc	Rnd rst	Wild Esc
4	3	7	609 680.3	3 444.7	3 444.7	3 444.7	3 444.7	3 444.7	3 444.7	0.18	0.20	0.20
12	7	9	1 023 745.5	149 692.6	154 832.9	149 692.4	149 692.4	149 692.4	149 692.4	0.39	0.51	0.46
35	20	22	2 682 067.9	10 639.1	10 849.5	10 350.9	10 404.9	10 358.6	10 025.1	2.31	1.61	2.49
80	45	45	6 422 128.6	22 262.2	25 802.9	21 377.4	20 761.2	21 777.4	20 831.3	15.89	8.05	14.97
150	80	85	12 059 380.3	40 667.2	38 313.0	35 705.7	34 316.0	34 345.0	34 282.3	88.21	48.92	77.78

The columns contains in order: the first three columns show the instance size in number of orders, vehicles and locations. Then follows three columns that contain the average improvement from runs with the different escape adjustments: no escape, random reset and our escape alorithm. The next three columns contain the best improvement for the same three escape adjustments, and the final three columns contain the average running time in seconds for the three escape adjustments.

a modification that resets the algorithm each time we get stuck and start from a new random solution. And lastly we ran our algorithm with our escape algorithm. Comparing these three options towards eachother will help us evaluate if our wild algorithm is helping us in general and see if our wild algorithm is different than doing a reset and just starting from a new solution somewhere.

Results of running ALNS

We ran our algorithm 10x3 times, ten for each escape adjustment, on instance set 1 from section 5.2 on page 21. We modified our escape algorithm on this run to ignore if a best solution is found while moving from one neighbourhood to the next. This way the extra iterations explained in section 4.6 on page 19, does not give any unfair advantage to our wild escape algorithm. We will simply be moving from one neighbourhood to the next without checking if we find a better solution on the way.

The result from running our algorithm with these three adjustments are summarized in table 5.1. It shows the objective values found on average during each of the 10 runs, the best solution found overall and the average running time, for each excape adjustment.

Observations of results from wild algorithm evaluation

The table 5.1 shows that on average using our wild algorithm clearly outperforms both the restart algorithm and not including a reset algorithm. We see that the average result for all instance sizes are lowest using our wild algorithm. In addition our wild algorithm outperforms the others for all instances in finding the best solution, except for the instance with 80 orders, where not using an escape algorithm ended up finding a better best solution. This is however probably due to the algorithm ending in a lucky neighbourhood and since the wild operator is vastly improving the solution on average we still think it is better to run our algorithm with the wild escape algorithm adjustment.

With regards to the running time we observe the following pattern for the largest two instances: the random restart is the fastest, followed by our wild escape algorithm, and lastly not using any escape algorithm. It is ecspected that the random reset here outperforms the others as we have generated the random solutions before starting our algorithm. The random reset also resets the weights of all of the heuristics and that could indicate that fast heuristics get more running time here than

they usually would get. Since our wild escape algorithm is clearly outperforming the others we have accepted the slight increase in runningtime.

5.3.2 Initial evaluation of heuristics

Different heuristics have different strengths and weaknesses. Some heuristics are not performing well on their own, but work very well in combination with other heuristics. Other heuristics are stron on their own, and their performance is prohibited by other heuristics, or not getting enough running time when too many heuristics are included.

To find the right combination of heuristics we have done an evaluation of their performance to search for the best possible combination of operators. To do this selected the instance set with a representative size of ... and ran our algorithm, without the wild algorithm adaptation, with each combination of operators. This results in $2^7 = 128$ each running 10 times. To be able to compare the results from differnt instances we calculated the improvements from the initial solution which resulted in the improvement from the best solution found during the 10 runs, and an average improvement from the initial solution from the 10 runs of each combination. Each observation was also multiplied by 1000 to make the analysis more readable.

