Modelling and Optimization

INF170

#4:General LP

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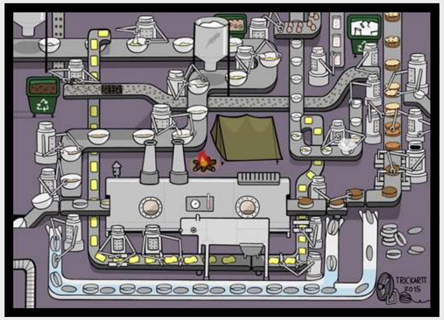
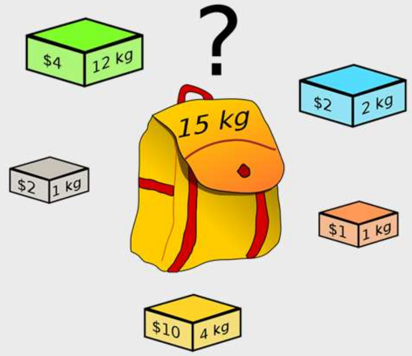
Dept. of Informatics
University of Bergen

Fall Semester
2018

Optimization Model





$$
\begin{aligned}
&\min \quad f(x) \\
&st \quad g(x) \le b \\
&x \in X
\end{aligned}
$$

Model Structure

1) Decision variables
2) Objective function
3) Constraints
4) Parameters
5) Sets

$$\min \; f(x)$$
$$st \quad g(x) \leq b$$
$$x \in \mathrm{X}$$

Model Structure

1) Decision variables (or just variables)

The unknown numerical value which is pursued to find through the resolution of the model.

Examples:

x = Amount of crude oil to be processed this week

y = Capacity of a new production plant

z = "Yes" or "No" decisions

The decision maker has control on the value that the decision variables will take.

2) Objective function

Measure which allows to evaluate the performance of the different values that the variables can take.

Examples:

Minimize cost function C

Maximize a profit function P

Minimize carbon emissions E

Maximize fairness F

3) Constraints

Conditions that restrict the values of the decision variables. These conditions define relations (≤ , = , ≥) the variables must satisfy.

Examples:

Total production ≤ Capacity

Number of carbon emissions ≤ Limit allowed

Output = Input * Conversion factor

4) Parameters

The parameters are fixed values given beforehand in the problem (the decision maker has no control on them when solving the optimization problem).

Examples:

Capacity

Budget

Demand…

5) Sets

The sets define the domain of variables and parameters.

Examples:

Set of customers

Set of production plants

$\mathbb{R}$

$\{0,1\}$

Diet Model

Variables:

x_{BEEF} : the number of packages of beef to be purchased

…

Objective function: Minimize $3.19\,x_{BEEF} + 2.59\,x_{CHK} + 2.29\,x_{FISH} + 2.89\,x_{HAM} + 1.89\,x_{MCH} + 1.99\,x_{MTL} + 1.99\,x_{SPG} + 2.49\,x_{TUR}$

Subject to (Constraints):

total percentage of vitamin "A" daily requirement met

$$60\,x_{BEEF} + 8\,x_{CHK} + 8\,x_{FISH} + 40\,x_{HAM} + 15\,x_{MCH} + 70\,x_{MTL} + 25\,x_{SPG} + 60\,x_{TUR} \geq 700$$

…

Diet Model

Minimize $3.19\, x_{beef} + 2.59\, x_{chk} + 2.29\, x_{fish} + 2.89\, x_{ham} + 1.89\, x_{mch} + 1.99\, x_{mtl} + 1.99\, x_{spg} + 2.49\, x_{tur}$

subject to A:

$$60\, x_{beef} + 8\, x_{chk} + 8\, x_{fish} + 40\, x_{ham} + 15\, x_{mch} + 70\, x_{mtl} + 25\, x_{spg} + 60\, x_{tur} >= 700$$

subject to C:

$$20\, x_{beef} + 0\, x_{chk} + 10\, x_{fish} + 40\, x_{ham} + 35\, x_{mch} + 30\, x_{mtl} + 50\, x_{spg} + 20\, x_{tur} >= 700$$

subject to B1:

$$10\, x_{beef} + 20\, x_{chk} + 15\, x_{fish} + 35\, x_{ham} + 15\, x_{mch} + 15\, x_{mtl} + 25\, x_{spg} + 15\, x_{tur} >= 700$$

subject to B2:

$$15\, x_{beef} + 20\, x_{chk} + 10\, x_{fish} + 10\, x_{ham} + 15\, x_{mch} + 15\, x_{mtl} + 15\, x_{spg} + 10\, x_{tur} >= 700$$

$x_{beef} >= 0;\ x_{chk} >= 0;\ x_{fish} >= 0;\ x_{ham} >= 0;\ x_{mch} >= 0;\ x_{mtl} >= 0;\ x_{spg} >= 0;\ x_{tur} >= 0;$

Summation and "For …" Notation

- Summation notation

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = \sum_{j=1}^{n} a_j\, x_j$$

- "for ..." notation

$$\left.\begin{array}{l} a_{1,1}x_1 + a_{1,2}x_2 \le b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 \le b_2 \\ a_{3,1}x_1 + a_{3,2}x_2 \le b_3 \end{array}\right\} \quad a_{i,1}x_1 + a_{i,2}x_2 \le b_i \qquad \mathit{for}\ i = 1,2,3$$

Diet Model

Sets:

F = *set of foods*

N = *set of nutrients*

Parameters:

a_{ij} = amount of nutrient j in food i, $\forall i \in F, \ \forall j \in N$

c_i = cost per serving of food i, $\forall i \in F$

$Fmin_i$ = minimum number of required servings of food i, $\forall i \in F$

$Fmax_i$ = maximum allowable number of servings of food i, $\forall i \in F$

$Nmin_j$ = minimum required level of nutrient j, $\forall j \in N$

$Nmax_j$ = maximum allowable level of nutrient j, $\forall j \in N$

Variables:

x_i = number of servings of food i to purchase/consume, $\forall i \in F$

Objective Function:

Minimize the total cost of the food

Diet Model

$$Minimize \ \sum_{i \in F} c_i \, x_i$$

#Constraint Set 1: For each nutrient $j \in N$, at least meet the minimum required level.

$$\sum_{i \in F} a_{ij} \, x_i \geq Nmin_j \, , \quad \forall j \in N$$

#Constraint Set 2:For each nutrient $j \in N$, do not exceed the maximum allowable level.

$$\sum_{i \in F} a_{ij} \, x_i \leq Nmax_j \, , \quad \forall j \in N$$

#Constraint Set 3:For each food $i \in F$, select at least the minimum required number of servings.

$$x_i \geq Fmin_i \, , \qquad \forall i \in F$$

#Constraint Set 4:For each food $i \in F$, do not exceed the maximum allowable number of servings.

$$x_i \leq Fmax_i \, , \qquad \forall i \in F$$

Maximizing Profits

```
var XB;

var XC;

maximize Profit: 25 * XB + 30 * XC;

subject to Time: (1/200) * XB + (1/140) * XC <= 40;

subject to B_limit: 0 <= XB <= 6000;

subject to C_limit: 0 <= XC <= 4000;
solve;
display XB, XC;
```

Maximizing Profits

Set: P, a set of products

Parameters: a_j = tons per hour of product j, for each $j \in P$

b = hours available at the mill

c_j = profit per ton of product j, for each $j \in P$

u_j = maximum tons of product j, for each $j \in P$

variables: x_j = tons of product j to be made, for each $j \in P$

Objective function:

$$\text{Maximize} \sum_{j \in P} c_j x_j$$

Subject to (Constraints):

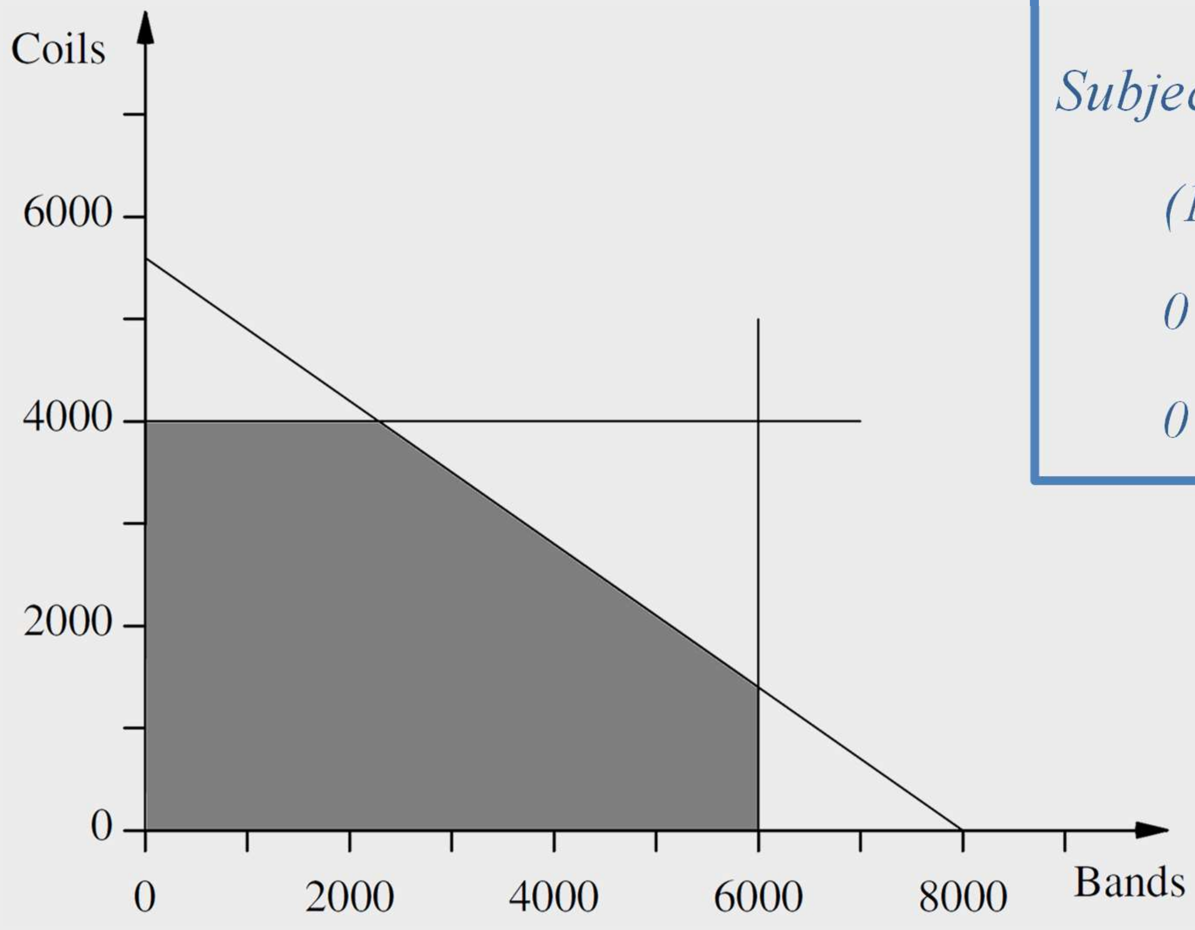
$$\sum_{j \in P} (1/a_j)\, x_j \leq b$$

$$0 \leq x_j \leq u_j, \;\; \text{for each } j \in P$$

Maximizing Profits

```
set P;
param a {j in P};
param b;
param c {j in P};
param u {j in P};
var X {j in P};
maximize Total_Profit: sum {j in P} c[j] * X[j];
subject to Time: sum {j in P} (1/a[j]) * X[j] <= b;
subject to Limit {j in P}: 0 <= X[j] <= u[j];
```

Feasible Region



Maximize $25\, x_B + 30\, x_C$

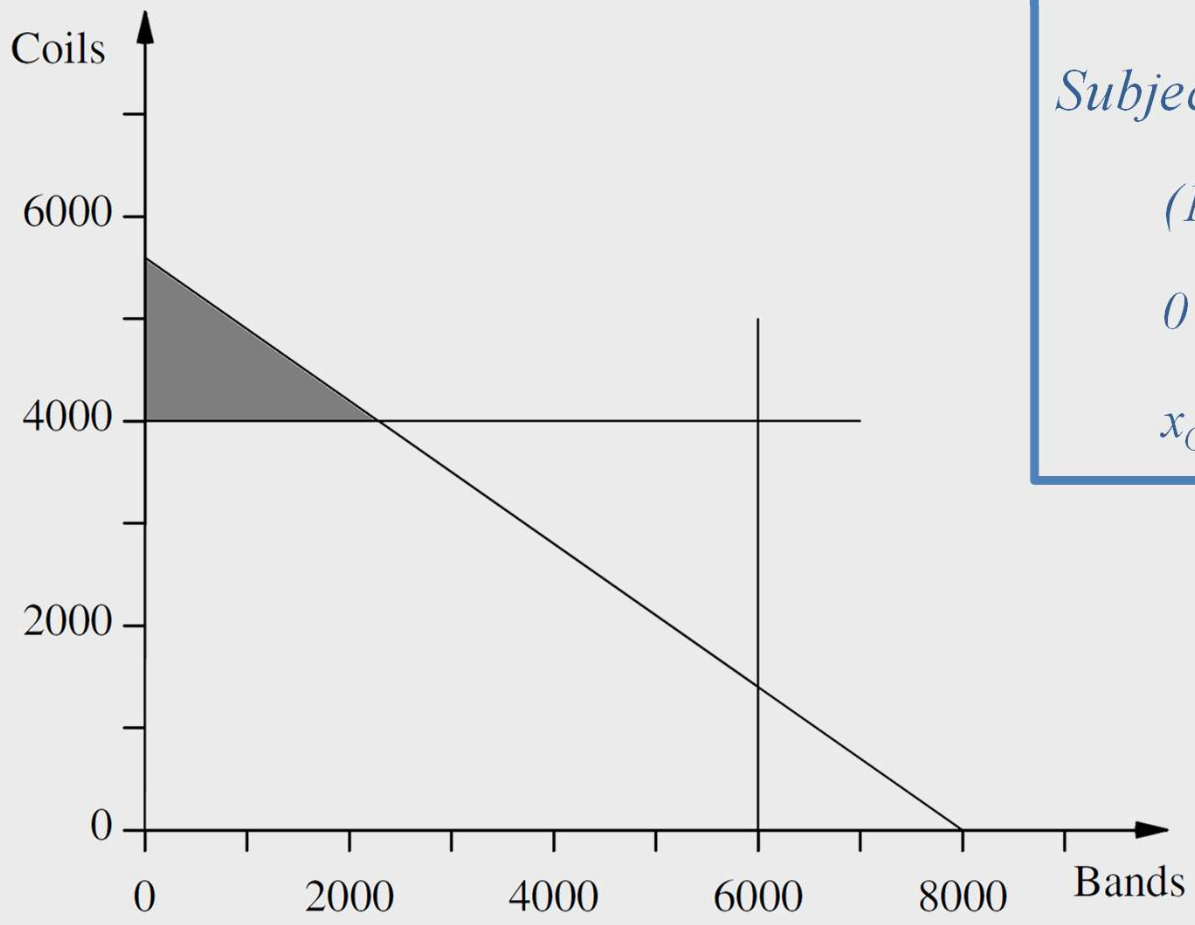
Subject to :

