

# Computer Vision

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University of Bucharest, 2<sup>nd</sup> semester, 2023-2024

# Course structure

## 1. Features and filters: low-level vision

Linear filters, color, texture, edge detection

## 2. Grouping and fitting: mid-level vision

Fitting curves and lines, robust fitting, RANSAC, Hough transform, segmentation

## 3. Multiple views

Local invariant feature and description, epipolar geometry and stereo, object instance recognition

## 4. Object Recognition: high – level vision

Object classification, object detection, part based models, bovw models

## 1. Video understanding

Object tracking, background subtraction, motion descriptors, optical flow

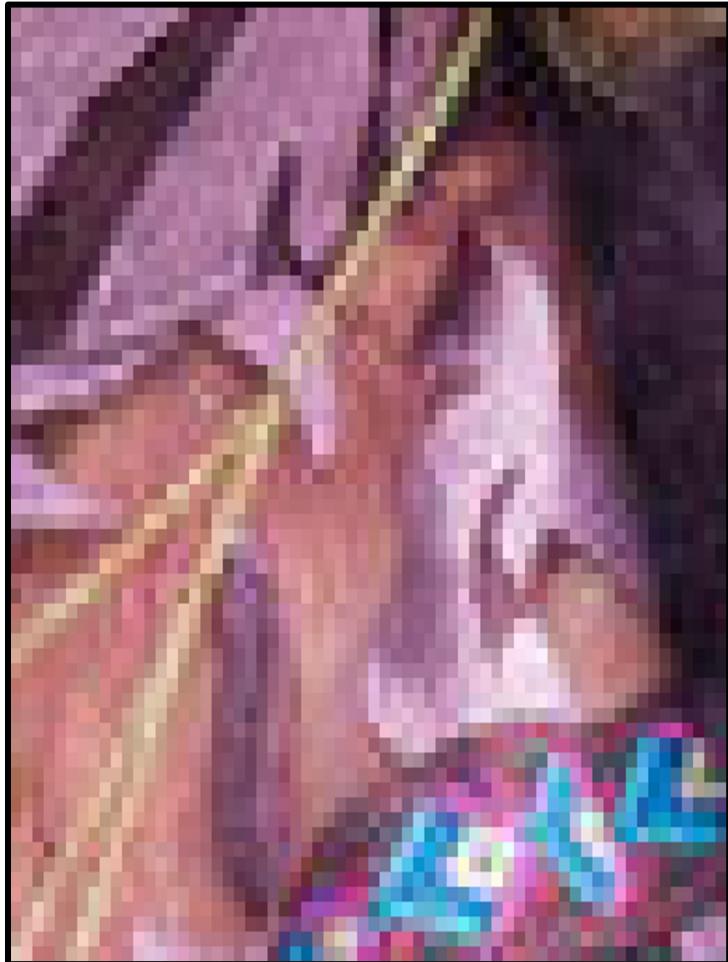
# Low-Level: Resizing



# Low-Level: Resizing



nearest neighbor interpolation



bilinear interpolation



# Low-Level: Grayscale



# Low-Level: Exposure



# Low-Level: Hue



# Low-Level: Saturation



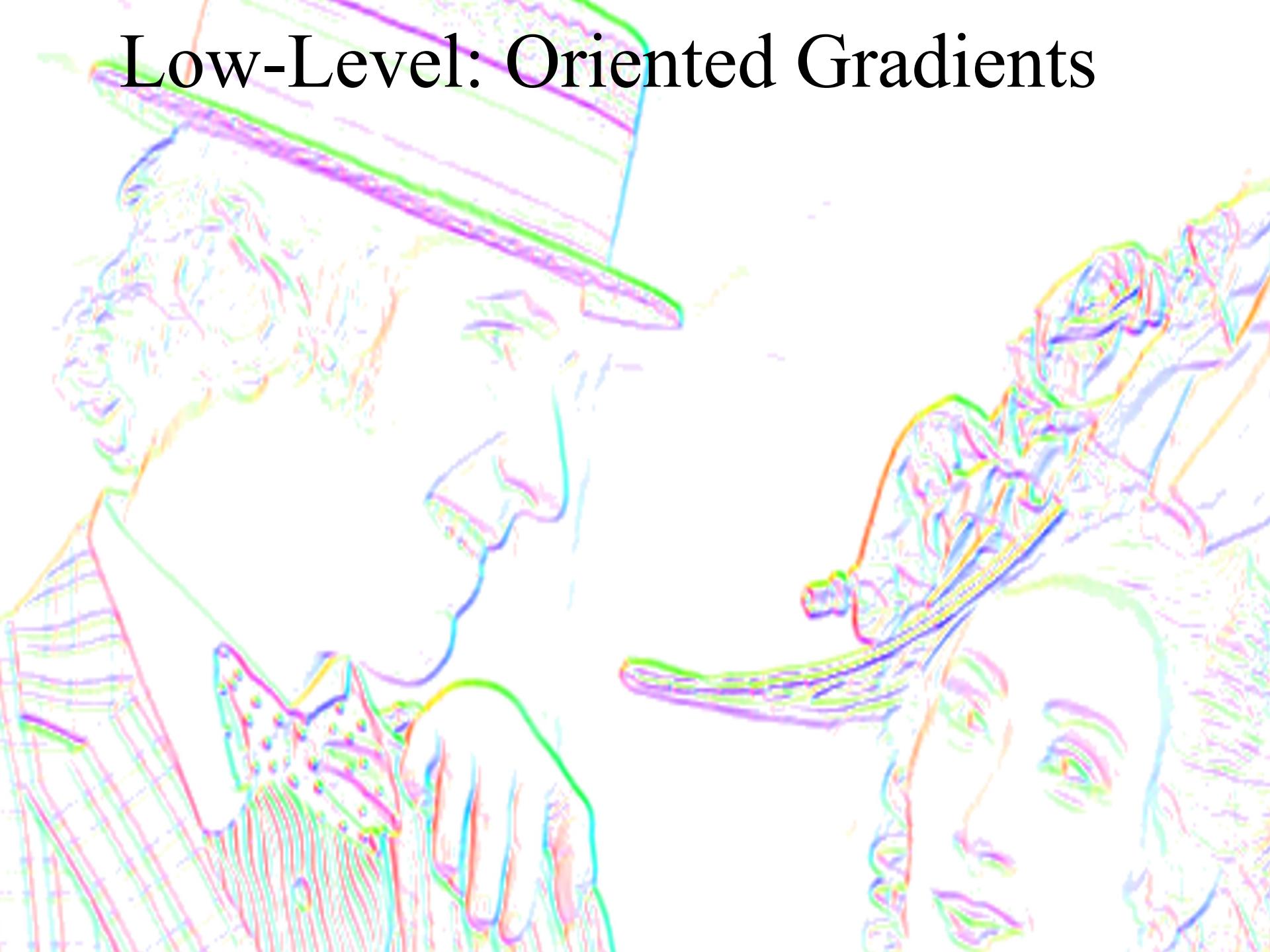
# Low-Level: Edges



# Low-Level: Oriented Gradients



# Low-Level: Oriented Gradients



# Low-Level: Segmentation (color)



# Low-Level Vision

Photo manipulation

- Size
- Color
- Exposure

Feature extraction

- Edges
- Oriented gradients
- Segments

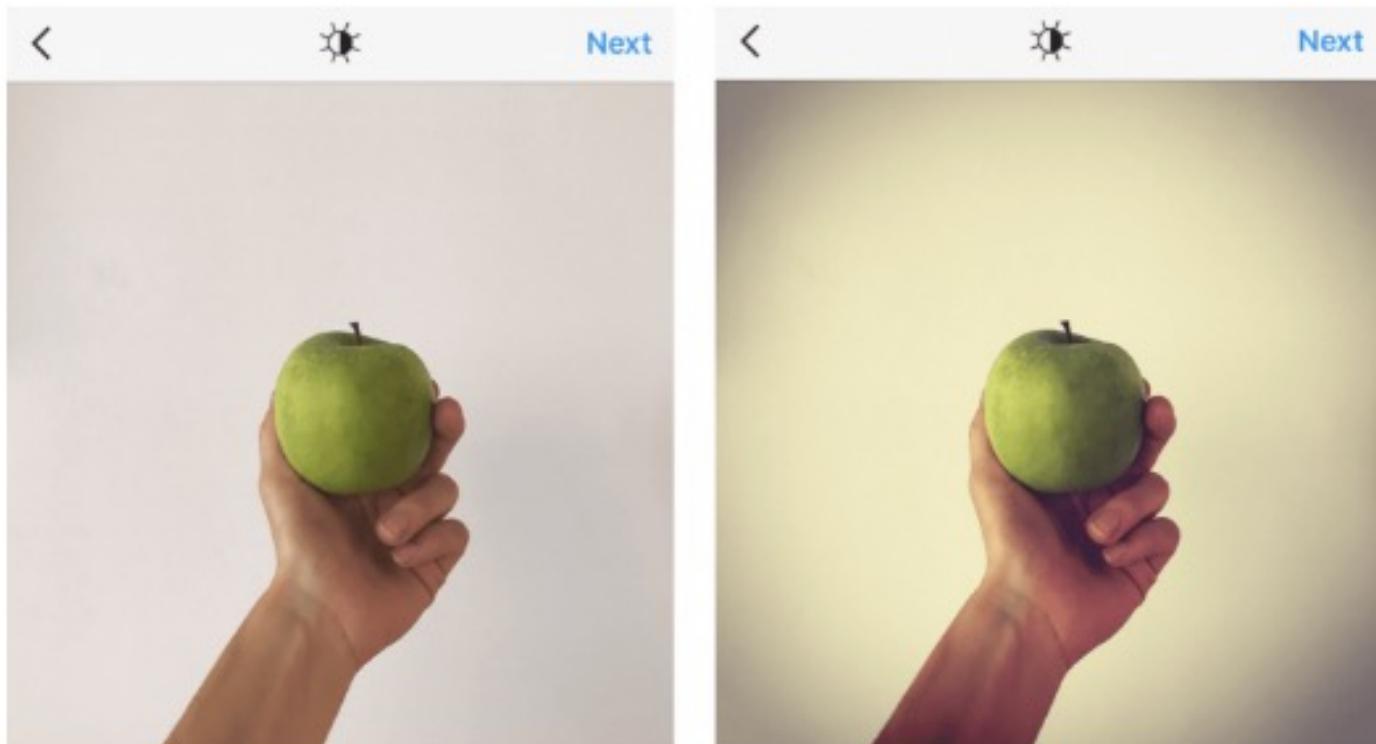
Low level vision is exciting!!!



## Instagram filter #10: X-Pro II

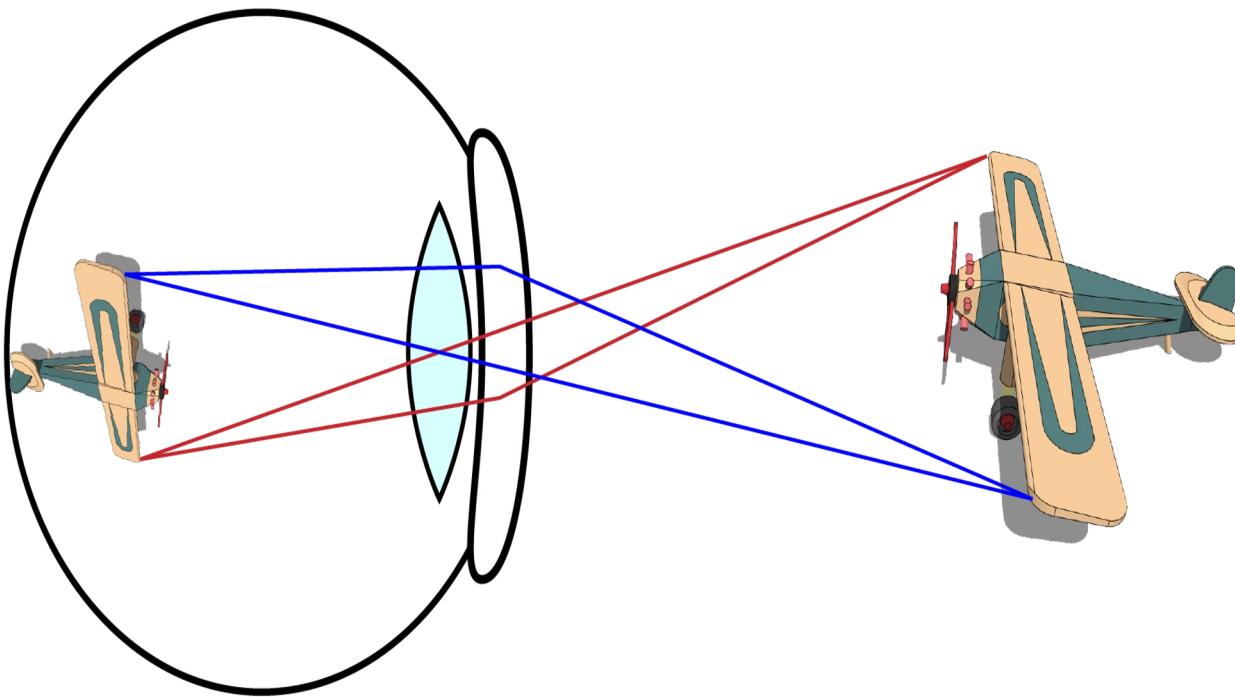
Last but not least, X-Pro II. This filter is the most high-contrast and probably the least discreet on the list: it adds a LOAD of shadow and darkness, not to mention a hefty vignette, which shades away the edges of the photo. X-PRO II is based on a photo-developing technique called “cross-processing”, where photos are processed in a chemical solution for different types of film. This is one of Instagram’s oldest filters, and was intended to be used to correct the below-average camera phones of 2010, when Instagram had just launched. Surprisingly, it is still one of the most popular filters on the platform 😊

**Good for:** Turning normal photos into very intense ones

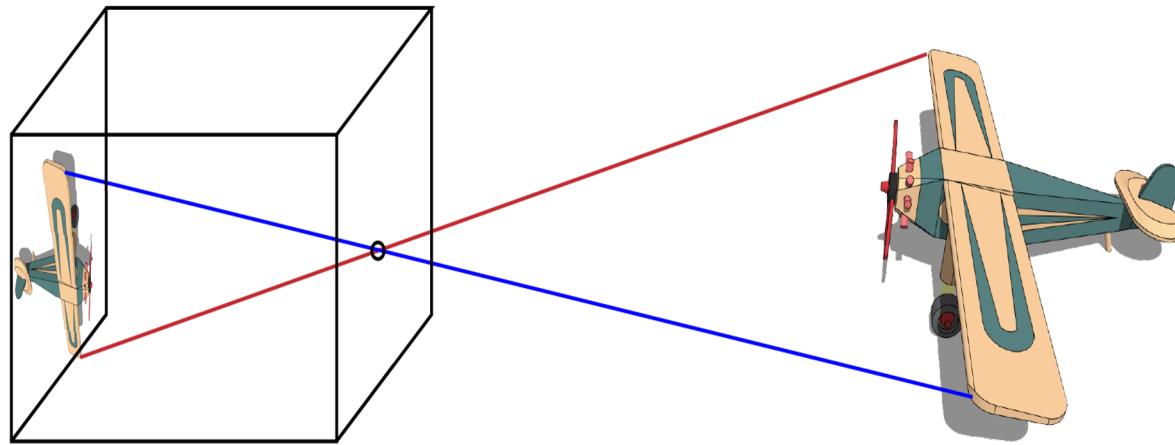


What is an image?

# Eyes: projection onto retina

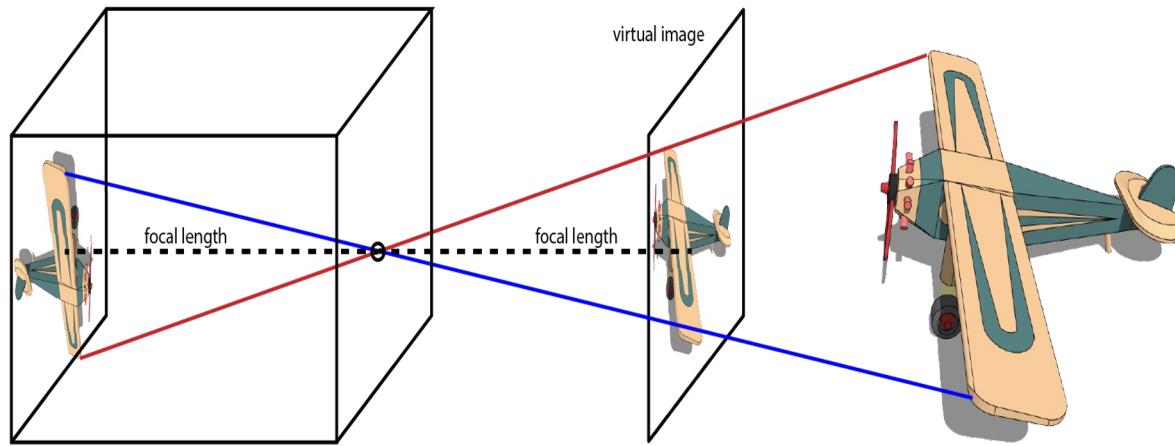


# Model: pinhole camera



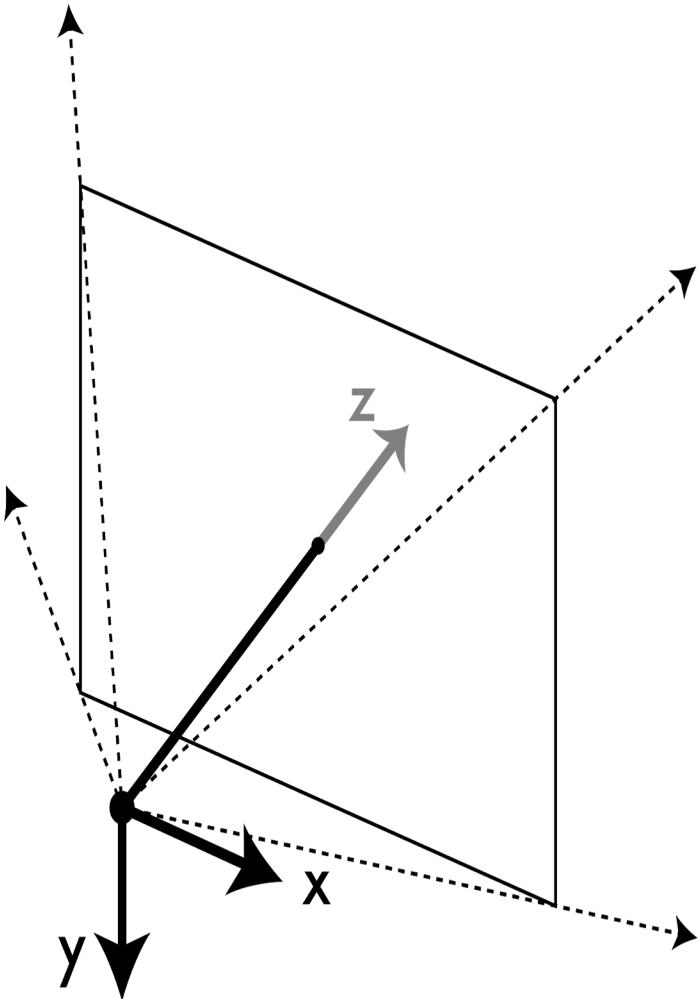
- camera aperture is described as a point
- no lenses are used to focus light

# Model: pinhole camera

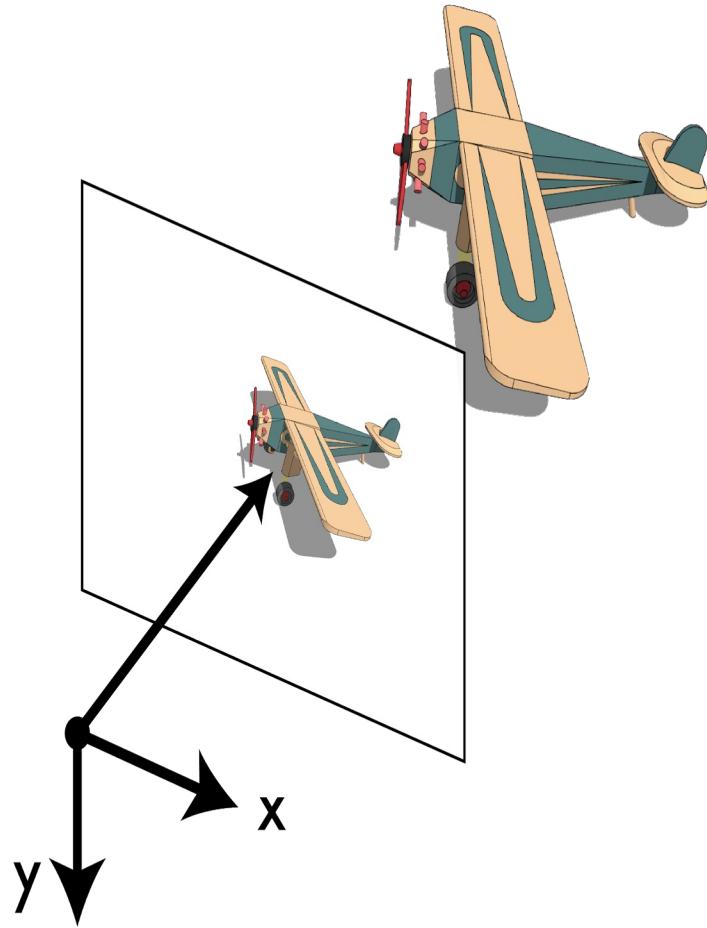


- the image plane is located at distance  $f$  (focal length) from the origin  $O$  (where the camera aperture is located)

