Modelling and Optimization

INF170

#8:MILP

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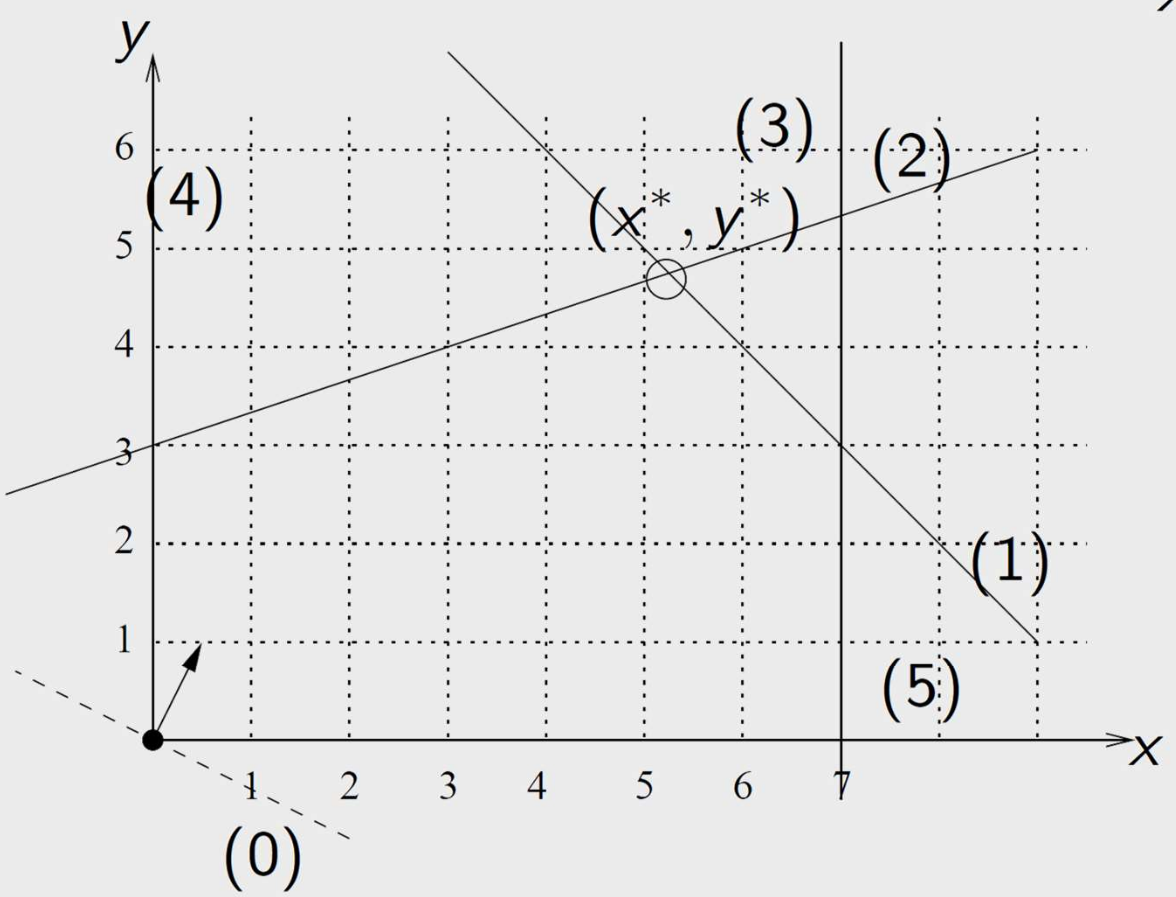
AGENDA

- Integer programming: IPs and MIPs
- Power of Binary Variables
- Facility Location Problem