$$(1/200)\, x_B + (1/140)\, x_C \leq 40$$

$$0 \leq x_B \leq 6000$$

$$0 \leq x_C \leq 4000$$

Feasible Region



Maximize $25\, x_B + 30\, x_C$

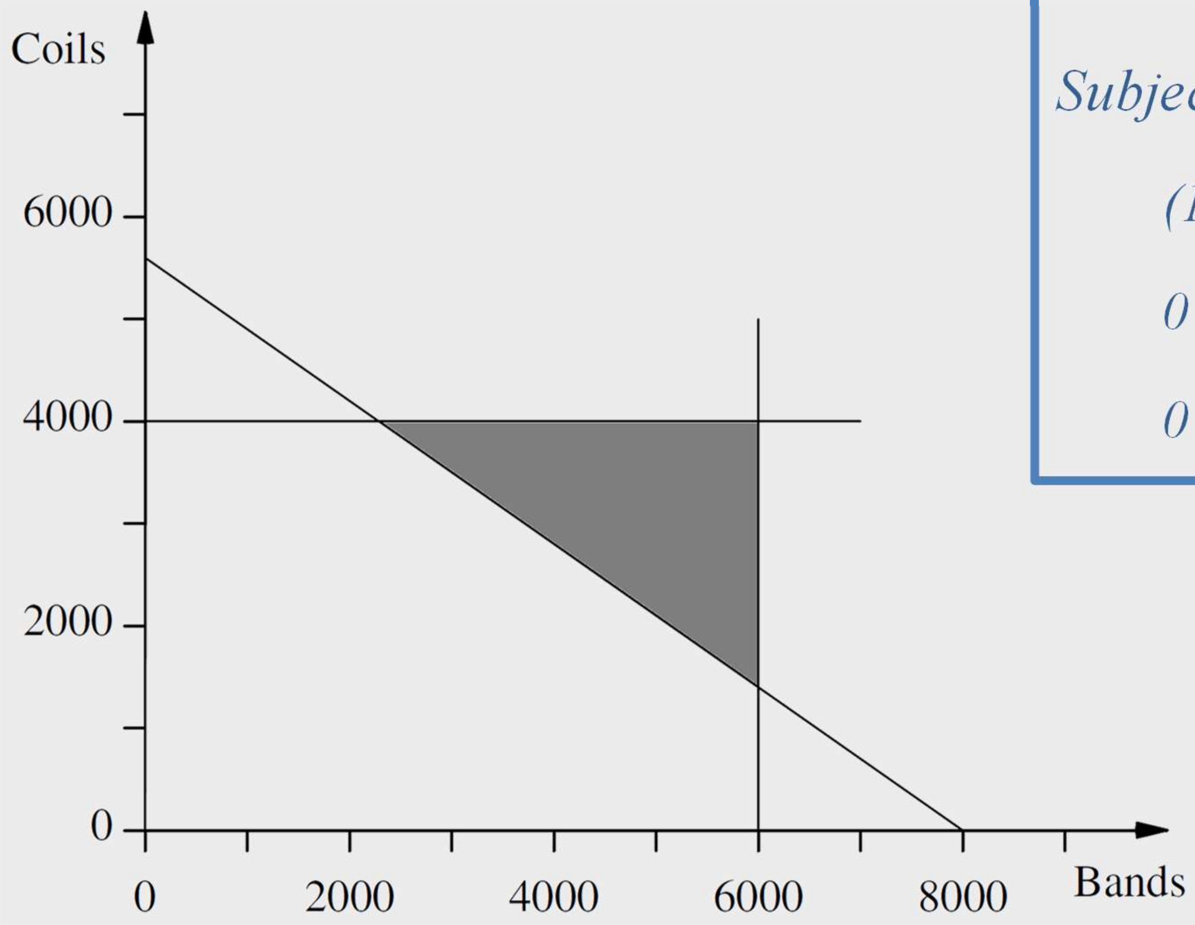
Subject to :

$(1/200)\, x_B + (1/140)\, x_C \leq 40$

$0 \leq x_B \leq 6000$

$x_C \geq 4000$

Feasible Region



Maximize $25\, x_B + 30\, x_C$

Subject to :

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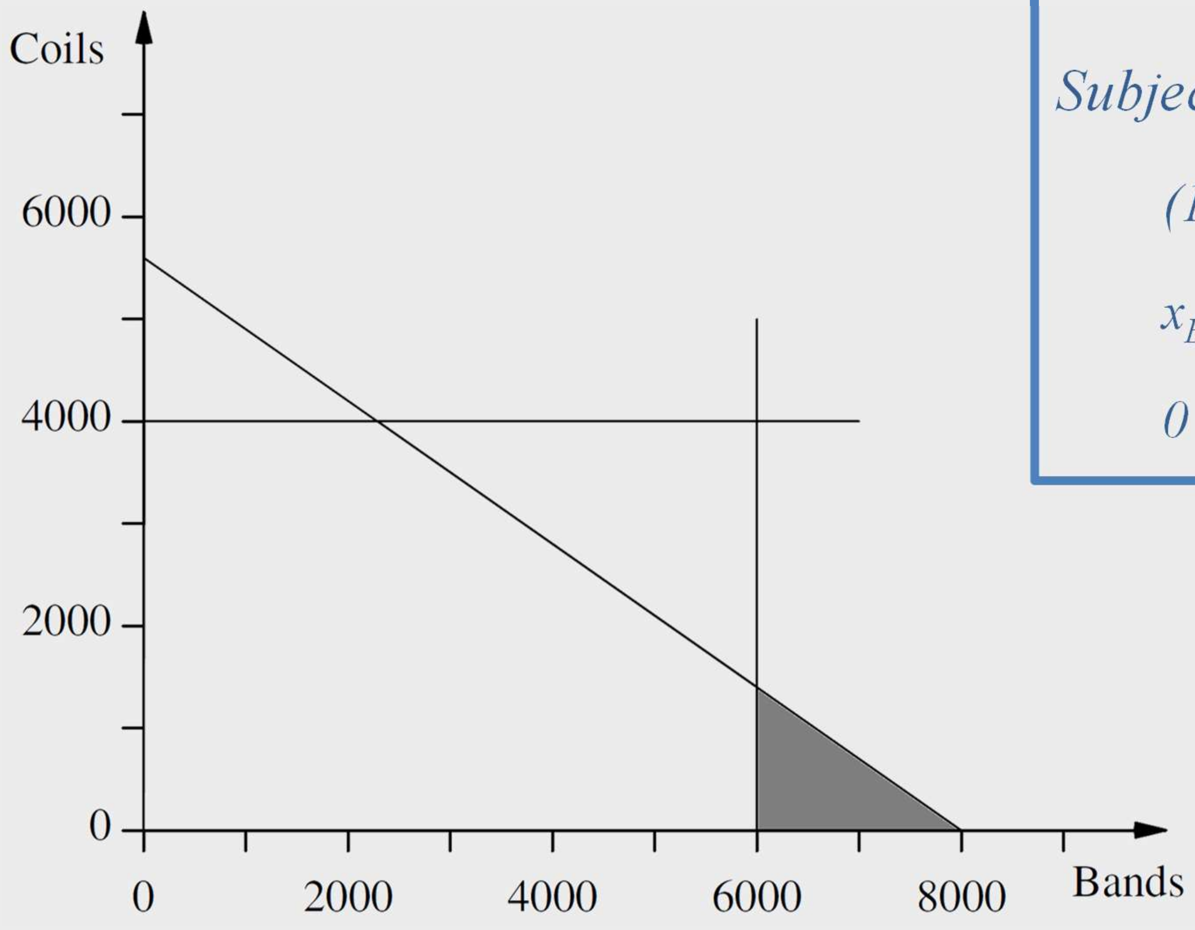

$$\text{Maximize} \quad 25\, x_B + 30\, x_C$$

Subject to :

$$(1/200)\, x_B + (1/140)\, x_C \le 40$$

$$x_B \ge 6000$$

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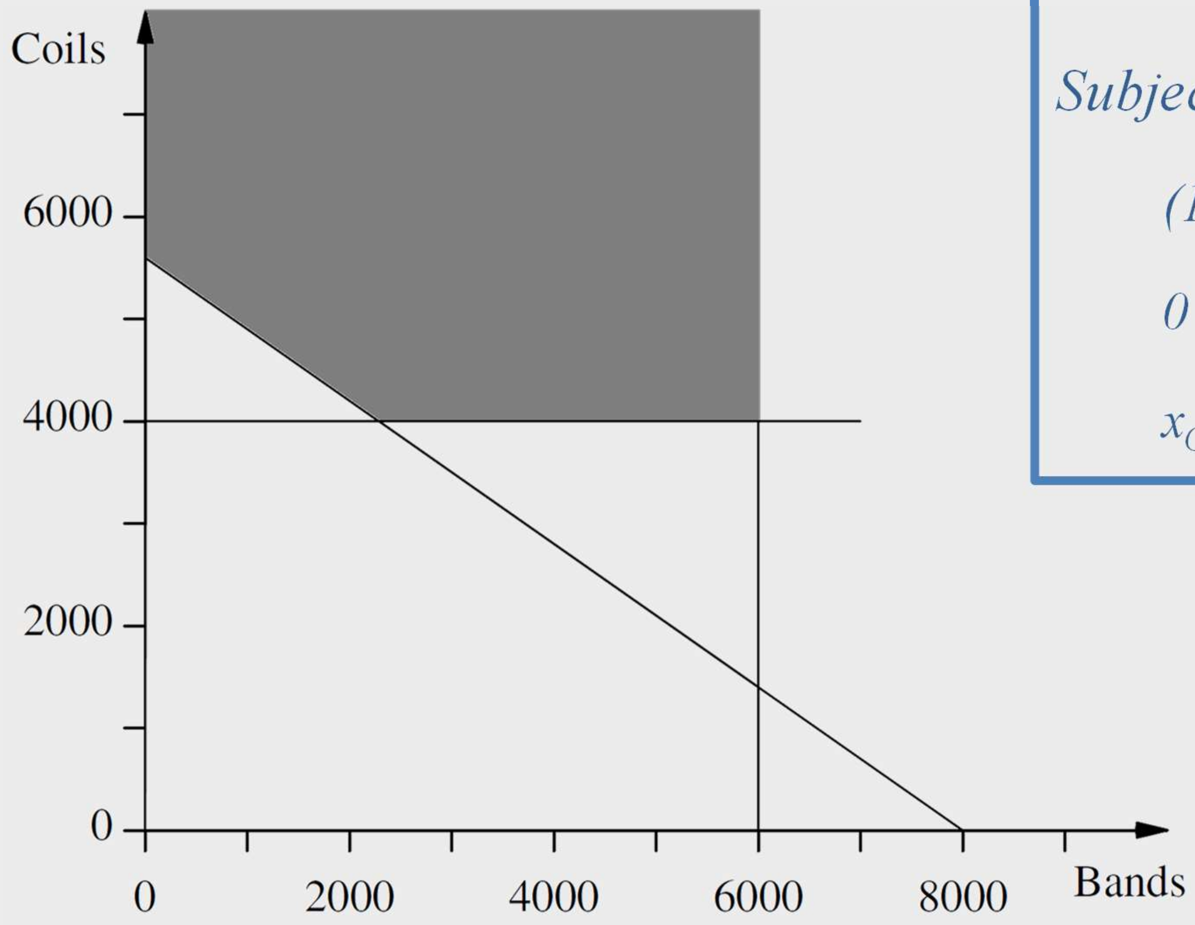
Feasible Region



