PION DECAY EXAMPLE

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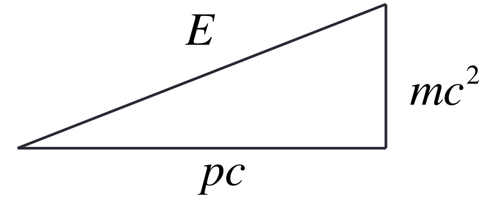
Phy 40
Fall 2025

Relativity and Units

Remember forever!



- Basic Relativity

$$\beta \equiv \frac{v}{c}$$

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

$$\text{momentum } p = \gamma m v$$

$$\text{total energy } E = \gamma m c^2$$

$$\text{kinetic energy } K = E - mc^2$$

$$E = \sqrt{\left(mc^2\right)^2 + \left(pc\right)^2}$$

Some Handy Relationships

$$\beta = \frac{pc}{E}$$

$$\gamma = \frac{E}{mc^2}$$

$$\beta\gamma = \frac{pc}{mc^2}$$

These units make these relationships really easy to calculate

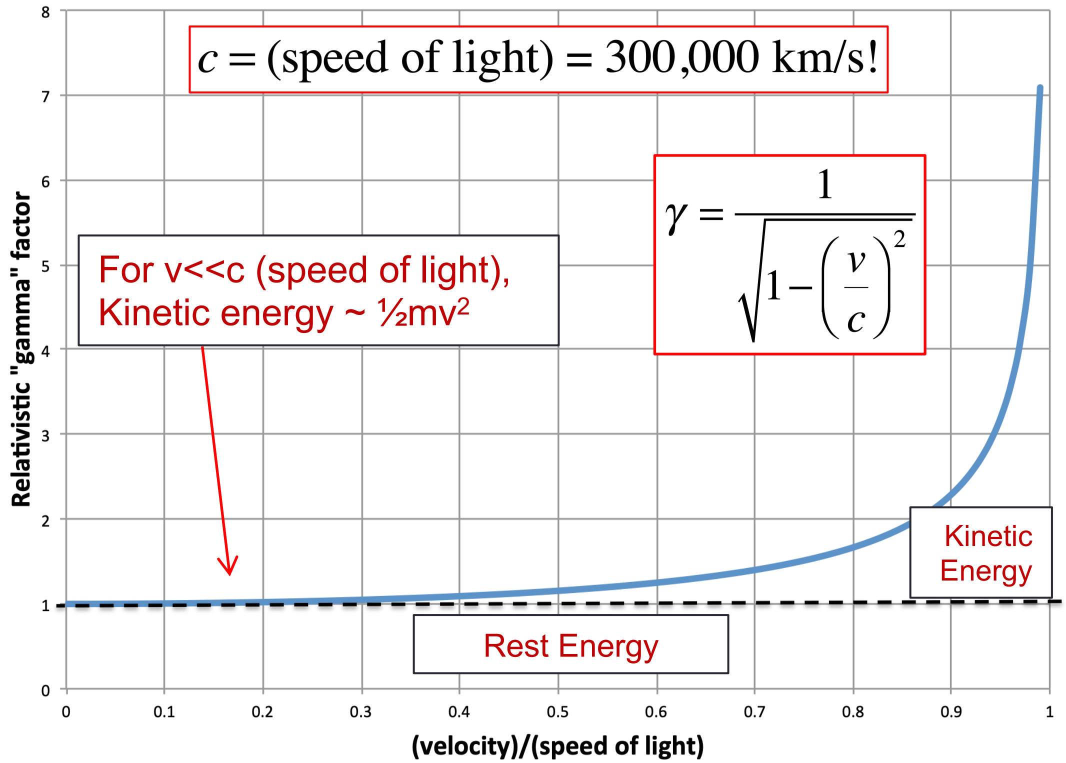
- Units
  - Energy: eV (keV, MeV, etc) [1 eV = 1.6×10^{-19} J]
  - Mass: eV/c^2 [proton = 1.67×10^{-27} kg = 938 MeV/c^2]
  - Momentum: eV/c [proton @ β=.9 = 1.94 GeV/c]