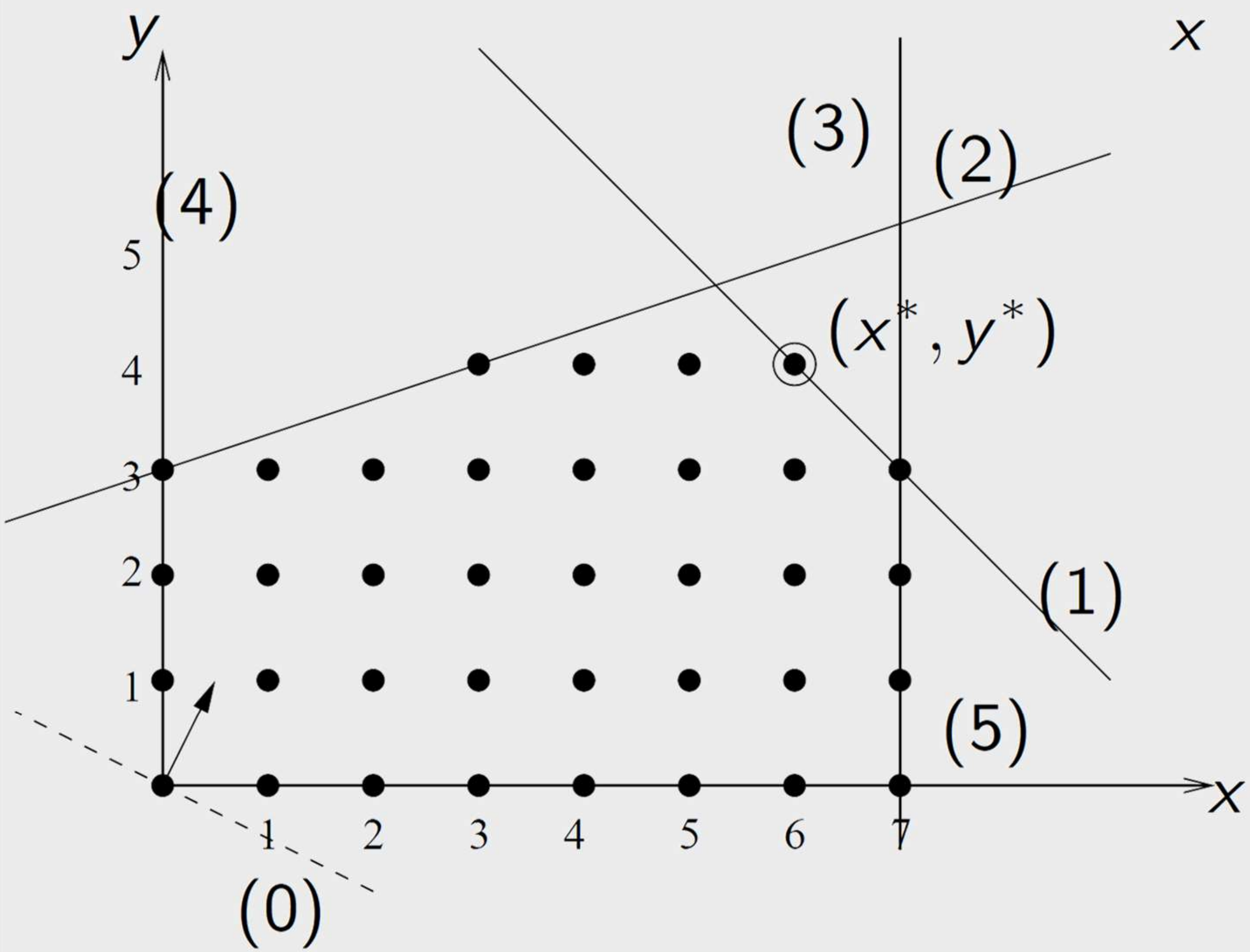
A Small Example

$$
\begin{array}{lrcrcrr}
\text{maximize} & x & + & 2y & & & (0) \\
\text{subject to} & x & + & y & \leq & 10 & (1) \\
 & -x & + & 3y & \leq & 9 & (2) \\
 & x & & & \leq & 7 & (3) \\
 & & & x, y & \geq & 0 & (4, 5)
\end{array}
$$



A Small Example

$$
\begin{array}{lrcrcrl}
\text{maximize} & x & + & 2y & & & (0)\\
\text{subject to} & x & + & y & \leq & 10 & (1)\\
& -x & + & 3y & \leq & 9 & (2)\\
& x & & & \leq & 7 & (3)\\
& & & x, y & \geq & 0 & (4,5)\\
& & & x, y & & \text{integer} &
\end{array}
$$



Diet Model

	cost	min	max	A	C	B1	B2	NA	CAL
BEEF	3.19	2	10	60	20	10	15	938	295
CHK	2.59	2	10	8	0	20	20	2180	770
FISH	2.29	2	10	8	10	15	10	945	440
HAM	2.89	2	10	40	40	35	10	278	430
MCH	1.89	2	10	15	35	15	15	1182	315
MTL	1.99	2	10	70	30	15	15	896	400
SPG	1.99	2	10	25	50	25	15	1329	370
TUR	2.49	2	10	60	20	15	10	1397	450

	min	max
A	700	20000
C	700	20000
B1	700	20000
B2	700	20000
NA	0	50000
CAL	16000	24000

```
Xbeef  = 5.36061
Xchk   = 2
Xfish  = 2
Xham   = 10
Xmch   = 10
Xmtl   = 10
Xspg   = 9.30605
Xtur   = 2
Objective: 118.0594032
```

	Diet
A	1956.29
B1	1036.26
B2	700
C	1682.51
NA	50000
CAL	19794.6

Diet Model

	cost	min	max	A	C	B1	B2	NA	CAL
BEEF	3.19	2	10	60	20	10	15	938	295
CHK	2.59	2	10	8	0	20	20	2180	770
FISH	2.29	2	10	8	10	15	10	945	440
HAM	2.89	2	10	40	40	35	10	278	430
MCH	1.89	2	10	15	35	15	15	1182	315
MTL	1.99	2	10	70	30	15	15	896	400
SPG	1.99	2	10	25	50	25	15	1329	370
TUR	2.49	2	10	60	20	15	10	1397	450

	min	max
A	700	20000
C	700	20000
B1	700	20000
B2	700	20000
NA	0	50000
CAL	16000	24000

```
Xbeef = 5
Xchk  = 2
Xfish = 2
Xham  = 10
Xmch  = 10
Xmtl  = 10
Xspg  = 9
Xtur  = 2
```