We evaluated the data resulting from the different combination in three steps:

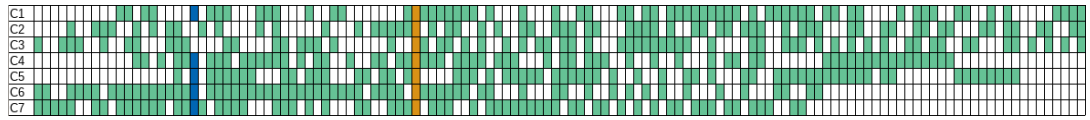
- In the first part we ranked the combinations based on the best improvement and the average improvement
- In the second part we ran ANOVA and regression analysis to see which heuristics have a significant impact on the result
- In the third part we run t-tests to see if certain heuristics in combination with others have a positive or a negative impact on the final result

Ranking the combinations

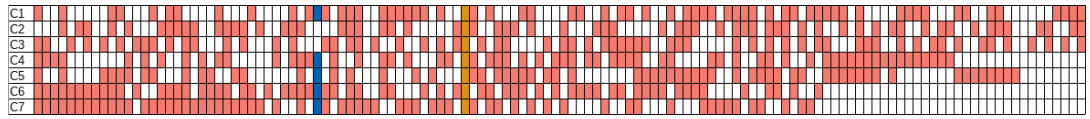
We shorten the names of each heuristic and will refer to them for here on as components. The components are assigned the following names:

- C1: this is the swap heuristic described in section 4.2.1 on page 16
- C2: exchange heuristic from section 4.2.2 on page 17
- C3: is the 2-opt heuristic in section 4.2.3 on page 17
- C4: is the random removal first fit insertion heuristic from section 4.2.4 on page 17
- C5: clustering heuristic described in section 4.2.5 on page 17
- C6: this is the worst removal and greedy insert heuristic in section 4.2.6 on page 18
- C7: is the shaw removal and regret-3 insertion heuristic described in section 4.2.7 on page 18

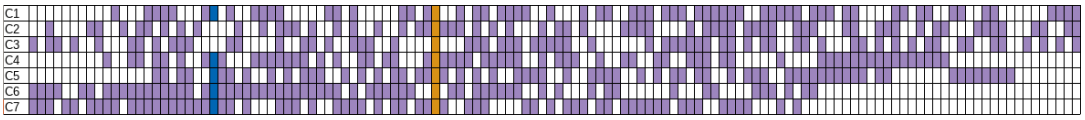
Figure 5.2: Ranking of improvements from initial solution



(a) Average improvement



(b) Best improvement



(c) Best+Average improvement

Highlighted tiles indicate use of a component. The blue highlighted tiles show the final composition. Yello highlighted tiles show the use of all operators.

Figure 5.2 shows the ranking of each combination of components from left to right. A colored tile indicates that the current component is in use. The further a combination is to the left, the higher improvement was from the initial solution compared to the other combinations.

The figure 5.2a shows the combination of components ranked based on the average improvment over the 10 runs for that combination. Then figure 5.2b shows the combination of components ranked based on the best improvement overall during the 10 runs. Finally figure 5.2c shows the ranking of the average improvement + the best improvement to see which combinations performs best overall.

ANOVA and regression analysis

The ranking gives us an overview over which components are working and is always part of a good combination. It also gives us information on which components are not performing well overall and which combination of components are not working well. However, ANOVA and regression analysis can help us evaluate which components have a significant positive impact on the result. Table 5.2 on the facing page shows the results of ANOVA (III) analysis performed using each component as a source of variance. We extended the model with the instances as random effects to give us a better explanation of the result and less noise in the model. We did one ANOVA analysis for the average improvement over 10 runs shown in figure 5.2a, and one for the best improvement found in figure 5.2b.

In addition to this we performed a multiple linear regression in table 5.3 on the next page on the same data as the ANOVA. The regressional analysis gives us insight into wether a component is positively or negatively influencing the result aswell as how well the components explain the result through the R^2. Also here we used the Instances as a random effect, and the final instance is when all Instance terms are 0.

