



ROOTS

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Finding Roots of an Equation

- The roots of a function $f(x)$ are all points (if any) for which

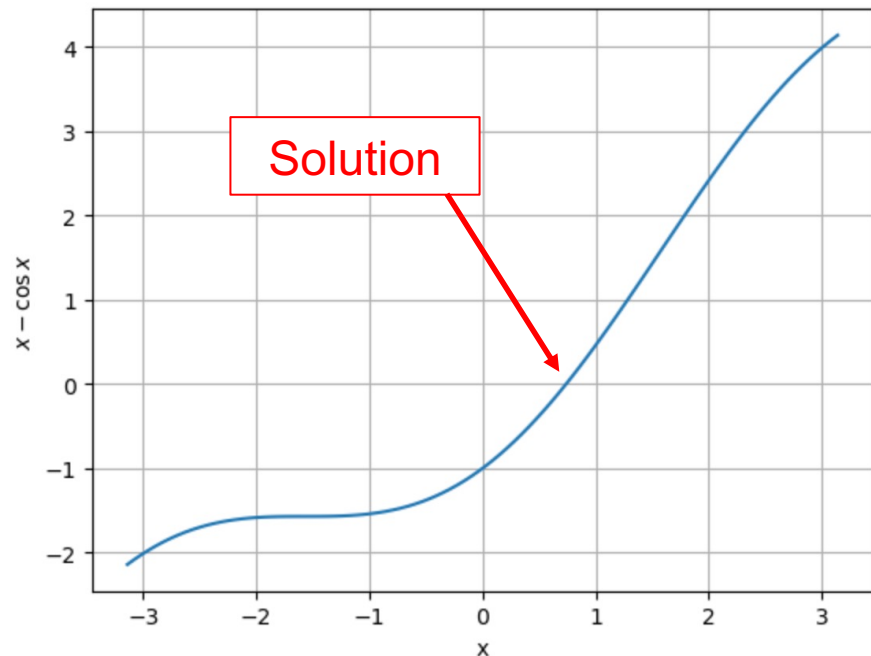
$$f(x) = 0$$

- I can express any explicit or non-explicit function as a root problem. For example,

$$x = \cos x$$

has no explicit solution for x , but it will be satisfied by the root of

$$f(x) = x - \cos x$$



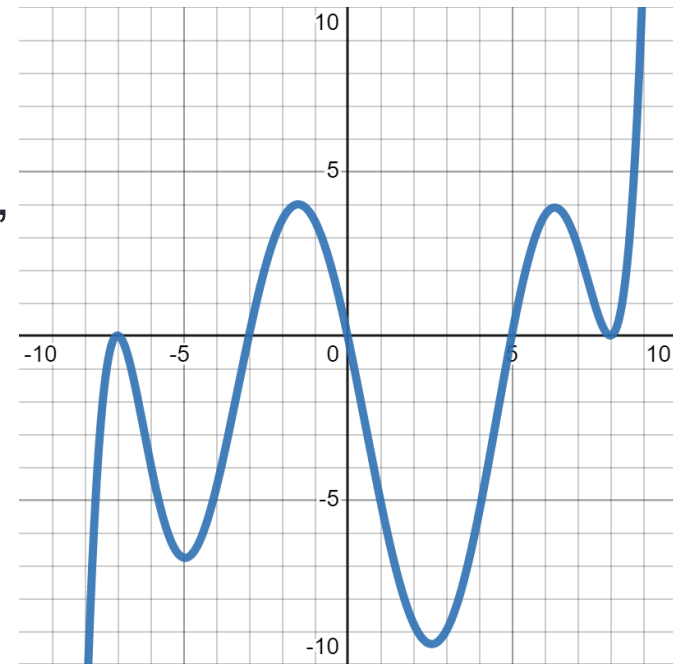


Iterations and Convergence

- In general, we will iterate multiple times, each time finding a new value of $x^{(i)}$, presumably getting closer to the true root x^*
- There's a lot of formalism for evaluating convergence, but we'll focus on simple tests such as

$$|f(x^{(i)})| \leq \epsilon \quad \text{or} \quad |x^{(i)} - x^{(i-1)}| \leq \epsilon$$

- Since many functions will have multiple roots, we will usually need to start with a bracket or at least an initial guess.