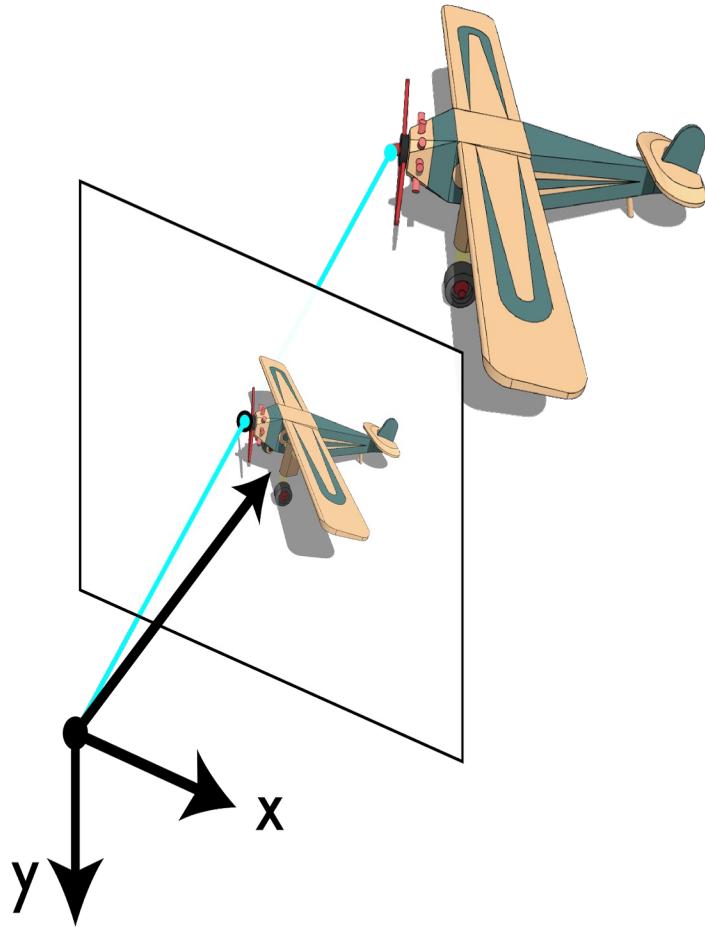
# Image: 2D projection of the 3D world



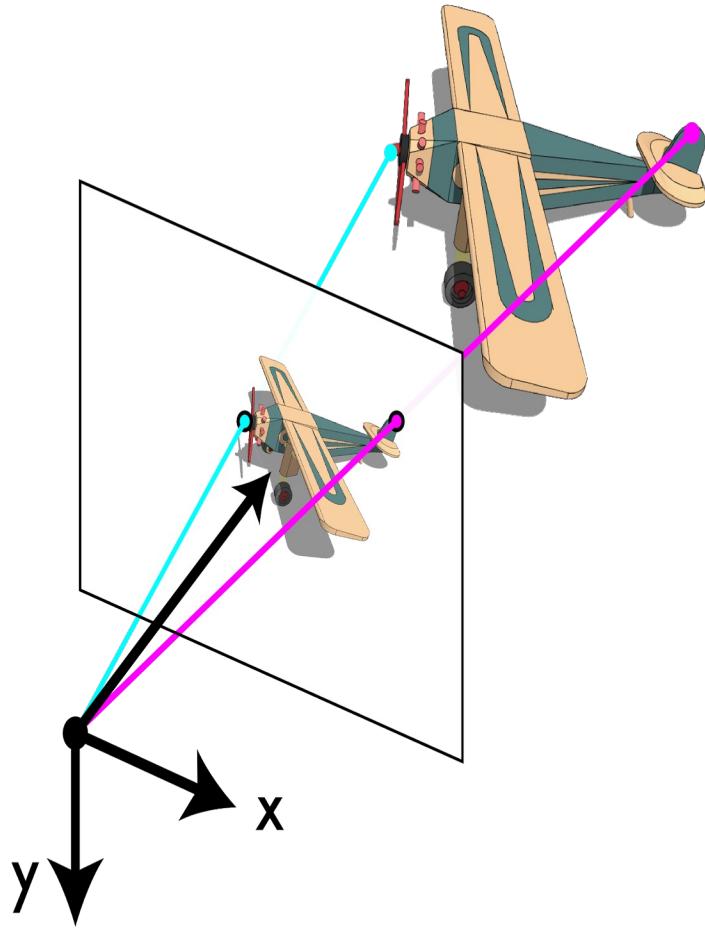
# Image: 2D projection of the 3D world



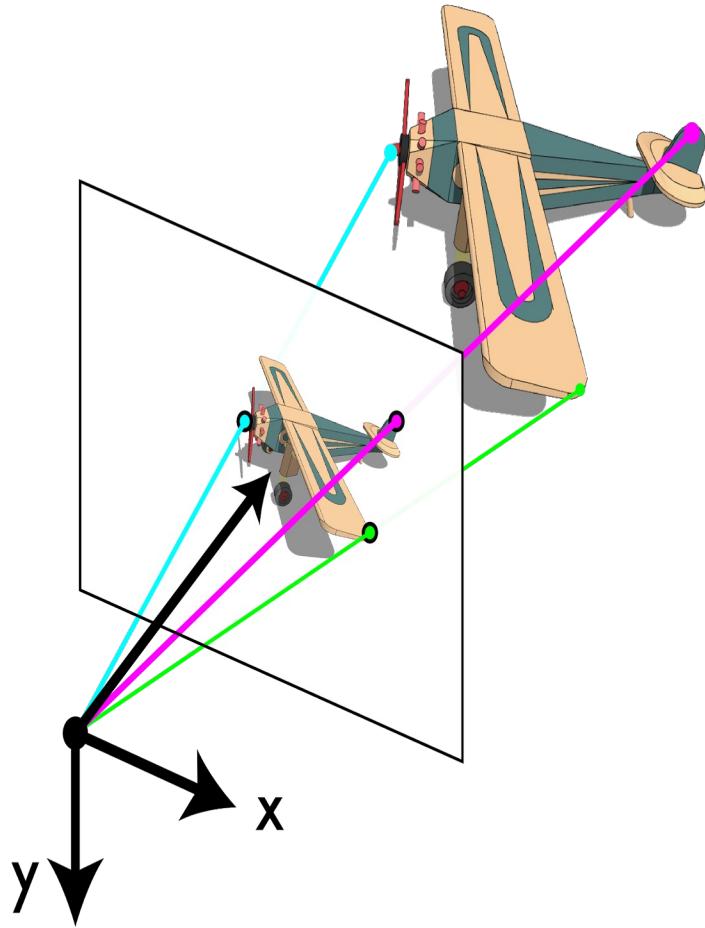
# Image: 2D projection of the 3D world



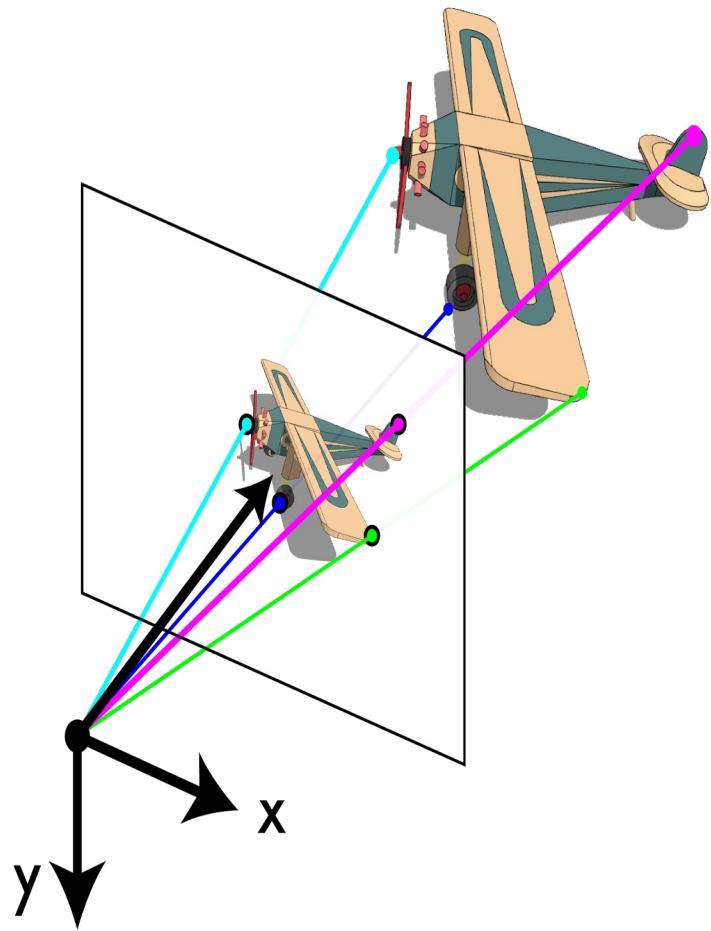
# Image: 2D projection of the 3D world



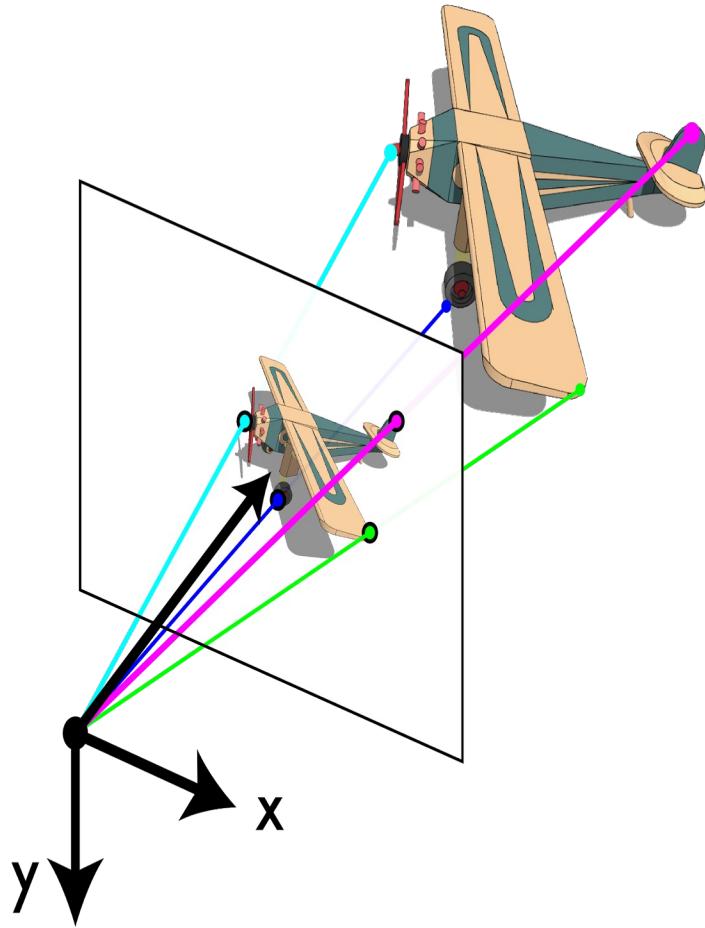
# Image: 2D projection of the 3D world



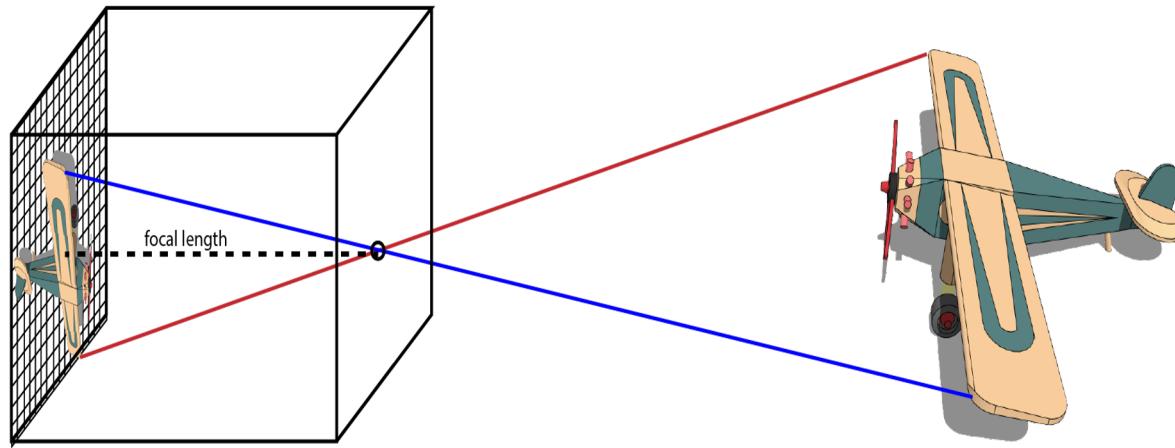
# Image: 2D projection of the 3D world



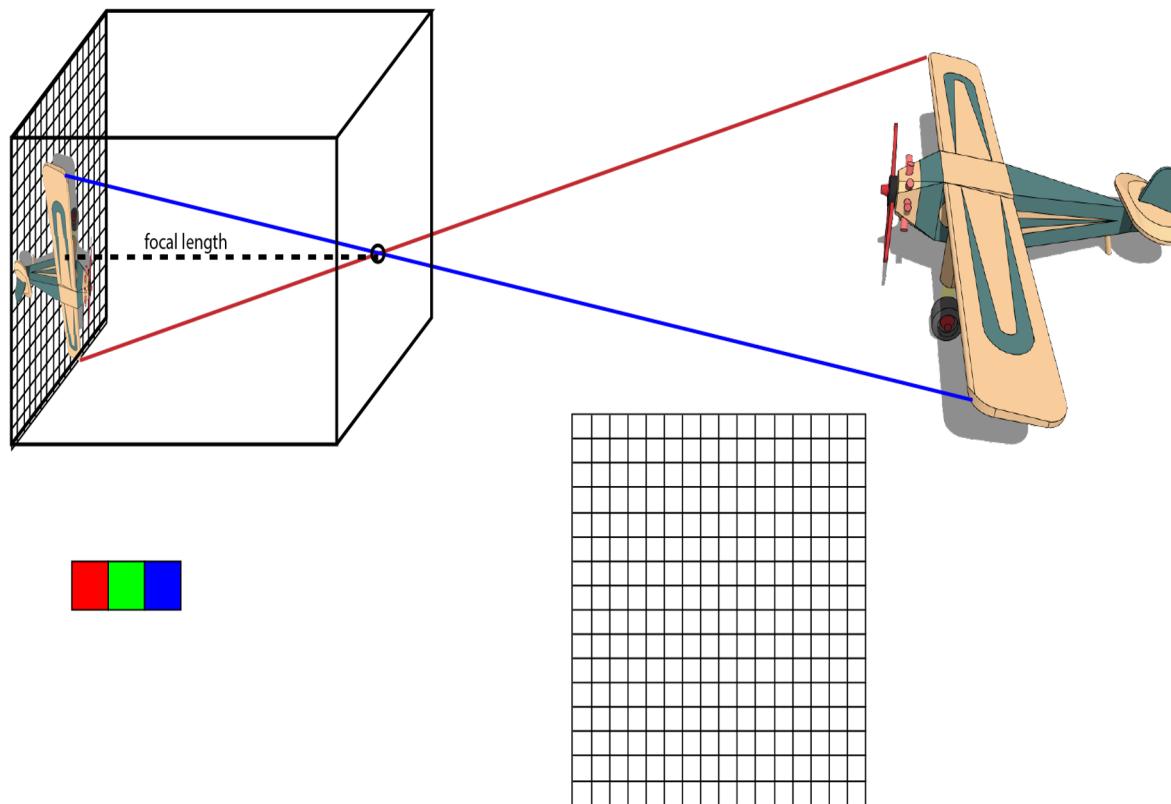
At each point we record incident light



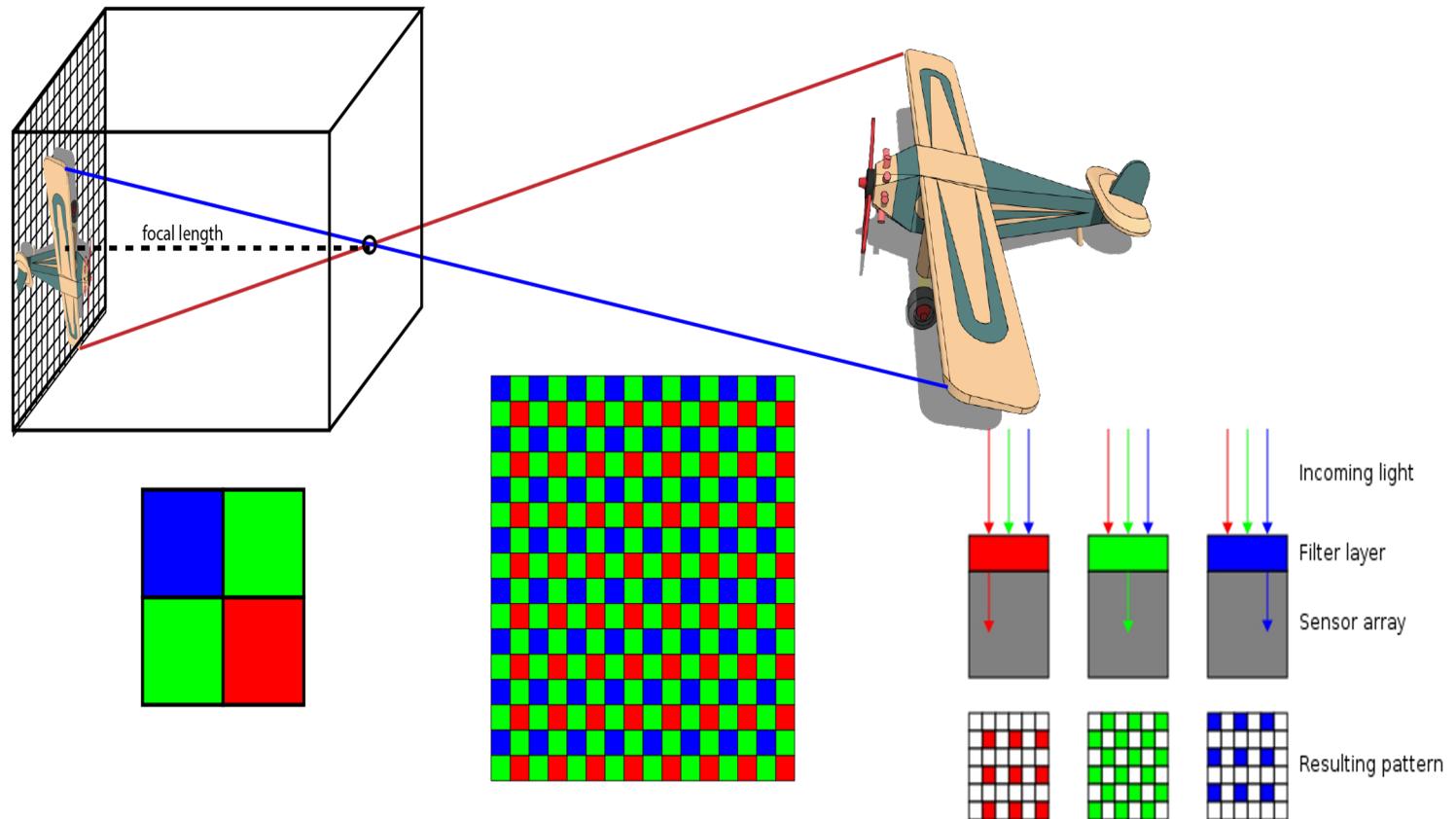
At each point we record incident light



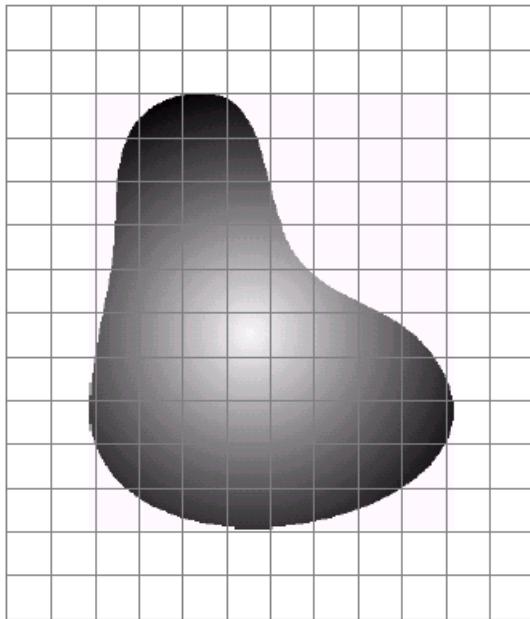
# How do we record color?



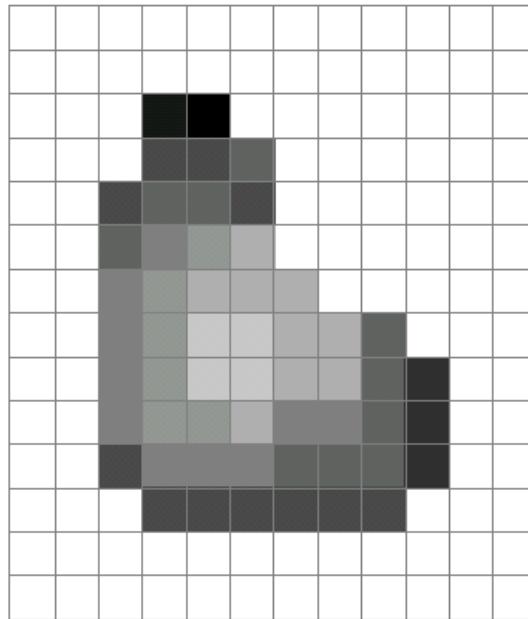
# Bayer pattern for CMOS sensors



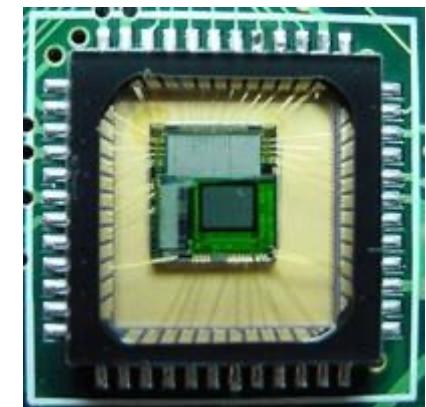
# Sensor Array



Continuous image projected  
onto a sensor array



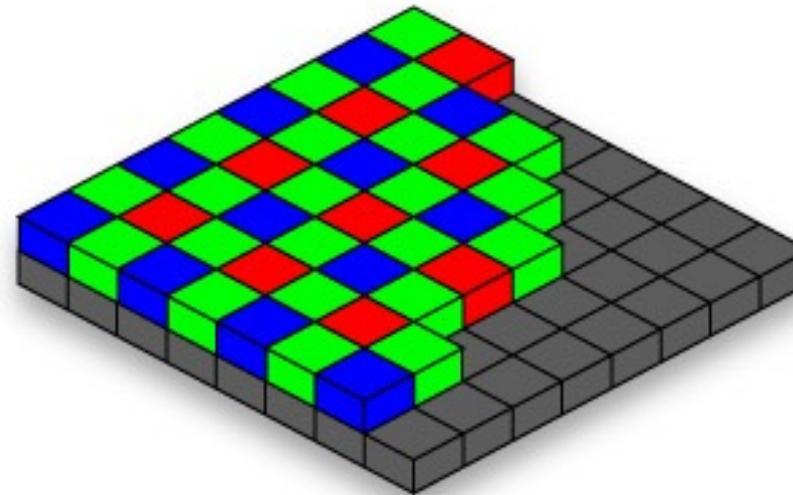
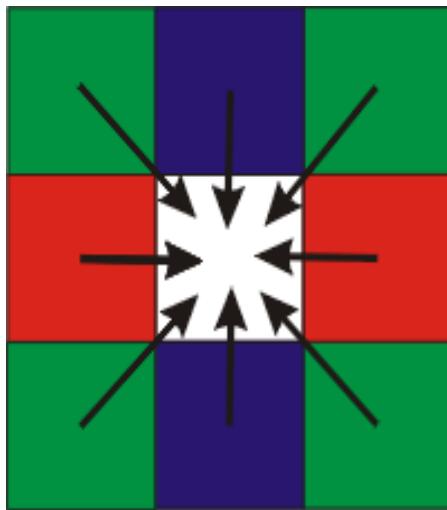
Result of image sampling  
and quantization



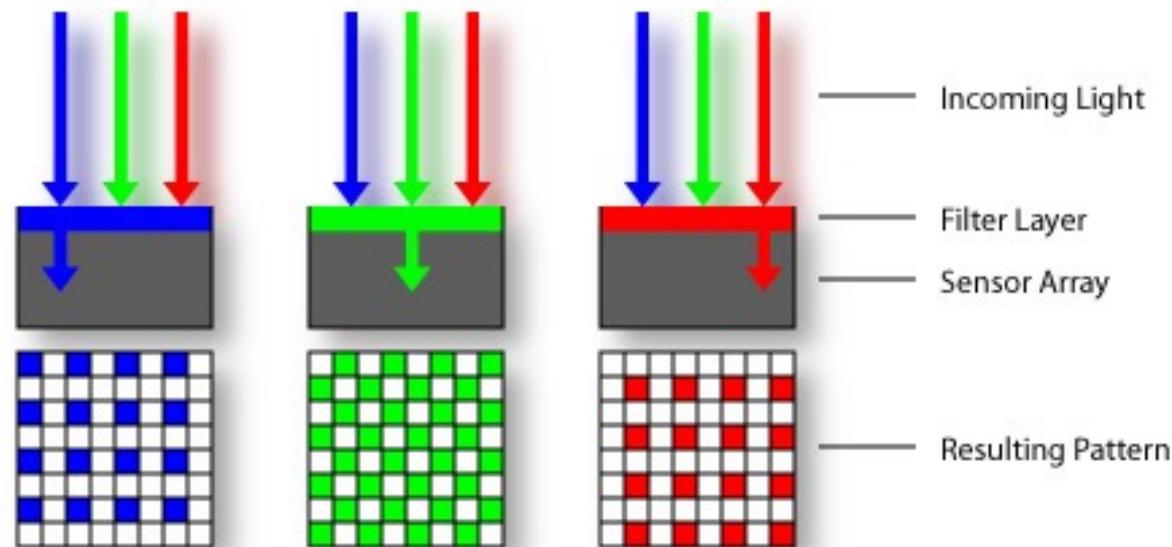
CMOS sensor

Each sensor cell records amount of light coming in at a small range of orientations

# Color Sensing: Bayer Grid



Estimate RGB at each cell from  
neighboring values



# Lab 1- demosaicing an image with the Bayer pattern

## Demosaicing

In this assignment, we are going to 'demosaic' an image encoded with the Bayer Pattern. There are some cameras that use the Bayer Pattern in order to save an image. Using this encoding only 50% of green pixels, 25% of red pixels and 25% of blue pixels are kept. The Bayer encoding takes a RGB image and encodes it as in the bellow image.