Kinetic Energy

- A body in motion will have a total energy given by

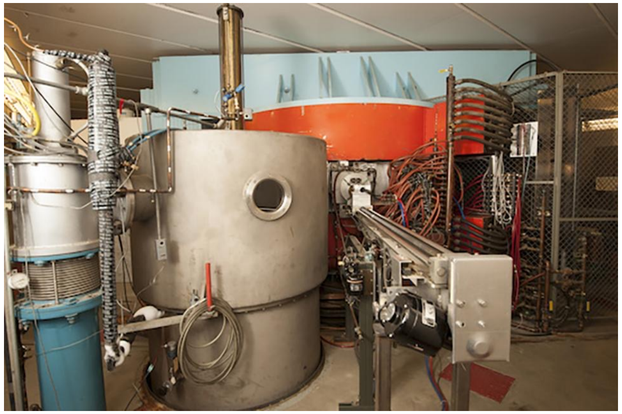
$$E = \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \equiv \gamma mc^2$$

- The difference between this and mc^2 is called the "kinetic energy"
- Here are some examples of kinetic energy

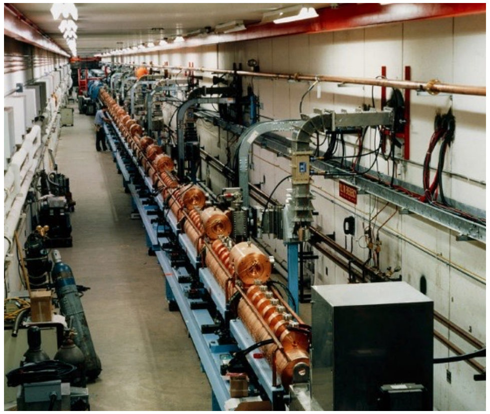


Example	**Velocity**	**Velocity/ Speed of light**	**Kinetic Energy/(mc^2)**
Race car	150 mph	.0000002	.000000000000025
Apollo 10 (fastest humans)	24,791 mph	.000037	.0000000068
Proton in the LHC (full energy)	Light minus 2.7 m/s	.999999991	7500
Electron in LEP	Light minus 3.6 mm/s	.999999999988	203,000

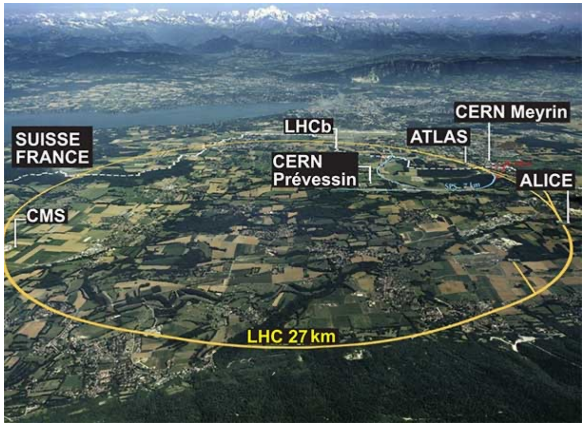
Example Beam Parameters



Crocker Nuclear Lab (across the path)
67 MeV protons



Fermilab LINAC
400 MeV protons



LHC at CERN
7 TeV

Parameter	Symbol	Equation	CNL	LINAC	LHC
Mass	m [GeV/c²]		0.938		
kinetic energy	K [GeV]		0.067	0.400	7000
total energy	E [GeV]	$K+mc^2$	1.005	1.3382	7000.938
momentum	p [GeV/c]	$\sqrt{E^2-(mc^2)^2}$	.3608	0.95426	7000.938
rel. beta	β	$(pc)/E$	.359	0.713	0.999999991
rel. gamma	γ	$E/(mc^2)$	1.071	1.426	7461.5

Note: the factors of “c” always cancel out!