	Diet
A	1927
B1	1025
B2	**690**
C	1660
NA	49255
CAL	19575

Diet Model

	cost	min	max	A	C	B1	B2	NA	CAL
BEEF	3.19	2	10	60	20	10	15	938	295
CHK	2.59	2	10	8	0	20	20	2180	770
FISH	2.29	2	10	8	10	15	10	945	440
HAM	2.89	2	10	40	40	35	10	278	430
MCH	1.89	2	10	15	35	15	15	1182	315
MTL	1.99	2	10	70	30	15	15	896	400
SPG	1.99	2	10	25	50	25	15	1329	370
TUR	2.49	2	10	60	20	15	10	1397	450

	min	max
A	700	20000
C	700	20000
B1	700	20000
B2	700	20000
NA	0	50000
CAL	16000	24000

```
Xbeef = 6
Xchk  = 2
Xfish = 2
Xham  = 10
Xmch  = 10
Xmtl  = 10
Xspg  = 10
Xtur  = 2
```

	Diet
A	2012
B1	1060
B2	720
C	1730
NA	**51522**
CAL	20240

Diet Model

	cost	min	max	A	C	B1	B2	NA	CAL
BEEF	3.19	2	10	60	20	10	15	938	295
CHK	2.59	2	10	8	0	20	20	2180	770
FISH	2.29	2	10	8	10	15	10	945	440
HAM	2.89	2	10	40	40	35	10	278	430
MCH	1.89	2	10	15	35	15	15	1182	315
MTL	1.99	2	10	70	30	15	15	896	400
SPG	1.99	2	10	25	50	25	15	1329	370
TUR	2.49	2	10	60	20	15	10	1397	450

	min	max
A	700	20000
C	700	20000
B1	700	20000
B2	700	20000
NA	0	50000
CAL	16000	24000

```
Xbeef  = 9
Xchk   = 2
Xfish  = 2
Xham   = 8
Xmch   = 10
Xmtl   = 10
Xspg   = 7
Xtur   = 2
Objective: 119.3
```

	Diet
A	2037
B1	945
B2	700
C	1560
NA	49793
CAL	19155

Integral Decision Variables

- A decision variable is <u>integral</u> if it is restricted to a specified interval of integers
  - Sometimes referred to as <u>discrete</u>
  - Example: <u>binary</u> (0 or 1) variables are integral

$$x_i \text{ binary} = x_i \in \{0,1\} = \begin{cases} x_i & \leq 1 \\ x_i & \geq 0 \\ x_i & \text{integer} \end{cases}$$

Integer Programs

- An optimization model is an integer program if at least one of its decision variables is integral
- Finer classification:
  - If all variables are integral → pure integer program
  - Otherwise → mixed integer program

MILP

- Advantages of integer programs
  - More realistic
  - More modeling power
- Disadvantages of integer programs
  - More difficult to model
  - Can be much more difficult to solve

Banner Chemicals

Situation

- Banner Chemicals manufactures specialty chemicals. One of their products comes in two grades, high and supreme. The capacity at the plant is 110 barrels per week.
- The high and supreme grade products use the same basic raw materials but require different ratios of additives. The high grade requires 3 gallons of additive A and 1 gallon of additive B per barrel while the supreme grade requires 2 gallons of additive A and 3 gallons of additive B per barrel.
- The supply of both of these additives is quite limited. Each week, this product line is allocated only 300 gallons of additive A per week and 280 gallons of additive B.
- A barrel of the high grade has a profit margin of $80 per barrel while the supreme grade has a profit margin of $200 per barrel.