'rggb'



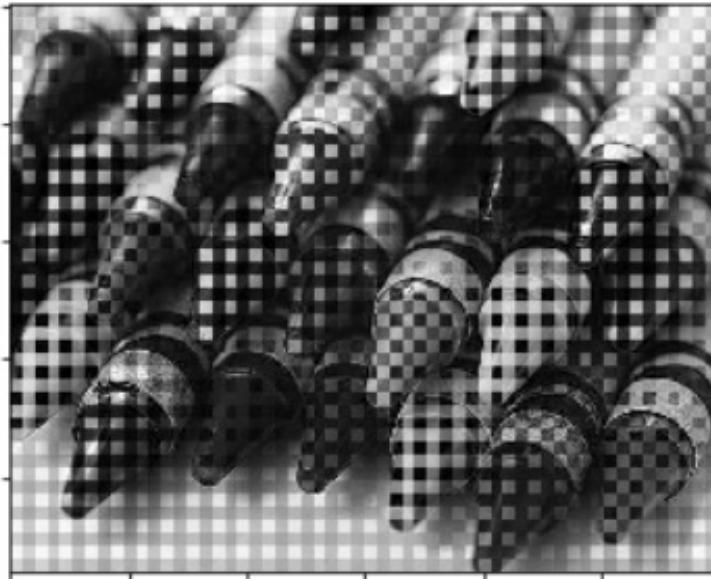
'bgrg'



'gbrg'



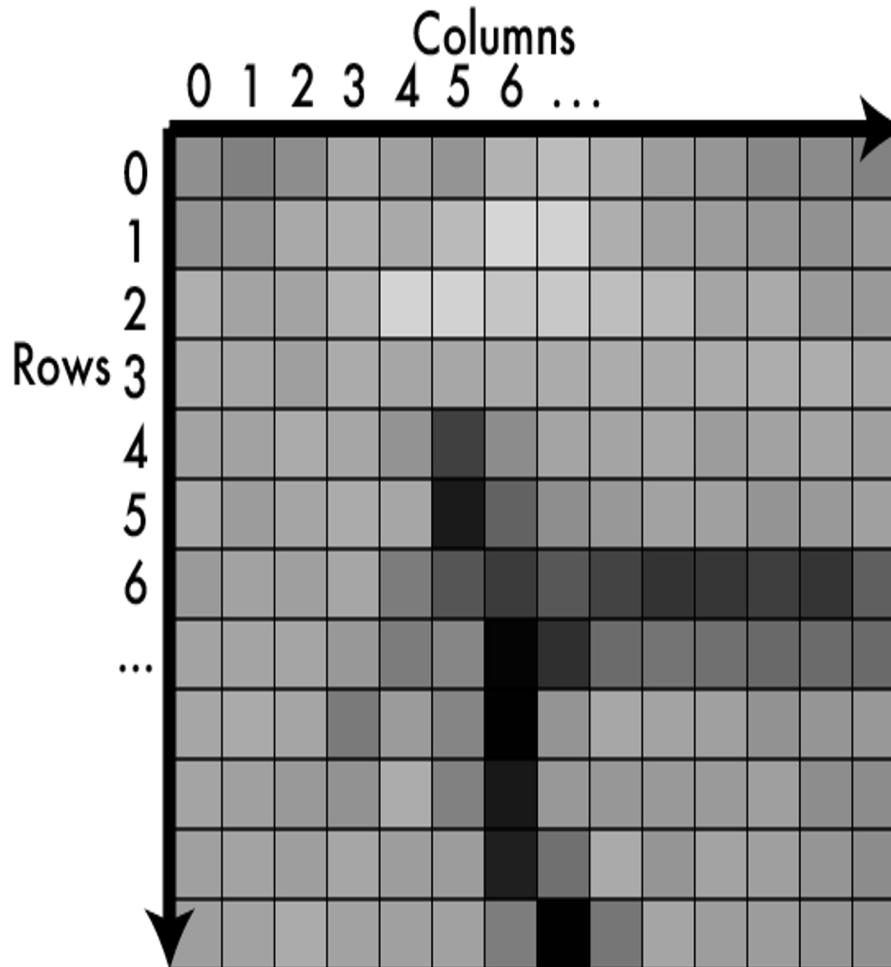
'grbg'



demosaicing



# An image is a matrix of light



# Values in matrix = how much light

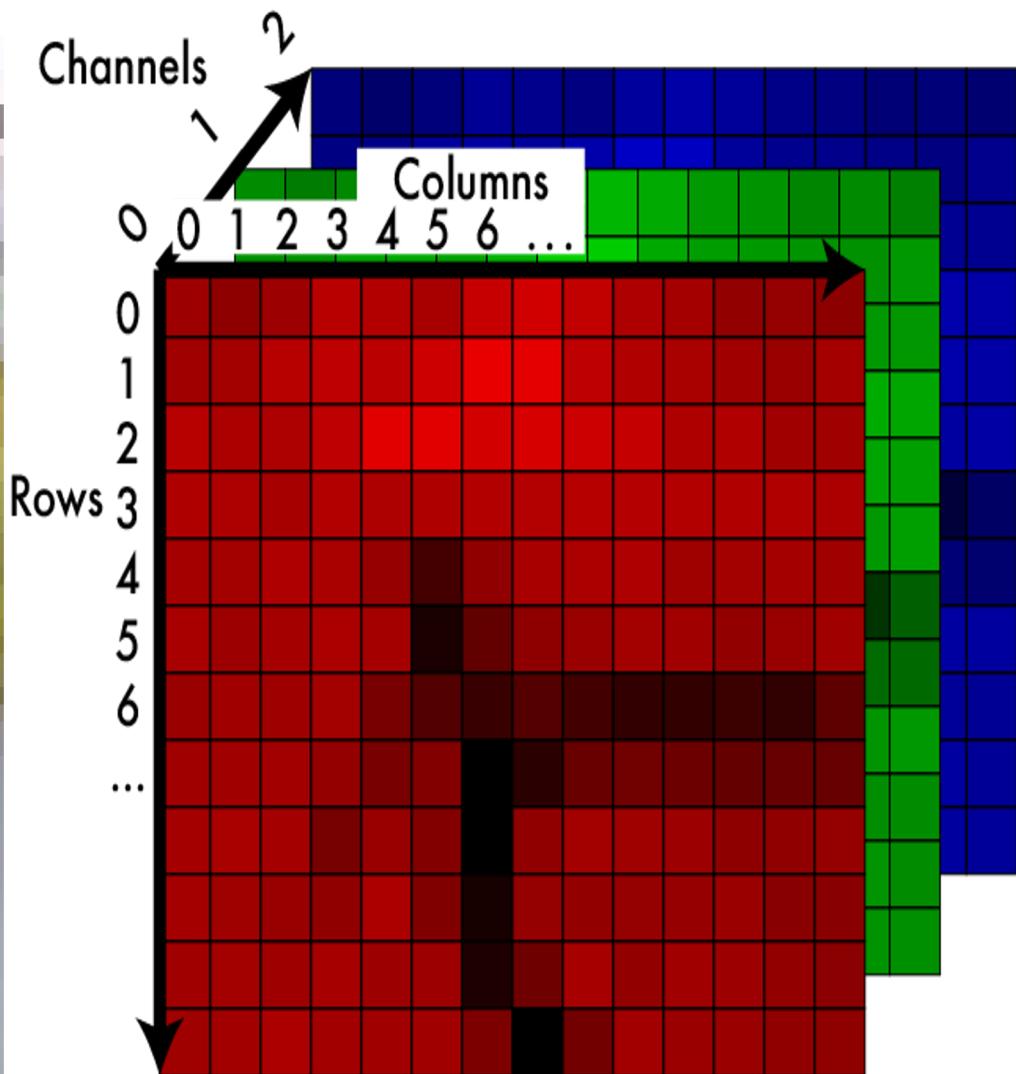
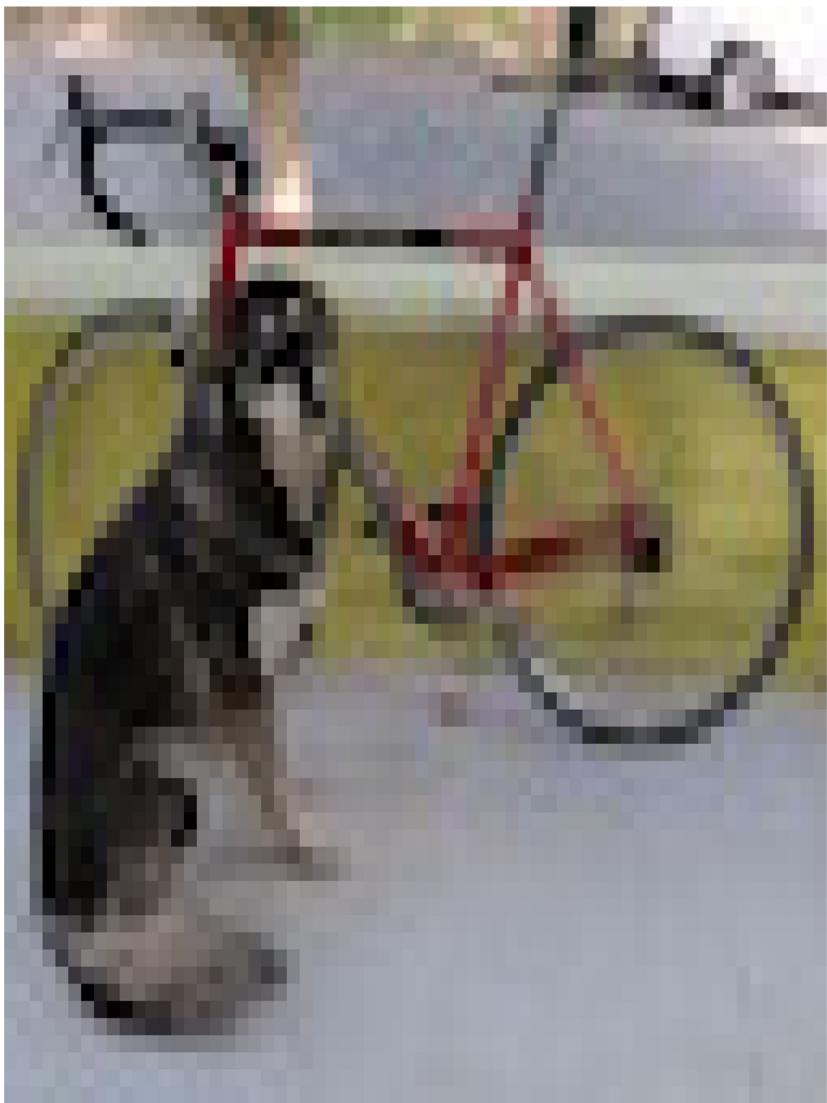
		Columns													
		0	1	2	3	4	5	6	...						
Rows	0	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	1	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	2	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	3	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	4	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	5	100	102	107	102	132	30	60	156	148	122	115	104	105	103
	6	100	102	107	102	132	40	20	50	32	20	20	24	30	62
	...	100	102	107	102	132	71		156	51	57	57	58	62	58
		100	102	107	102	132	69		156	148	122	115	104	105	103
		100	102	107	102	132	89	12	156	148	122	115	104	105	103
		100	102	107	102	132	146	13	45	148	122	115	104	105	103
		100	102	107	102	132	146	46		42	122	115	104	105	103

# Values in matrix = how much light

- Higher = more light
- Lower = less light
- Bounded
  - No light = 0
  - Sensor/device limit = max
  - Typical ranges:
    - [0-255], fit into a byte
    - [0-1], floating point
- Called pixels

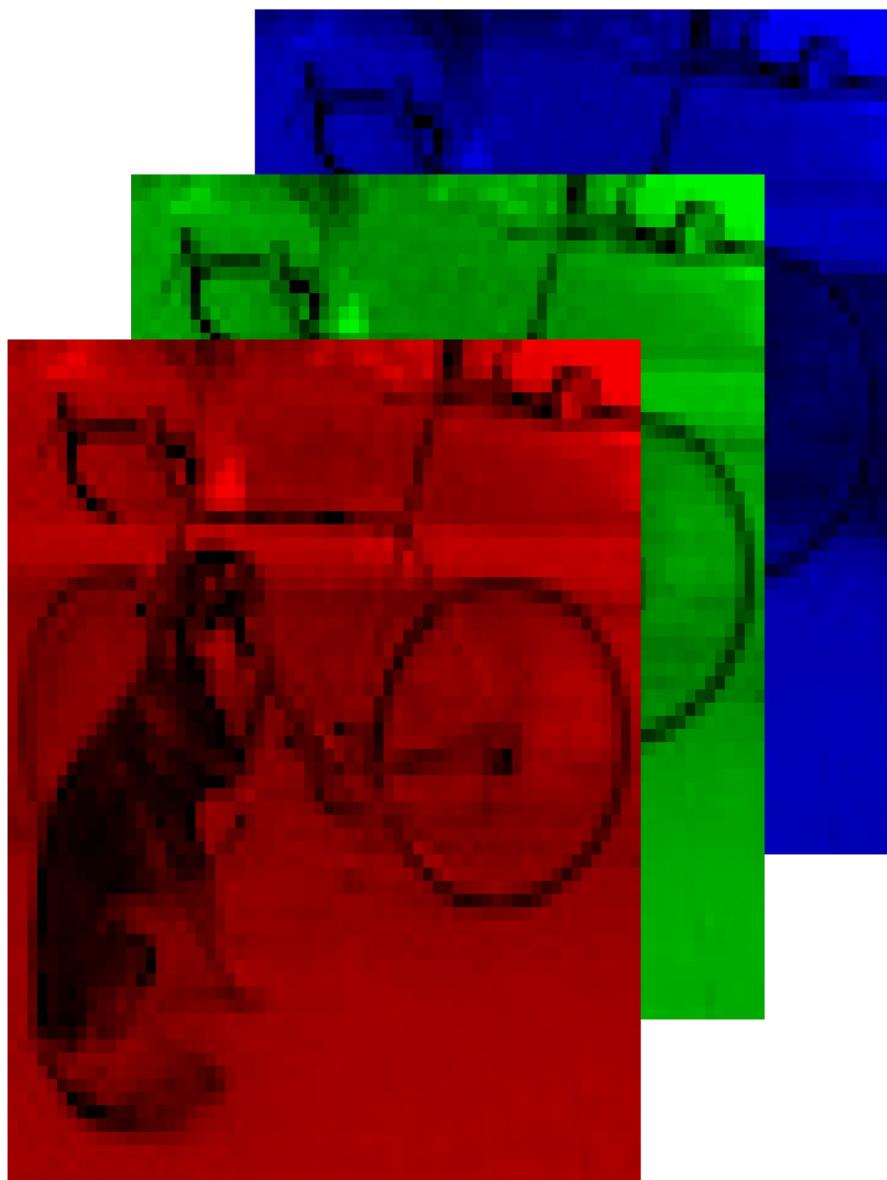
		Columns													
		0	1	2	3	4	5	6	...						
Rows	0	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	1	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	2	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	3	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	4	100	102	107	102	132	146	136	156	148	122	115	104	105	103
	5	100	102	107	102	132	30	60	156	148	122	115	104	105	103
	6	100	102	107	102	132	40	20	50	32	20	20	24	30	62
	...	100	102	107	102	132	71	156	51	57	57	58	62	58	
	8	100	102	107	102	132	69	156	148	122	115	104	105	103	
	9	100	102	107	102	132	89	12	156	148	122	115	104	105	103
	10	100	102	107	102	132	146	13	45	148	122	115	104	105	103
	11	100	102	107	102	132	46	42	122	115	104	105	103		

# Color image: 3d tensor in colorspace

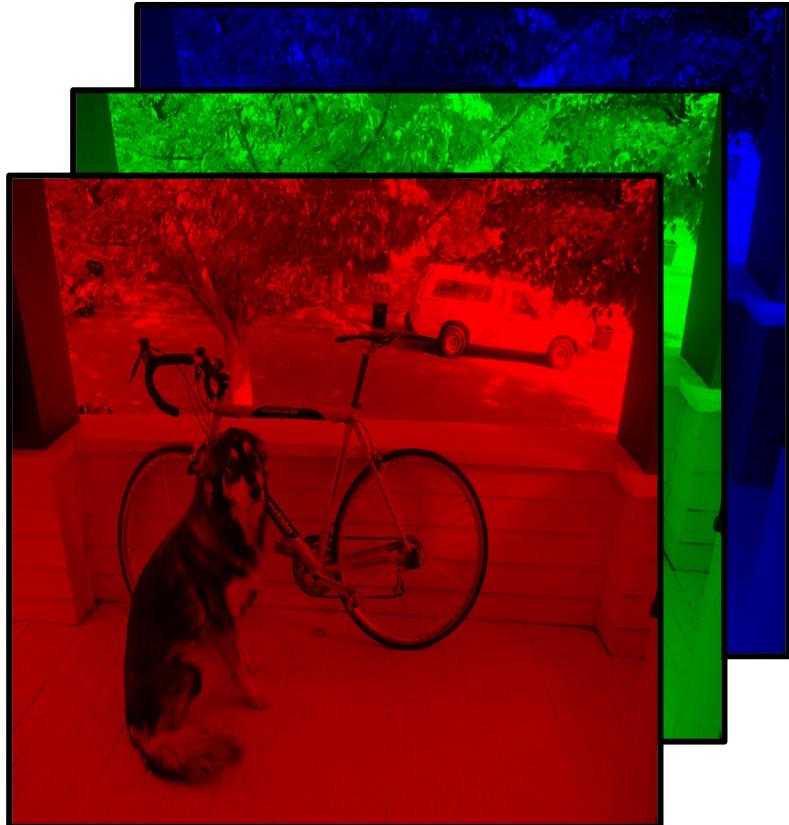


# RGB information in separate “channels”

- each channel encodes one primary, use RGB for now.
- we can match “real” colors using a mix of primaries.
- adding the light produced from each primary mimics the original color.

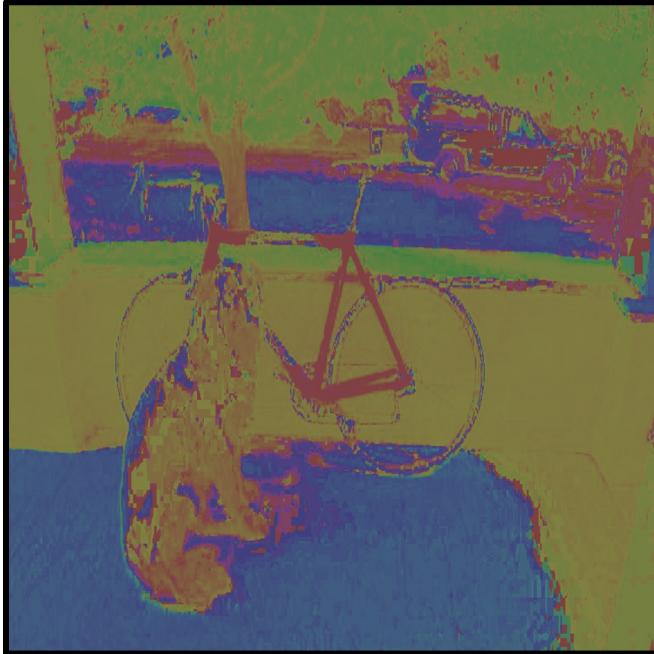


# HSV Color image: 3d tensor, different info



# Saturation

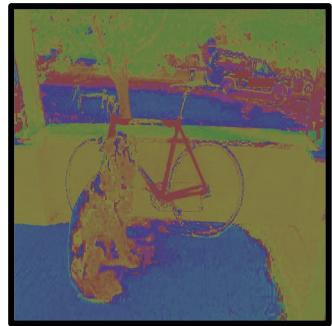
Hue



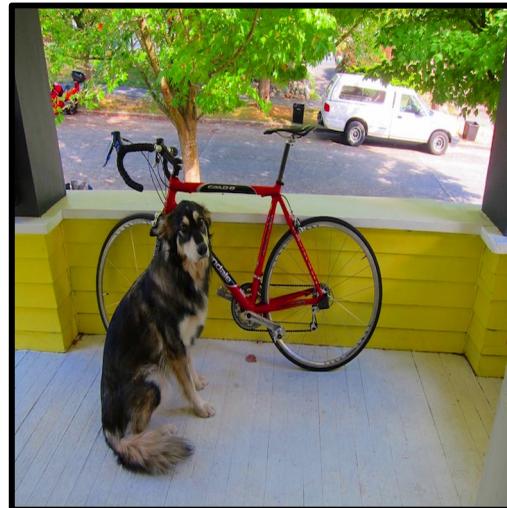
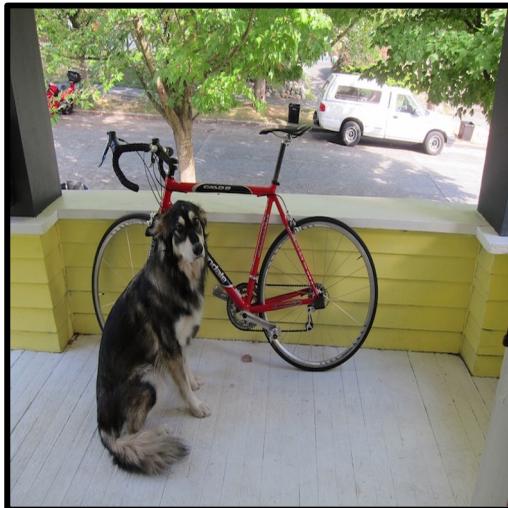
Value



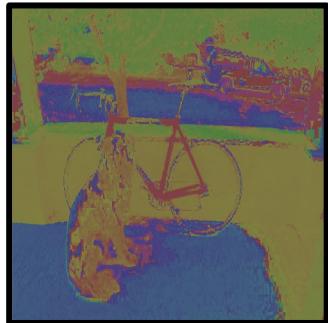
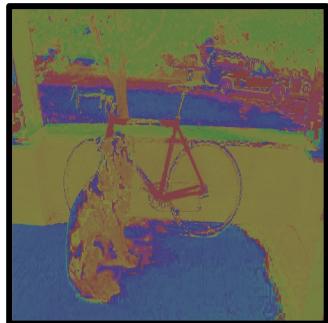
# More saturation = intense colors



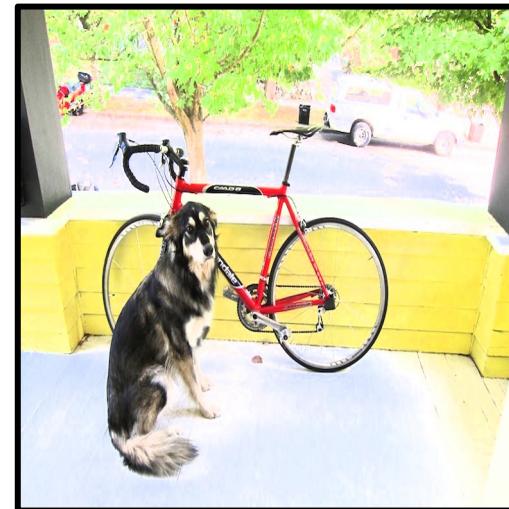
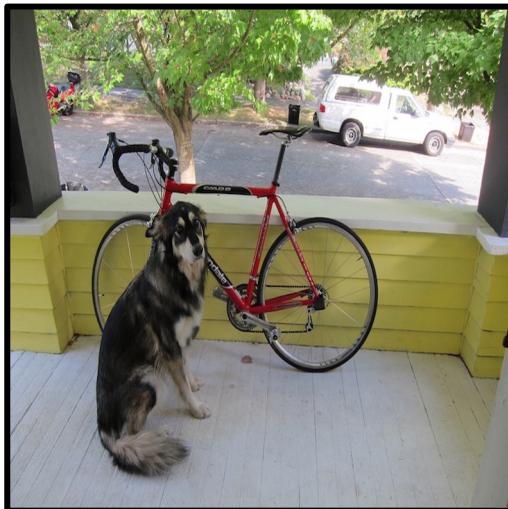
2x



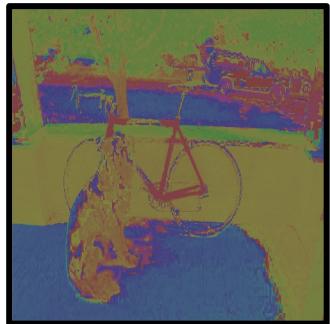
# More value = lighter image



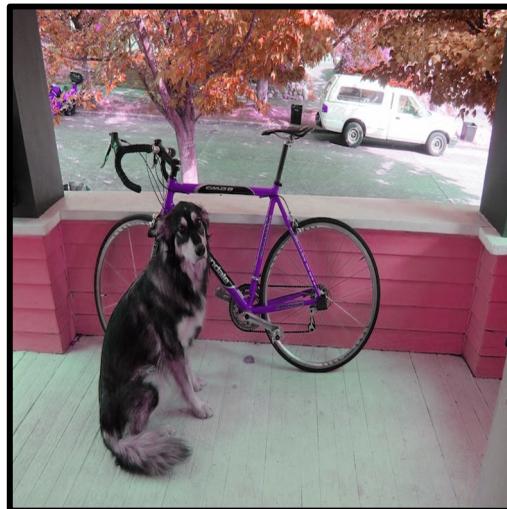
2x



# Shift hue = shift colors



- .2

A large black curved arrow pointing from the original color image down to the color image with the negative hue shift.

