



# ROOTS

---

Eric Prebys



# Finding Roots of an Equation

- The roots of a function  $f(x)$  are all points (if any) for which

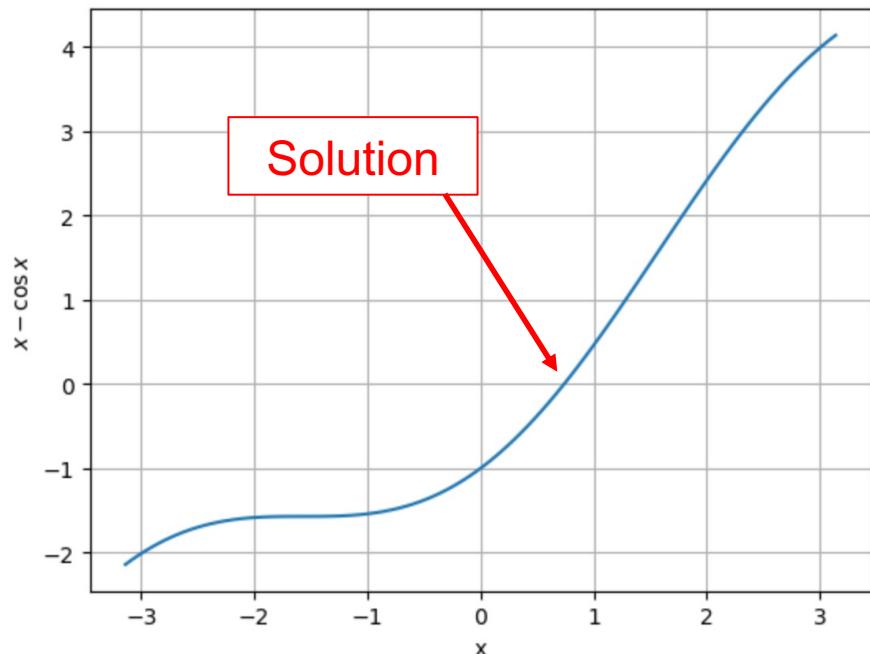
$$f(x) = 0$$

- I can express any explicit or non-explicit function as a root problem.  
For example,

$$x = \cos x$$

has no explicit solution for  $x$ , but it will be satisfied by the root of

$$f(x) = x - \cos x$$



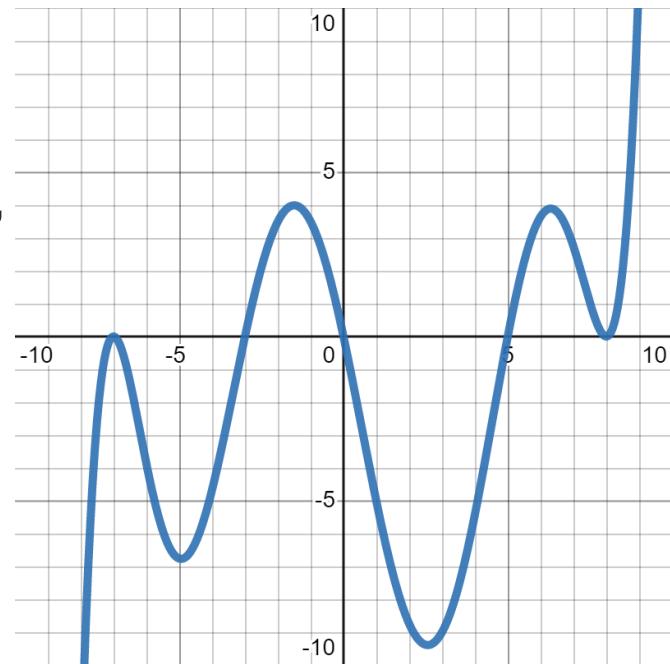


# Iterations and Convergence

- In general, we will iterate multiple times, each time finding a new value of  $x^{(i)}$ , presumably getting closer to the true root  $x^*$
- There's a lot of formalism for evaluating convergence, but we'll focus on simple tests such as

$$|f(x^{(i)})| \leq \epsilon \quad \text{or} \quad |x^{(i)} - x^{(i-1)}| \leq \epsilon$$

- Since many functions will have multiple roots, we will usually need to start with a bracket or at least an initial guess.