Table 5.2: Analysis of variance

Source	Sum sq.	df	Mean sq.	F	P>F
C1	0.052	1	0.052	0.09	0.7682
C2	0.748	1	0.748	1.26	0.2628
C3	0.038	1	0.038	0.06	0.8017
C4	22.432	1	22.432	37.66	0
C5	12.734	1	12.734	21.38	0
C6	117.945	1	117.945	198	0
C7	112.406	1	112.406	188.7	0
Instance	371.704	4	92.926	156	0
Error	350.257	588	0.596		
Total	975.207	599			

(a) Average improvement statistics

Source	Sum sq.	df	Mean sq.	F	P>F
C1	0.075	1	0.0745	0.23	0.6306
C2	0.195	1	0.1949	0.61	0.4369
C3	0	1	0.0001	0	0.988
C4	8.372	1	8.3718	26	0
C5	3.679	1	3.6789	11.43	0.0008
C6	66.236	1	66.2357	205.71	0
C7	67.081	1	67.0813	208.33	0
Instance	344.971	4	86.2428	267.84	0
Error	189.331	588	0.322		
Total	671.378	599			

(b) Best improvement statistics

The columns contains in order: the source of the variability and for each source the sum of the sqares, the degrees of freedom, the mean squares, the F-statistic and the p-value.

Table 5.3: Results of multiple linear regression model

Term	Estimate	SE	tStat	pValue
Intercept	994.58	0.11713	8491.5	0
C1	-0.018582	0.063017	-0.29487	0.7682
C2	-0.07063	0.063017	1.1208	0.26283
C3	-0.01583	0.063017	-0.2512	0.80175
C4	0.39108	0.063729	6.1366	1.5502e-09
C5	-0.29466	0.063729	-4.6236	4.6407e-06
C6	0.89676	0.063729	14.071	5.7098e-39
C7	0.87544	0.063729	13.737	1.923e-37
Inst1	0.50928	0.099639	5.1113	4.3337e-07
Inst2	-0.051052	0.099639	-0.51237	0.60859
Inst3	1.6601	0.099639	16.662	2.4375e-51
Inst4	1.755	0.099639	17.614	4.4128e-56

(a) Average improvement statistics, $R^2 = 0.641$

Term	Estimate	SE	tStat	pValue
Intercept	994.97	0.086114	11554	0
C1	0.022291	0.046332	0.48112	0.63061
C2	-0.036047	0.046332	-0.77801	0.43687
C3	-0.00069693	0.046332	-0.015042	0.988
C4	0.23891	0.046855	5.099	4.6112e-07
C5	-0.15838	0.046855	-3.3801	0.00077249
C6	0.67202	0.046855	14.342	3.2e-40
C7	0.67629	0.046855	14.434	1.2062e-40
Inst1	0.64176	0.073257	8.7605	2.0732e-17
Inst2	0.16783	0.073257	2.291	0.022316
Inst3	1.837	0.073257	25.077	6.2887e-95
Inst4	1.6686	0.073257	22.778	8.2582e-83

(b) Best improvement statistics, $R^2 = 0.718$

The columns contains in order: the term and for each term the coefficient estimate, the standard error of the coefficients, t-statistics to test if the term is significant, and the p-value

Observations from initial evaluation of heuristics

The results from table 5.2 and table 5.3 on the preceding page split our components into two groups. The first obvious group are the significant heuristics. The ANOVA results from table 5.2a and table 5.2b on the previous page tells us that four components, C4-C7, have a significant impact on the result. However from 5.3a and

Regarding the remaining heursitics, C1, C2, and C3, we can safely conclude that they are not contributing singificantly on the result on their own. However we cannot conclude that they do not have a positive effect in combination with the significant heuristics. We therefore observe that we need further testing to know if these heuristics have a positive or negative influence on the result. We refer to the heuristics C1,C2,C3 and C5 as the undecided group, or G in our further evaluation of the heuristics.