Maximize $25\, x_B + 30\, x_C$

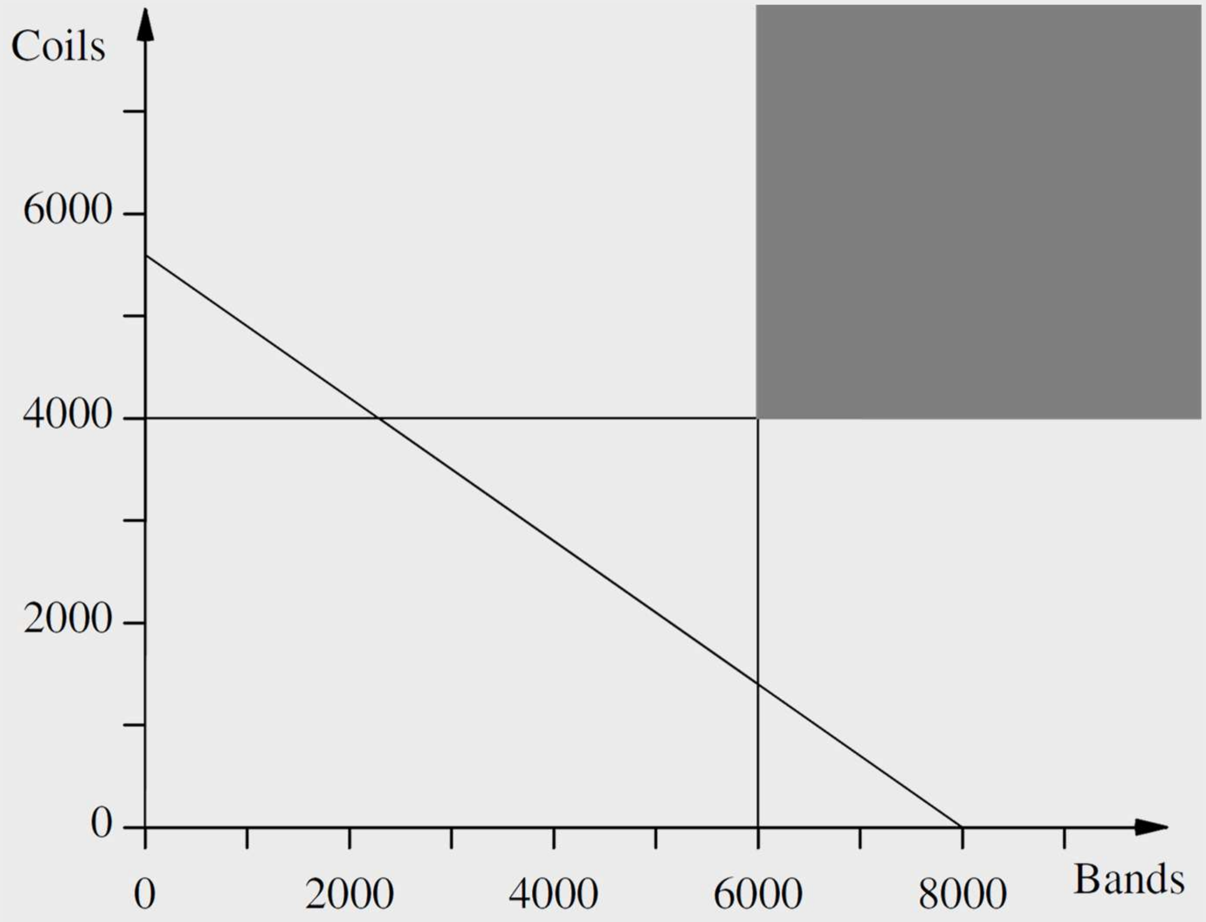
Subject to :

$(1/200)\, x_B + (1/140)\, x_C \geq 40$

$0 \leq x_B \leq 6000$

$x_C \geq 4000$

Feasible Region



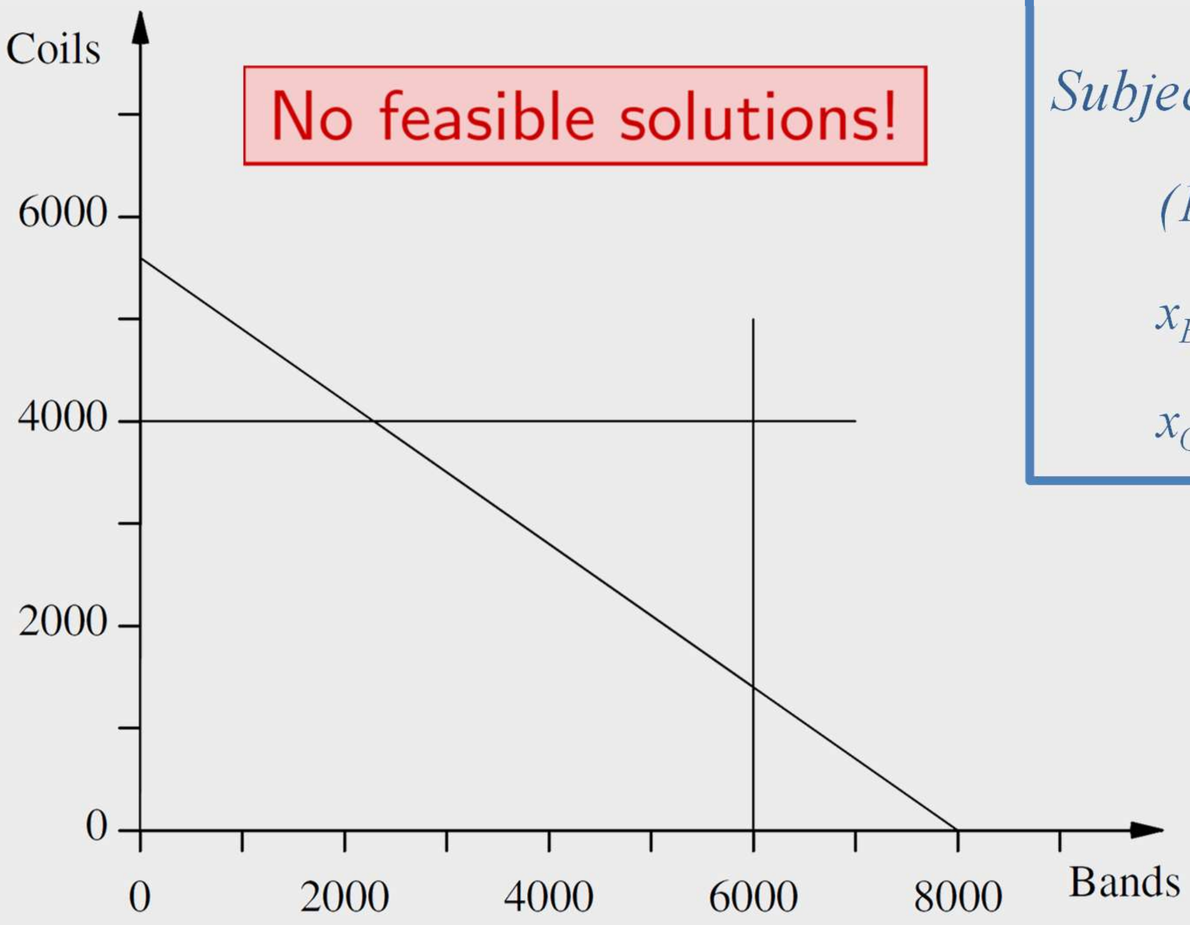
Maximize $25\,x_B + 30\,x_C$

Subject to :

$x_B \geq 6000$

$x_C \geq 4000$

FEASIBLE REGION



Maximize $25\,x_B + 30\,x_C$

Subject to :

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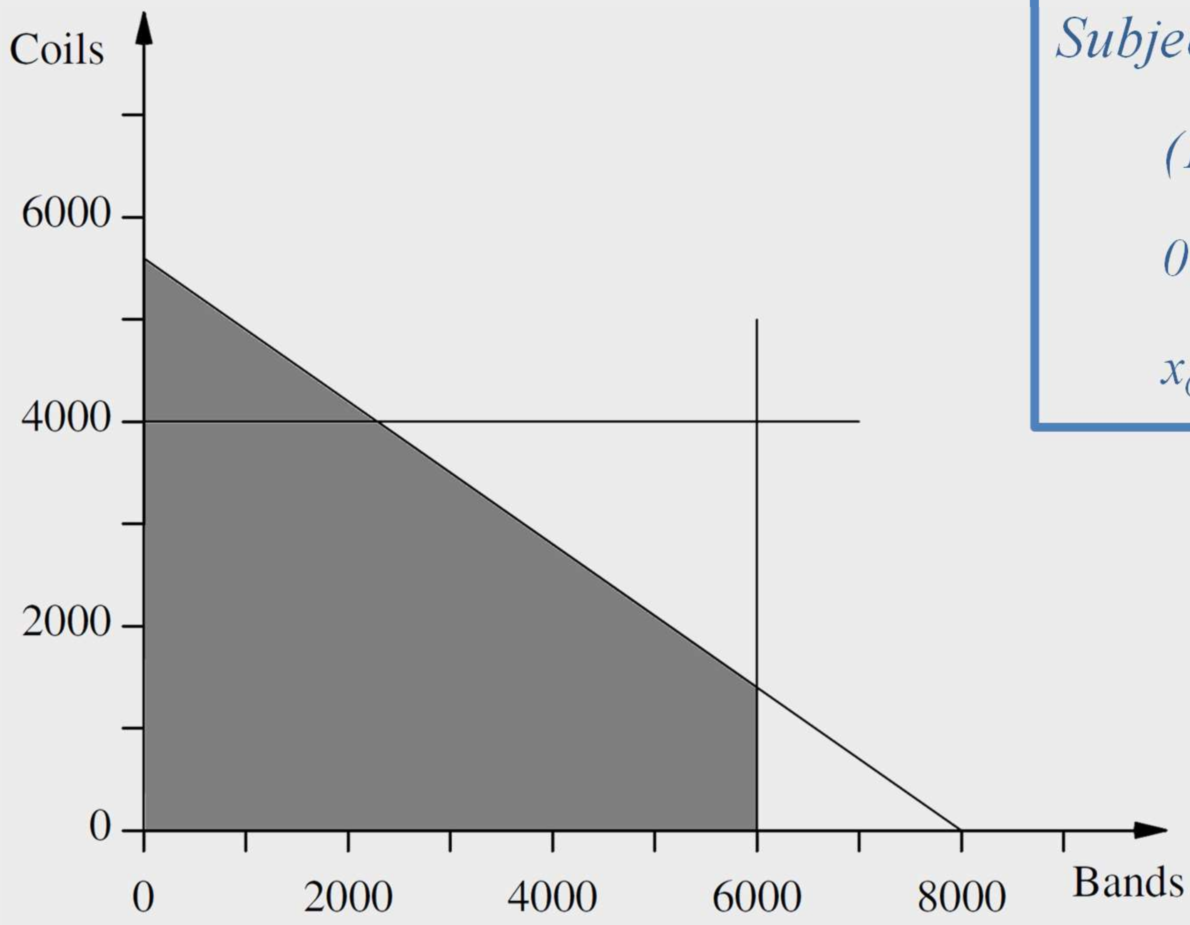
$$x_B \geq 6000$$

$$x_C \geq 4000$$

FEASIBLE REGION



$$\text{Maximize} \quad 25\,x_B + 30\,x_C$$

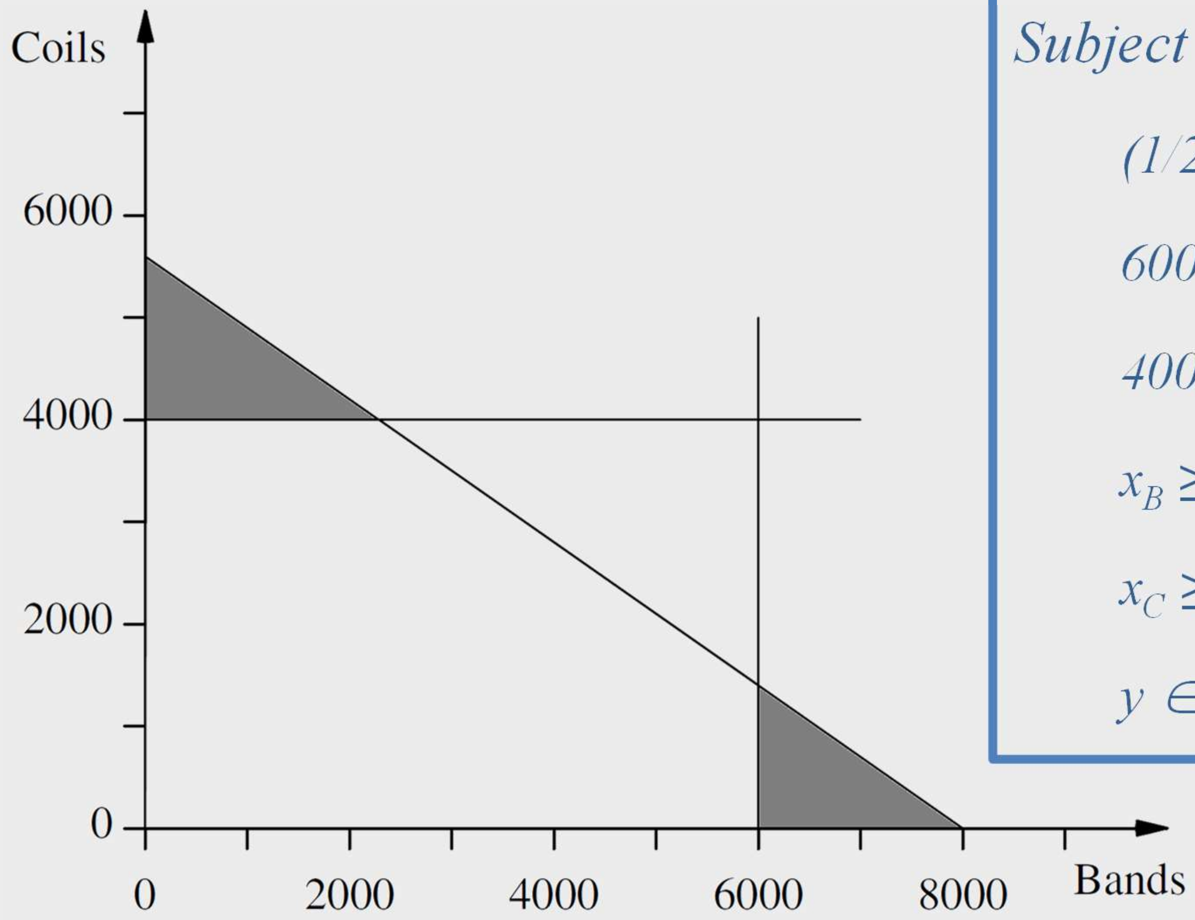
Subject to :

$$(1/200)\,x_B + (1/140)\,x_C \le 40$$

$$0 \le x_B \le 6000$$

$$x_C \ge 0$$

Feasible Region



Maximize $25\, x_B + 30\, x_C$

Subject to :