# Image interpolation and resizing

# An image is kinda like a function

An image is a mapping from indices to pixel values:

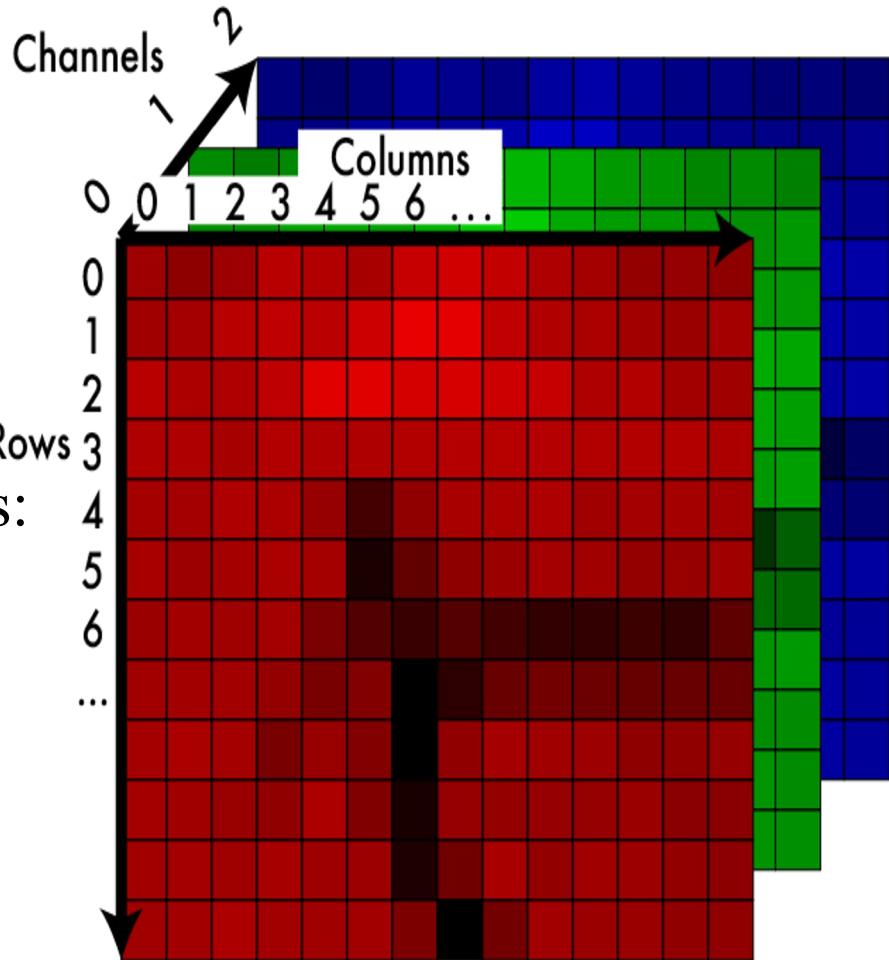
$$\text{Im} : I \times I \times I \rightarrow R$$

height	width	channel	value
indices of rows	indices of columns		

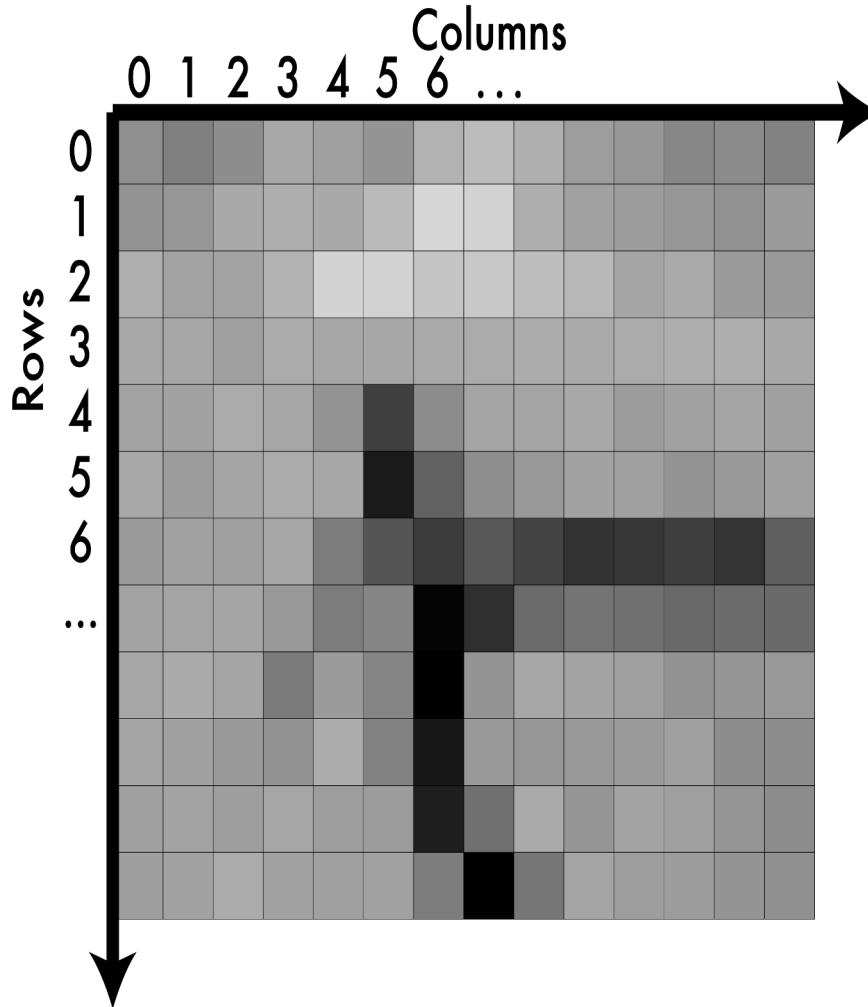
We may want to pass in non-integers:

$$\text{Im}' : R \times R \times I \rightarrow R$$

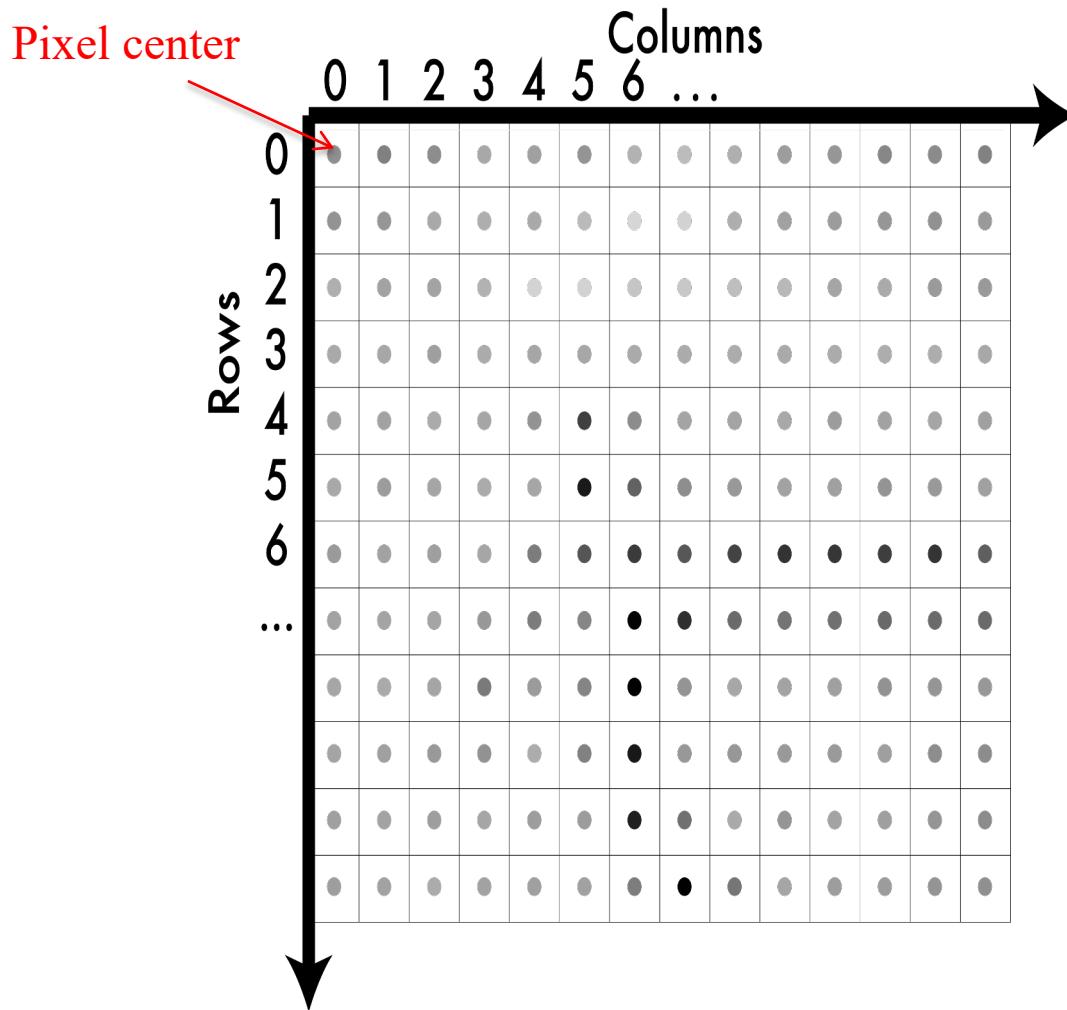
height	width	channel	value
--------	-------	---------	-------



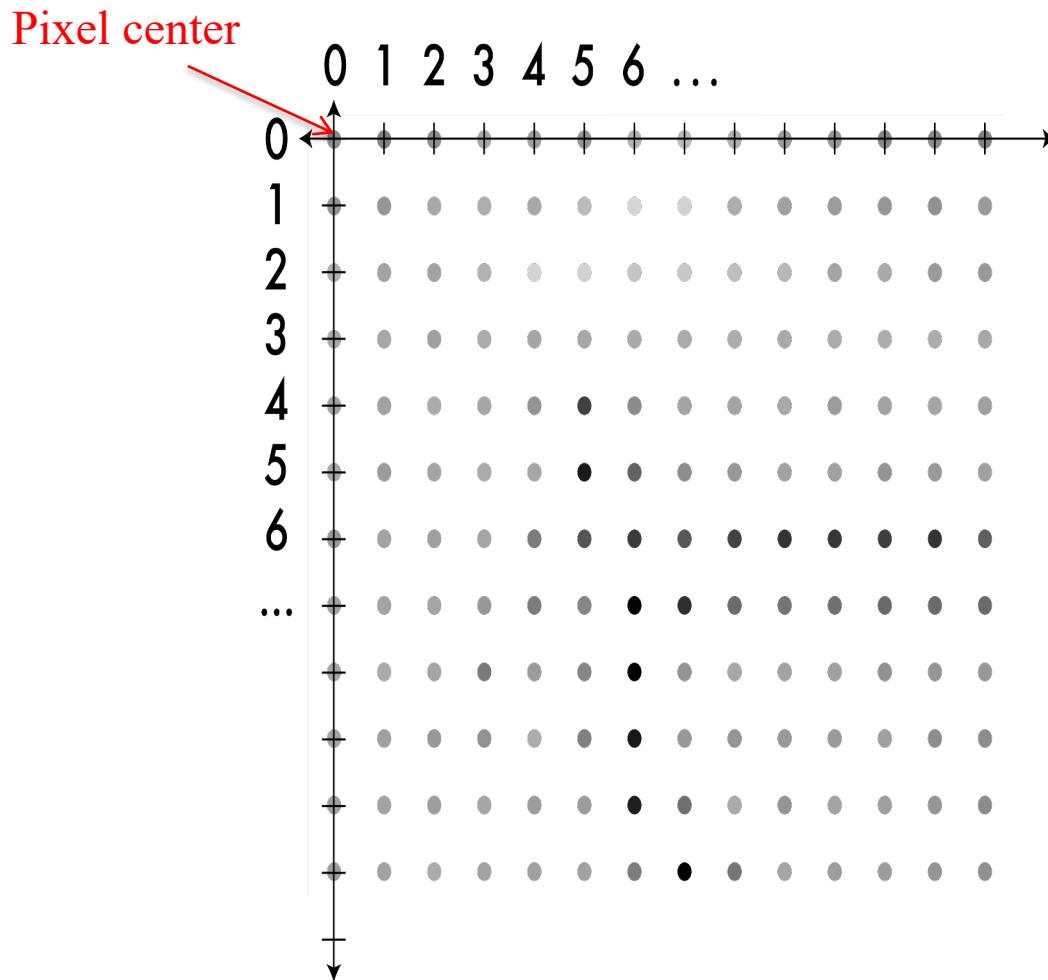
# A note on coordinates in images



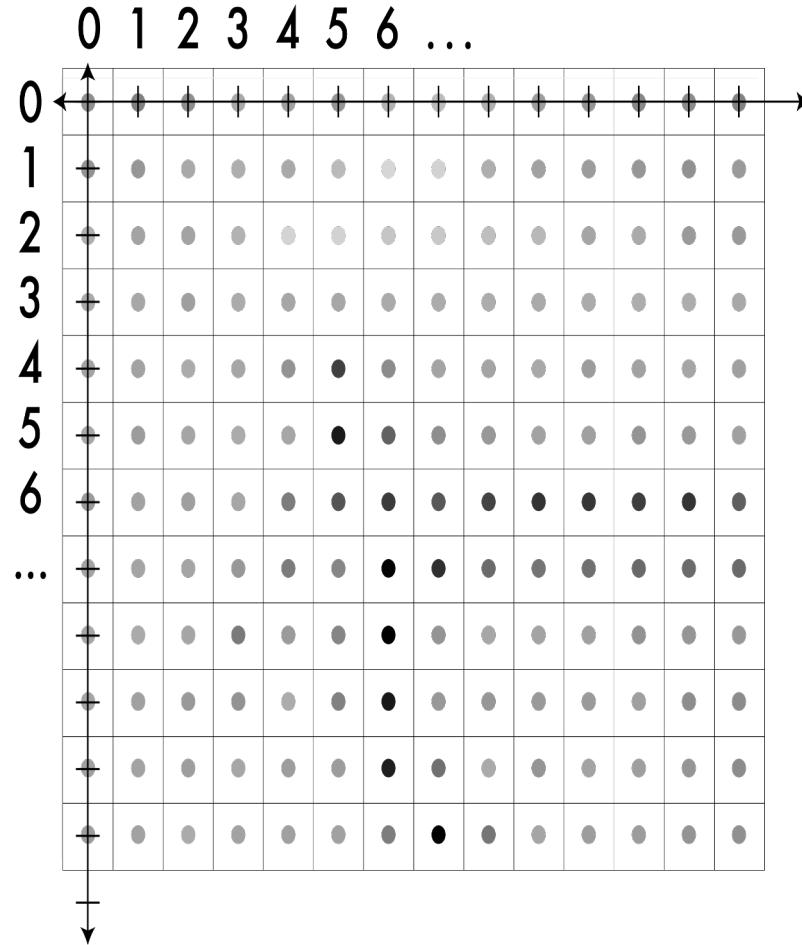
# A note on coordinates in images



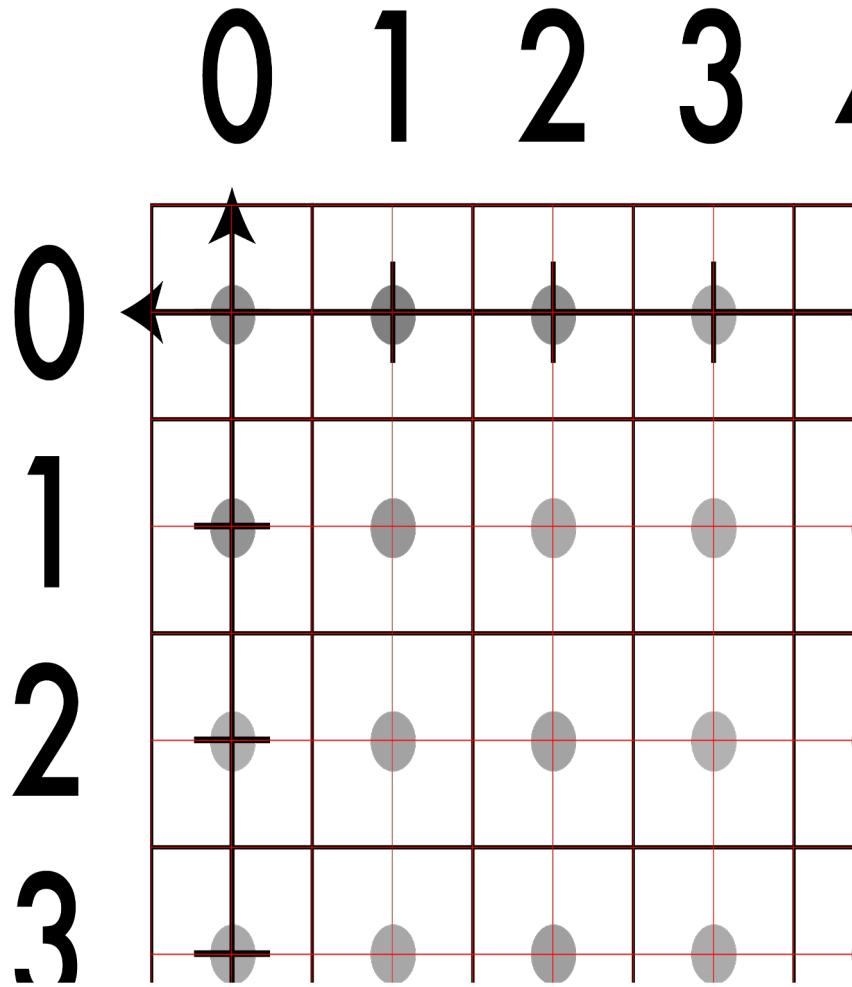
# A note on coordinates in images



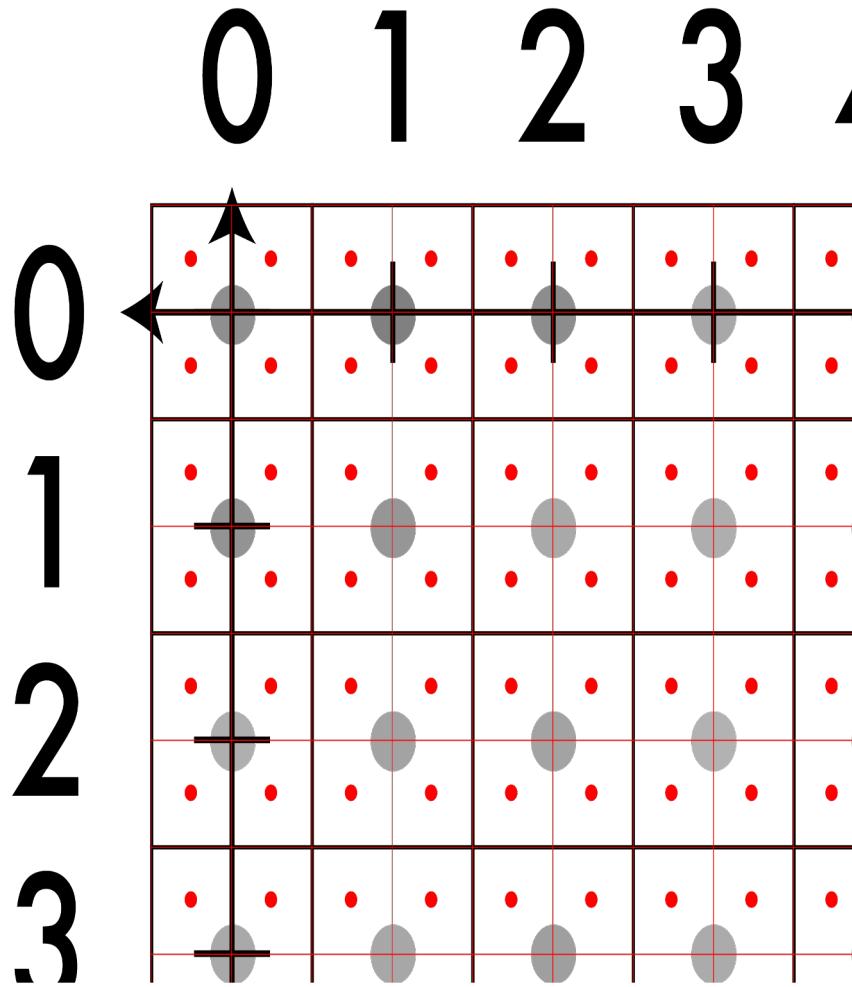
# A note on coordinates in images



# A note on coordinates in images

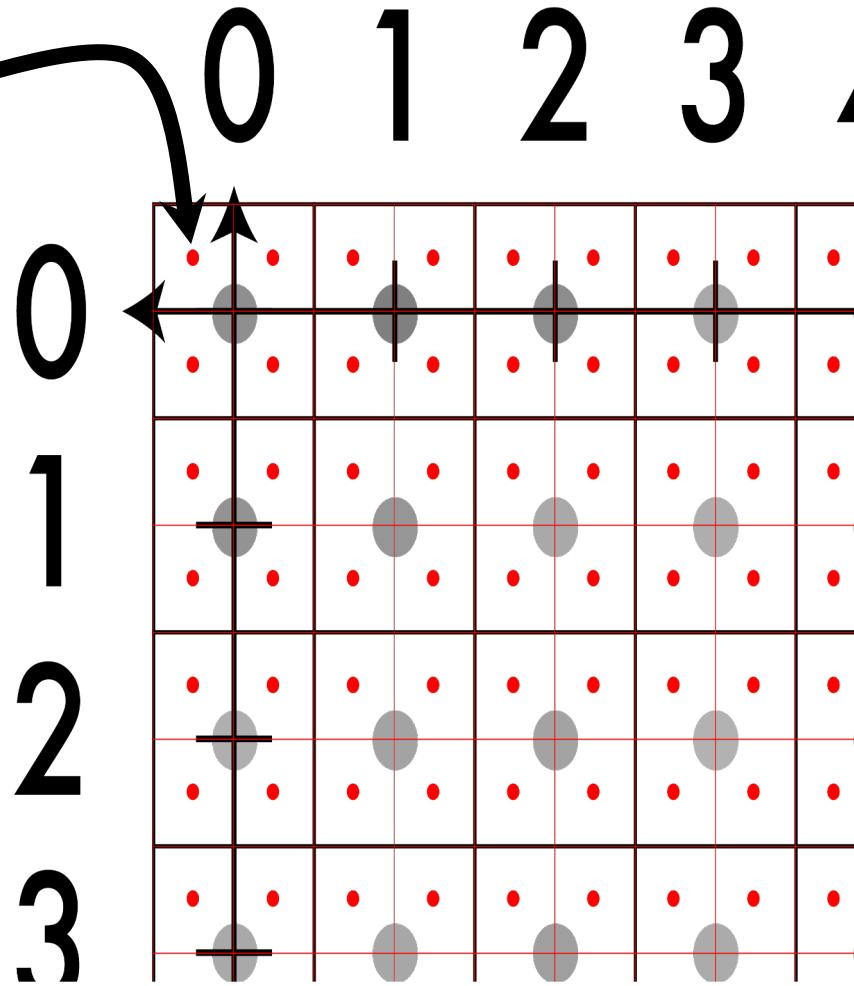


# A note on coordinates in images



# A note on coordinates in images

This point is:  
 $(-.25, -.25)$



# Nearest neighbor interpolation

$$\text{Im} : I \times I \times I \rightarrow R$$

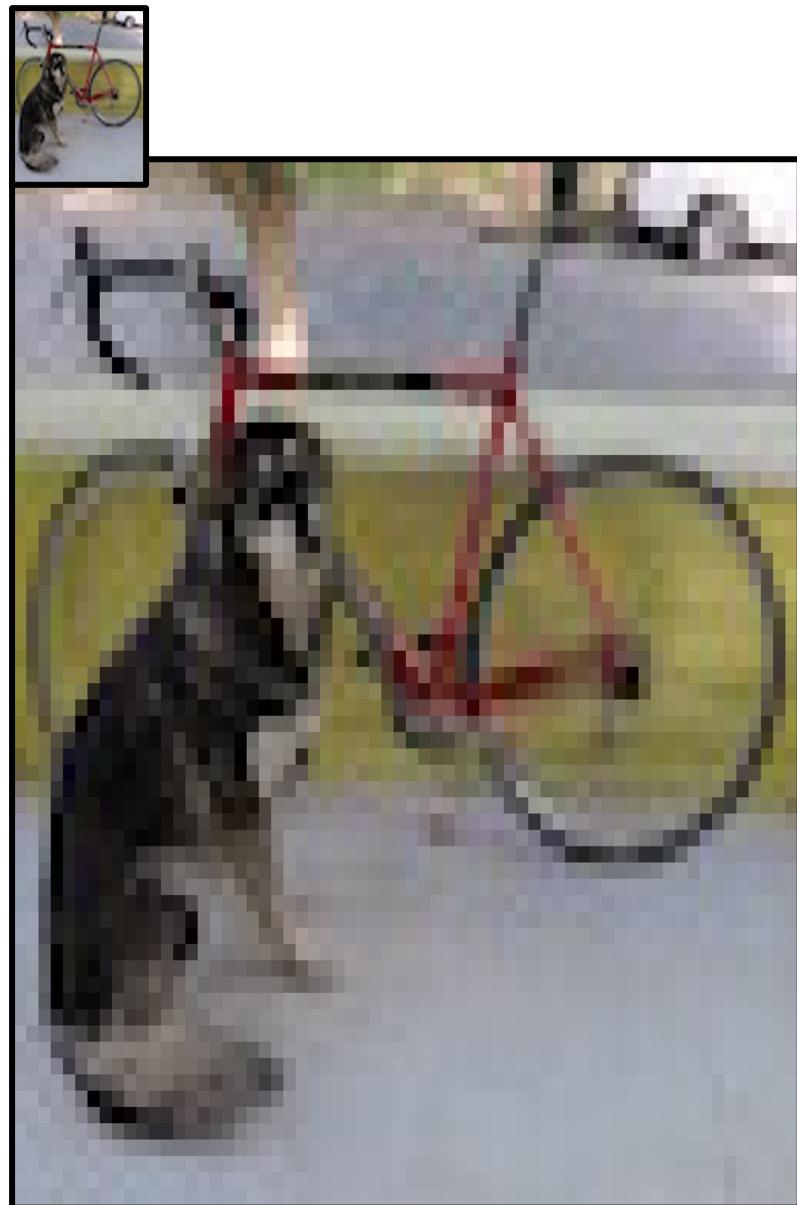
height      width      channel      value