5.3.3 Further evaluation of heuristics

To further analyse the performance of the heuristics we want to analyse how the heuristics are performing as a group (G) to see if they have an effect on the results. The result from section 5.3.2 on page 25 tells us that it is out of the question to use any combination of components where only undecided components are used. These components will alone have a negative impact on the result but it is still possible that using them combined with the significant components could have a positive impact on the result We have therefore removed the observations where the undecided group appear alone in the following testing. We want to test if using one or several of the undecided heuristics in combination with other heuristics have a positive impact on the result.

We have done this in two parts. We first wanted to see if the components as a group has a positive or negative influence on the result. To test this we have done 2-sample t-tests to compare the mean of the population where we combine the undecided heuristics with some significant heuristic, to the mean of the pooulation when we are not using an undecided heuristic. Then we wanted to see if using the undecided group with specific combinations of the significant components have significant impact on the model. We did this using further ANOVA (III) statistical analysis and multi linear regression model.

Evaluation of undecided components as a group

The first thing we did was to run a T-test on that compares the mean of the population which include some combination of the undecided group heuristics and the significant heuristics, towards the population that does not contain any heuristic from the undecided group. To represent the population without any of the undecided heuristics we have the used the parameter P_N and to represent the population with some combination of the undecided heuristics and the significant heuristics we have used the parameter P_H. The results from the T-test are summarized in

We continued the testing of the undecided group by performing ANOVA (III) analysis on different combinations of the undecided group and the significant heuristics. We did this to see if a combination of the undecided group and the significant

Table 5.4: Results of T-tests on the undecided group mean vs no undecided heuristics

Populations	Impr type	Tail	H-stat	p-value	tStat	conf-int
$P_H - P_N$	Avrg	both	0	0.5187	-0.6458	-0.3785 – 0.1912
$P_H - P_N$	Avrg	right	0	0.7407	-0.6458	-0.3326 – inf
$P_H - P_N$	Avrg	left	0	0.2593	-0.6458	-inf – 0.1453
$P_H - P_N$	Best	both	0	0.6903	0.3986	-0.2174 – 0.3281
$P_H - P_N$	Best	right	0	0.3452	0.3986	-0.1734 – inf
$P_H - P_N$	Best	left	0	0.6548	0.3986	-inf – 0.2841

The columns contain in order: the populations tested agains eachother, type of data tested(average or best improvement), tail which determines the alternative hypothesis, h-value (1 rejects the null hypothesis, 0 failure to reject), p-value of the test, t-Statistic of the test, confidence interval for the true population mean

Table 5.5: Analysis of variance with the undecided group in combination with the significant heuristics

Source	Sum sq.	df	Mean sq.	F	P>F
G+C4	19.024	1	19.0236	538.34	0
G+C6	0	1	0	0	0.9859
G+C7	0.005	1	0.0045	0.13	0.7204
G+C4+C6	0.059	1	0.0589	1.67	0.1973
G+C4+C7	0.001	1	0.0009	0.03	0.8722
G+C6+C7	0.22	1	0.2202	6.23	0.0128
G+C4+C6+C7	0.166	1	0.1657	4.69	0.0308
Inst	309.022	4	77.2555	2186.21	0
Error	19.365	548	0.0353		
Total	385.274	559			

(a) Average improvement statistics

Source	Sum sq.	df	Mean sq.	F	P>F
G+C4	12.879	1	12.8793	609.42	0
G+C6	0.131	1	0.1307	6.19	0.0132
G+C7	0.239	1	0.2389	11.31	0.0008
G+C4+C6	0.223	1	0.2227	10.54	0.0012
G+C4+C7	0.153	1	0.1525	7.22	0.0074
G+C6+C7	0.42	1	0.4195	19.85	0
G+C4+C6+C7	0.395	1	0.395	18.69	0
Inst	295.738	4	73.9346	3498.45	0
Error	11.581	548	0.0211		
Total	352.547	559			

(b) Best improvement statistics

The columns contains in order: the source of the variability and for each source the sum of the sqares, the degrees of freedom, the mean squares, the F-statistic and the p-value.

heuristics could help explain the variations in the result and to see if a combination of some or all of the significant heuristics work better than others.