$(1/200)\, x_B + (1/140)\, x_C \leq 40$

$6000 - M\,(1 - y) \leq x_B \leq 6000 + M\, y$

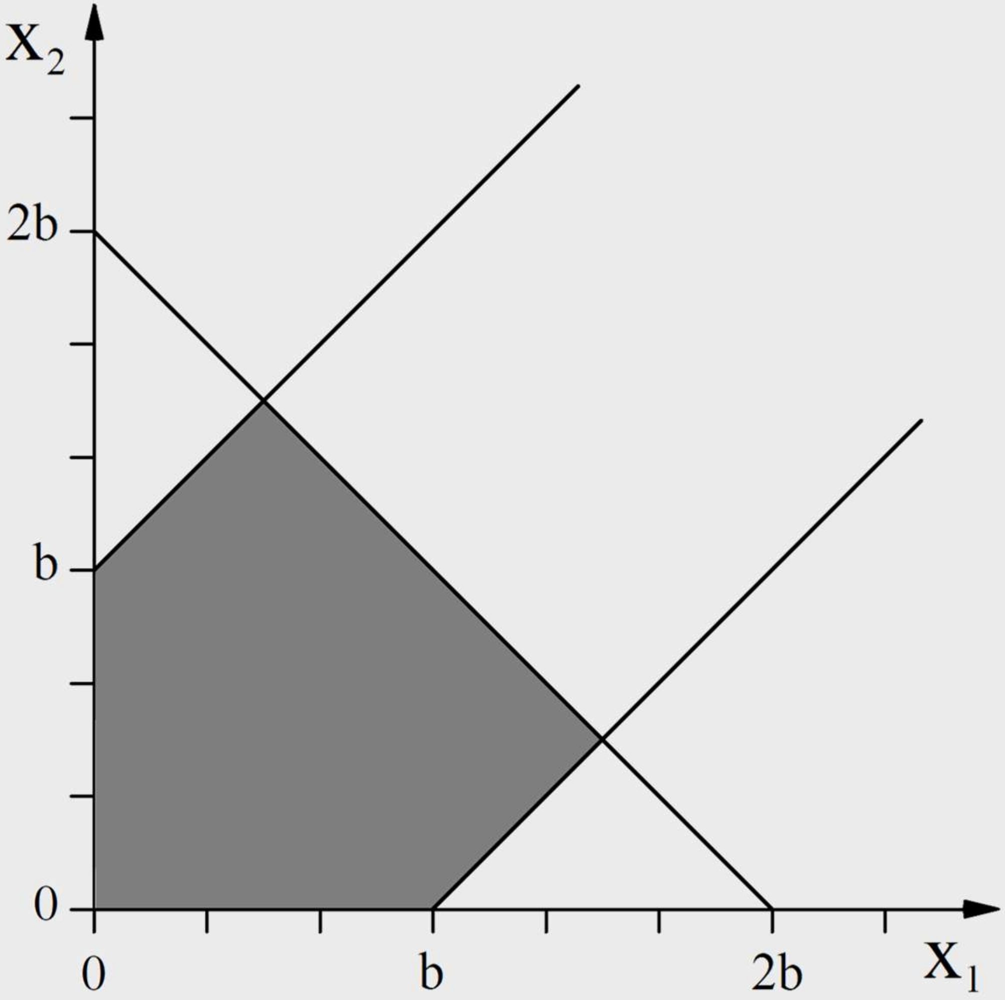
$4000 - M\, y \leq x_C \leq 4000 + M\,(1 - y)$

$x_B \geq 0$

$x_C \geq 0$

$y \in \{0, 1\}$

FEASIBLE REGION



$$\begin{aligned}
&\textit{Maximize} && c_1 x_1 + c_2 x_2 \\
&\textit{Subject to :} && x_1 + x_2 \leq 2b \\
& && |\, x_1 - x_2 \,| \leq b \\
& && x_1 \,, x_2 \geq 0
\end{aligned}$$

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&\textit{Subject to :} && x_1 + x_2 \leq 2b \\
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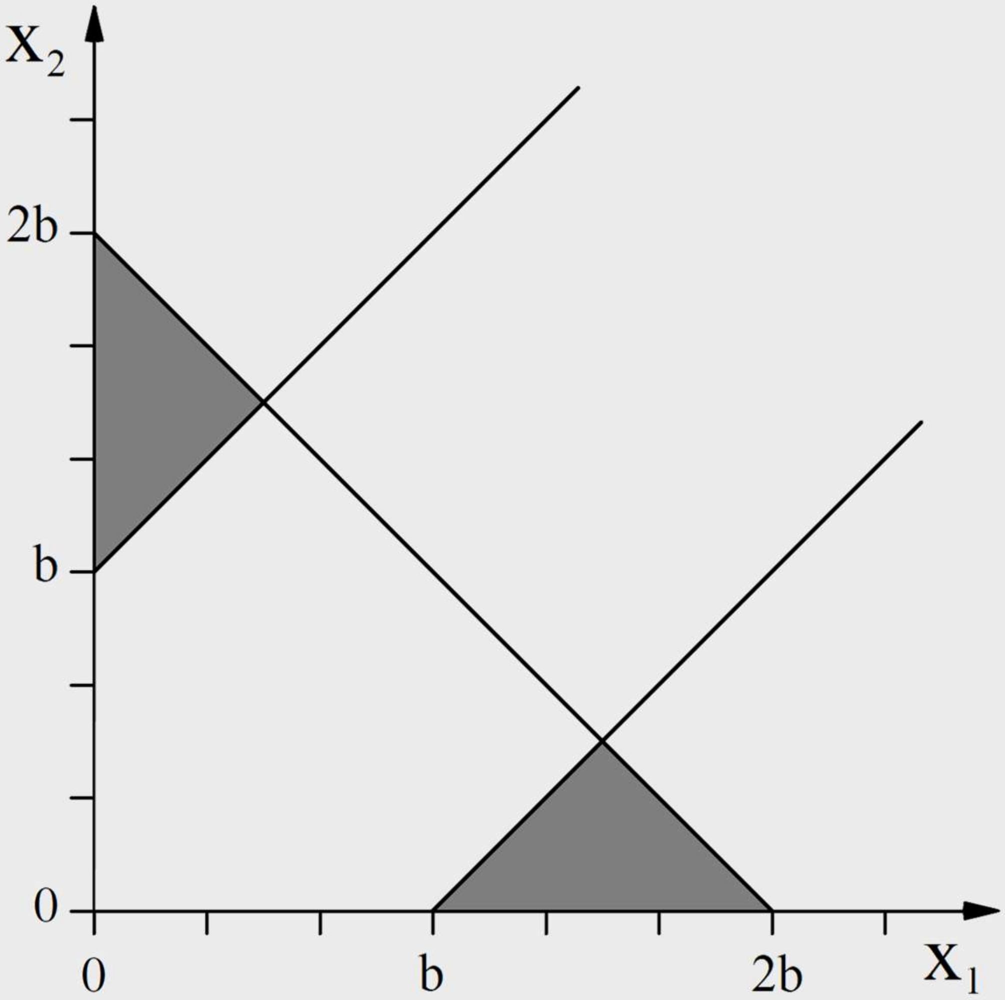
& && x_1 - x_2 \geq - b \\
& && x_1 \,, x_2 \geq 0
\end{aligned}$$

Feasible Region



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&\text{Subject to :} && x_1 + x_2 \leq 2b \\
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\begin{aligned}
&\text{Maximize} && c_1 x_1 + c_2 x_2 \\
&\text{Subject to :} && x_1 + x_2 \leq 2b \\
& && x_1 - x_2 \geq b - My \\
& && x_1 - x_2 \leq - b + M\,(1 - y) \\
& && x_1 ,\, x_2 \geq 0
\end{aligned}
$$

If–Then Condition

$$\begin{aligned}
&\text{Minimize} \quad && c_1 x + c_2 y \\
&\text{st:} && g(x) \geq b \\
& && x \geq 0 \text{ and } y \in \{0, 1\}
\end{aligned}$$

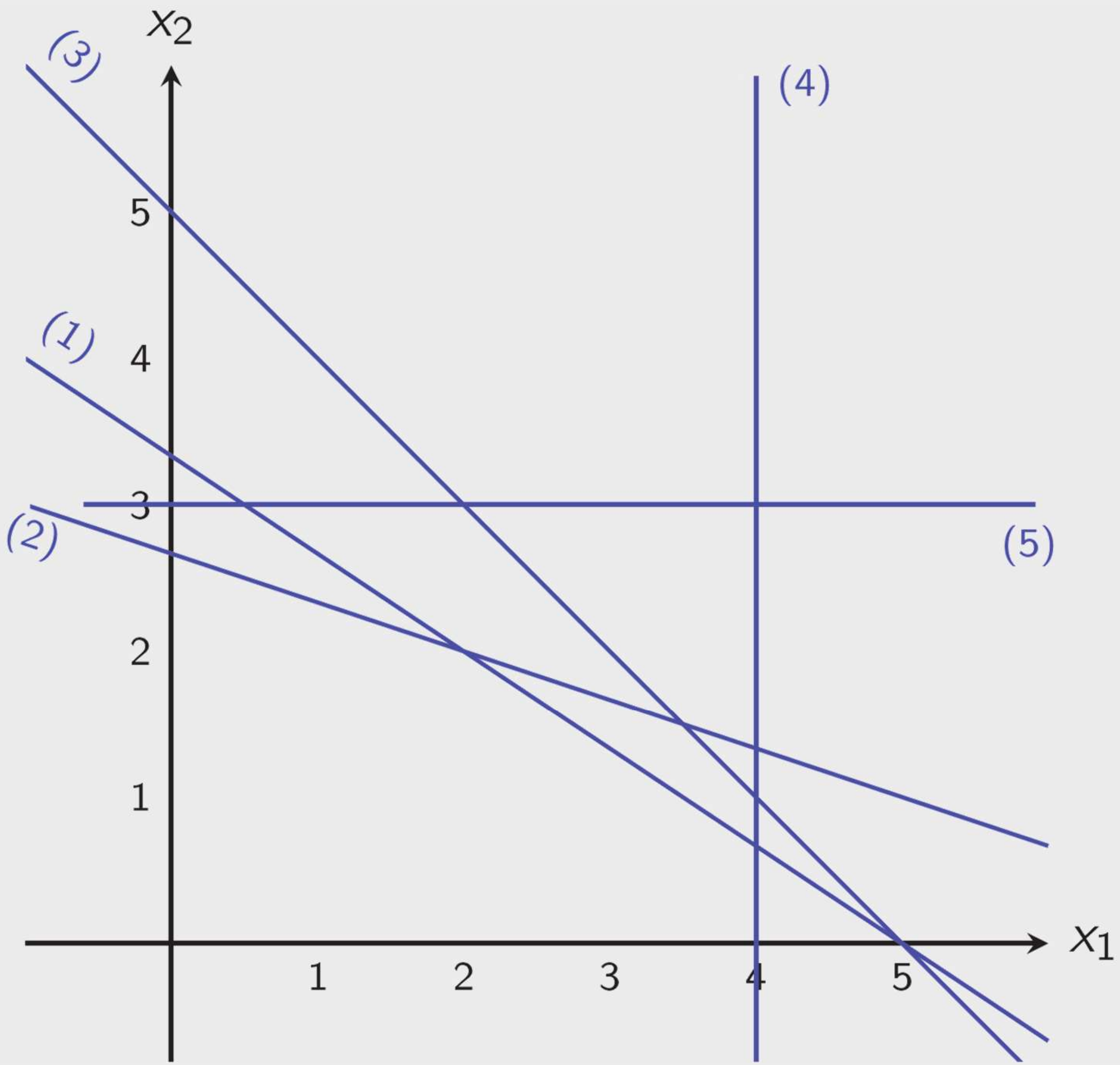
$$\begin{aligned}
&\text{If } x > 0 \leftrightarrow y = 1 \\
&\text{If } x = 0 \leftrightarrow y = 0
\end{aligned}
\quad \Longrightarrow \quad
\boxed{x \leq M y}$$

	$y = 0$	$y = 1$
$x = 0$	✓	✗
$x > 0$	✗	✓

Feasible Region

Feasible region:

$$
\begin{aligned}
2x_1 + 3x_2 &\leq 10 \qquad (1)\\
x_1 + 3x_2 &\leq 8 \qquad (2)\\
x_1 + x_2 &\leq 5 \qquad (3)\\
x_1 &\leq 4 \qquad (4)\\
x_2 &\geq 3 \qquad (5)\\
x_1 &\geq 0 \qquad (6)\\
x_2 &\geq 0 \qquad (7)
\end{aligned}
$$



No feasible solutions!