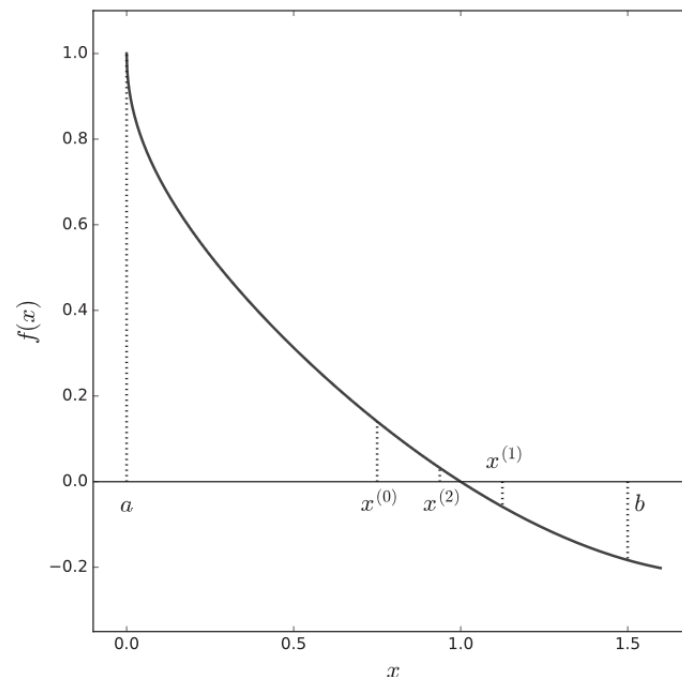




Bisection Method

- We start with two points a_0 and b_0 that bracket the root; ie

$$f(a_0) \cdot f(b_0) < 0$$



- Now calculate the midpoint and see which side of the root it's on

$$c_0 = \frac{1}{2}(a_0 + b_0)$$

$$f(a_0) \cdot f(c_0) > 0 \rightarrow a_1 = c_0; \quad b_1 = b_0$$

$$f(a_0) \cdot f(c_0) < 0 \rightarrow a_1 = a_0; \quad b_1 = c_0$$

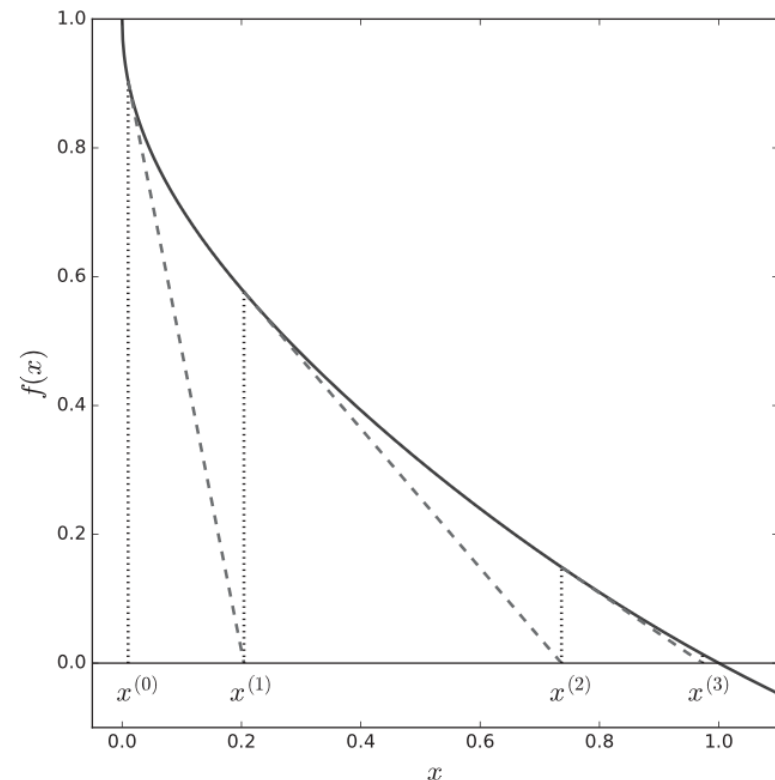
- Repeat until convergence criterion is met.



Newton's Method

- Newton's method requires that we know the derivative of the function $f'(x)$
- Each iteration uses the local slope to project to the next iteration.

$$x^{(i+1)} = x^{(i)} - \frac{f(x^{(i)})}{f'(x^{(i)})}$$



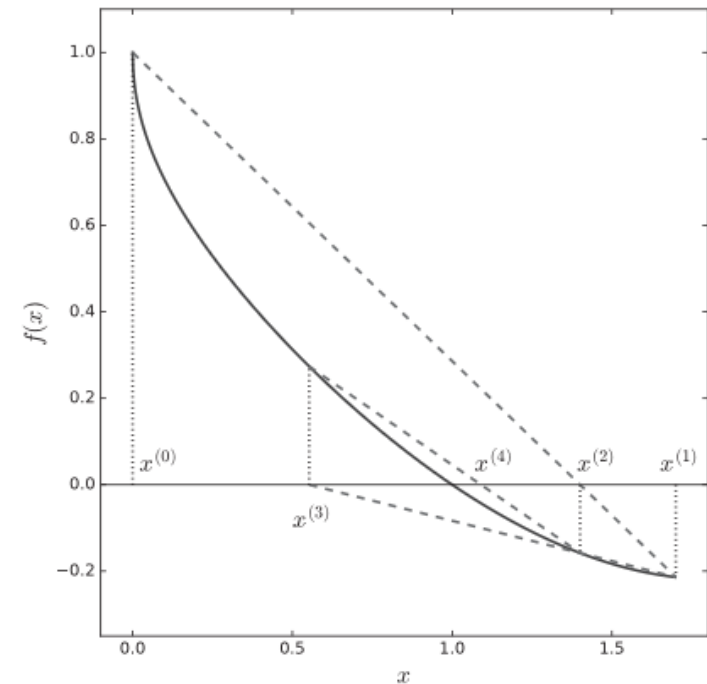


Secant Method

- Similar to the Newton Method, but use two initial guesses to approximate the derivative

$$f'(x^{(k-1)}) \approx \frac{f(x^{(k-1)}) - f(x^{(k-2)})}{x^{(k-1)} - x^{(k-2)}}$$

$$\rightarrow x^{(k)} = x^{(k-1)} - f(x^{(k-1)}) \frac{x^{(k-1)} - x^{(k-2)}}{f(x^{(k-1)}) - f(x^{(k-2)})}$$



- You'll do algorithmic examples of these three methods in lab next week.