The results are summarized in table 5.5 and table 5.6 on the next page. As an example a source of G+C4, contains all observations where at least one of the undecided components from G are combined exclusively with C4.

Observations from results of the heuristics further evaluation

The results from table 5.4 tells us that we cannot reject the H_0 null hypothesis that the mean of the two populations are different using a 95% confidence interval, for neither average or best improvement. This tells us that the components from the undecided group either have no significant impact on the result, or that the effect from some are nulling out the others. Further testing is needed to make any further conclusions.

Table 5.5 and table 5.6 on the next page gives us further insight into our model. First of all the result from table 5.5a tells us that only two combinations of the undecided group and the significant variables have a significant impact on the result using a 95% confidence interval. Using some heuristics from the undecided group combined with component C6 and C7, aswell as C4 has a significant impact on the average improvement result. From table 5.6a on the next page we also see that the combinations are also getting a positive coefficient. We also observe that the

Table 5.6: Results of multiple linear regression model with the undecided group in combination with the significant heuristics

Term	Estimate	SE	tStat	pValue
Intercept	995.85	0.035525	28032	0
G+C4	-0.89285	0.038481	-23.202	1.8003e-83
G+C6	0.00068095	0.038481	0.017696	0.98589
G+C7	0.013782	0.038481	0.35815	0.72037
G+C4+C6	0.049672	0.038481	1.2908	0.19731
G+C4+C7	-0.0061945	0.038481	-0.16097	0.87217
G+C6+C7	0.096062	0.038481	2.4963	0.012841
G+C4+C6+C7	0.083317	0.038481	2.1651	0.030808
Inst1	0.53458	0.02512	21.281	1.0604e-73
Inst2	0.031671	0.02512	1.2608	0.20793
Inst3	1.71	0.02512	68.073	1.6221e-269
Inst4	1.5959	0.02512	63.531	6.3233e-255

(a) Average improvement statistics, $R^2 = 0.95$

Term	Estimate	SE	tStat	pValue
Intercept	995.88	0.027473	36249	0
G+C4	-0.73464	0.029759	-24.686	5.012e-91
G+C6	0.07402	0.029759	2.4873	0.013167
G+C7	0.10006	0.029759	3.3625	0.00082635
G+C4+C6	0.096599	0.029759	3.246	0.0012417
G+C4+C7	0.079948	0.029759	2.6865	0.0074396
G+C6+C7	0.13259	0.029759	4.4554	1.0157e-05
G+C4+C6+C7	0.12865	0.029759	4.3232	1.8269e-05
Inst1	0.70773	0.019426	36.432	1.6371e-148
Inst2	0.27728	0.019426	14.273	1.5823e-39
Inst3	1.873	0.019426	96.415	0
Inst4	1.5781	0.019426	81.237	8.6749e-308

(b) Best improvement statistics, $R^2 = 0.967$

The columns contains in order: the term and for each term the coefficient estimate, the standard error of the coefficients, t-statistics to test if the term is significant, and the p-value

$R^2 = 0.95$ is very high so this model explains the results very well and this supports the use of these heuristic combination in regards to the average improvement.

The same combinations are significant and positive in reagards to the best improvement tabels table 5.5b on the preceding page and table 5.6b. And even though more combinations are significant for best improvement the combinations with highest positive estimated coefficients are still the combinations with C6 and C7, and C6 C7 and C4. We also observe here the increased $R^2 = 0.967$ which supports our conclusion that this should consistently lead to good results on best and average improvement.

The results from the further testing tells us that there are combinations of the undecided group and the significant components that have a positive significant impact on the result of both average and best improvement. It supports our results from section 5.3.2 on page 25 of significant components C6, C7 and C4 and leads us to conclude that there might be a positive influence from the undecided group components, however further testing is nessecary to determine which components should be included.

5.3.4 Evaluation of individual components

Until now, the heuristic components C4, C6 and C7 have been proven significant. The components C1, C2, C3 and C5 have been proven significant in combination with the significant components but not alone. We continue refering to them as the undecided group components or G.