Question

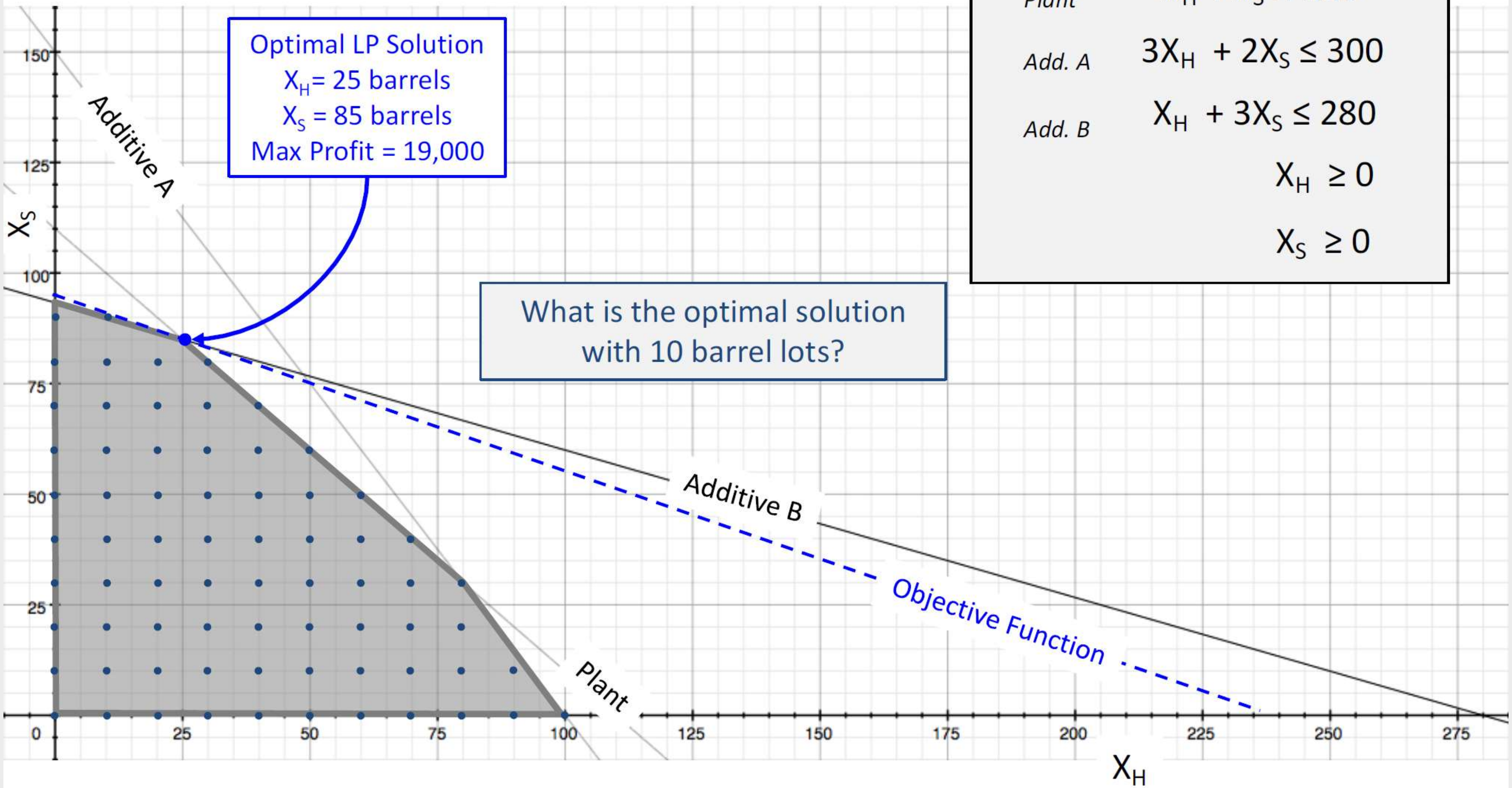
- How many barrels of High and Supreme grade should Banner Chemicals produce each week assuming you can only produce in 10 barrel lots?

Banner Chemicals

$$\text{Max } z(X_H, X_S) = 80X_H + 200X_S$$

s.t.

Plant	$X_H + X_S \leq 110$
Add. A	$3X_H + 2X_S \leq 300$
Add. B	$X_H + 3X_S \leq 280$
	$X_H \geq 0$
	$X_S \geq 0$



- Let's try "rounding" the solution to the closest acceptable integer values?
  - LP Solution:
    - X_H=25 barrels X_S= 85 barrels
  - Rounding to closest "10 barrel" solution for (X_H, X_S):
    1. $z_{LOT}(30, 90) = \$20,400$ but it is infeasible (Plant constraint)
    2. $z_{LOT}(30, 80) = \$18,400$ feasible
    3. $z_{LOT}(20, 90) = \$19,600$ but it is infeasible (Additive B constraint)
  - So, using this approach $z^*_{LOT} = \$18,400$ with X_H=30, X_S=80
  - But, is it the best?
- Let's solve all of the points to make sure!

  This approach is called *Mass Enumeration*.

Banner Chemicals

$$\text{Max } z(X_H, X_S) = 80X_H + 200X_S$$

s.t.

Plant	$X_H + X_S \leq 110$
Add. A	$3X_H + 2X_S \leq 300$
Add. B	$X_H + 3X_S \leq 280$
	$X_H \geq 0$
	$X_S \geq 0$

Optimal IP Solution
X_H= 10 barrels
X_S = 90 barrels
Max Profit = 18,800

Optimal LP Solution
X_H= 25 barrels
X_S = 85 barrels
Max Profit = 19,000

Closest "rounded" LP Solution
X_H= 30 barrels
X_S = 80 barrels
Max Profit = 18,400

Barrels of Xs											
100	x	x	x	x	x	x	x	x	x	x	x
90	$ 18,000	$ 18,800	x	x	x	x					
80	$ 16,000	$ 16,800	$ 17,600	$ 18,400	x	x					
70	$ 14,000	$ 14,800	$ 15,600	$ 16,400	$ 17,200	x					
60	$ 12,000	$ 12,800	$ 13,600	$ 14,400	$ 15,200	$ 16,000	x	x	x	x	x
50	$ 10,000	$ 10,800	$ 11,600	$ 12,400	$ 13,200	$ 14,000	$ 14,800	x	x	x	x
40	$ 8,000	$ 8,800	$ 9,600	$ 10,400	$ 11,200	$ 12,000	$ 12,800	$ 13,600	x	x	x
30	$ 6,000	$ 6,800	$ 7,600	$ 8,400	$ 9,200	$ 10,000	$ 10,800	$ 11,600	$ 12,400	x	x
20	$ 4,000	$ 4,800	$ 5,600	$ 6,400	$ 7,200	$ 8,000	$ 8,800	$ 9,600	$ 10,400	x	x
10	$ 2,000	$ 2,800	$ 3,600	$ 4,400	$ 5,200	$ 6,000	$ 6,800	$ 7,600	$ 8,400	$ 9,200	x
0	$ -	$ 800	$ 1,600	$ 2,400	$ 3,200	$ 4,000	$ 4,800	$ 5,600	$ 6,400	$ 7,200	$ 8,000
	0	10	20	30	40	50	60	70	80	90	100
	Barrels of Xh										