Feasible region:

$$
\begin{aligned}
2x_1 + 3x_2 &\leq 10 \quad &(1)\\
x_1 + 3x_2 &\leq 8 \quad &(2)\\
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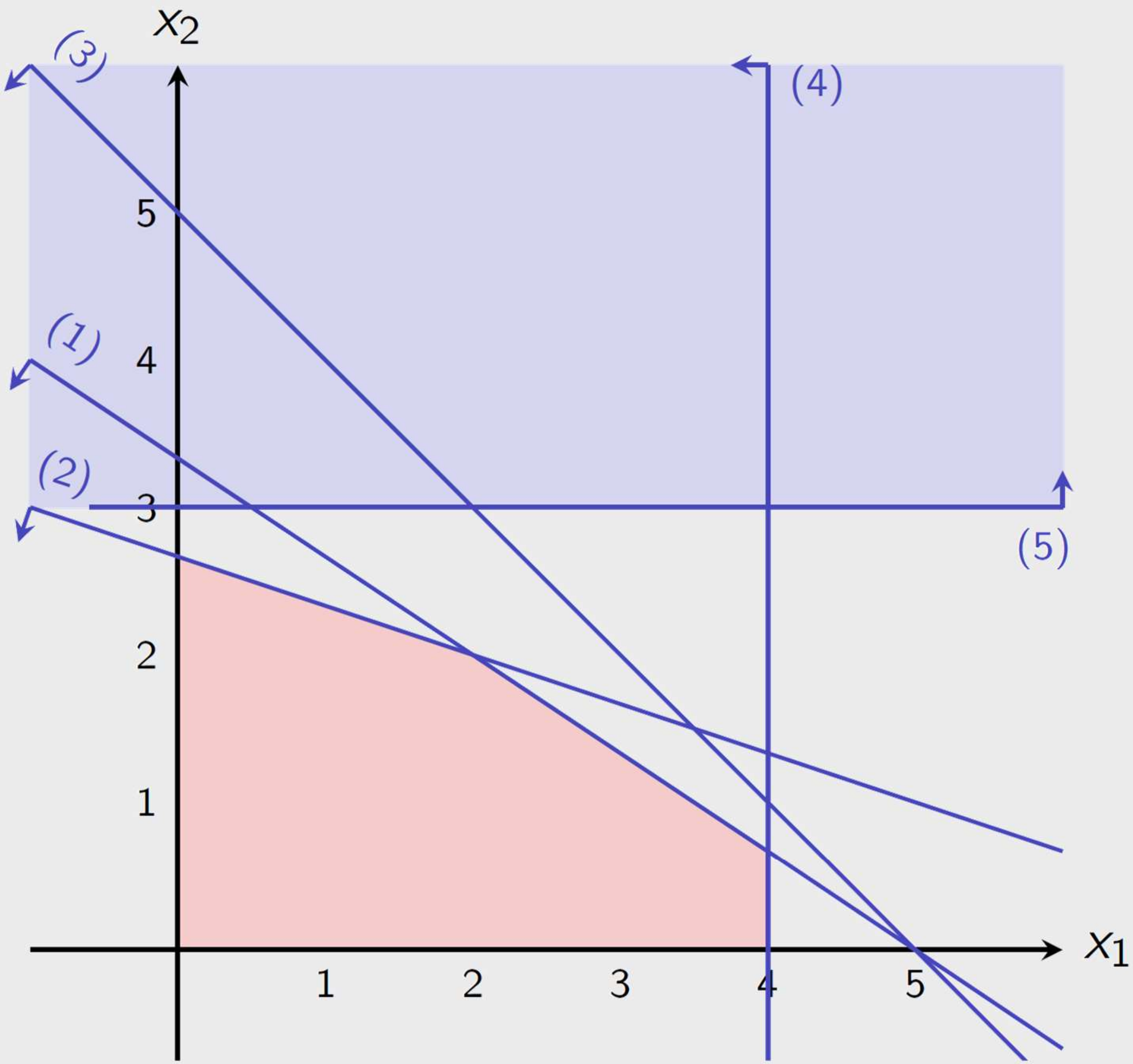
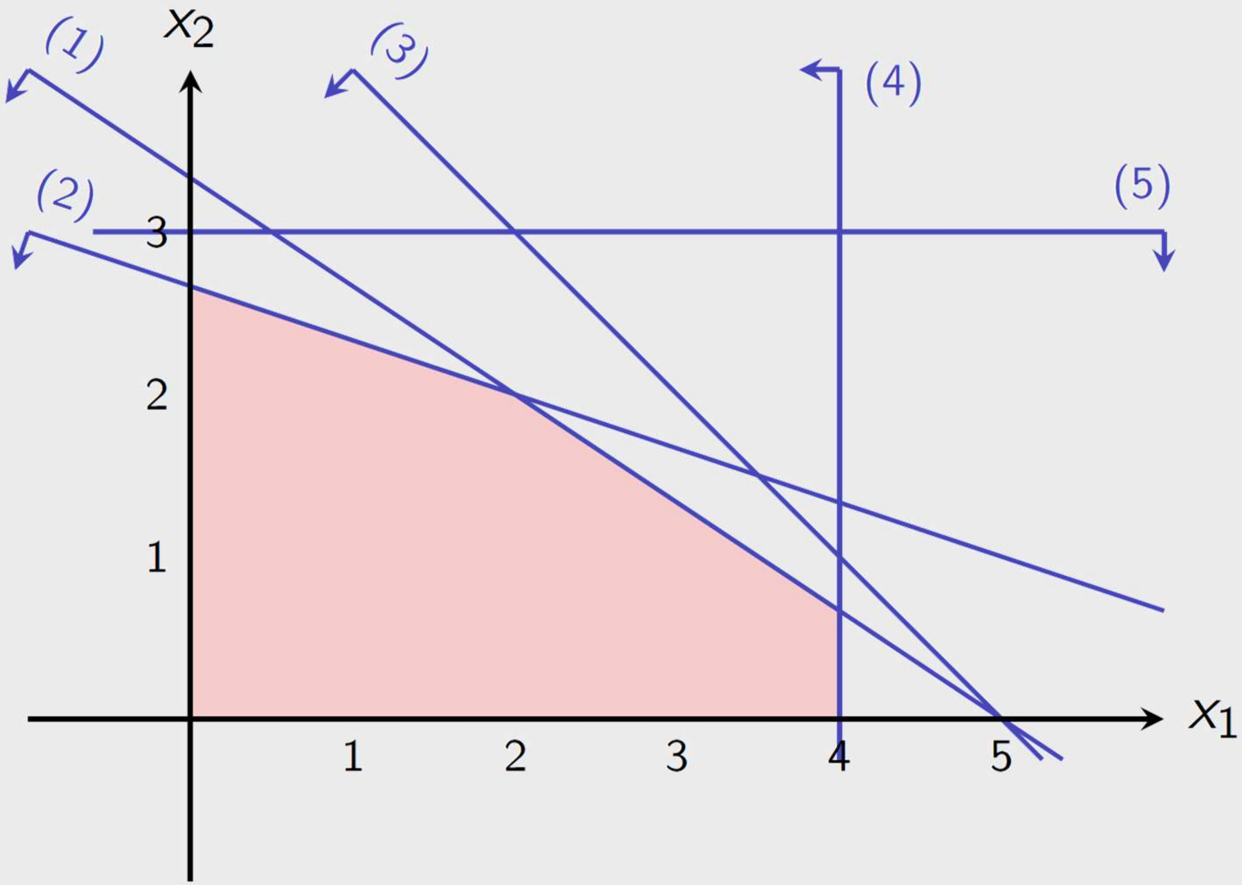


Objective Function

$$
\begin{array}{llll}
\max & 3x_1 + 5x_2 & & \\
\text{s.t.} & 2x_1 + 3x_2 \leq 10 & (1) \\
 & x_1 + 3x_2 \leq 8 & (2) \\
 & x_1 + x_2 \leq 5 & (3) \\
 & x_1 \leq 4 & (4) \\
 & x_2 \leq 3 & (5) \\
 & x_1 \geq 0 & (6) \\
 & x_2 \geq 0 & (7)
\end{array}
$$



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& x_1 \geq 0 & (6) \\
& x_2 \geq 0 & (7)
\end{array}
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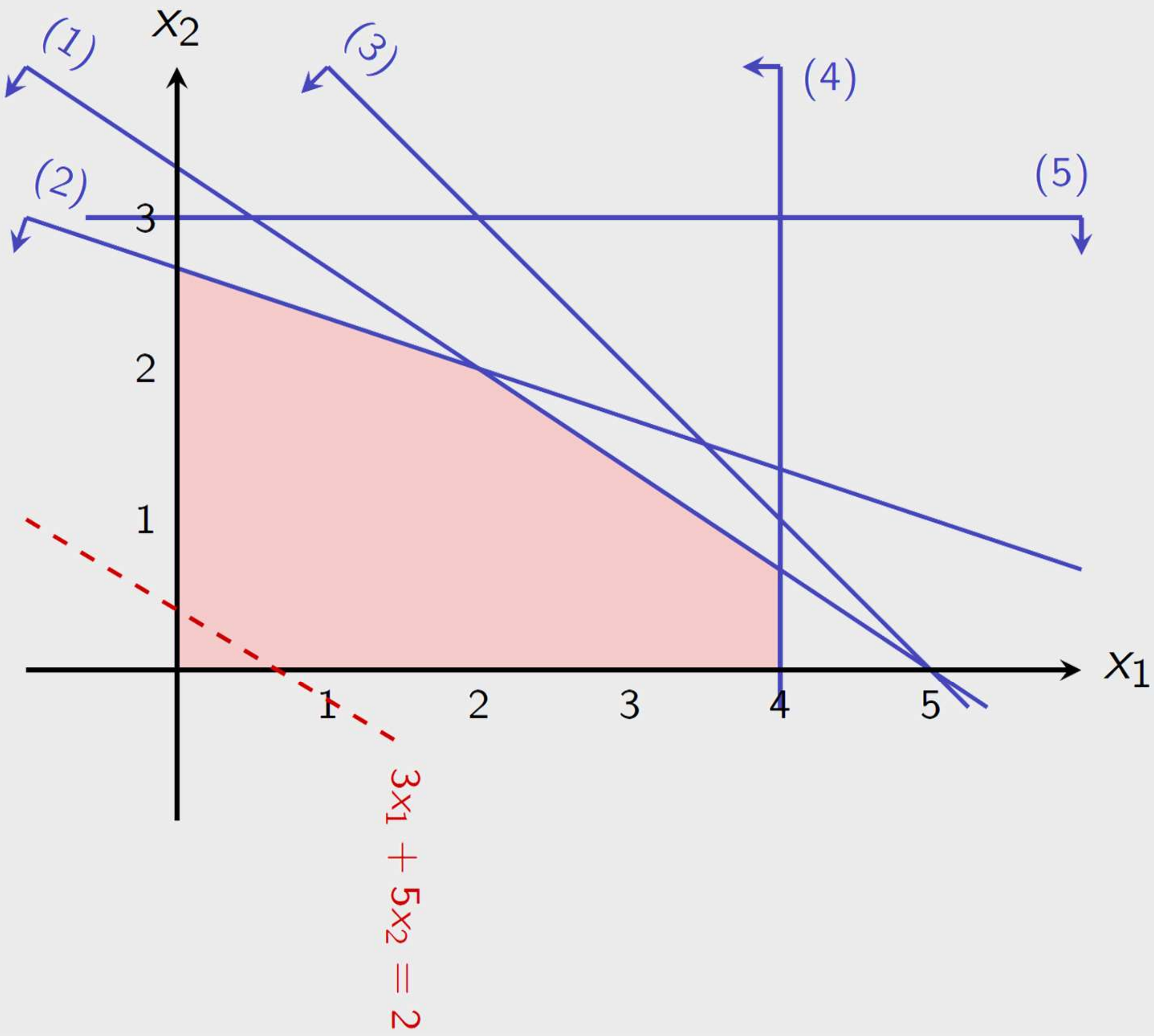

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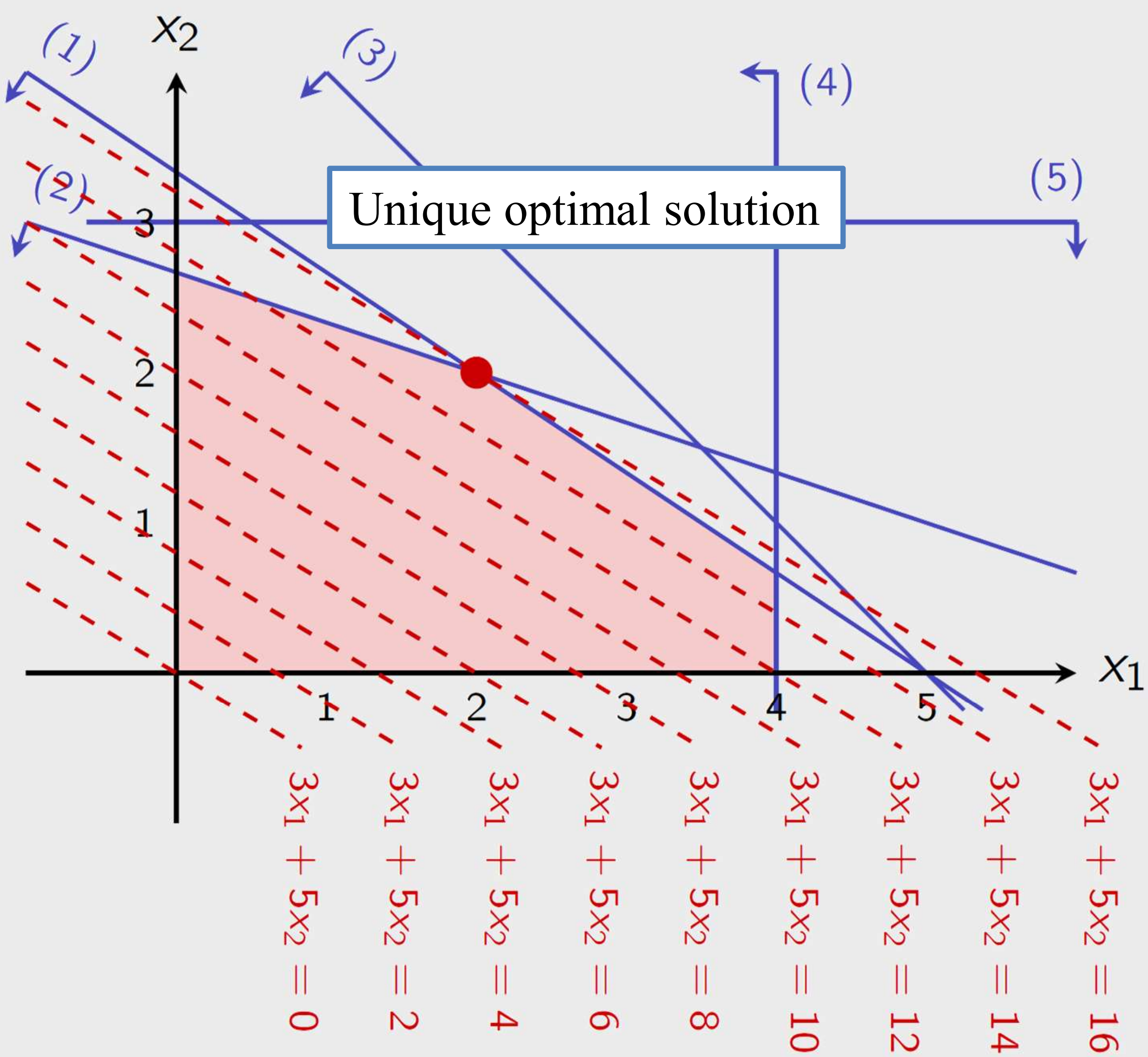