We want to figure out which of the components in the undecided group, if any, have a positive, or negative, influence on the result. To do this we performed pairwise t-tests to check if an undecided component is significantly improving or decreasing the best and average improvement. Like in section 5.3.3 on page 28 it is out of the question to use any combination of heuristics where only undecided heuristics are used, so we remove these observations from the data also in these tests. In our tests we test if the mean of the populations where we combine the undecided heuristics with some significant heuristic is significantly different than the pooulation when we are not using an undecided heuristic.

Table 5.7: Results of T-tests on individual components

Populations	Tail	H-stat	p-value	tStat	conf-int
$P_{HA}^{C_1} - P_{NA}^{C_1}$	both	0	0.5427	-0.6090	-0.1580 – 0.0727
$P_{HA}^{C_1} - P_{NA}^{C_1}$	right	0	0.7286	-0.6090	-0.1325 – inf
$P_{HA}^{C_1} - P_{NA}^{C_1}$	left	0	0.2714	-0.6090	-inf – 0.0472
$P_{HB}^{C_1} - P_{NB}^{C_1}$	both	0	0.9240	-0.0955	-0.1428 – 0.1296
$P_{HB}^{C_1} - P_{NB}^{C_1}$	right	0	0.5380	-0.0955	-0.1208 – inf
$P_{HB}^{C_1} - P_{NB}^{C_1}$	left	0	0.1076	-0.0955	-inf – 0.1076
$P_{HA}^{C_2} - P_{NA}^{C_2}$	both	1	3.4853e-10	-6.5098	-0.0692 – 0.0412
$P_{HA}^{C_2} - P_{NA}^{C_2}$	right	0	1.0000	-6.5098	-0.0661 – inf
$P_{HA}^{C_2} - P_{NA}^{C_2}$	left	1	1.7426e-10	-6.5098	-inf – 0.0443
$P_{HB}^{C_2} - P_{NB}^{C_2}$	both	1	4.5425e-10	-3.5486	-0.0370 – 0.0106
$P_{HB}^{C_2} - P_{NB}^{C_2}$	right	0	0.9998	-3.5486	-0.0349 – inf
$P_{HB}^{C_2} - P_{NB}^{C_2}$	left	1	2.2712e-04	-3.5486	-inf – 0.0127
$P_{HA}^{C_3} - P_{NA}^{C_3}$	both	1	0.0244	-2.2635	-0.0276 – -0.0043
$P_{HA}^{C_3} - P_{NA}^{C_3}$	right	0	0.9878	-2.2635	-0.0250 – inf
$P_{HA}^{C_3} - P_{NA}^{C_3}$	left	1	0.0122	-2.2635	-inf – -0.0069
$P_{HB}^{C_3} - P_{NB}^{C_3}$	both	0	0.6271	-0.4864	-0.0134 – 0.0081
$P_{HB}^{C_3} - P_{NB}^{C_3}$	right	0	0.6865	-0.4864	-0.0116 – inf
$P_{HB}^{C_3} - P_{NB}^{C_3}$	left	0	0.3135	-0.4864	-inf – 0.0063
$P_{HA}^{C_5} - P_{NA}^{C_5}$	both	0	0.4702	-0.7231	-0.0301 – 0.0118
$P_{HA}^{C_5} - P_{NA}^{C_5}$	right	0	0.7649	-0.7231	-0.0255 – inf
$P_{HA}^{C_5} - P_{NA}^{C_5}$	left	0	0.2351	-0.7231	-inf – 0.0071
$P_{HB}^{C_5} - P_{NB}^{C_5}$	both	1	1.7956e-04	-3.7972	-0.0167 – 0.0528
$P_{HB}^{C_5} - P_{NB}^{C_5}$	right	1	8.9779e-05	-3.7972	-0.0197 – inf
$P_{HB}^{C_5} - P_{NB}^{C_5}$	left	0	0.9999	-3.7972	-inf – 0.0499