Each cell shows $z = 80X_H + 200X_S$
x indicates infeasible solution

Banner Chemicals

In order to solve in integer values of "lots of ten", we need to:

- ✓ Convert Decision Variables
  - XHL = XH /10 XSL = XS /10
- ✓ Scale the coefficients and constraint RHS
  - e.g. 110 barrels becomes 11 lots of ten
- ✓ Indicate that the new variables are Integers

$$
\begin{array}{ll}
\text{Max } z(X_H, X_S) = 80X_H + 200X_S & \\
\quad s.t. & \\
\textit{Plant} & X_H + X_S \le 110 \\
\textit{Add. A} & 3X_H + 2X_S \le 300 \\
\textit{Add. B} & X_H + 3X_S \le 280 \\
 & X_H \ge 0 \\
 & X_S \ge 0
\end{array}
$$

→

$$
\begin{array}{ll}
\text{Max } z(X_{HL}, X_{SL}) = 800X_{HL} + 2000X_{SL} & \\
\quad s.t. & \\
\textit{Plant} & X_{HL} + X_{SL} \le 11 \\
\textit{Add. A} & 3X_{HL} + 2X_{SL} \le 30 \\
\textit{Add. B} & X_{HL} + 3X_{SL} \le 28 \\
 & X_{HL}, X_{SL} \ge 0 \text{ Integers}
\end{array}
$$

Integral Decision Variables

Why use them?

- When its physically impossible to have fractional solutions
  - For example; number of people to hire, number of ships to make
  - However, in some cases when we are dealing with large numbers, continuous is fine
- Modeling logical conditions (Binary)
  - ✓ If Then:
    - If we have product leaving plant A then we must open it
  - ✓ Either Or:
    - We can either produce ≥1000 units or none at all.
  - ✓ Select From:
    - We must select ≥ 4 DCs to open from the 10 possible
    - We must select ≤ 5 products to make from the 15 available

Modeling Mutually Exclusive Choices

- Sometimes decisions are mutually exclusive
- e.g. "At most one of these projects should be executed."
- Let

$$x_i = \begin{cases} 1 & \text{if element } i \text{ is picked} \\ 0 & \text{otherwise} \end{cases}$$

- Let E be a set of elements, from which we want to make sure at most one is picked
- Can model these types of constraints as:

$$\sum_{i \in E} x_i \leq 1$$

$$x_i \in \{0,1\} \quad \text{for } i \in E$$

Modeling Dependencies between Decisions

- Sometimes one decision depends on another being made (or not being made)
- Suppose activity *j* depends on activity *k* being performed
- Can model these types of constraints as:

$$x_j \leq x_k$$

- x_j cannot be equal to 1 unless x_k is equal to 1 also

Covering Constraints

- Sometimes we want to make sure at least one element from a set of elements is picked
- Let E be a set of elements, from which we want to make sure at least one is picked
- Can model these types of constraints as:

$$\sum_{i \in E} x_i \geq 1$$

$$x_i \in \{0,1\} \quad \text{for } i \in E$$

- "We've covered E"

Partitioning Constraints

- Sometimes we want to make sure exactly one element from a set of elements is picked
- Let E be a set of elements, from which we want to make sure exactly one is picked
- Can model these types of constraints as:

$$\sum_{i \in E} x_i = 1$$

$$x_i \in \{0,1\} \quad \text{for } i \in E$$

Fixed Charges

- Linear programming: easy to model linear (per-unit) costs
- Sometimes we need to model fixed costs
- Example: inventory management
  - holding cost h_j per jacket in period j
  - setup cost f_j must be paid before warehouse can be used in period j

$\rightarrow$ cost of holding x_j jackets in inventory in period j

$$\theta(x_j) = \begin{cases} f_j + h_j x_j & \text{if } x_j > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Nonlinear costs!
- How to model as linear costs using binary variables?

Fixed Charges

$$\theta(x_j) = \begin{cases} f_j + h_j x_j & \text{if } x_j > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Introduce binary variable y_j

$$y_i = \begin{cases} 1 & \text{if } x_j > 0 \ \text{(if there is inventory held in period } j\text{)} \\ 0 & \text{otherwise} \end{cases}$$

- Objective (for x_j and y_j): $f_j \ y_j + h_j x_j$
- Link between x_j and y_j : $0 \le x_j \le M \, y_j$
    - M is large enough number
    - Can use upper bound on x_j (if it is available) instead of M
    - $y_j = 0 \ \rightarrow \ x_j = 0, \ y_j = 1 \rightarrow x_j \le M$

Sanity Check

- When writing constraints with binary variables, it is sometimes helpful to do a sanity check by checking all possible combinations of decision variable values
- For example, "If I get (x_5), I should definitely get a (x_6)."

x_5	x_6	OK?
0	0	Yes
1	0	No
0	1	Yes
1	1	Yes

- The constraint we formulated

$$x_5 \leq x_6$$

works with this table

FACILITY LOCATION

- Willy Wonka is opening new chocolate factories to meet demand in new markets
- Data:
  - C = set of markets (or customers) that need to be served
  - F = set of possible factory locations
  - d_j = demand of customer j , for each $j \in C$
  - $c_{i,j}$ = unit cost of fulfilling j's demand from factory location i
  - f_i = fixed cost of opening factory at location i
  - u_i = capacity of factory at location i
- Which factories should Willy Wonka open in order to meet demand in these new markets at minimum cost?