4-Vectors and Lorentz Transformations

- Usual conventions

$$\mathbf{X} \equiv (x, y, z, ct)$$

$$\mathbf{P} \equiv \left(p_x, p_y, p_z, \frac{E}{c} \right)$$

$$\mathbf{A}' = \Lambda \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & -\beta \\ 0 & 0 & -\beta & \gamma \end{pmatrix} \mathbf{A} \quad \text{(velocity along z axis)}$$

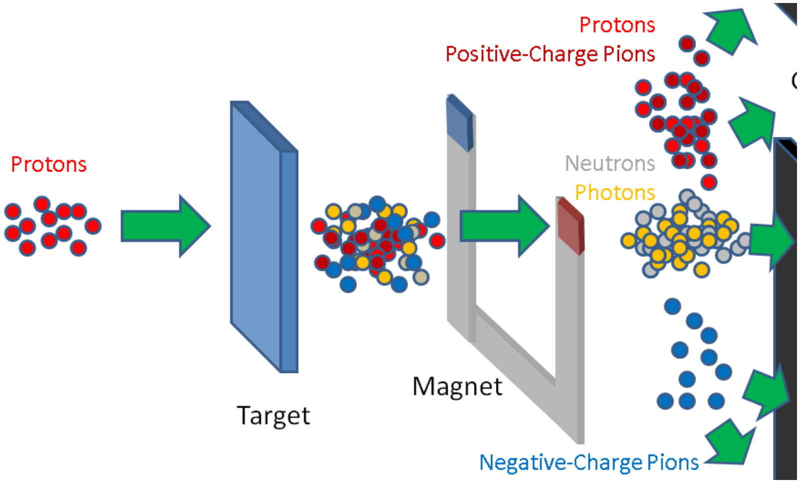
$$|\mathbf{X}|^2 = (ct)^2 - x^2 - y^2 - z^2 \equiv (c\tau)^2$$

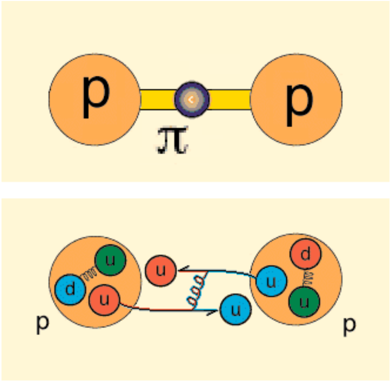
$$|\mathbf{P}|^2 = \left(\frac{E}{c} \right)^2 - p_x^2 - p_y^2 - p_z^2 \equiv (mc^2)^2$$

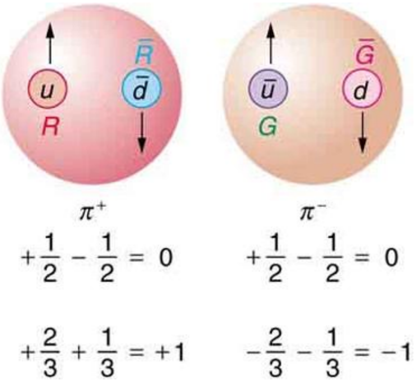
- Note that for a system of particles $\left| \sum \mathbf{P}_i \right|^2 = (M_{eff} c^2)^2 \equiv s$

Charged Pions

- Pions can be thought of as mediating the strong force that hold nuclei together.
- A π^+ consists of an up quark and an anti-down quark while a down π^- consists of an anti-up quark and a down quark.
- When a high energy proton beam hits a target, it creates mostly pions.
  - We can make a beam out of the pions
  - We can also use their decay products muons and neutrinos.