The columns contain in order: the populations tested agains eachother, type of data tested(average or best improvement), tail which determines the alternative hypothesis, h-value (1 rejects the null hypothesis, 0 failure to reject), p-value of the test, t-Statistic of the test, confidence interval for the true population mean

T-tests of individual components

Similar to the previous section we use the parameter $P_{NA}^{C_i}$ to refer to the population without a specific component C_i for the average improvement, indicated by A, and the parameter $P_{HB}^{C_i}$ as the population including the component C_i for the best improvement, indicated by B. Here C_i represents one of the component heuristics described in the previous section. We did the test for all 4 of the undecided heuristics from the previous section C_1, C_2, C_3 and C_5. The results from the pairwise t-tests are summarized in 5.7.

Observations from the results of the individual component evaluations

The first thing we see from the t-tests is that we were right in assuming that the effect of the components were cancelling eachother out. We go through each of the components results from table 5.7 here:

- C1: The population mean of both best and average improvement is not signif-

icantly different using a 95% confidence interval. It follows then that the tails are also not significantly different.

- C2: We reject the null hypothesis that the means are equal using a 95% significanse interval for the average and best improvement regarding this component. The left tail alternative hypothesis' that the means of $P_{HA}^{C_2}$ and P_{HB}^{C2} is lower than $P_{NA}^{C_2}$ and $P_{NB}^{C_2}$ are accepted, while the right tail alternative hypothesis is rejected for both best and average improvement.

- C3: The null hypothesis that the population mean of $P_{HA}^{C_3}$ is equal to the population mean of $P_{NA}^{C_3}$ is rejected and accepted for $P_{HB}^{C_3}$ and $P_{NB}^{C_3}$. The left tail alternative hypothesis that the means of $P_{HA}^{C_3}$ is with 95% confidence lower than $P_{NA}^{C_3}$, is accepted.

- C5: The null hypothesis is rejected for $P_{HB}^{C_5}$ and $P_{NB}^{C_5}$ and accepted for $P_{HA}^{C_5}$ and $P_{NA}^{C_5}$. The alternative hypothesis of the right tail of the best improvement, that the mean is significantly higher in $P_{HB}^{C_5}$ than $P_{NB}^{C_5}$ is accepted.

The results summarized above tells us that using C1 will have no effect on the outcome of the result. Also C2 and C3 are significantly decreasing the average for both components and also for best for C2. This indicates that it is not beneficial to include these operators in a model. Finally C5 is not affecting the average improvement however it is positively effecting the best improvement, indicating that it could be beneficial to include this component.

5.3.5 Deciding on the final model composition

The results from section 4.6 on page 19 tells us that our final model should include our wild algorithm. This will influence our running time a little but give us much more reliable results.

As for selecting heuristics or components to include, the results from section 5.3.2 and section 5.3.3 on page 28, indicates that we need to include C4, C6 and C7 in our final model. We observed from our testing of the wild algorithm that the running time of C1 is significantly lower than our other operators figure **??** on page ??. Even though section 5.3.4 on page 30 showed us that C1 had no effect on the model, the low running time is leading us to rather include this operator than not to get us a little bit better running times. It will as section 5.3.4 on page 30 also showed not negatively effect our results. C5 is also not effecting the average improvement of our model, but it also has a good running time, and it significantly effects the result of the best improvement found. This tells us that we should include this operator in our final composition.

The results from section 5.3.3 on page 28 indicated that a combination of C4, C6 and C7 + some of combination of the undecided group components is significantly effecting both the average and best improvement. Therefore our final model composition will be the ALNS algorithm with the wild algorithm and heuristic components C1, C4, C5, C6 and C7.