$$\text{Im}' : R \times R \times I \rightarrow R$$

height      width      channel      value

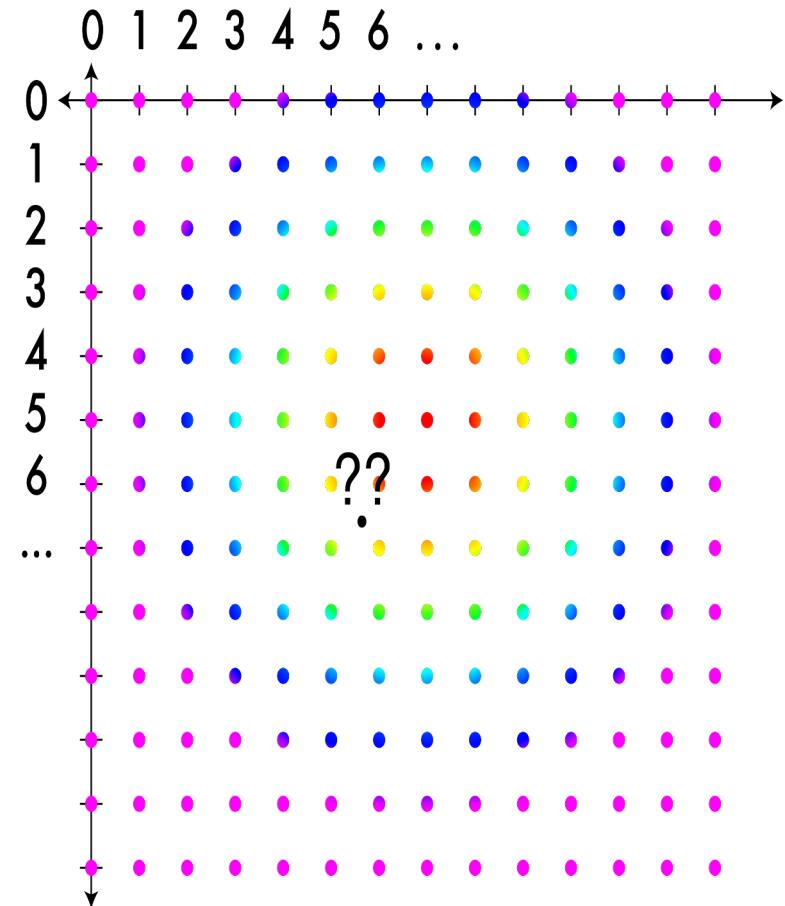
$$\text{Im}'(y, x, c) = \text{Im}(\text{round}(y), \text{round}(x), c)$$

- Looks blocky



# Bilinear interpolation

Find the closest pixels in a box



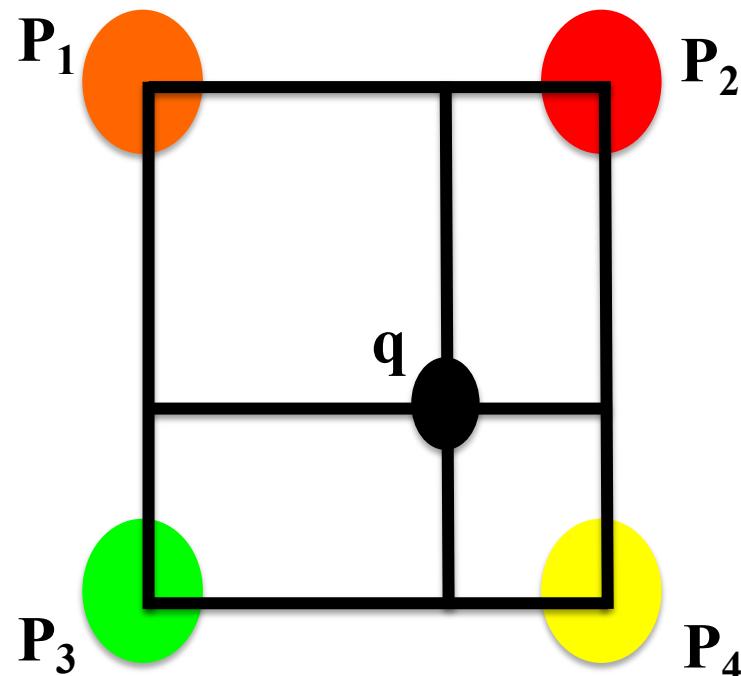
# Bilinear interpolation

Find the closest pixels in a box

Determine coefficients  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$   
such that:

$$q = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 P_4$$

Closer pixels should have a higher coefficient (weight)



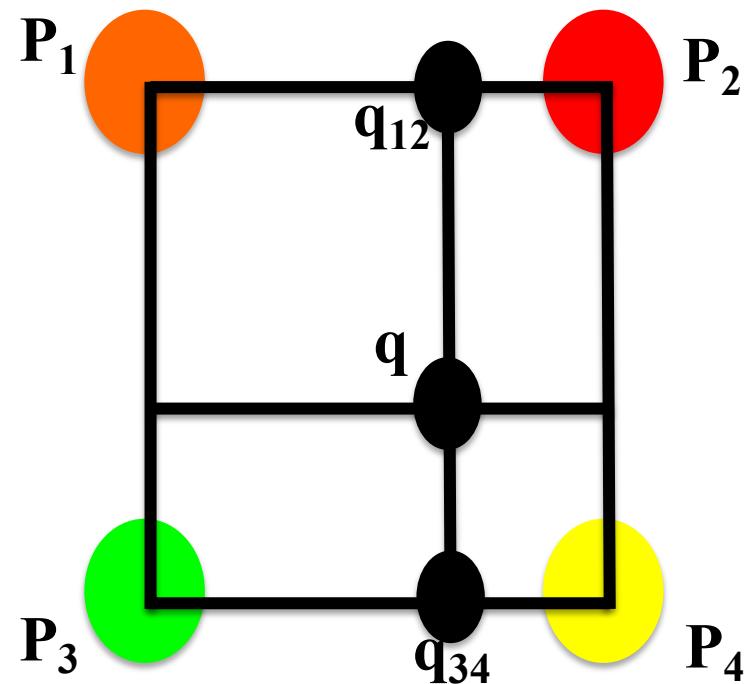
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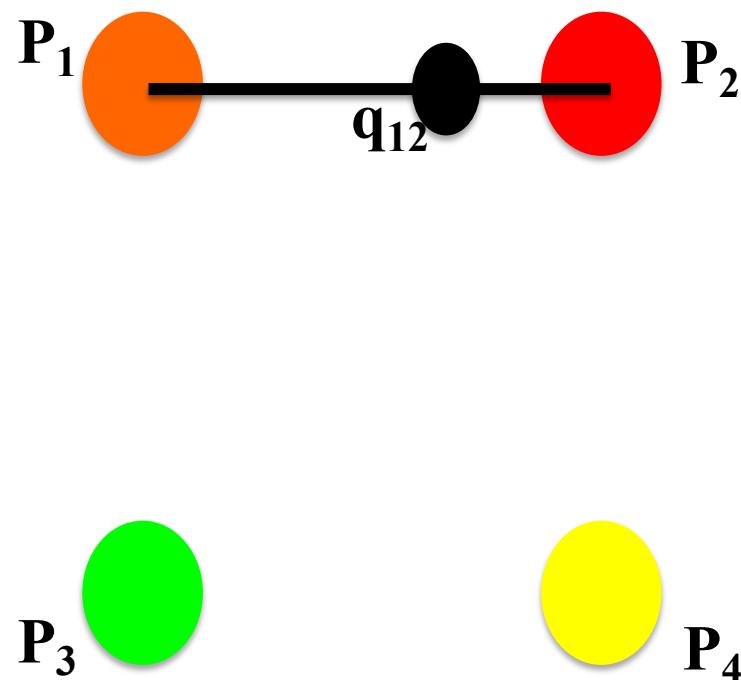
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$q_{12}$  between  $P_1$  and  $P_2$



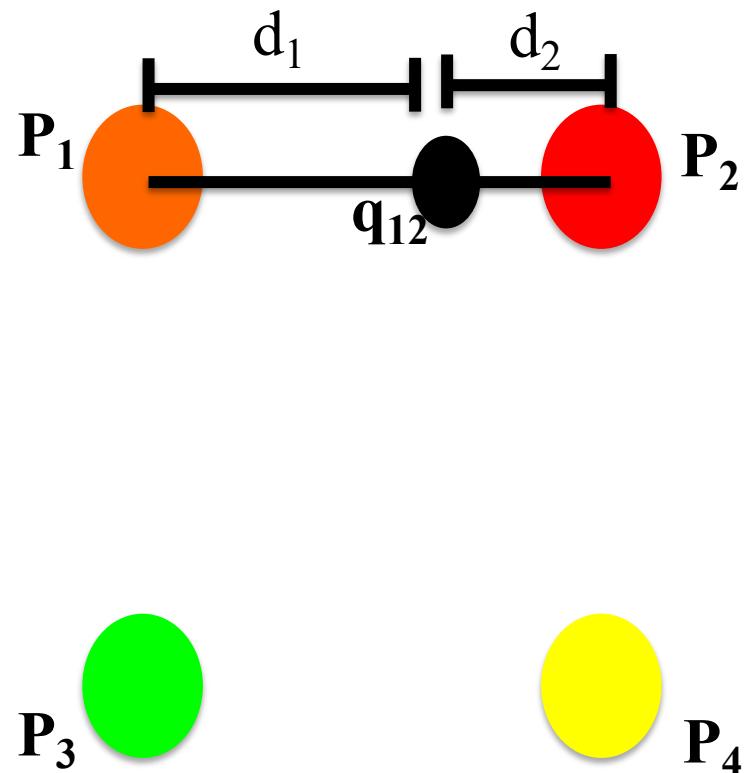
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Find the closest pixels in a box

Determine coefficients  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$   
such that:

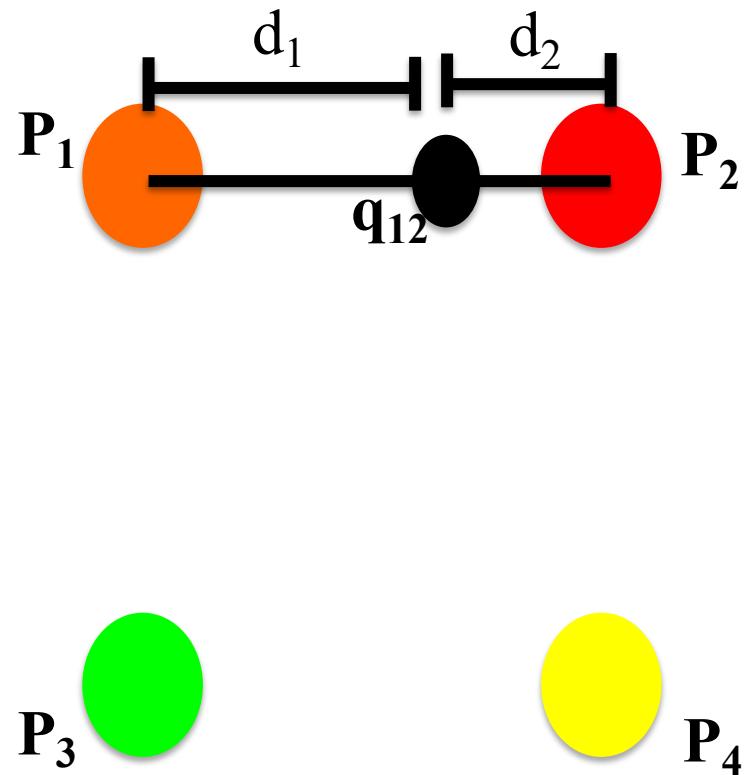
$$q = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 P_4$$

$q_{12}$  between  $P_1$  and  $P_2$

$$q_{12} = \frac{d_2}{d_1 + d_2} P_1 + \frac{d_1}{d_1 + d_2} P_2$$

But  $d_1 + d_2 = 1$ , so:

$$q_{12} = d_2 P_1 + d_1 P_2$$



# Bilinear interpolation

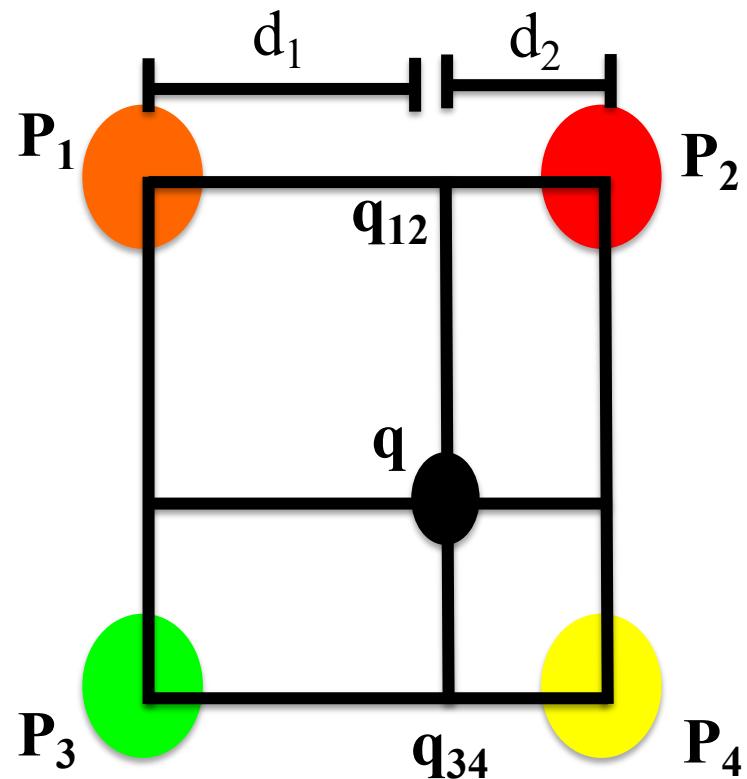
Find the closest pixels in a box

Determine coefficients  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$   
such that:

$$q = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 P_4$$

$$q_{12} = d_2 P_1 + d_1 P_2$$

$$q_{34} = d_2 P_3 + d_1 P_4$$



# Bilinear interpolation

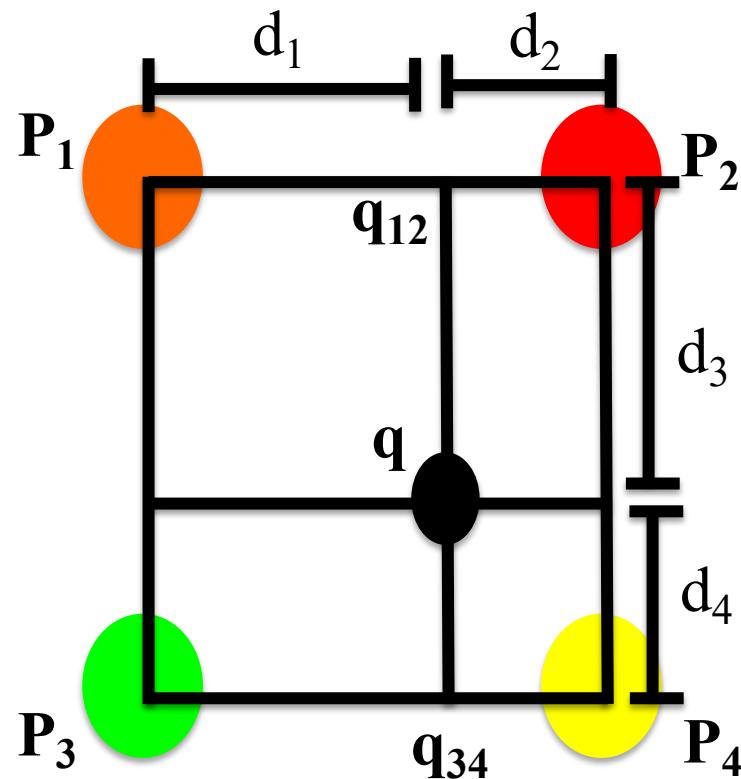
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Determine coefficients  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$   
such that:

$$q = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 P_4$$

$$q_{12} = d_2 P_1 + d_1 P_2$$

$$q_{34} = d_2 P_3 + d_1 P_4$$



# Bilinear interpolation

Find the closest pixels in a box

Determine coefficients  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$   
such that:

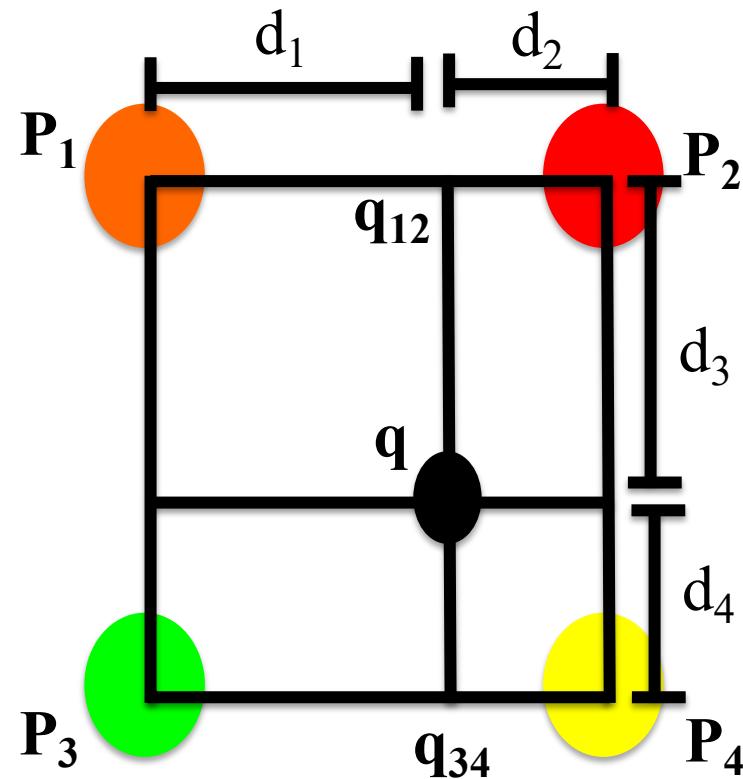
$$q = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 P_4$$

$$q_{12} = d_2 P_1 + d_1 P_2$$

$$q_{34} = d_2 P_3 + d_1 P_4$$

Pixel  $q$  between  $q_{12}$  and  $q_{34}$ , so:

$$q = d_4 q_{12} + d_3 q_{34} = d_4(d_2 P_1 + d_1 P_2) + d_3(d_2 P_3 + d_1 P_4)$$



# Bilinear interpolation

Find the closest pixels in a box

Determine coefficients  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$   
such that:

$$q = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 P_4$$

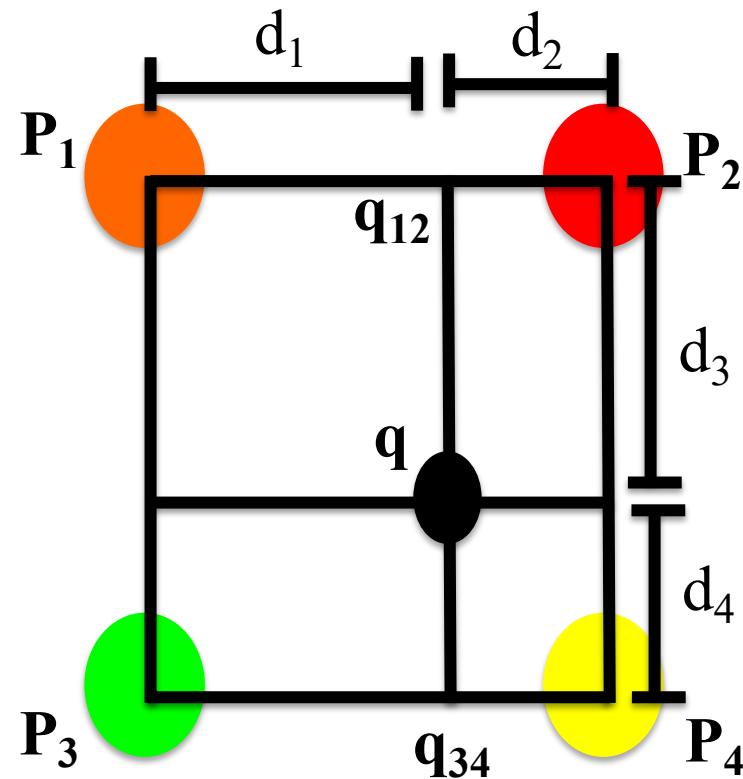
$$q_{12} = d_2 P_1 + d_1 P_2$$

$$q_{34} = d_2 P_3 + d_1 P_4$$

Pixel  $q$  between  $q_{12}$  and  $q_{34}$ , so:

$$q = d_4 q_{12} + d_3 q_{34} = d_4(d_2 P_1 + d_1 P_2) + d_3(d_2 P_3 + d_1 P_4)$$

$$\text{So, } \alpha_1 = d_2 d_4, \alpha_2 = d_1 d_4, \alpha_3 = d_2 d_3, \alpha_4 = d_1 d_3$$