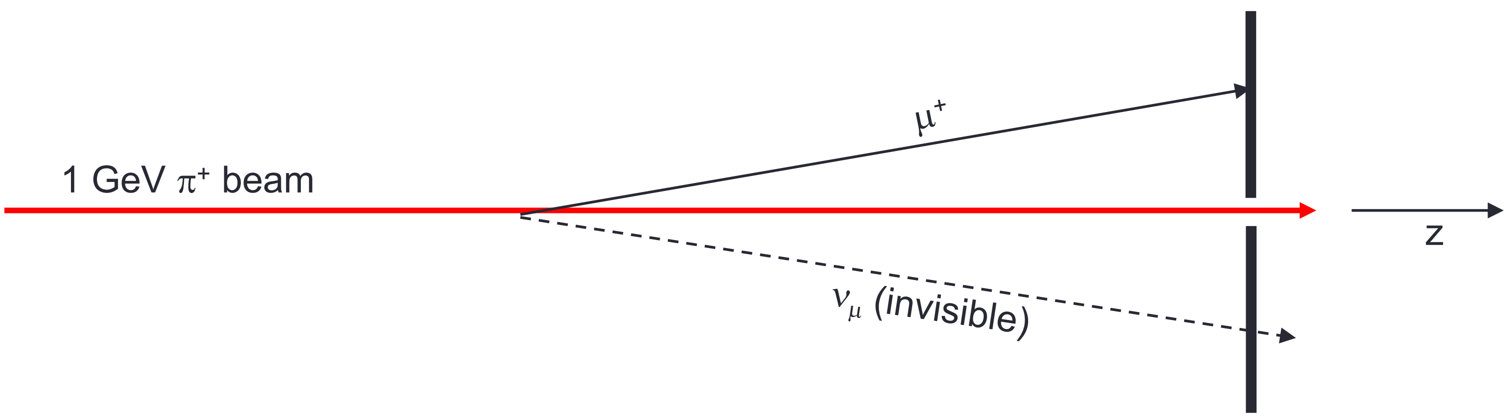




$$m_{\pi^\pm} = 139.57\ \mathrm{MeV}/c^2$$
$$\approx 280 m_e \approx \frac{1}{7} m_p$$
$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\pi^- \to \mu^- + \overline{\nu_\mu}$$
$$\tau = 26.0\ \mathrm{ns}$$

Our Example Problem

- We're going to start with a charged pion beam with 1 GeV of kinetic energy.
- 200 downstream of the source will be a tracking detector 6m on a side.
  - There will also be a 5cm diameter hole so the pions that haven't decayed can pass through



- Our goal is to calculate the fraction of the pions that will produce a decay muon that hits the tracking chamber.

Steps of the Problem

1. Generate a beam of beam of 1 GeV pions, moving in the *z*-direction
2. Calculate the decay position based on the lifetime, corrected for Lorentz time dilation.
3. Isotropically decay each pion to a muon and neutrino in its own rest frame.
4. Project the muon to the detector location.
   - The neutrino will be invisible.
5. Determine whether the decay muons
   - Hit the tracker
   - Miss the tracker
   - Go down the hole.

Vector Package

- The vector package is a powerful set of routines to deal with 2D, 3D, and Lorenitz 4-vectors.
- Can work in Cartesian, cylindrical, or polar coordinates.
- Includes standard geometric operations
- For 4-vectors, includes Lorentz boosting methods.
- There is a ”particle” class in Scikit-HEP, which is better suited to more complicated operations, but vector will be find for this example.

Decay Parameters

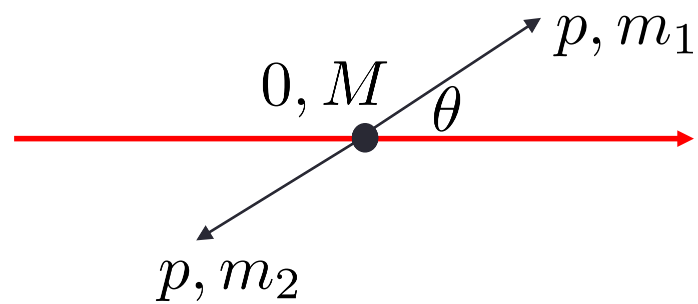
- The lifetime will be extended by time dilation by

$$\tau_{lab} = \gamma\tau$$

- The length it will travel will be

$$L = v\tau_{lab} = c\beta\gamma\tau$$

- In the center of mass frame, the particles will go out back to back with equal momentum