Decision variables:

- Which factories to open?

$$y_i = \begin{cases} 1 & \text{if facility } i \text{ is opened} \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } i \in F$$

- How to allocate demand to factories?

- $x_{i,j}$ = fraction of customer j demand satisfied by facility i

  for $i \in F$, $j \in C$

Facility Location

Objective:

$$min \sum_{i \in F} \sum_{j \in C} (c_{i,j} d_j) x_{i,j} + \sum_{i \in F} f_i y_i$$

Demand satisfied for each customer $j \in C$:

$$\sum_{i \in F} x_{i,j} = 1 \qquad \text{for } j \in C$$

Capacity for each open factory $i \in F$:

$$\sum_{j \in C} d_j x_{i,j} \leq u_i y_i \qquad \text{for } i \in F$$

Variable-type constraints:

$$x_{i,j} \geq 0 \text{ for } i \in F \text{ , } j \in C$$

$$y_i \in \{0,1\} \text{ for } i \in F$$

GoNuts Juice Company: Model 1

GoNuts manufactures different juices made entirely of various exotic nuts. Their primary market is China and they operate three plants located in Ethiopia, Tanzania, and Nigeria. You have been asked to help them determine where to manufacture the two newest juices they offer, Gingko Nut and Kola Nut. Each plant has a different variable cost structure and capacity for manufacturing the different juices. Also, each juice has an expected demand.

Cost/Unit	Ginko	Kola
Ethiopia	¥21.00	¥22.50
Tanzania	¥22.50	¥24.50
Nigeria	¥23.00	¥25.50

Capacity	Units/Month
Ethiopia	425
Tanzania	400
Nigeria	750

Demand	Units/Month
Ginko	550
Kola	450

How much of each juice should be made at each plant in order to minimize total cost while meeting demand and adhering to plant capacity?

Gonuts Juice Company: Model 1

Decision variables:

$x_{G,E}$ = Number of *Ginko Juice* units made in *Ethiopia* plant

x_{ij} = Number of units of product i made in plant j

$x_{ij} \geq 0$ for all i,j

Objective Function:

$$min\, z = \sum_i \sum_j c_{ij} x_{ij}$$

c_{ij} = Cost per unit of product i made at plant j

GONUTS JUICE COMPANY: MODEL 1

Constraints:

Plant capacity:

$$\sum_i x_{ij} \leq C_j \qquad \forall j$$

C_j = Capacity in units at plant j

Product demand:

$$\sum_j x_{ij} \geq D_i \qquad \forall i$$

D_i = Demand for product i in units

$$\min z = \sum_i \sum_j c_{ij} x_{ij}$$

s.t.

$$\sum_i x_{ij} \le C_j \qquad \forall j$$

$$\sum_j x_{ij} \ge D_i \qquad \forall i$$

$$x_{ij} \ge 0 \qquad \forall\, i,j$$

Where:

x_{ij} = Number of units of product i made in plant j

c_{ij} = Cost per unit of product i made at plant j

C_j = Capacity in units at plant j

D_i = Demand for product i in units

Optimal Solution

	Ginko	Kola
Ethiopia	0	425
Tanzania	375	25
Nigeria	175	0

Total min cost = ¥ 22,637.50

GONUTS JUICE COMPANY: MODEL 2

GoNuts manufactures different juices made entirely of various exotic nuts. Their primary market is China and they operate three plants located in Ethiopia, Tanzania, and Nigeria. You have been asked to help them determine where to manufacture the two newest juices they offer, Gingko Nut and Kola Nut. Each plant has a different fixed and variable cost structure and capacity for manufacturing the different juices. The fixed cost only applies if the plant produces any juice. Also, each juice has an expected demand.

Cost/Unit	Ginko	Kola
Ethiopia	¥21.00	¥22.50
Tanzania	¥22.50	¥24.50
Nigeria	¥23.00	¥25.50

Capacity	Units/Month	Fixed (¥/Month)
Ethiopia	425	¥1,500
Tanzania	400	¥2,000
Nigeria	750	¥3,000

Demand	Units/Month
Ginko	550
Kola	450

How much of each juice should be made at each plant in order to minimize total cost while meeting demand and adhering to plant capacity?