# Bilinear interpolation

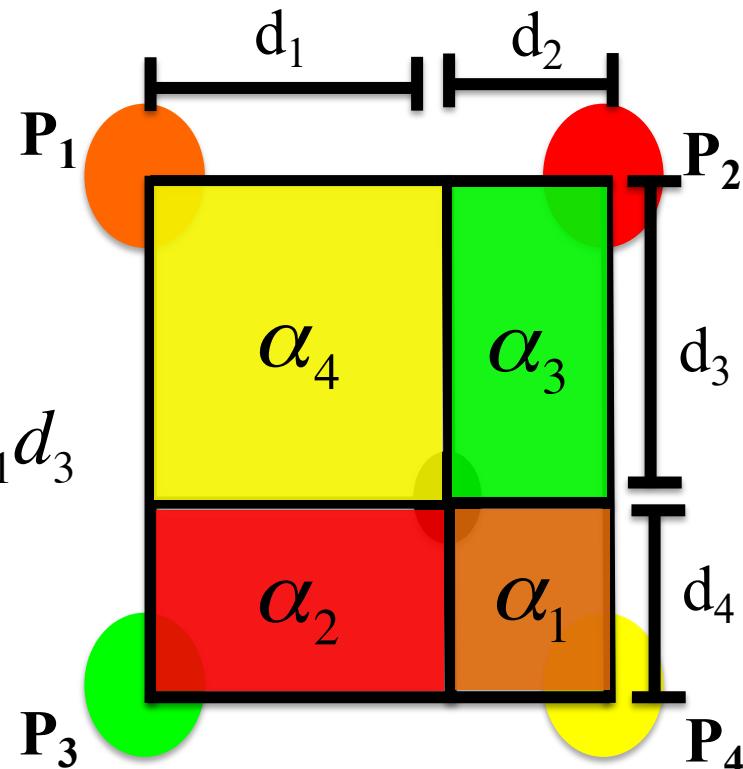
Find the closest pixels in a box

Determine coefficients  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$   
such that:

$$q = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 P_4$$

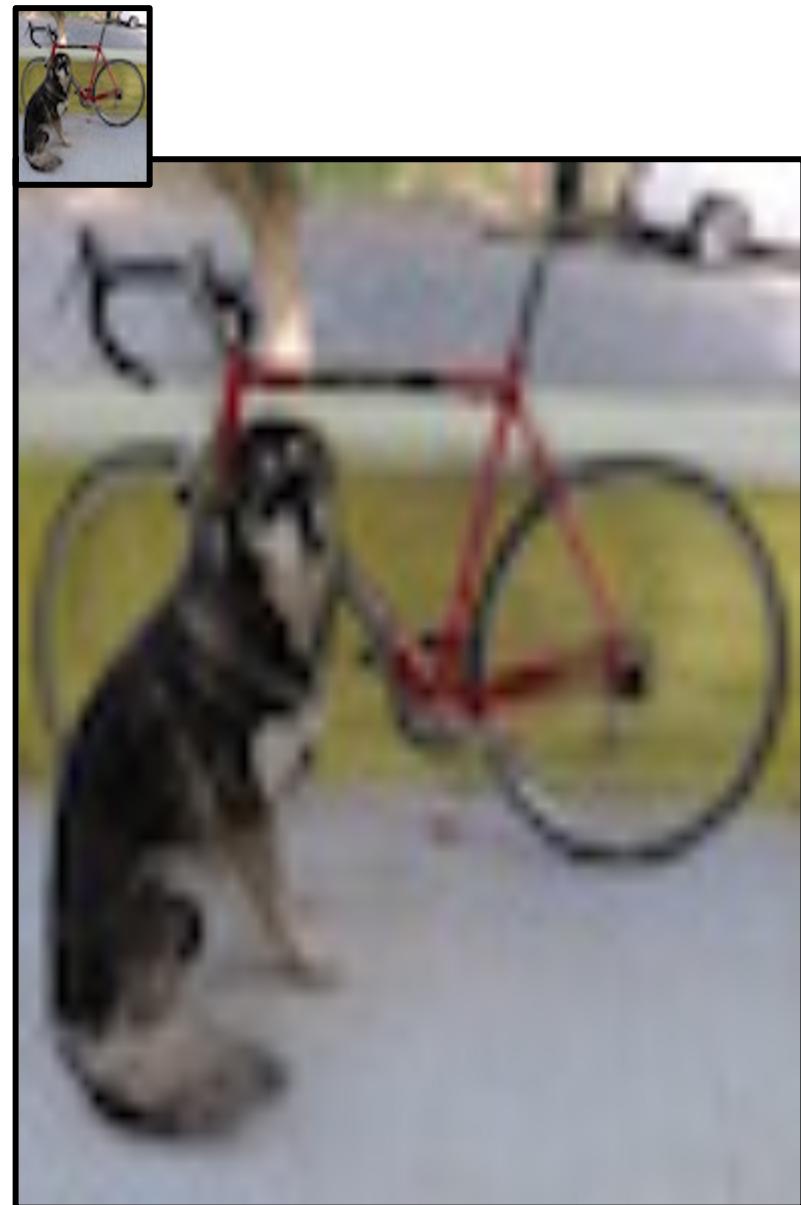
$$\alpha_1 = d_2 d_4, \alpha_2 = d_1 d_4, \alpha_3 = d_2 d_3, \alpha_4 = d_1 d_3$$

$\alpha_i$  - area of the opposite rectangle to  
pixel  $P_i$



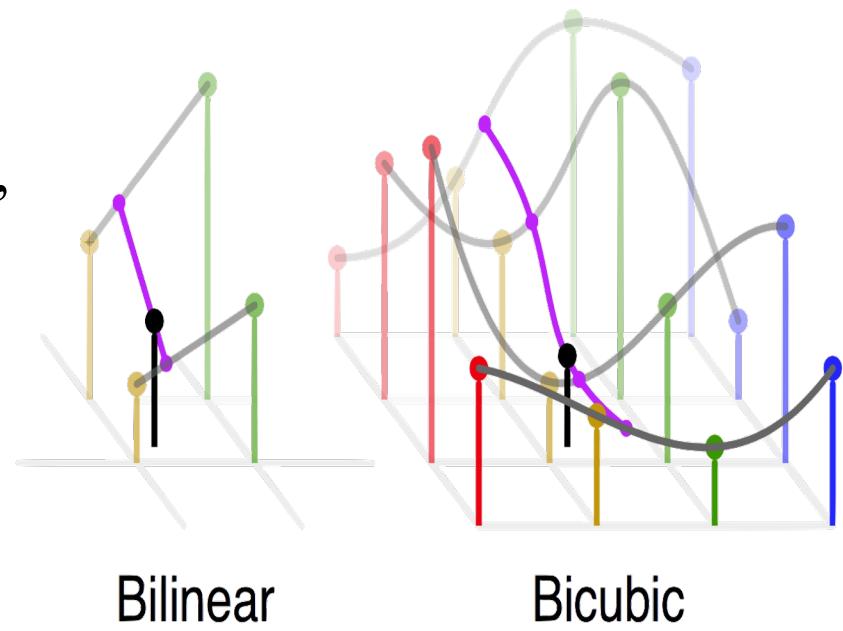
# Bilinear interpolation

- Smoother than NN
- More complex
  - 4 lookups
  - some math
- Often the right tradeoff of speed vs final result

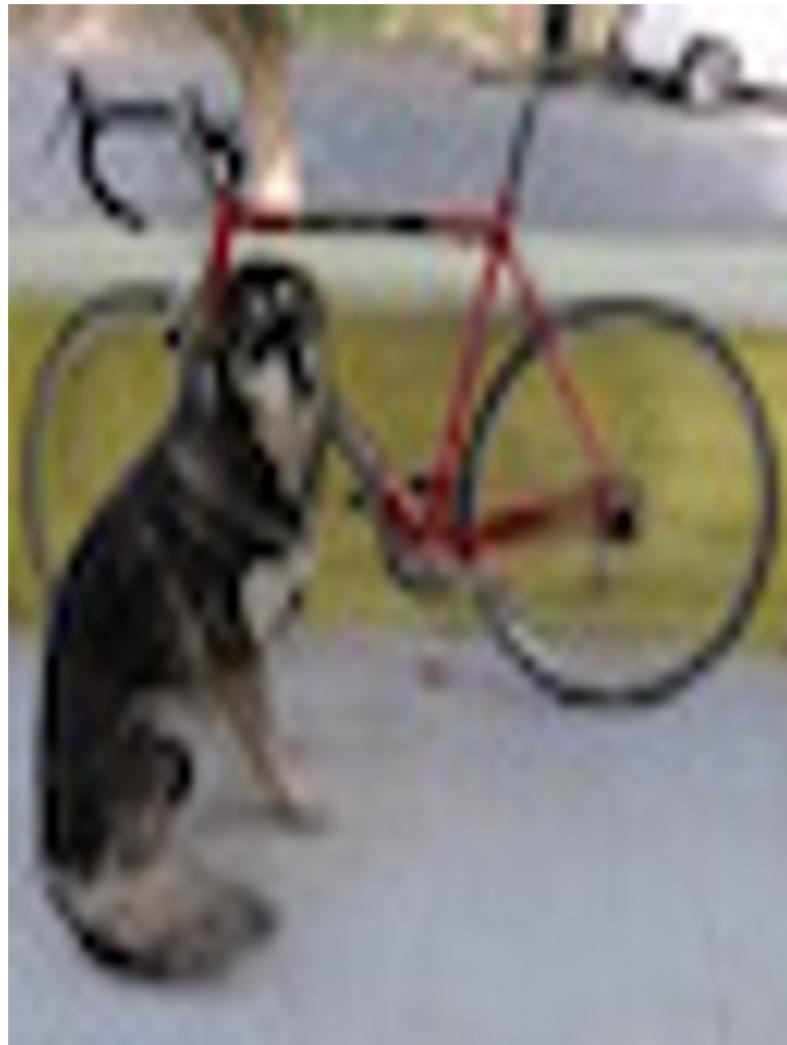


# Bicubic interpolation

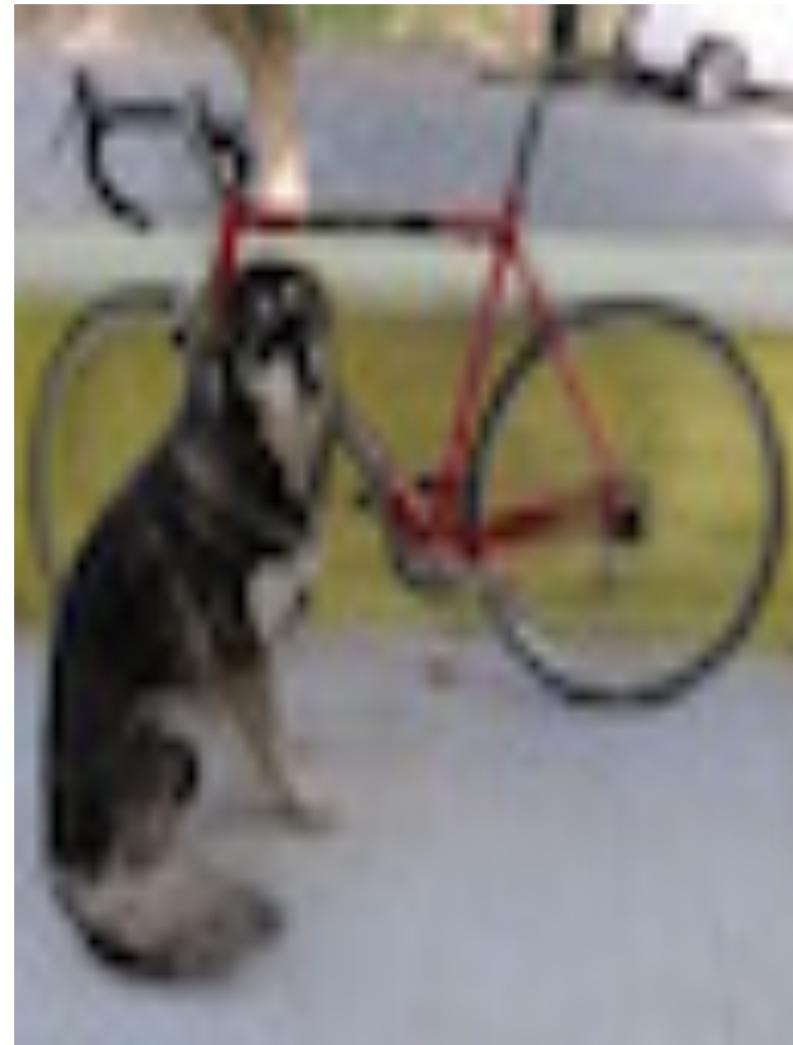
- A cubic interpolation of 4 cubic interpolations
- Smoother than bilinear, no “star”
- 16 nearest neighbors
- Use 4 points to fit 3<sup>rd</sup> order poly:
  - $f(x) = a + bx + cx^2 + dx^3$
- Interpolate along axis
- Fit another poly to interpolated values
- More complex
- Maybe better?



# Bicubic vs bilinear

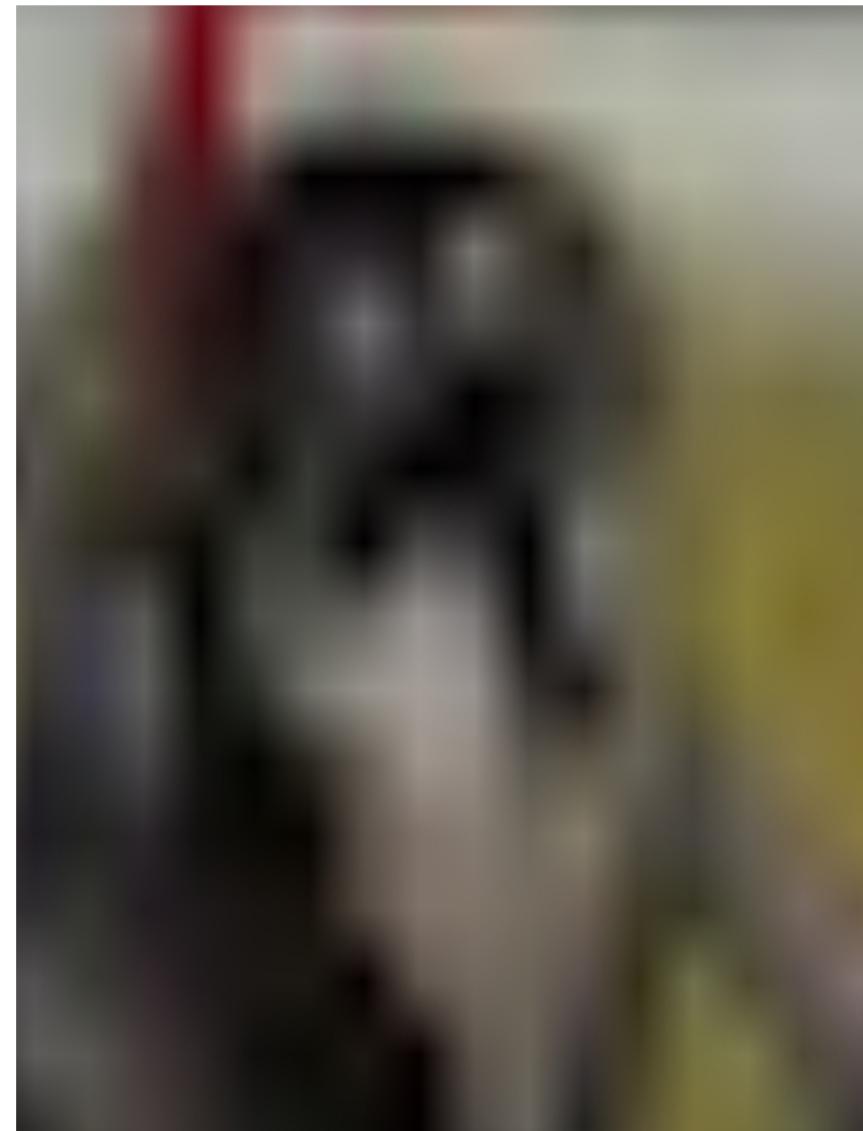
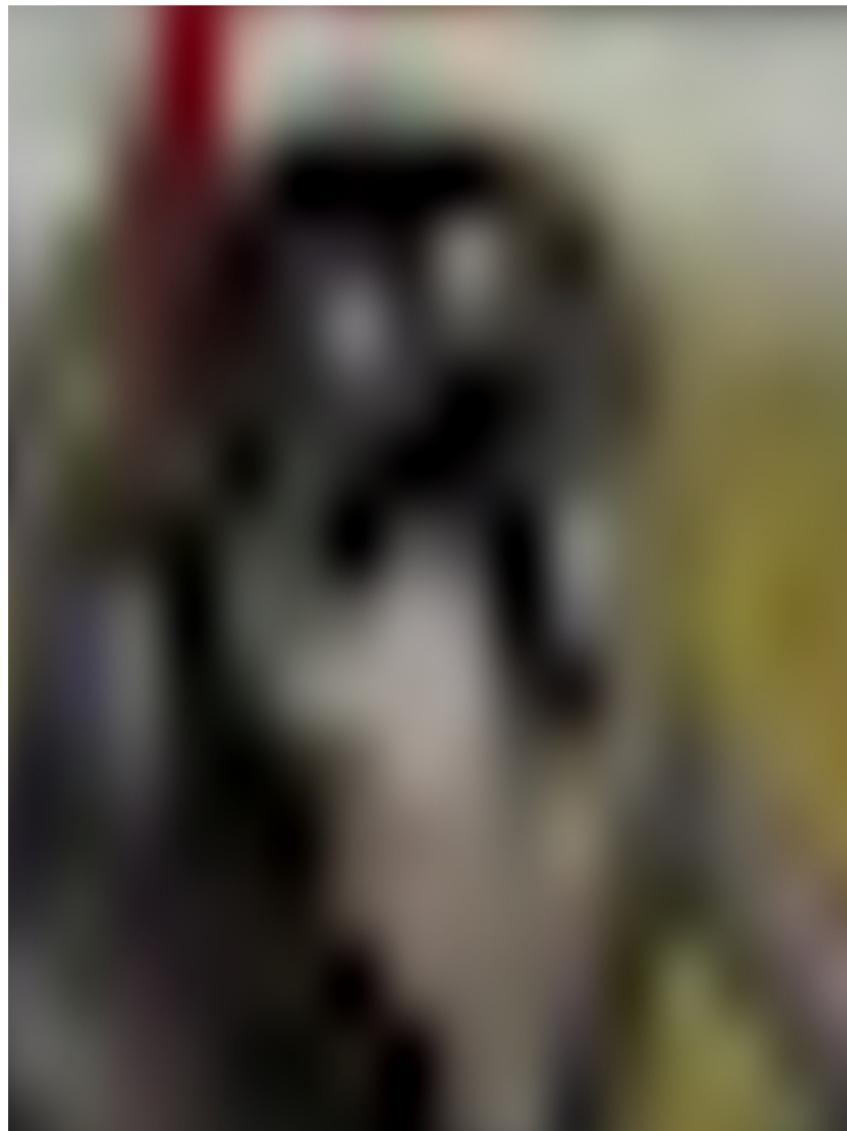


bicubic



bilinear

# Bicubic vs bilinear

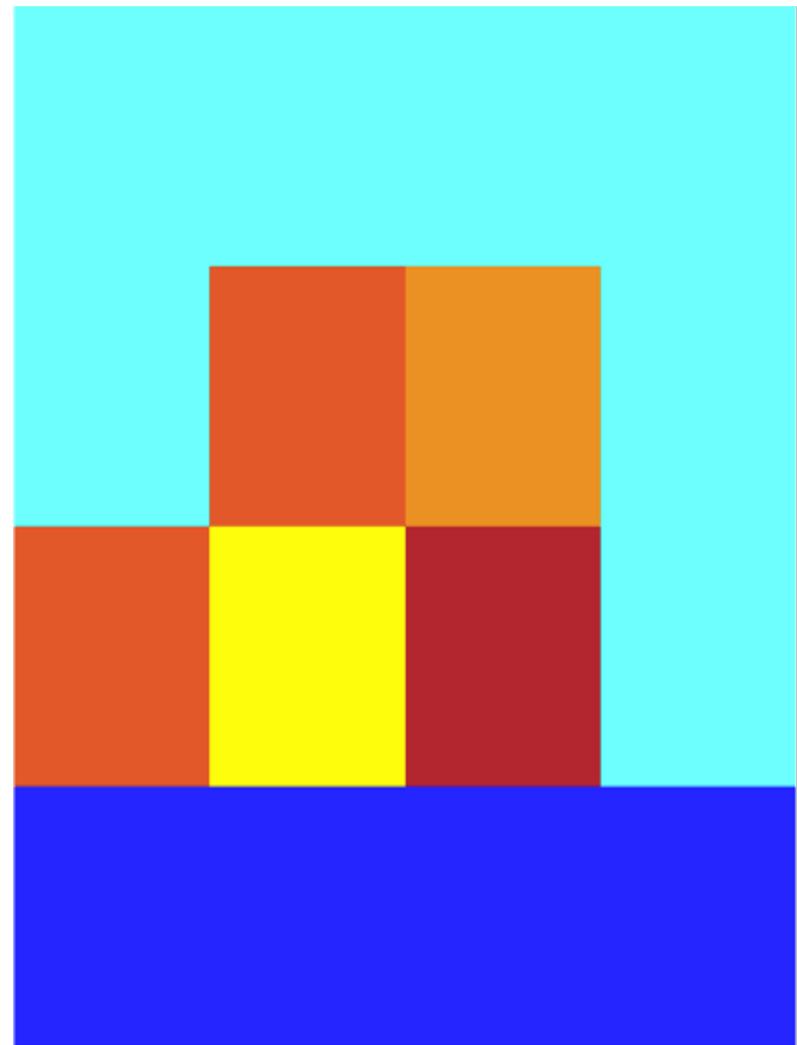


# Using interpolation for Image Resizing

# Image resizing!

Say we want to increase the size of an image...

This is a beautiful image of a sunset... it's just very small...

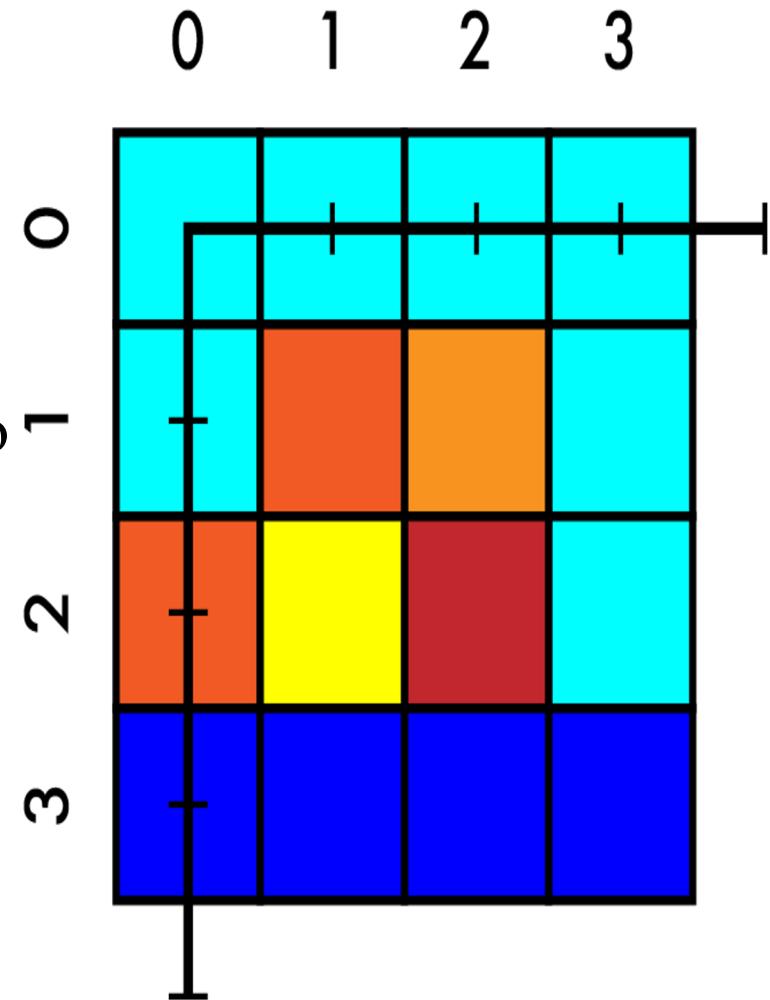


# Image resizing!

Say we want to increase the size of an image...

This is a beautiful image of a sunset... it's just very small...