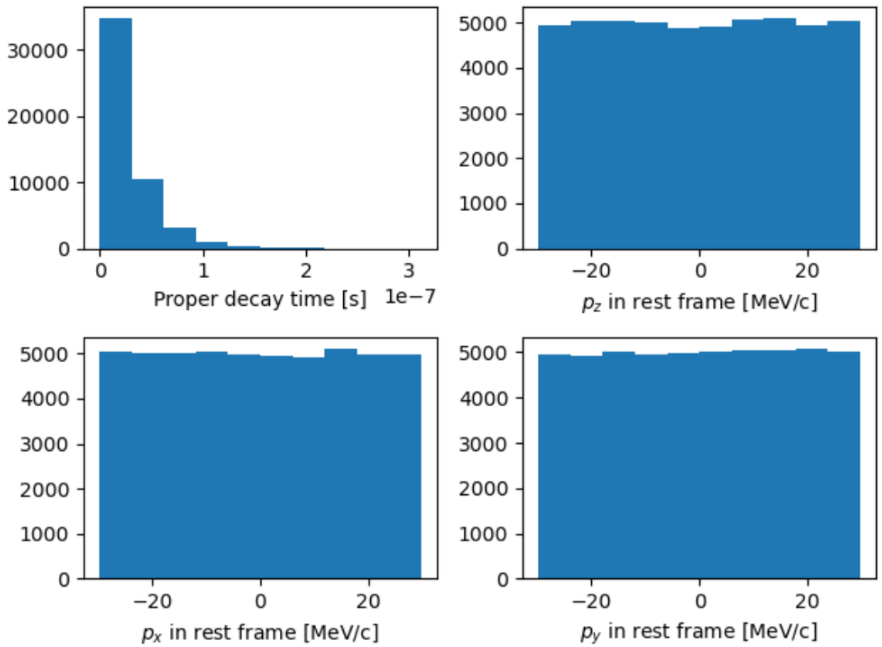
Flat in cosθ and ϕ

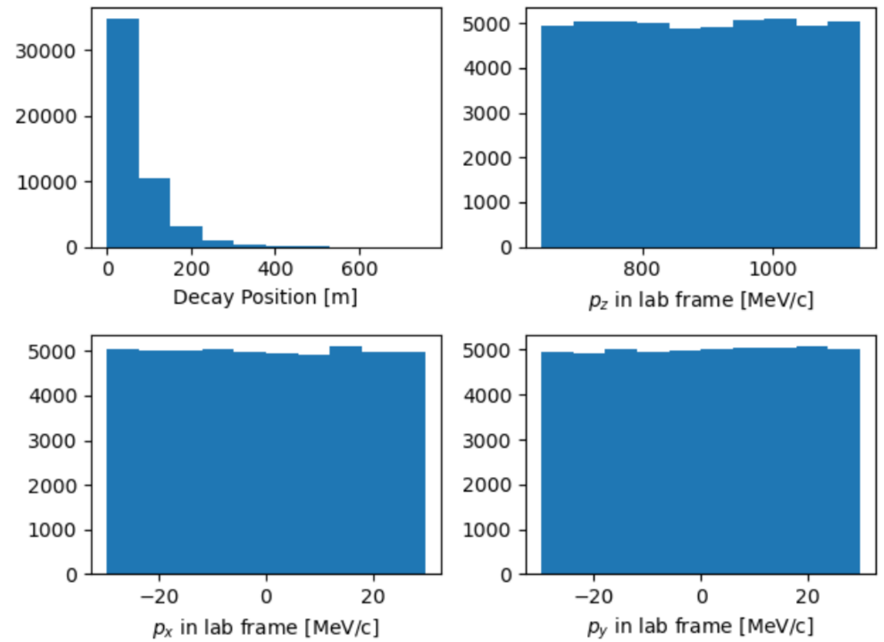
- The momentum is given by

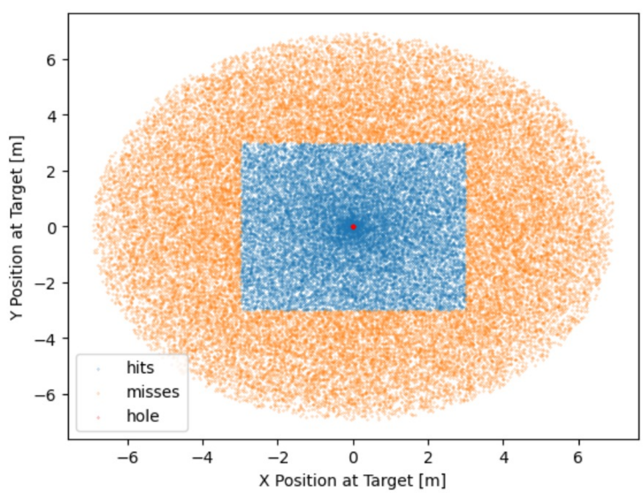
$$p = \frac{\sqrt{\left(M^2 - (m_1 + m_2)^2\right)\left(M^2 - (m_1 - m_2)^2\right)}}{2M}c$$

(go to notebook)

Results





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95.59% decayed upstream of the detector. Of these:
37.64% hit the detector.
62.20% were outside the detector.
0.16% went down the hole detector.
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