GONUTS JUICE COMPANY: MODEL 2

Decision variables:

x_{ij} = Number of units of product i made in plant j

y_i = 1 if plant j is opened; = 0 otherwise

$x_{ij} \geq 0$ for all i,j

Objective Function:

$$min\, z = \sum_i \sum_j c_{ij} x_{ij} + \sum_j f_j y_j$$

c_{ij} = Cost per unit of product i made at plant j

f_j = Fixed cost per month if plant j is used

Gonuts Juice Company: Model 2

- You need to ensure that if a plant produces product, then it is actually opened!
- If Then conditions require both a
  - ✓ Binary Variable
  - ✓ Linking Constraint

New Constraints:

If-Then Condition:

$$\sum_{i} x_{ij} \leq M y_j \qquad \forall j$$

M = a big number (such as C_j in this case)

C_j = Capacity in units at plant j

Gonuts Juice Company: Model 2

$$\min z = \sum_i \sum_j c_{ij} x_{ij} + \sum_j f_j y_j$$

s.t.

$$\sum_i x_{ij} \le C_j \qquad \forall j$$

$$\sum_j x_{ij} \ge D_i \qquad \forall i$$

$$\sum_i x_{ij} - M y_j \le 0 \qquad \forall j$$

$$x_{ij} \ge 0 \qquad \forall\, i,j$$

$$y_j = \{0,1\} \qquad \forall\, j$$

Where:

x_{ij} = Number of units of product i made in plant j

y_i = 1 if plant j is opened; = 0 otherwise

c_{ij} = Cost per unit of product i made at plant j

f_j = Fixed cost per month if plant j is used

C_j = Capacity in units at plant j

D_i = Demand for product i in units

M = a big number (such as C_j in this case)

	Ginko	Kola
Ethiopia	0	425
Tanzania	0	0
Nigeria	550	25

$z^* =$ ¥ 27,350.00

GONUTS JUICE COMPANY: MODEL 3

GoNuts manufactures different juices made entirely of various exotic nuts. Their primary market is China and they operate three plants located in Ethiopia, Tanzania, and Nigeria. You have been asked to help them determine where to manufacture the two newest juices they offer, Gingko Nut and Kola Nut. Each plant has a different fixed and variable cost structure and both minimum and maximum capacities for manufacturing the different juices if the plant opens. The fixed cost only applies if the plant produces any juice. Also, each juice as an expected demand.

Cost/Unit	Ginko	Kola
Ethiopia	¥21.00	¥22.50
Tanzania	¥22.50	¥24.50
Nigeria	¥23.00	¥25.50

Demand	Units/Month
Ginko	550
Kola	450

Capacity (units/Month)	Max Capacity	Min Capacity	Fixed (¥/Month)
Ethiopia	425	100	¥1,500
Tanzania	400	250	¥2,000
Nigeria	750	600	¥3,000

If the Nigeria plant opens, it must produce at least 600 units

How much of each juice should be made at each plant in order to minimize total cost while meeting demand and adhering to plant capacity?

GONUTS JUICE COMPANY: MODEL 3

- We need to add a constraint that ensures that if we DO use plant j, that the volume is between the minimum allowable level, L_j, and the maximum capacity, C_j. This is sometimes called an Either-Or condition.

New Constraints:

Either-Or Condition:

$$\sum_i x_{ij} \geq L_j y_j \qquad \forall j$$

L_j = Minimum level of production at plant j

GONUTS JUICE COMPANY: MODEL 3

$$\min z = \sum_i \sum_j c_{ij} x_{ij} + \sum_j f_j y_j$$

s.t.

$$\sum_i x_{ij} \le C_j \qquad \forall j$$

$$\sum_j x_{ij} \ge D_i \qquad \forall i$$

$$\sum_i x_{ij} - M y_j \le 0 \qquad \forall j$$

$$\sum_i x_{ij} - L_j y_j \ge 0 \qquad \forall j$$

$$x_{ij} \ge 0 \qquad \forall\, i,j$$

$$y_j = \{0,1\} \qquad \forall\, j$$

Where:

x_{ij} = Number of units of product i made in plant j

y_i = 1 if plant j is opened; = 0 otherwise

c_{ij} = Cost per unit of product i made at plant j

f_j = Fixed cost per month if plant j is used

C_j = Capacity in units at plant j

L_j = Minimum level of production at plant j

D_i = Demand for product i in units

M = a big number (such as C_j in this case)

	Ginko	Kola
Ethiopia	0	425
Tanzania	375	25
Nigeria	175	0

$z^* = ¥\,27{,}425.00$

GONUTS JUICE COMPANY: MODEL 4

GoNuts manufactures different juices made entirely of various exotic nuts. Their primary market is China and they operate three plants located in Ethiopia, Tanzania, and Nigeria. You have been asked to help them determine where to manufacture the two newest juices they offer, Gingko Nut and Kola Nut. Each plant has a different variable cost structure and a maximum capacity. GoNuts can only operate 2 plants at a maximum. Also, each juice has an expected demand.

Cost/Unit	Ginko	Kola
Ethiopia	¥21.00	¥22.50
Tanzania	¥22.50	¥24.50
Nigeria	¥23.00	¥25.50

Capacity	Units/Month
Ethiopia	425
Tanzania	400
Nigeria	750

Demand	Units/Month
Ginko	550
Kola	450

How much of each juice should be made at each plant in order to minimize total cost while meeting demand and adhering to plant capacity?

Gonuts Juice Company: Model 4

- We need to add a constraint that ensures that only N plants are used! We will use the Binary Variables, y_j, the Linking Constraints, and a new constraint that says the sum of the Binary Variables must not exceed N. This is sometimes called an Select-From condition.