Objective Function

$$
\begin{array}{llr}
\max & 2x_1 + 3x_2 & \\
\text{s.t.} & 2x_1 + 3x_2 \le 10 & (1) \\
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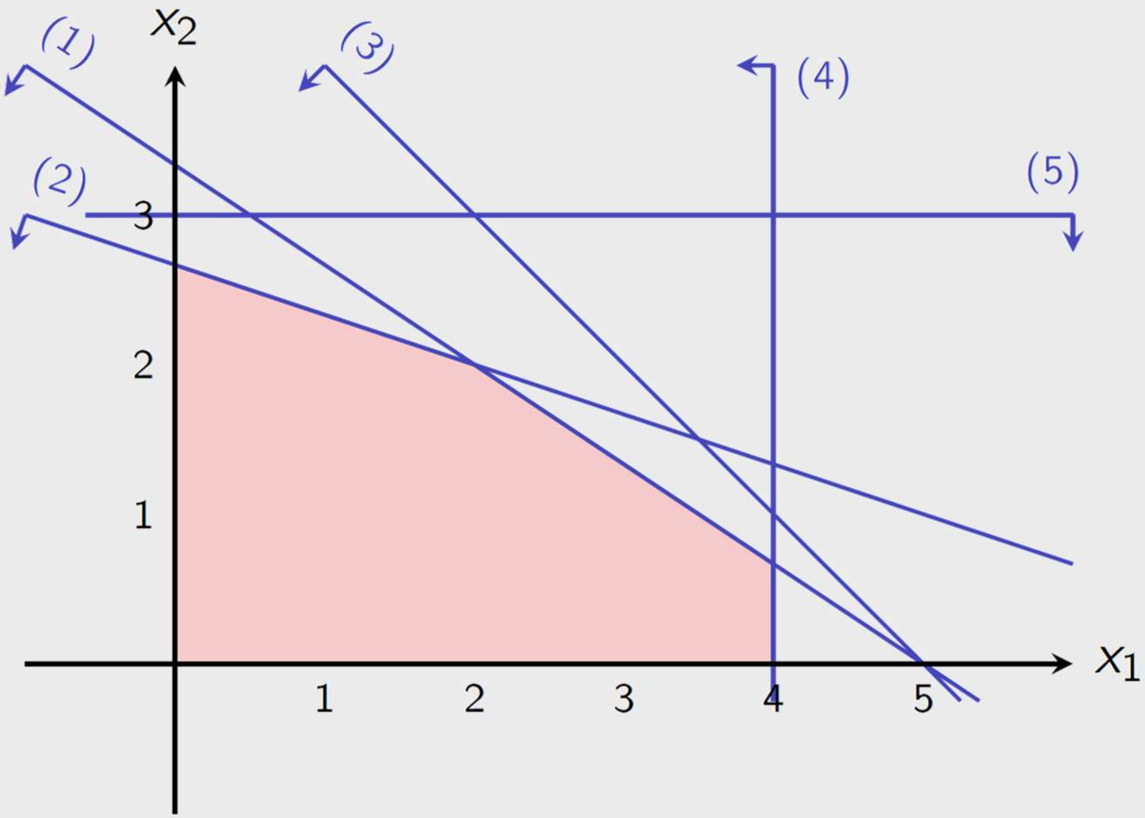
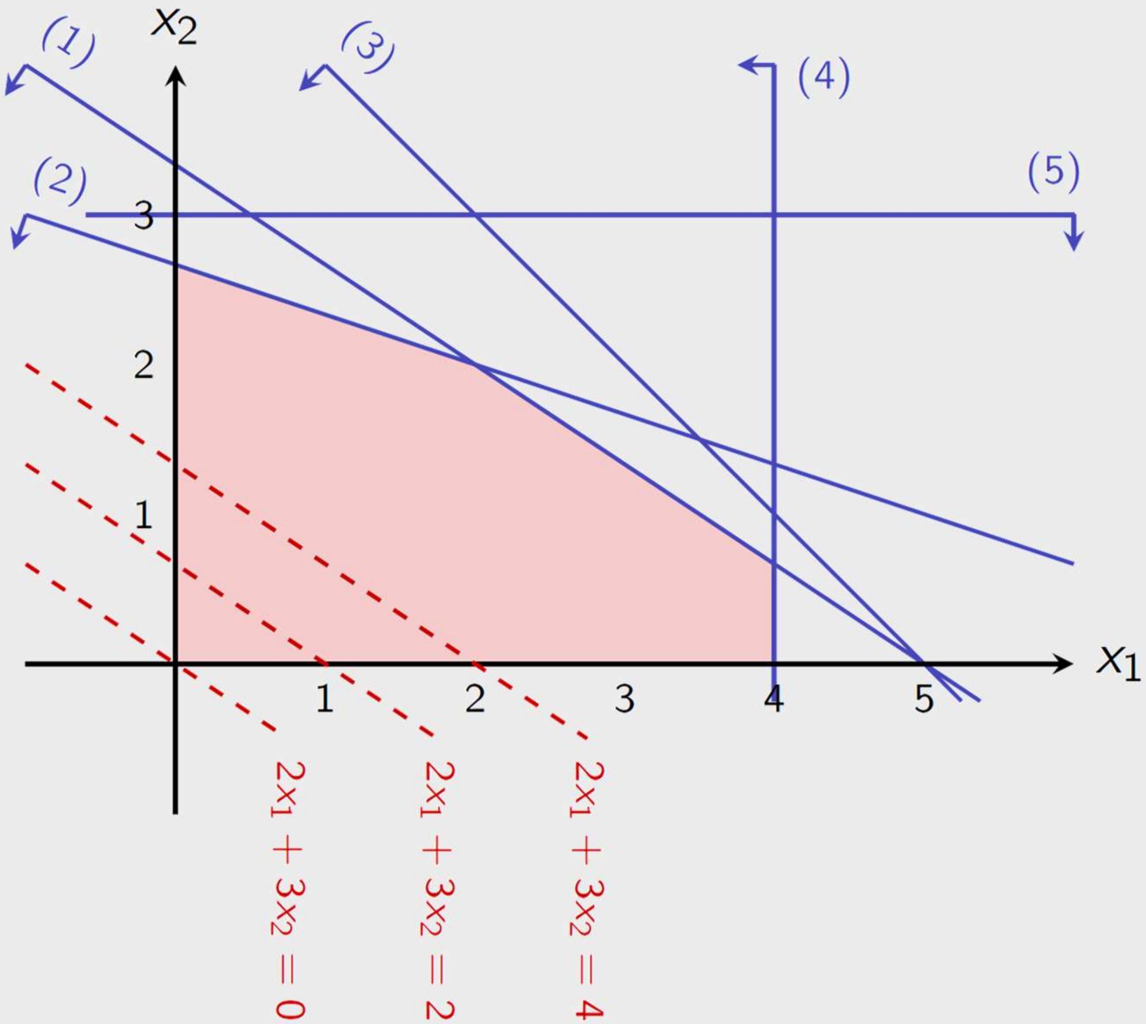
& x_1 + x_2 \le 5 & (3) \\
& x_1 \le 4 & (4) \\
& x_2 \le 3 & (5) \\
& x_1 \ge 0 & (6) \\
& x_2 \ge 0 & (7)
\end{array}
$$



Objective Function

$$
\begin{array}{lrcll}
\max & 2x_1 + 3x_2 & & & \\
\text{s.t.} & 2x_1 + 3x_2 & \leq & 10 & (1) \\
 & x_1 + 3x_2 & \leq & 8 & (2) \\
 & x_1 + x_2 & \leq & 5 & (3) \\
 & x_1 & \leq & 4 & (4) \\
 & x_2 & \leq & 3 & (5) \\
 & x_1 & \geq & 0 & (6) \\
 & x_2 & \geq & 0 & (7)
\end{array}
$$



Objective Function

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\begin{array}{lrcl}
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\text{s.t.} & 2x_1 + 3x_2 & \leq 10 & (1) \\
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\end{array}
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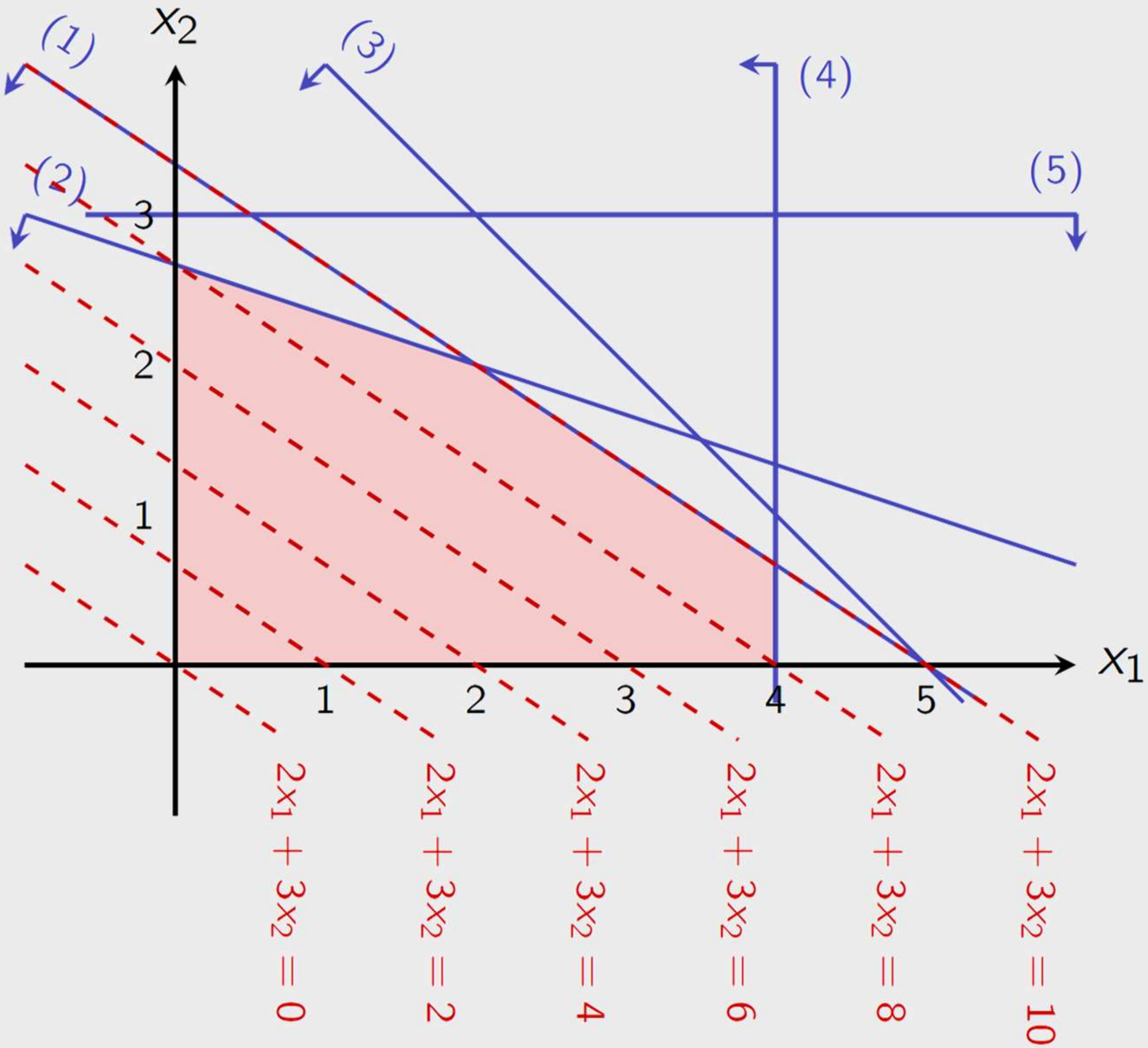