Table 5.8: Model performance in european instance set 1

				Mathematical model			SCALNS			
Inst Set	Ord	Veh	Loc	Solution objective	Optimaily gap	Run time(sec)	Average objective	Best objective	Avrg run Time	delta
Set 1	4	3	7	3 444.7	0.00%	0.1	3 444.7	3 444.7	0.1	0.00%
	12	7	9	149 692.3	0.00%	9963.7	149 692.4	149 692.3	0.5	0.00%
	35	20	22	2 091 776.5	99.91%	10000.2	10 323.2	9 997.9	2.8	99.52%
	80	45	45	6 422 128.6	99.99%	10000.0	21 170.5	20 911.5	21.1	99.67%
	150	80	85	NA	NA	NA	34 479.1	32 798.0	100.8	NA
Set 2	4	3	7	2 501.0	0.00%	0.1	2 501.0	2 501.0	0.1	0.00%
	12	7	9	5 987.8	0.00%	1041.8	5 987.8	5 987.8	0.6	0.00%
	35	20	22	1 985 165.5	99.90%	10000.3	14 382.4	14 272.1	2.6	99.28%
	80	45	45	6 809 899.5	100.0%	10000.0	25 736.6	24 760.8	16.8	99.64%
	150	80	85	NA	NA	NA	36 927.2	36 932.1	112.1	NA
Set 3	4	3	7	1 404.0	0.00%	0.3	1 404.0	1 404.0	0.1	0.00%
	12	7	9	5 862.5	33.16%	10000.0	5 862.5	5 862.5	0.9	0.00%
	35	20	22	679 594.8	99.71%	10000.4	6 334.7	6 267.7	3.7	99.08%
	80	45	45	5 748 613.6	99.99%	10000.5	12 609.0	12 347.6	32.2	99.79%
	150	80	85	NA	NA	NA	19 771.9	19 149.8	126.1	NA
Set 4	4	3	7	1 696.1	0.00%	0.7	1 696.1	1 696.1	0.1	0.00%
	12	7	9	437 572.4	0.13%	10000.2	434 722.4	434 722.4	0.6	0.01%
	35	20	22	547 881.6	99.72%	10000.2	4 652.8	4 494.5	5.1	99.18%
	80	45	45	6 201 301.5	99.99%	10000.1	15 540.0	15 290.6	21.4	99.75%
	150	80	85	NA	NA	NA	22 508.6	22 252.6	103.4	NA
Set 5	4	3	7	5 154.9	0.00%	0.4	5 154.9	5 154.9	0.1	0.00%
	12	7	9	3 716.2	22.93%	10000.2	3 716.2	3 716.2	0.6	0.00%
	35	20	22	1 757 079.6	99.90%	10000.2	13 138.9	12 944.2	2.1	99.26%
	80	45	45	5 909 616.9	100.00%	10000.0	24 034.0	23 855.5	9.1	99.60%
	150	80	85	NA	NA	NA	41 343.4	39 847.0	82.6	NA

The columns contains in order:Instance set, number of orders, number of vehicles and number of locations in the instances. The next three columns contain the result from running our mathematical model (MM) run in AMPL; solution objective, optimality gap and the running time. Then follows 3 columns with the results of our model, average objective found, best objective found and average running time. The final column contains the following function: (solution objective MM - SCALNS best objective)/solution objective MM.

5.4 Final results

After deciding on a model best suited to our problem we did a final run of our algorithm using composition described in the previous section. The algorithm was run using 5 instance sets of each 5 representative sizes.

5.4.1 Evaluation of final model

For each instance we also used the Windows computer to let AMPL try for 10 000 seconds to find an optimal solution for each instance. The results from the runs are summarized in table 5.8.

We also haphazardly selected some runs from different instances to analyse how the operators are performing. The results are shown in figure **??** on page ??. TODO: finish figures with operator performances

5.4.2 Observations of results the final composition

Table **??** on page ?? shows that our model is TODO: finish observations part.

However, from the previous sections result we also cannot conclude that they are not significant in combination with other heuristics or if they are leading to a worse solution than not including them, see **??**.

Chapter 6

Conclusions

This is a test Shaw (1997). Hopefully it works..

Appendix A

Appendix

This is a test Shaw (1997). Hopefully it works..

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