New Constraints:

Max Plants (Select-From condition):

$$\sum_j y_j \leq N$$

N = Number of plants allowed to be opened

Gonuts Juice Company: Model 4

$$\min z = \sum_i \sum_j c_{ij} x_{ij}$$

s.t.

$$\sum_i x_{ij} \le C_j \qquad \forall j$$

$$\sum_j x_{ij} \ge D_i \qquad \forall i$$

$$\sum_i x_{ij} - M y_j \le 0 \qquad \forall j$$

$$\sum_j y_j \le N$$

$$x_{ij} \ge 0 \qquad \forall\, i,j$$

$$y_j = \{0,1\} \qquad \forall\, j$$

Where:

x_{ij} = Number of units of product i made in plant j

y_i = 1 if plant j is opened; = 0 otherwise

c_{ij} = Cost per unit of product i made at plant j

C_j = Capacity in units at plant j

D_i = Demand for product i in units

M = a big number (such as C_j in this case)

N = Number of plants allowed to be opened

	Ginko	Kola
Ethiopia	0	425
Tanzania	0	0
Nigeria	550	25

z^*= ¥ 22,850.50

ASSIGNMENT #5:

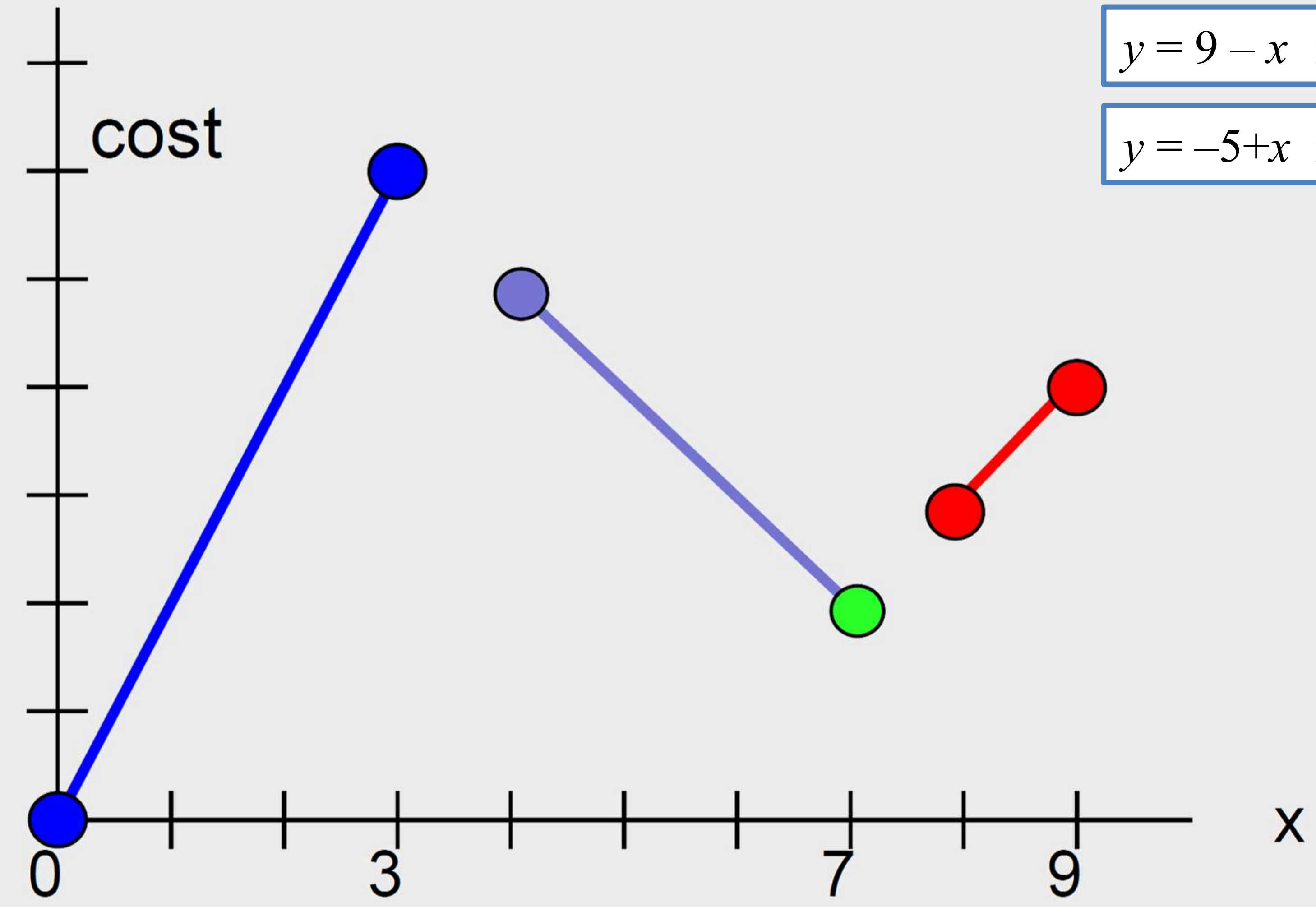
Write the linear form of the following questions.

Question 1:

$y = 2x \quad \text{if } 0 \le x \le 3$

$y = 9 - x \quad \text{if } 4 \le x \le 7$

$y = -5 + x \quad \text{if } 8 \le x \le 9$



Question 2:

x is an integer variable

$$\text{If} \quad 2x_1 + x_2 \leq 5 \quad \text{then} \quad 2x_3 - x_4 \geq 2$$

LECTURE #9:

IP/MILP MODELS