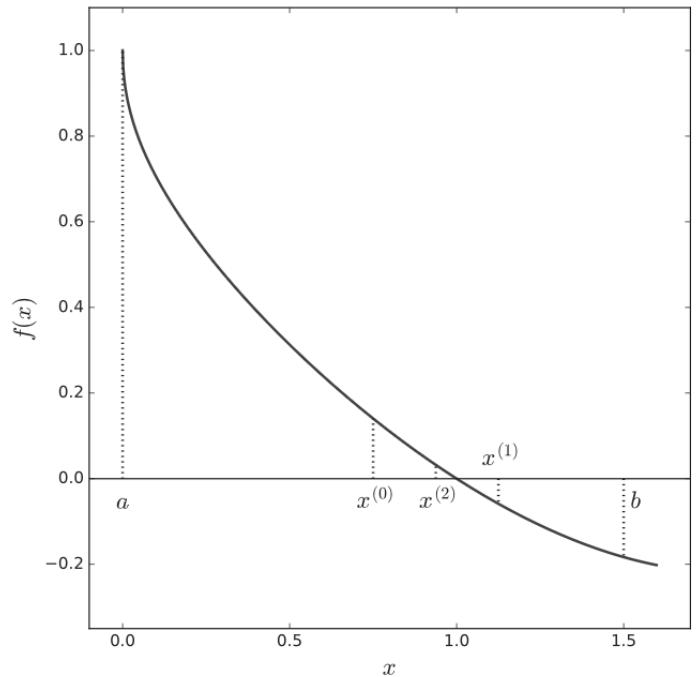




# Bisection Method

- We start with two points  $a_0$  and  $b_0$  that bracket the root; ie

$$f(a_0) \cdot f(b_0) < 0$$



- Now calculate the midpoint and see which side of the root it's on

$$c_0 = \frac{1}{2}(a_0 + b_0)$$

$$f(a_0) \cdot f(c_0) > 0 \rightarrow a_1 = c_0; b_1 = b_0$$

$$f(a_0) \cdot f(c_0) < 0 \rightarrow a_1 = a_0; b_1 = c_0$$

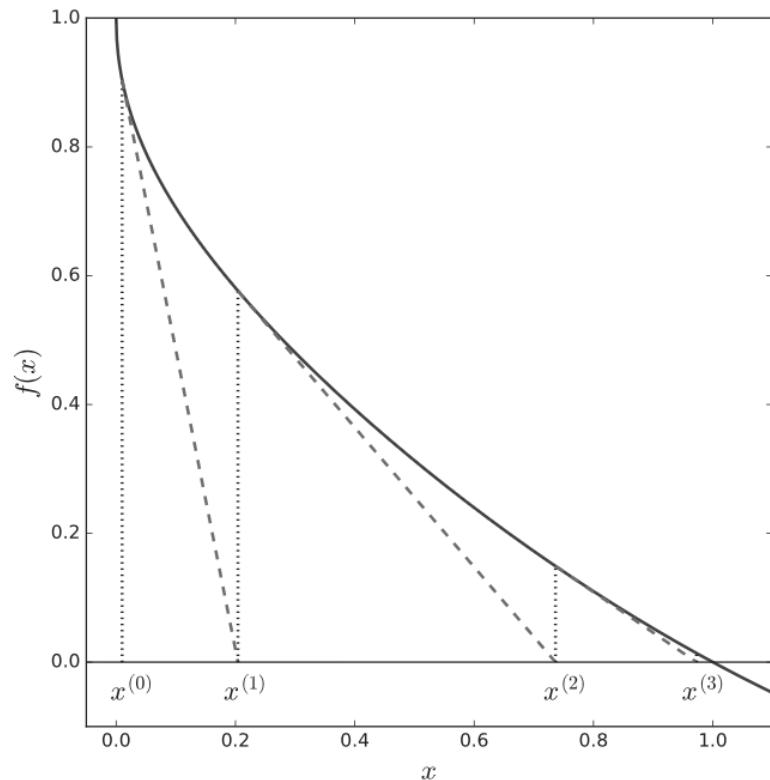
- Repeat until convergence criterion is met.



# Newton's Method

- Newton's method requires that we know the derivative of the function  $f'(x)$
- Each iteration uses the local slope to project to the next iteration.

$$x^{(i+1)} = x^{(i)} - \frac{f(x^{(i)})}{f'(x^{(i)})}$$



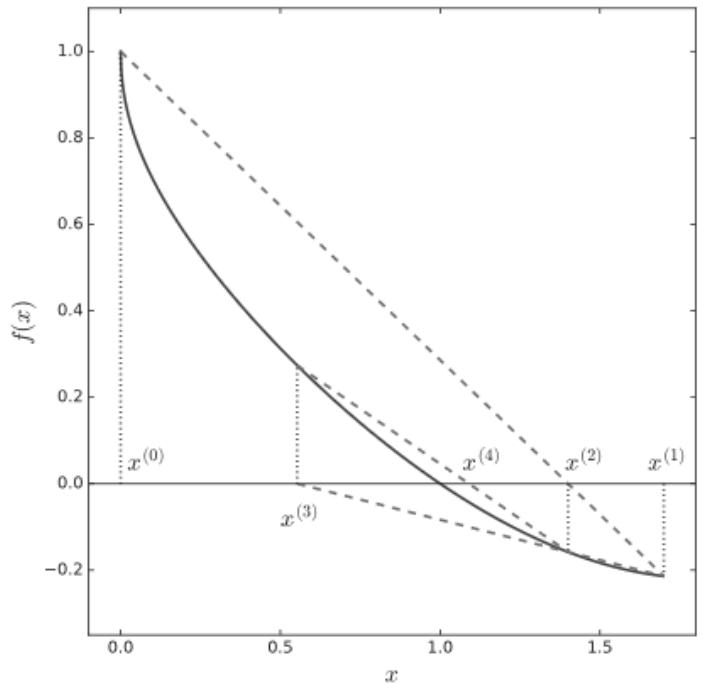


# Secant Method

- Similar to the Newton Method, but use two initial guesses to approximate the derivative

$$f'(x^{(k-1)}) \approx \frac{f(x^{(k-1)}) - f(x^{(k-2)})}{x^{(k-1)} - x^{(k-2)}}$$

$$\rightarrow x^{(k)} = x^{(k-1)} - f(x^{(k-1)}) \frac{x^{(k-1)} - x^{(k-2)}}{f(x^{(k-1)}) - f(x^{(k-2)})}$$



- You'll do algorithmic examples of these three methods in lab next week.