Placement of undead defense units

For the undead defense units there are four areas to place them along the path. These areas have approximately the same distance to each other and are numbered from 1 (the one at the entry point of the path) to 4 (the one at the end).

To decide about the position of one individual defense unit, we use a roulette wheel selection with changing probabilities over time. That means, in each round r we have four positive values $p_{1,r}$, $p_{2,r}$, $p_{3,r}$ and $p_{4,r}$ that add up to 100. Then we get some random number x between 0 and 100 and if $x \leq p_{1,r}$, the unit is placed in area 1, if $p_{1,r} < x \leq p_{1,r} + p_{2,r}$, the unit is placed in area 2 and so on. The crucial question is, how to get suitable values for the round specific probabilities.

Therefor we look at every position individually and how the number of units in this area should develop over time. In area 1 for instance we should have a lot of units in the early rounds and less later. In area 4 it is the other way around with a peak in round 20 (the number of rounds to play) and in area 2 and 3 the number of units increases first and then decreases with a peak in round 7 or 8 and in round 13 or 14 respectively.

To model these curves we use the density function of four different gaussian distributions, because of their smooth behaviour and the basic assumption, that the appearance of enemies in sevearal areas could be described by gaussian distributions too. The distribution for each area $a \in \{1, 2, 3, 4\}$ has a specific mean value μ_a, that represents the peak round when the highest number of units should appear in this area. Clearly the mean value for area 1 has to be 1, for area 4 it has to be 20 and for area 2 and 3 it has to be approximately 7 and 14 respectively. A second parameter of each distribution is the standard deviation $\sigma_a > 0$, that stretches the curve and determines, how the number of units in this area increases and decreases over time. This parameter can be modified freely to control the course of the game and the actual choice is a result of experimentation. Accordingly the complete formula for area $a \in \{1, \dots, 4\}$ in round r looks like this:

$$f_{\mu_a,\sigma_a}(r) = \frac{1}{\sigma_a\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{r-\mu_a}{\sigma_a}\right)^2} \qquad , 1 \leq r \leq 20$$

with $\mu_a = \frac{1}{3}(19a - 16)$ and $\sigma_a > 0$.

With these four distributions we get four numbers in each round r and they only have to be divided by their sum and multiplied by 100 to get the four probability values $p_{1,r}$, $p_{2,r}$, $p_{3,r}$ and $p_{4,r}$ we need for our roulette wheel selection. Doing this selection for every defense unit results in a well controlled random placement for each round.