Objective Function

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 & x_1 \leq 4 & (4) \\
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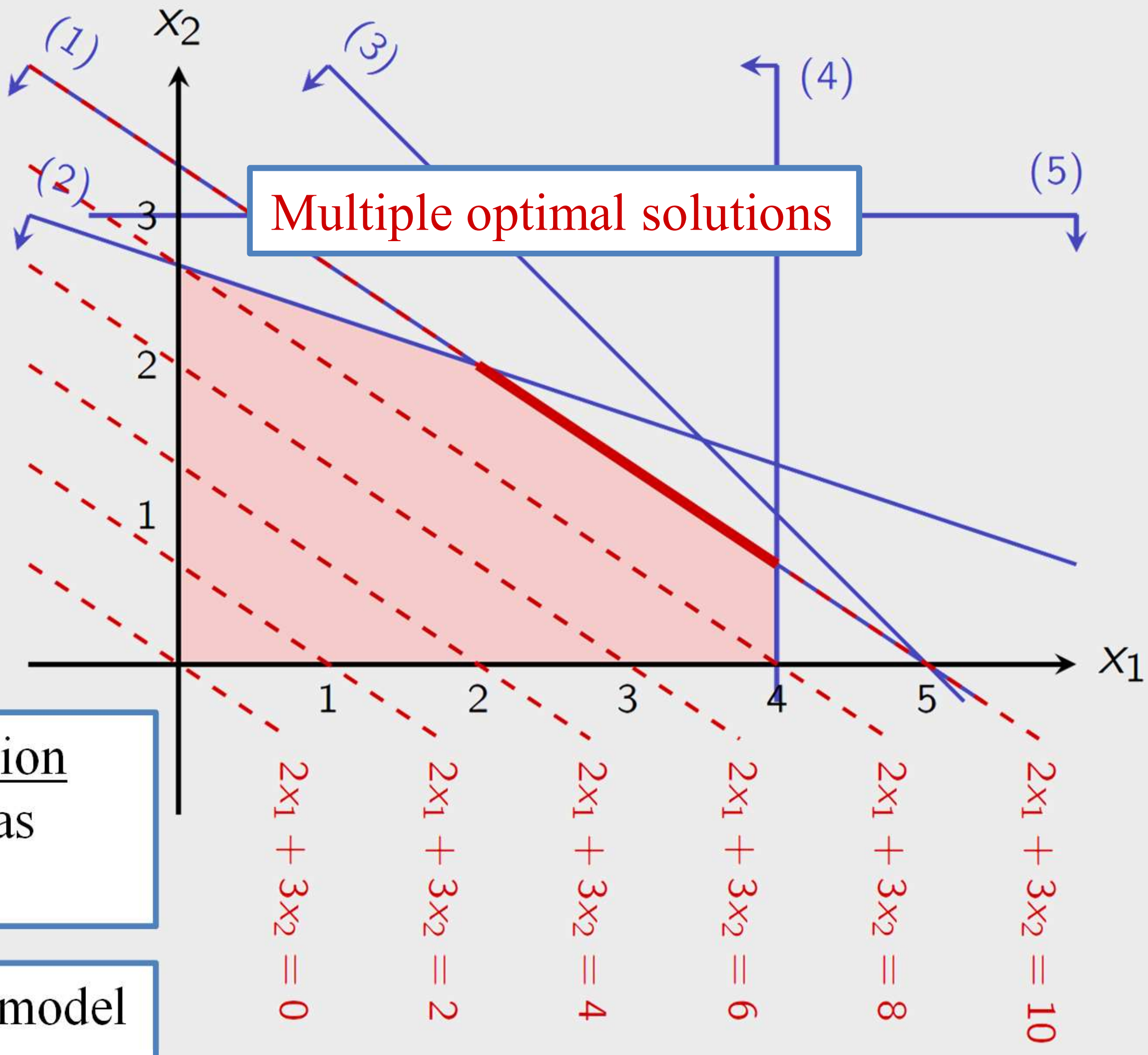
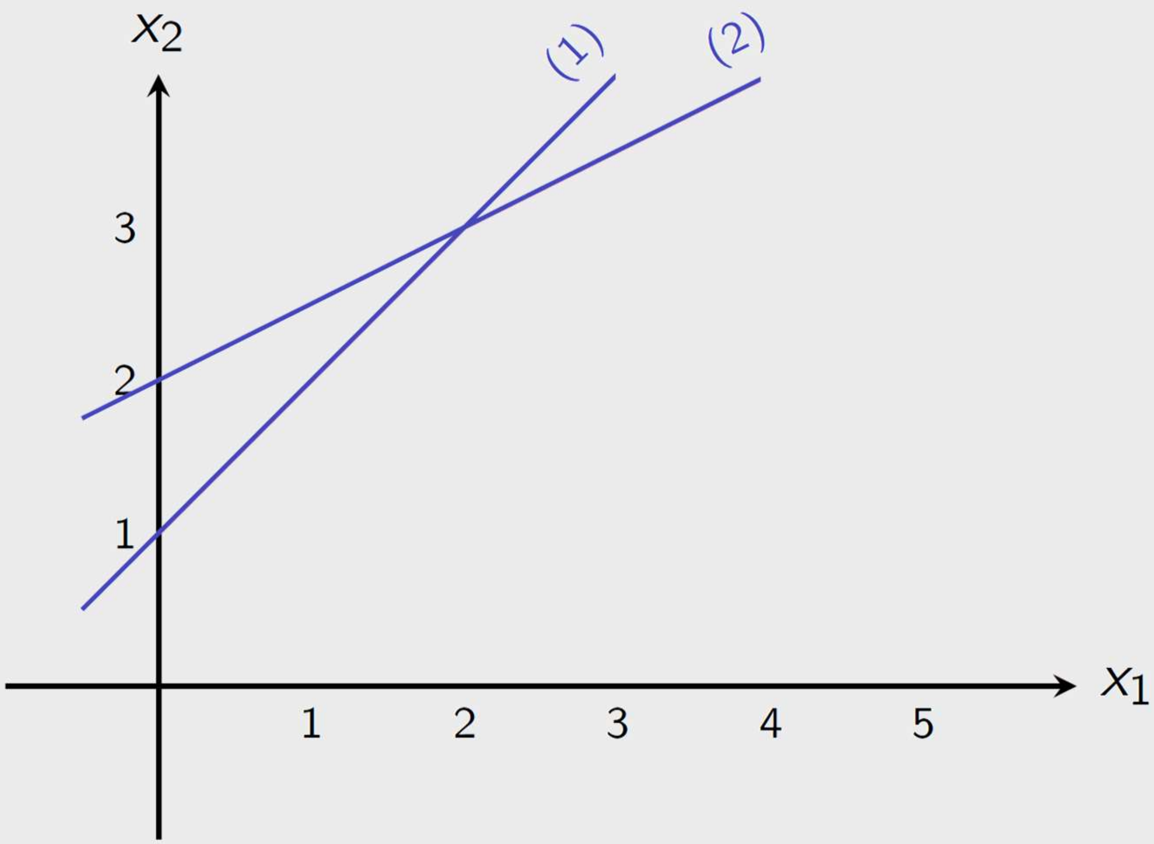


An optimal solution is a feasible solution with objective function value at least as good as any other feasible solution

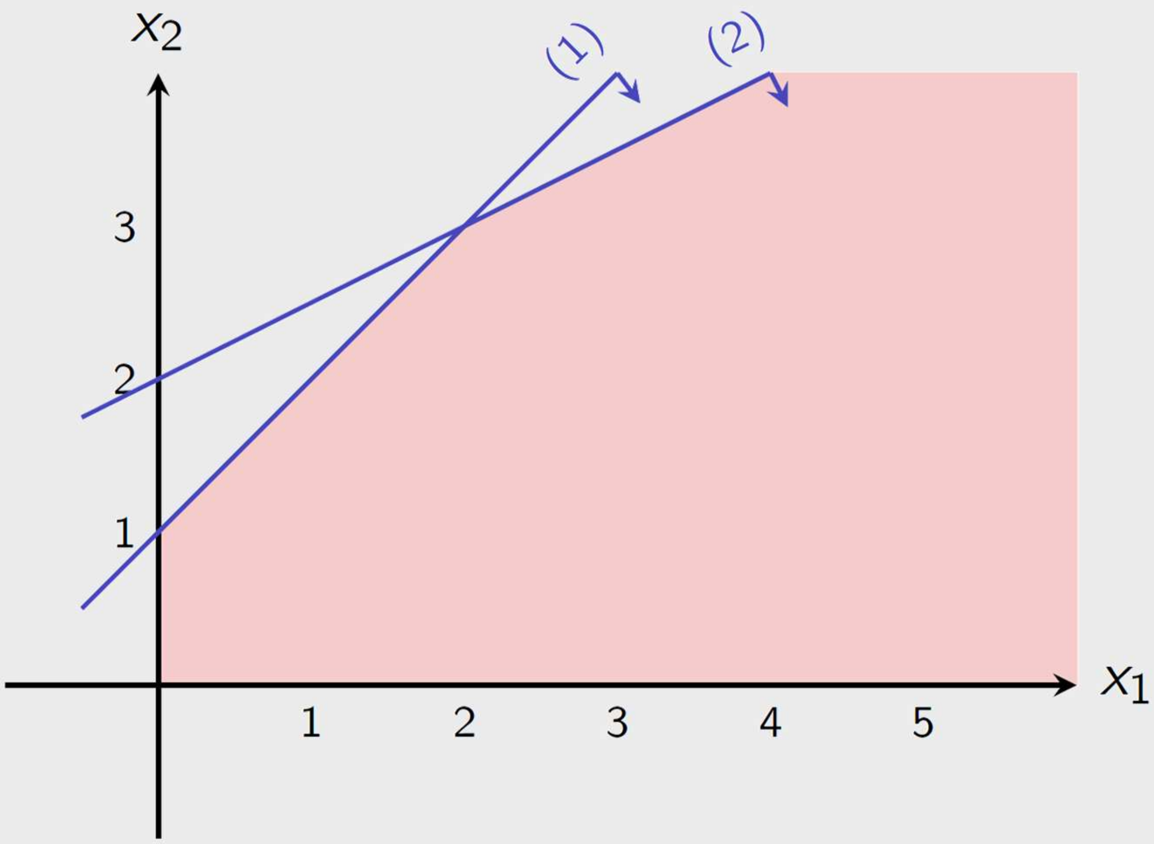
The optimal value of an optimization model is the objective function value of any optimal solution

$$
\begin{array}{lrcl}
\max & 3x_1 + x_2 & & \\
\text{s.t.} & -x_1 + x_2 & \leq 1 & (1) \\
& x_1 - 2x_2 & \geq -4 & (2) \\
& x_1 & \geq 0 & (3) \\
& x_2 & \geq 0 & (4)
\end{array}
$$

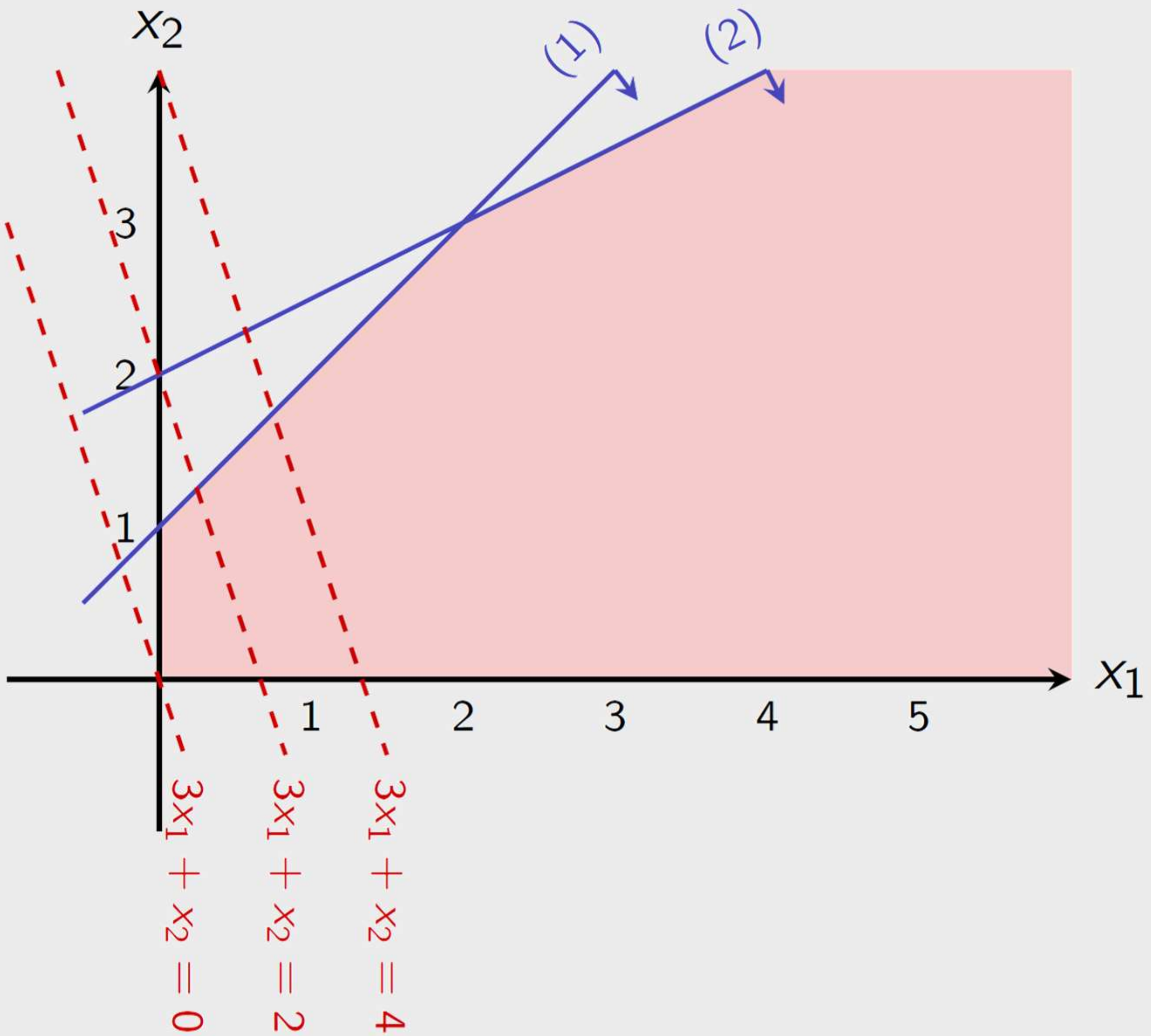


Objective Function

$$
\begin{array}{llcllr}
\max & 3x_1 & + & x_2 & & \\
\text{s.t.} & -x_1 & + & x_2 & \leq 1 & (1) \\
 & x_1 & - & 2x_2 & \geq -4 & (2) \\
 & x_1 & & & \geq 0 & (3) \\
 & & & x_2 & \geq 0 & (4)
\end{array}
$$



$$
\begin{array}{llll}
\max & 3x_1 + x_2 & & \\
\text{s.t.} & -x_1 + x_2 \leq 1 & & (1) \\
& x_1 - 2x_2 \geq -4 & & (2) \\
& x_1 \geq 0 & & (3) \\
& x_2 \geq 0 & & (4)
\end{array}
$$



Objective Function

$$
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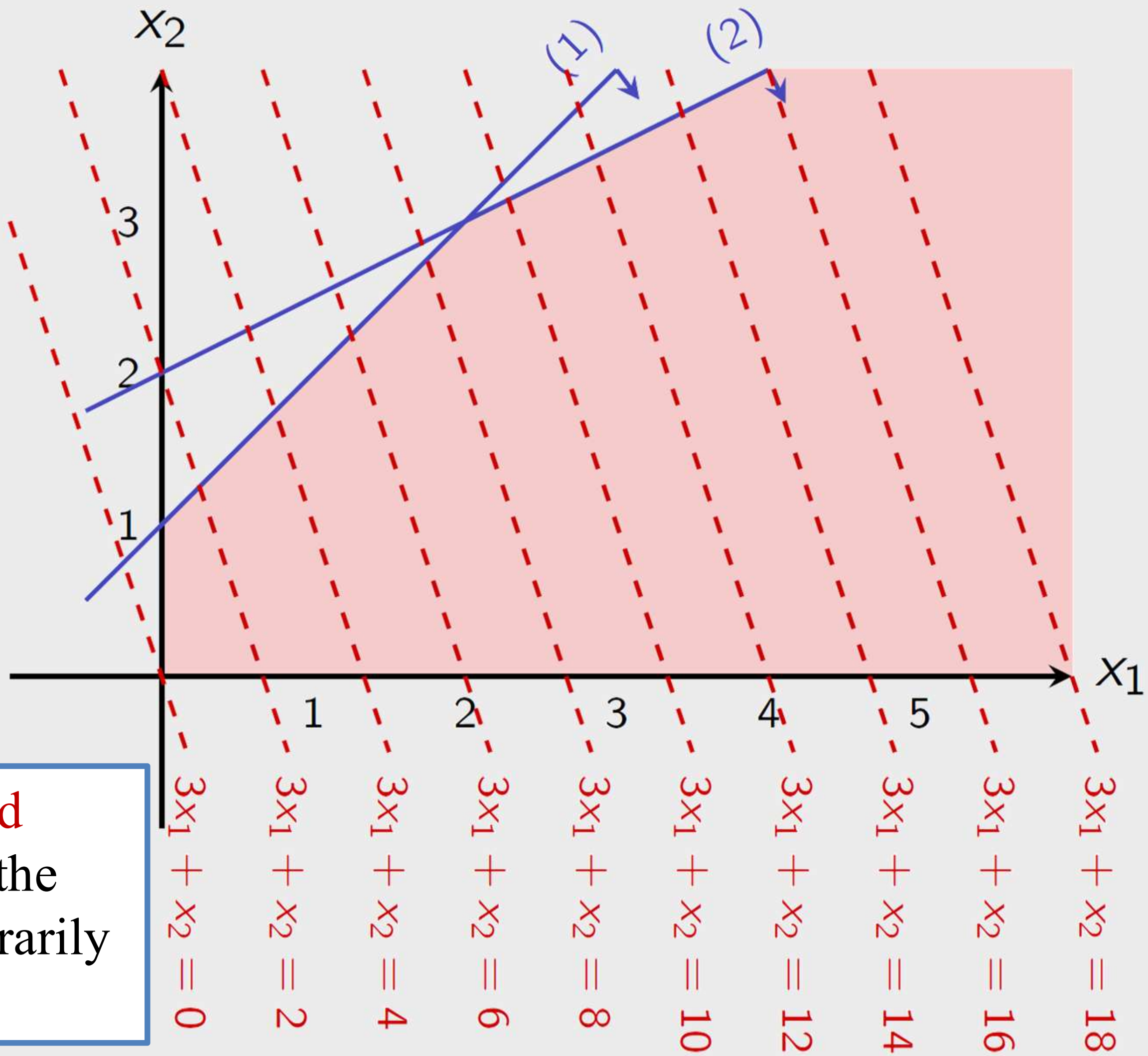
\text{s.t.} & -x_1 + x_2 \leq 1 & (1) \\
& x_1 - 2x_2 \geq -4 & (2) \\
& x_1 \geq 0 & (3) \\
& x_2 \geq 0 & (4)
\end{array}
$$

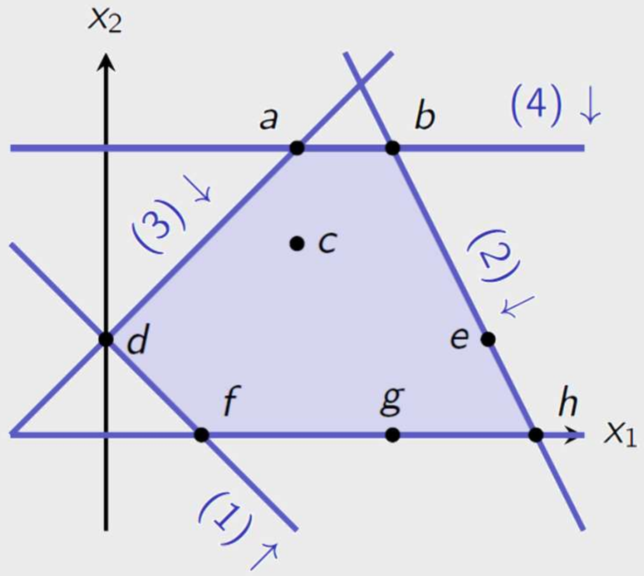
The model is "unbounded"



An optimization model is unbounded when feasible choices of values for the decision variables can produce arbitrarily good objective function values

Consider the following LP. Note that c_1 and c_2 are unspecified.

$$
\begin{array}{lll}
\min & c_1x_1 + c_2x_2 & \\
\text{s.t.} & x_1 + x_2 \geq 1 & (1) \\
 & 2x_1 + x_2 \leq 9 & (2) \\
 & x_1 - x_2 \geq -1 & (3) \\
 & x_2 \leq 3 & (4) \\
 & x_1 \geq 0, x_2 \geq 0 &
\end{array}
$$



- Suppose we have a unique optimal solution. Which points could it be, if any?
- Suppose " e " is an optimal solution. Which other points must also be optimal, if any?
- Suppose " f " and " h " are both optimal solutions. Which other points must also be optimal, if any?

Based on characteristics of:

- decision variables
- constraints
- objective function

Classification of Optimization Models

- A decision variable is continuous if it can take on any value in a specified interval
  - Example: a nonnegative variable is continuous, since it can take on any value in $[0,+\infty)$
- A decision variable is integral if it is restricted to a specified interval of integers
  - Sometimes referred to as discrete
  - Special example: binary (0 or 1) variables are integral

Classification of Optimization Models

- A function $f\,(x_1, \ldots, x_n)$ is linear (in $x_1, \ldots, x_n$) if it is a constant-weighted sum of $x_1, \ldots, x_n$
- Put another way, f can be written in the form

$$f\,(x_1, \ldots, x_n) = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

  where $c_1, \ldots, c_n$ are constants
- Otherwise, a function is nonlinear
- An objective function can be linear or nonlinear

Classification of Optimization Models

$$f(x_1, x_2, x_3) = 9x_1 - 17x_3$$

linear

$$f(x_1, x_2, x_3) = \frac{5}{x_1} + 3x_2 - 6x_3$$

nonlinear

$$f(x_1, x_2, x_3) = \sum_{j=1}^{3} x_j$$

linear

$$f(x_1, x_2, x_3) = \frac{x_1 - x_2}{x_2 + x_3}$$

nonlinear

$$f(x_1, x_2, x_3) = x_1 x_2 + 3x_3$$

nonlinear

Classification of Optimization Models

- A constraint can be written in the form

$$g(x_1, \ldots, x_n) \left\{ \begin{array}{c} \leq \\ = \\ \geq \end{array} \right\} b$$

where $g(x_1, \ldots, x_n)$ is a function of the decision variables $x_1, \ldots, x_n$ and b is a specified constant

- The Constraint is
  - linear if $g(x_1, \ldots, x_n)$ is linear in $x_1, \ldots, x_n$
  - nonlinear if $g(x_1, \ldots, x_n)$ is nonlinear in $x_1, \ldots, x_n$
- We don't allow strict inequalities (< or >) in the optimization models we study

Classification of Optimization Models

- An optimization model is a linear program (LP) if
  - the decision variables are continuous,
  - the objective function is linear, and
  - the constraints are linear
- An optimization model is a nonlinear program (NLP) if
  - the decision variables are continuous
  - the objective function is nonlinear or at least one of the constraints is nonlinear

Classification of Optimization Models

$$\begin{aligned} \min \quad & 12z_1 + 4z_3 \\ \text{s.t.} \quad & z_1 z_2 z_3 = 1 \\ & z_1, z_2 \geq 0 \end{aligned}$$

nonlinear

$$\begin{aligned} \max \quad & 3z_1 + 14z_2 + 7z_3 \\ \text{s.t.} \quad & 10z_1 + 5z_2 \leq 25 - 18z_3 \\ & z_j \geq 0 \quad \text{for } j = 1, \ldots, 3 \end{aligned}$$

linear

$$\begin{aligned} \min \quad & 7z_1 z_2 + 4z_1 z_3 + 2z_1 z_3 \\ \text{s.t.} \quad & \sum_{j=1}^{3} z_j = 2 \\ & z_j \leq 0 \quad \text{for } j = 1, \ldots, 3 \end{aligned}$$

nonlinear

Classification of Optimization Models

- An optimization model is an integer program if at least one of its decision variables is integral
- Finer classification:
  - If all variables are integral → pure integer program
  - Otherwise → mixed integer program

Classification of Optimization Models

- An optimization model is an (mixed) integer linear program (MILP/ILP) if
  - (At least one/All) of its decision variables is integral,
  - the objective function is linear, and
  - the constraints are linear
- An integer program is an integer nonlinear program (INLP) if
  - at least one of its decision variables is integral,
  - the objective function is nonlinear *or* at least one of the constraints are nonlinear

Classification of Optimization Models

$$\begin{aligned}
\min \quad & 3w_1 + 14w_2 - w_3 \\
\text{s.t.} \quad & w_1 w_2 \leq 1 \\
& w_1 + w_2 + w_3 = 10 \\
& w_j \geq 0 \quad \text{for } j = 1, \ldots, 3 \\
& w_1 \text{ integer}
\end{aligned}$$

(mixed) integer nonlinear

$$\begin{aligned}
\max \quad & 19w_1 \\
\text{s.t.} \quad & w_1 \leq w_2 \\
& w_2 + w_3 = 10 + w_1 \\
& w_2 \geq 1, w_3 \geq 1 \\
& w_1 \geq 0
\end{aligned}$$

linear

Classification of Optimization Models

- In general:
  - linear constraints and objective functions are preferred to nonlinear ones
  - continuous variables are preferred to integral variables
- Rule of thumb: when the optimal solution has values that are likely to be large enough so that fractions have no practical importance, using continuous variables is generally preferred.
- For example:
  - 2,000,000 pounds of rice vs 2,000,001 pounds of rice
  - 12 aircraft vs 13 aircraft

- Do **not** follow this rule in the exam/mandatory assignment unless it is clearly mentioned!

Classification of Optimization Models

- Sometimes, "tricks of the trade" can be used to convert certain nonlinear programs into linear programs
- Some nonlinear programs can be solved "as quickly" as a linear program
- Some integer linear programs can be solved "as quickly" as a linear program
- Linear programs are "the holy grail"

Classification of Optimization Models

- Classification of optimization models via characteristics of variables, constraints, and objective functions
  - linear programs
  - nonlinear programs
  - (pure and mixed) integer linear programs
  - (pure and mixed) integer nonlinear programs
- Linear better than nonlinear
- Continuous better than integral

Model Complexity

- Model size
  - Number of variables
  - Number of constraints
- Mathematical formulations
  - Linear/ Nonlinear formulations
- Variables
  - Continuous/ Discrete
- Data
  - Deterministic/ Stochastic
  - Aggregated / Disaggregated

A Good Model

- Ease of understanding the model
- Ease of detecting errors
- Ease of computing the solution
- Units of measurement / scaling
- Modelling languages/ Solvers

There is no recipe…

Assignment #2:

AMPL Book
Chapter 2. Exercises(1-8)

Next Lecture

Lecture #5:

LP Models