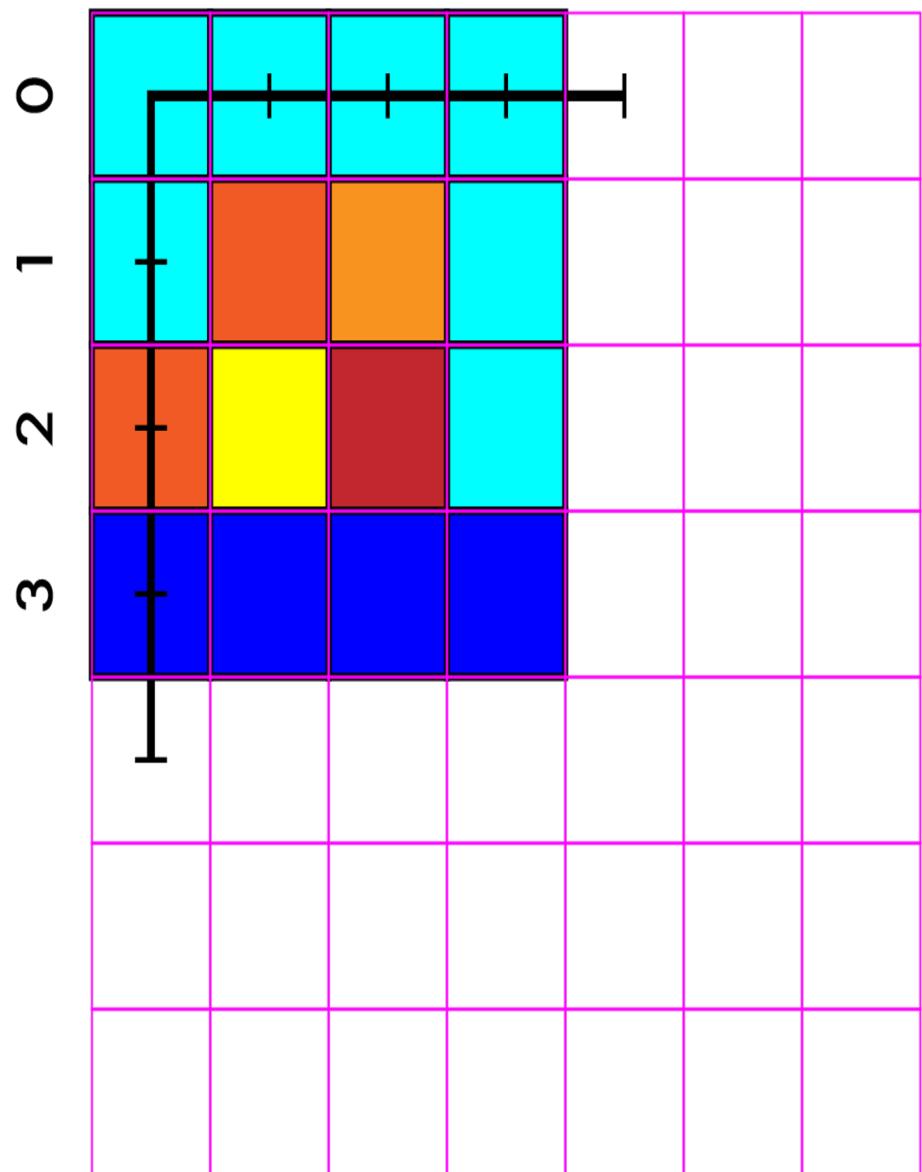
Say we want to increase size  $4 \times 4$  to  $7 \times 7$



# Resize 4 x 4 to 7 x 7

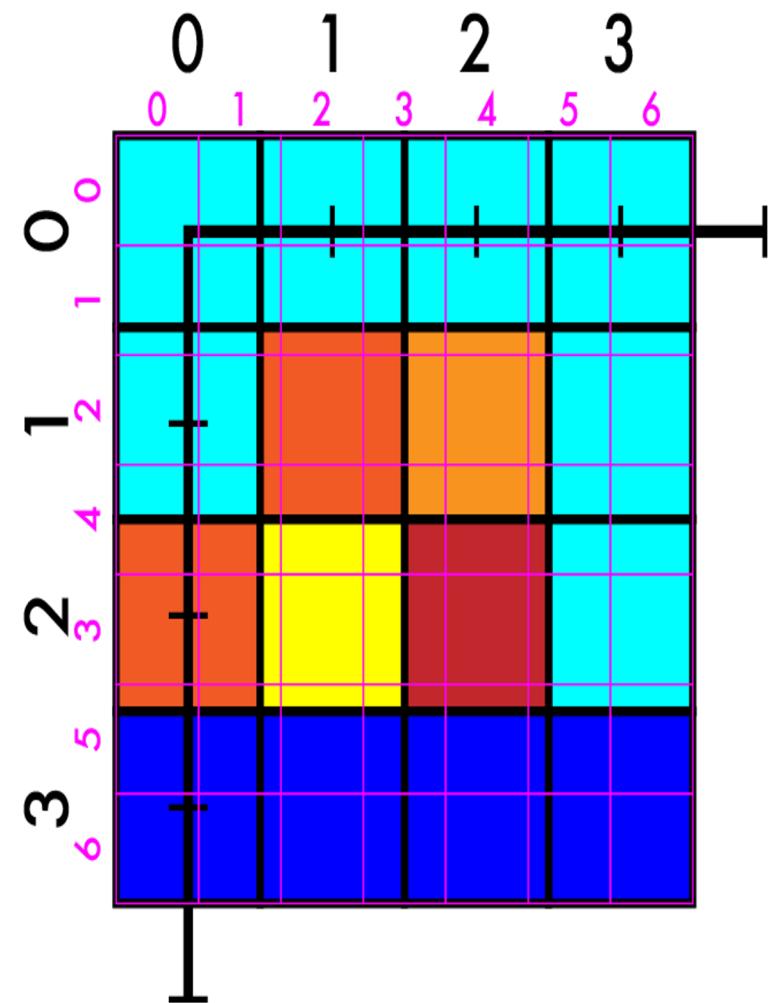
0 1 2 3

- Create our new image



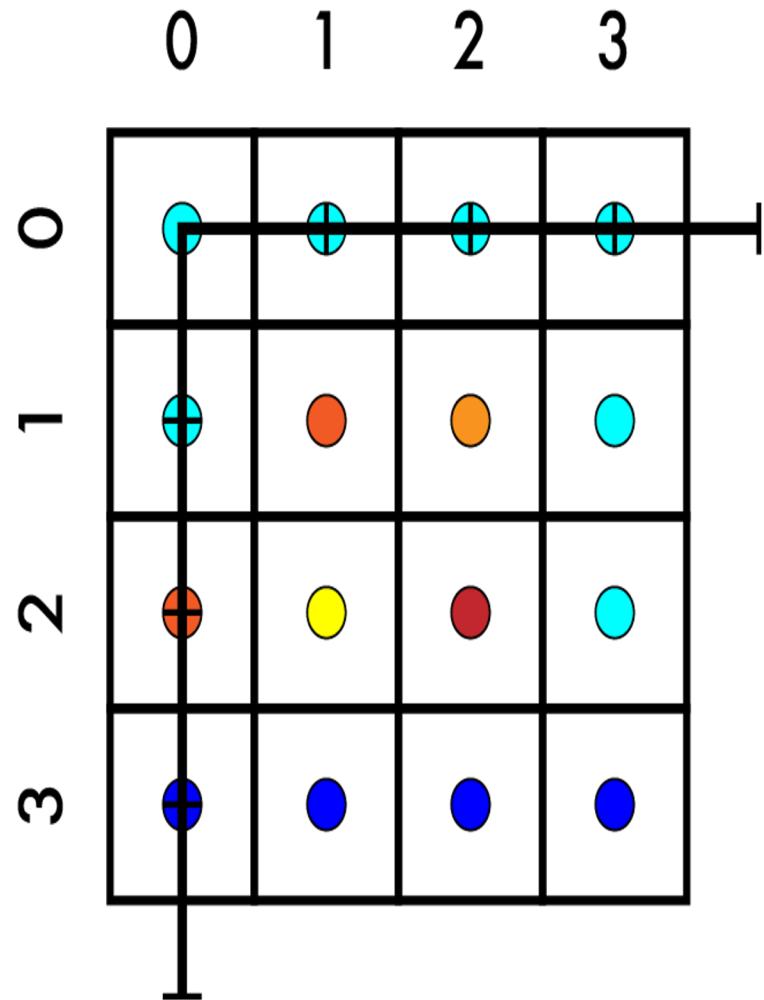
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates



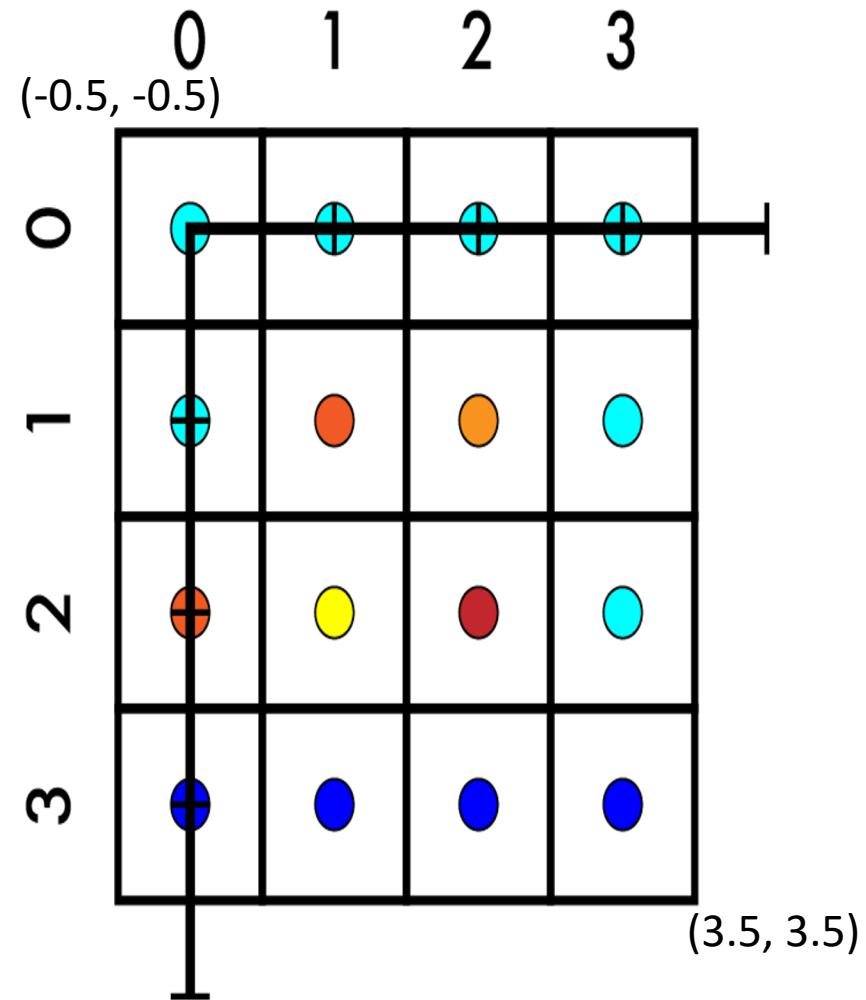
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates



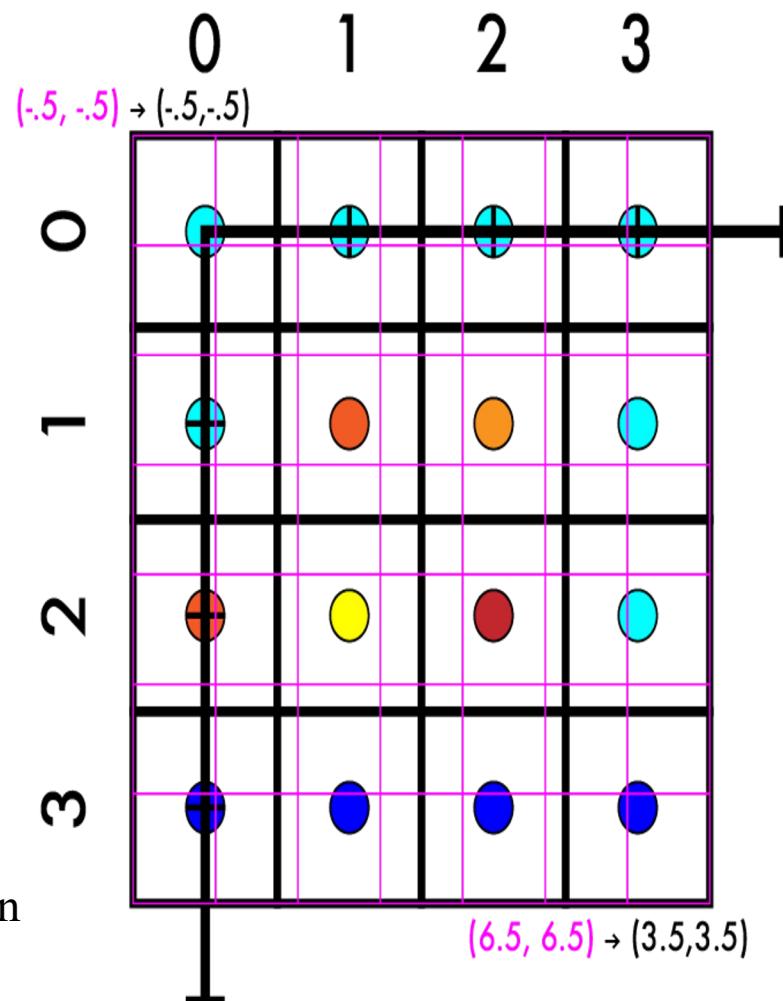
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates



# Match up coordinates

- Find the 2D transform that maps the left corner of the first image to the left corner of the second image + the right corner of the first image to the right corner of the second image (or the other way round)



# Match up coordinates

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

scaling                          translation

$$\begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

Left corner                          Left corner  
1<sup>st</sup> image                          2<sup>nd</sup> image

$$\begin{pmatrix} 3.5 \\ 3.5 \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} 6.5 \\ 6.5 \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

Right corner                          Right corner  
1<sup>st</sup> image                          2<sup>nd</sup> image

$-s_x * 0.5 + t_x = -0.5 \quad (1)$

$-s_y * 0.5 + t_y = -0.5 \quad (2)$

$s_x * 6.5 + t_x = 3.5 \quad (3)$

$s_y * 6.5 + t_y = 3.5 \quad (4)$

4 equations with 4 unknowns

# Match up coordinates

$$\left\{ \begin{array}{l} -s_x * 0.5 + t_x = -0.5 \quad (1) \\ -s_y * 0.5 + t_y = -0.5 \quad (2) \\ s_x * 6.5 + t_x = 3.5 \quad (3) \\ s_y * 6.5 + t_y = 3.5 \quad (4) \end{array} \right.$$

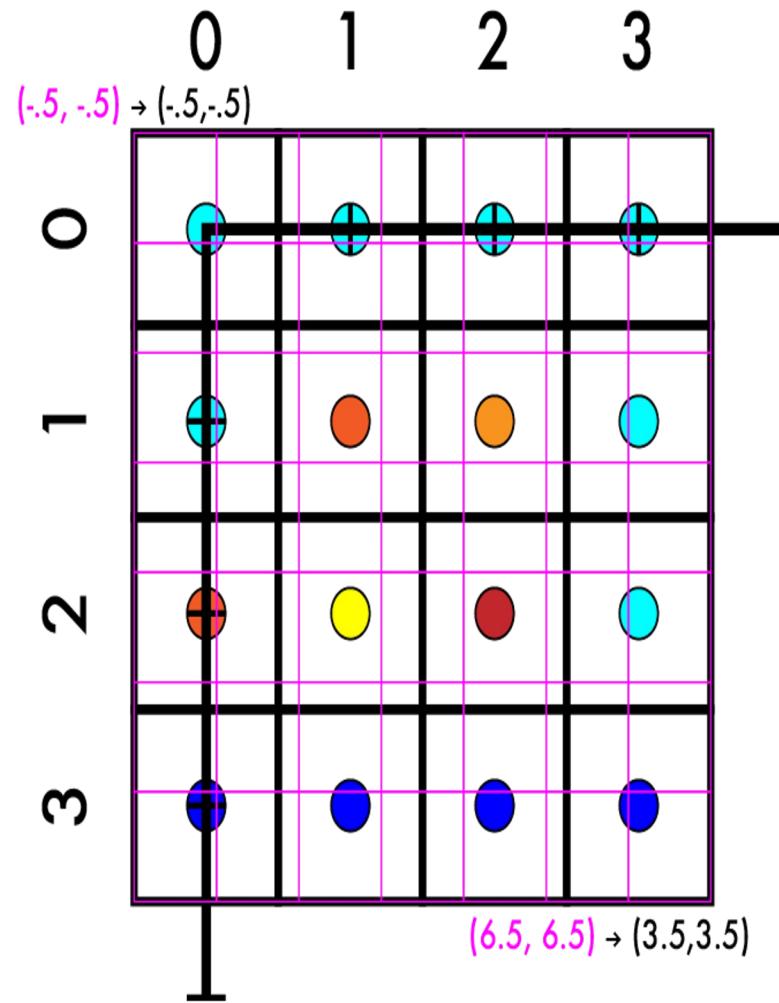
$$(3) - (1): 7*s_x = 4, \text{ so } s_x = 4/7$$

$$\text{from (1): } t_x = -0.5 + 4/7 * 0.5, \text{ so } t_x = -3/14$$

$$(4) - (2): 4*s_y = 7, \text{ so } s_y = 4/7$$

$$\text{from (2): } t_y = -0.5 + 4/7 * 0.5, \text{ so } t_y = -3/14$$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & 0 \\ 0 & \frac{4}{7} \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} -\frac{3}{14} \\ -\frac{3}{14} \end{pmatrix}$$

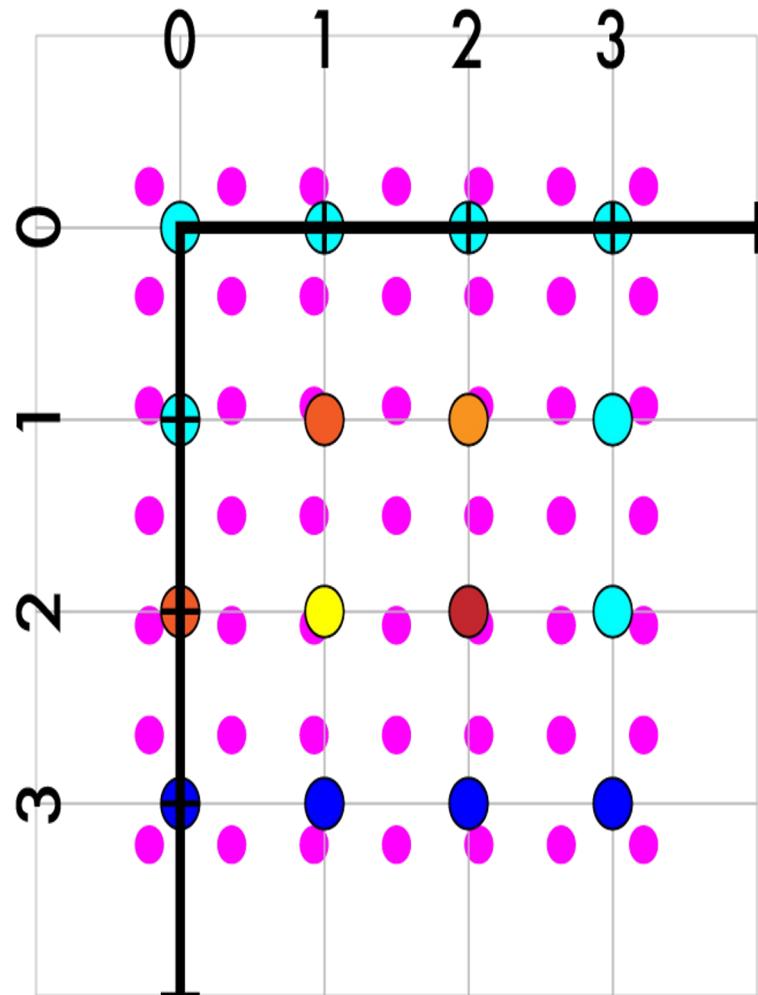


# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & 0 \\ 0 & \frac{4}{7} \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} -\frac{3}{14} \\ -\frac{3}{14} \end{pmatrix}$$

- Iterate over new points

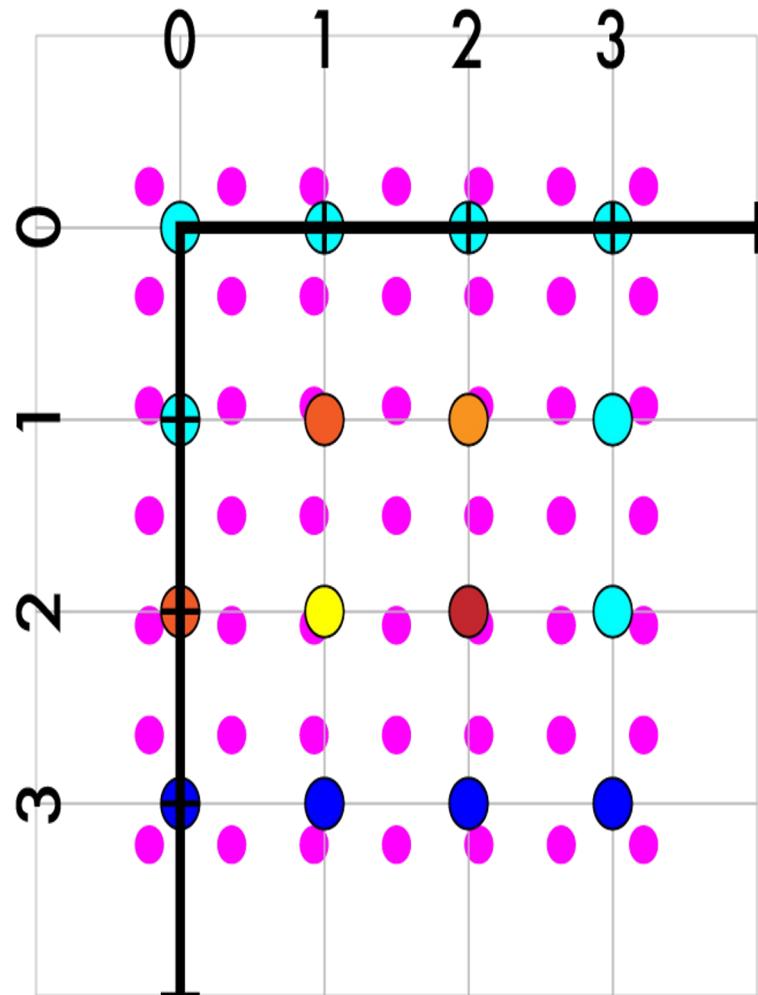


# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & 0 \\ 0 & \frac{4}{7} \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} -\frac{3}{14} \\ -\frac{3}{14} \end{pmatrix}$$

- Iterate over new points
  - Map to old coordinates

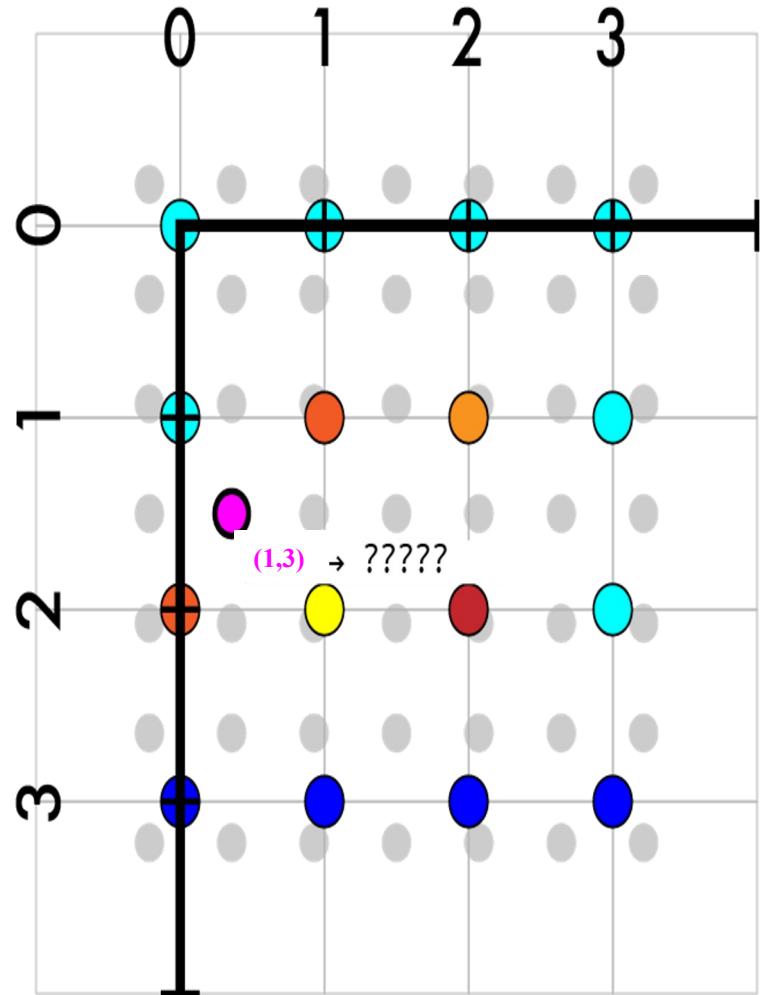


# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & 0 \\ 0 & \frac{4}{7} \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} -\frac{3}{14} \\ -\frac{3}{14} \end{pmatrix}$$

- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$



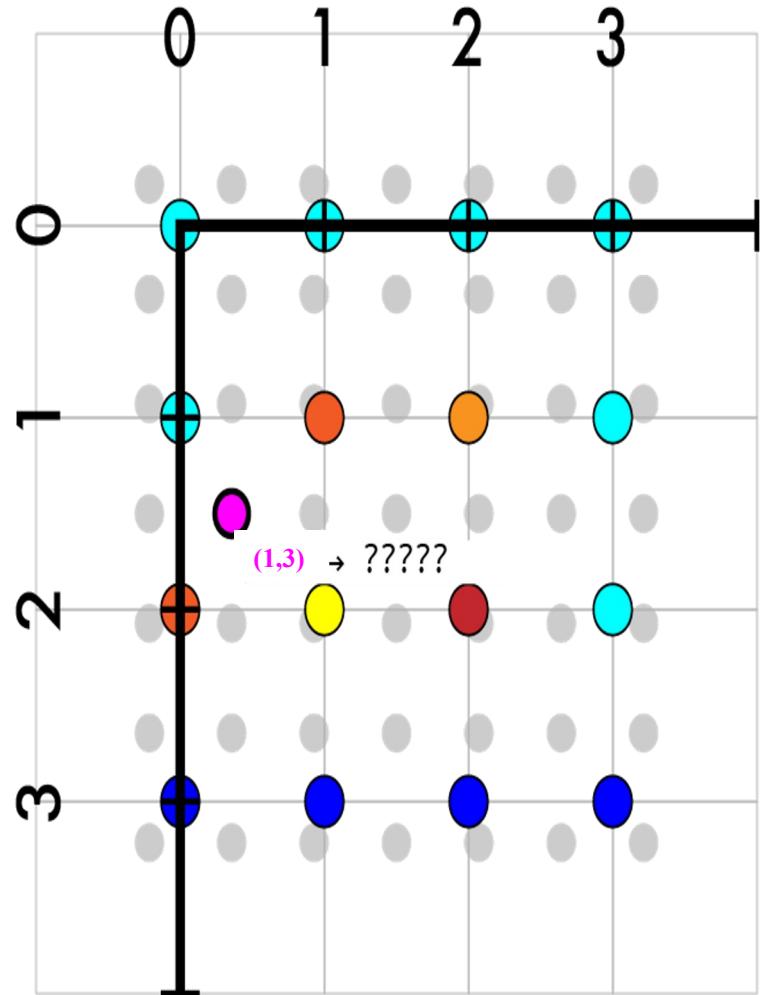
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & 0 \\ 0 & \frac{4}{7} \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} -\frac{3}{14} \\ -\frac{3}{14} \end{pmatrix}$$

- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & 0 \\ 0 & \frac{4}{7} \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} -\frac{3}{14} \\ -\frac{3}{14} \end{pmatrix} = \begin{pmatrix} \frac{5}{14} \\ \frac{21}{14} \end{pmatrix}$$

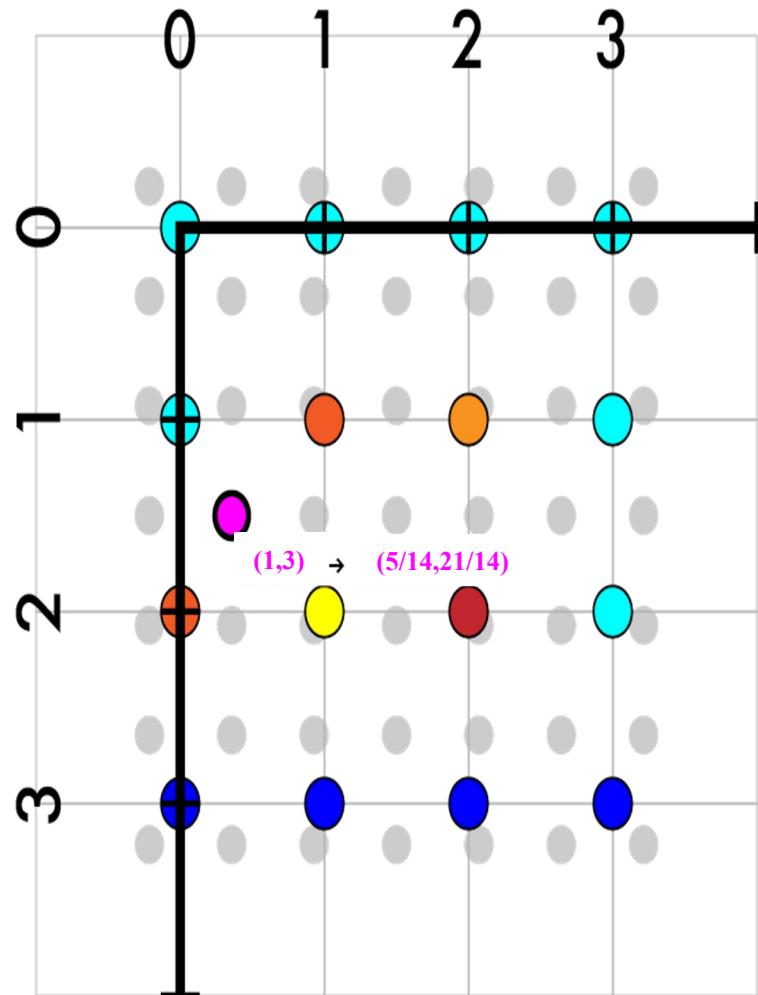


# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates

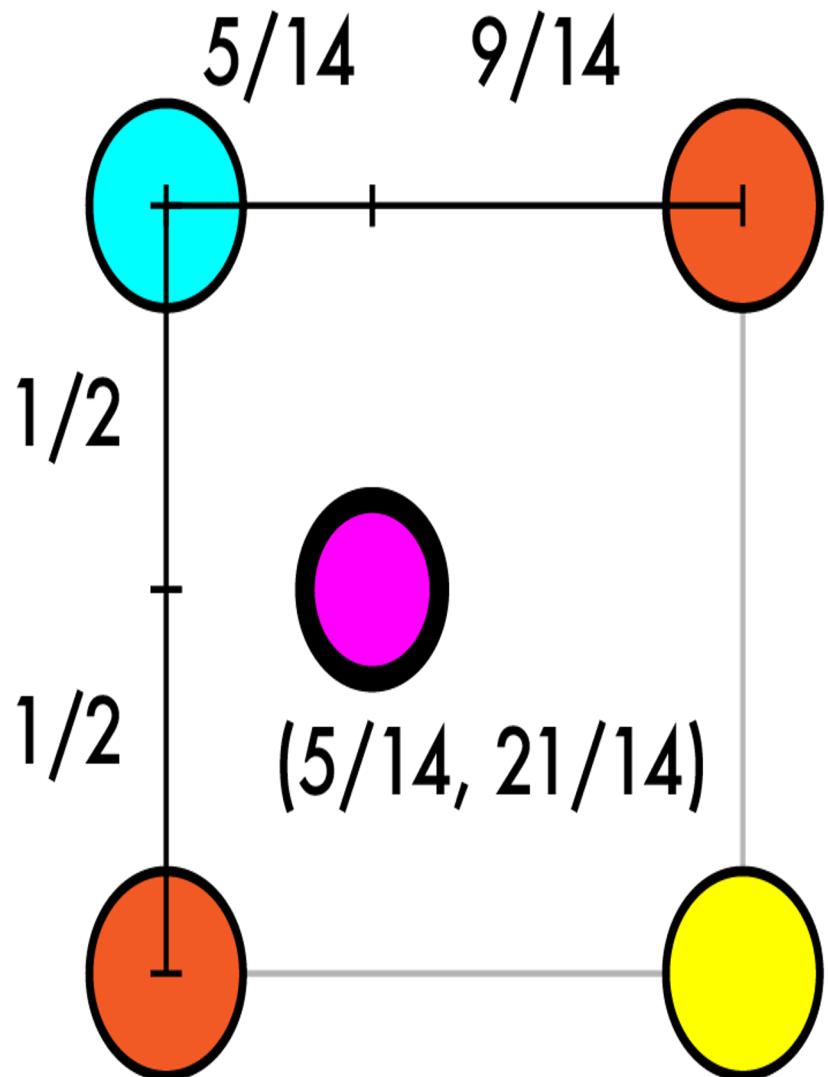
$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & 0 \\ 0 & \frac{4}{7} \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} -\frac{3}{14} \\ -\frac{3}{14} \end{pmatrix}$$

- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$  (new image)
  - Map it to  $(5/14, 21/14)$  (old image)
  - Interpolate old values (bilinear)



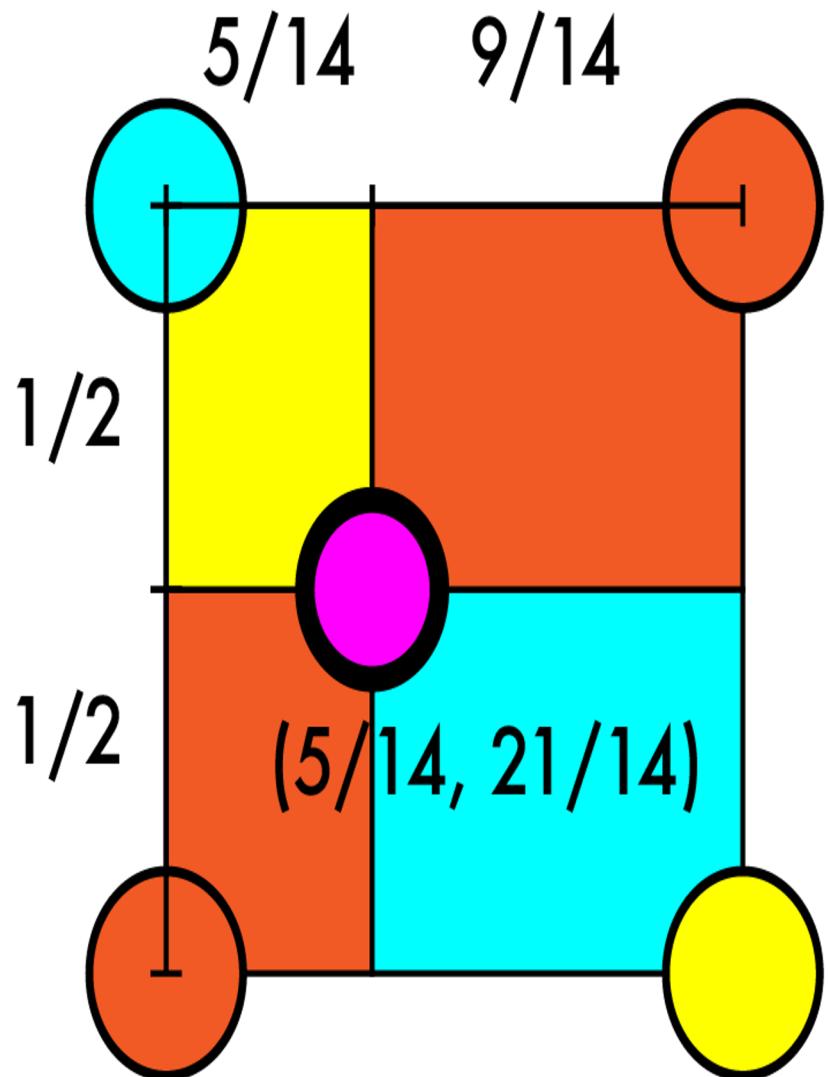
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values



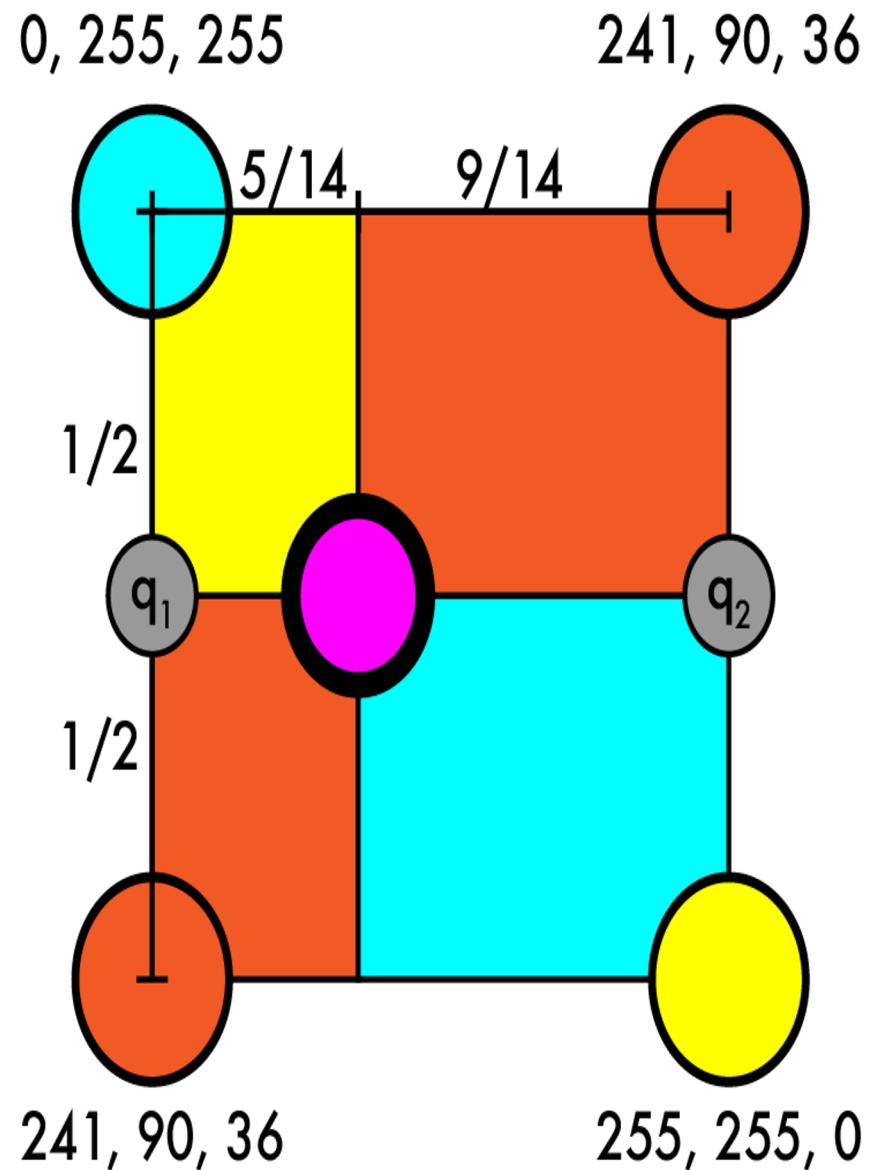
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values
    - Size of opposite rectangles



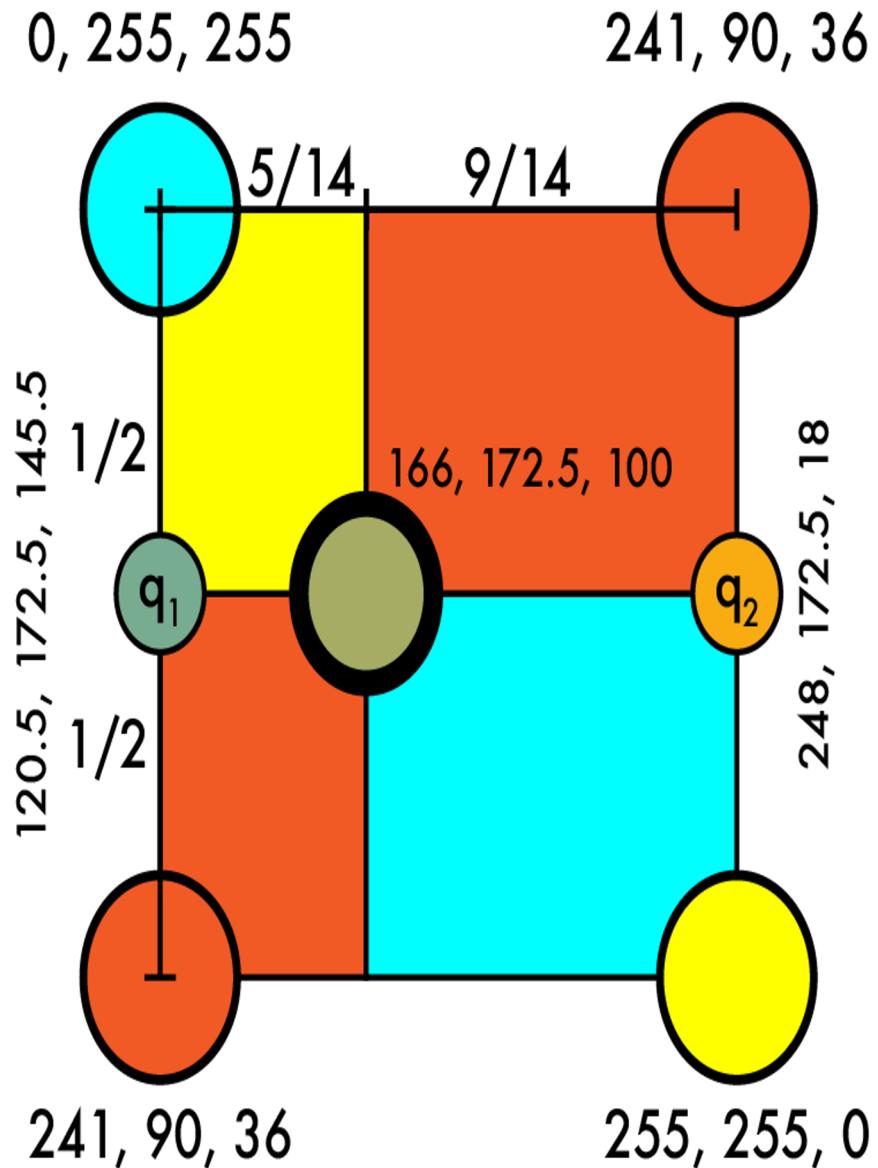
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values
    - Size of opposite rectangles
    - OR find  $q_1$  and  $q_2$ , then interpolate between them



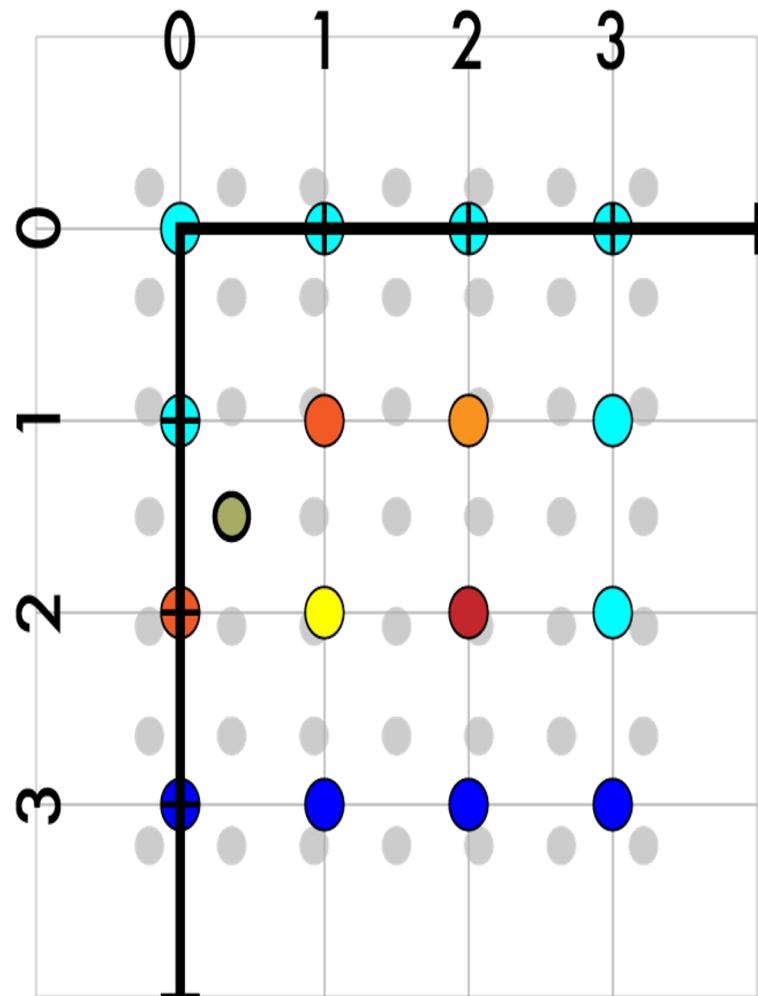
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values
    - Size of opposite rectangles
    - OR find  $q_1$  and  $q_2$ , then interpolate between them
    - $q_1 = (120.5, 172.5, 145.5)$
    - $q_2 = (248, 172.5, 18)$
    - $q = (166, 172.5, 100)$



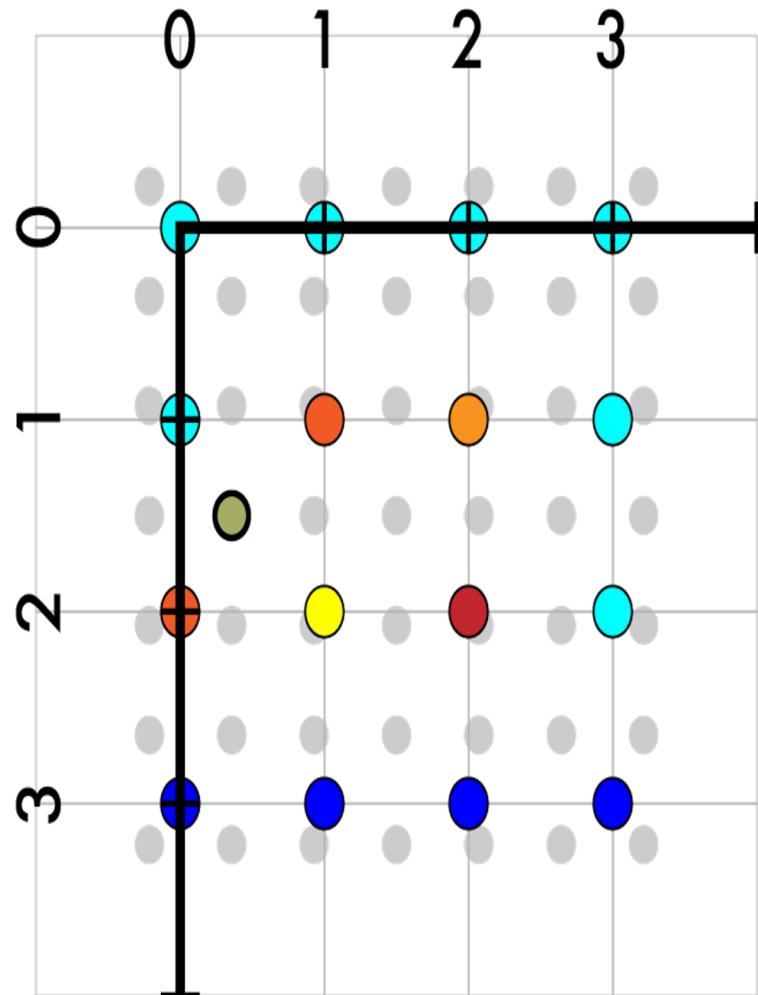
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values
    - $q = (166, 172.5, 100)$



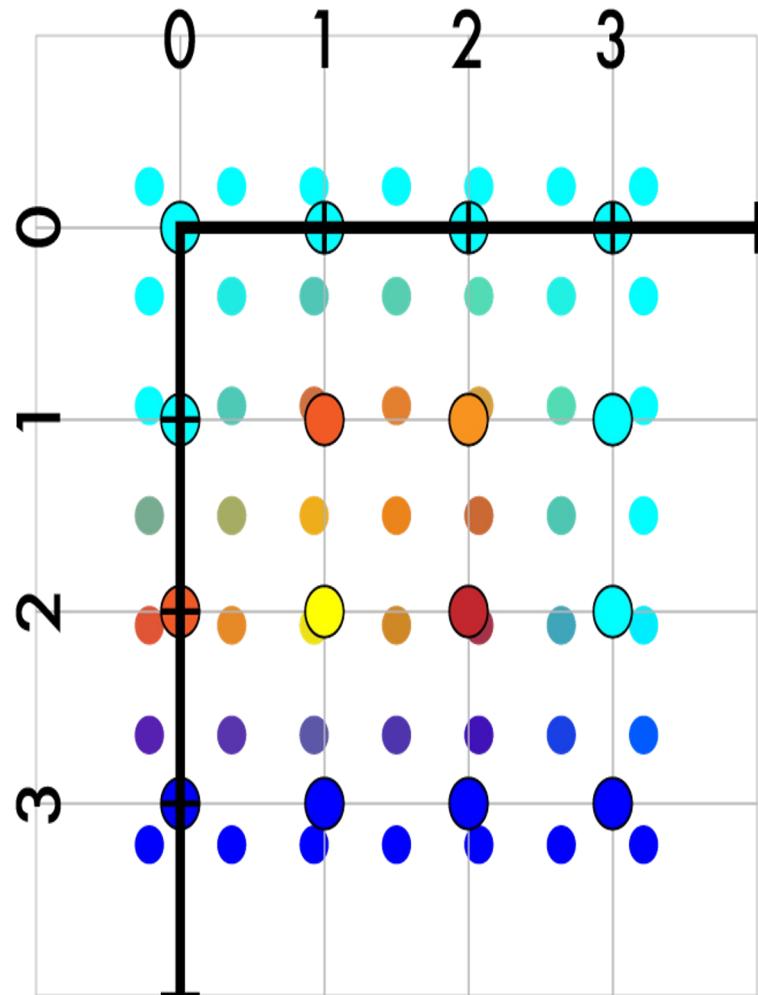
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values
    - $q = (166, 172.5, 100)$
- Fill in the rest



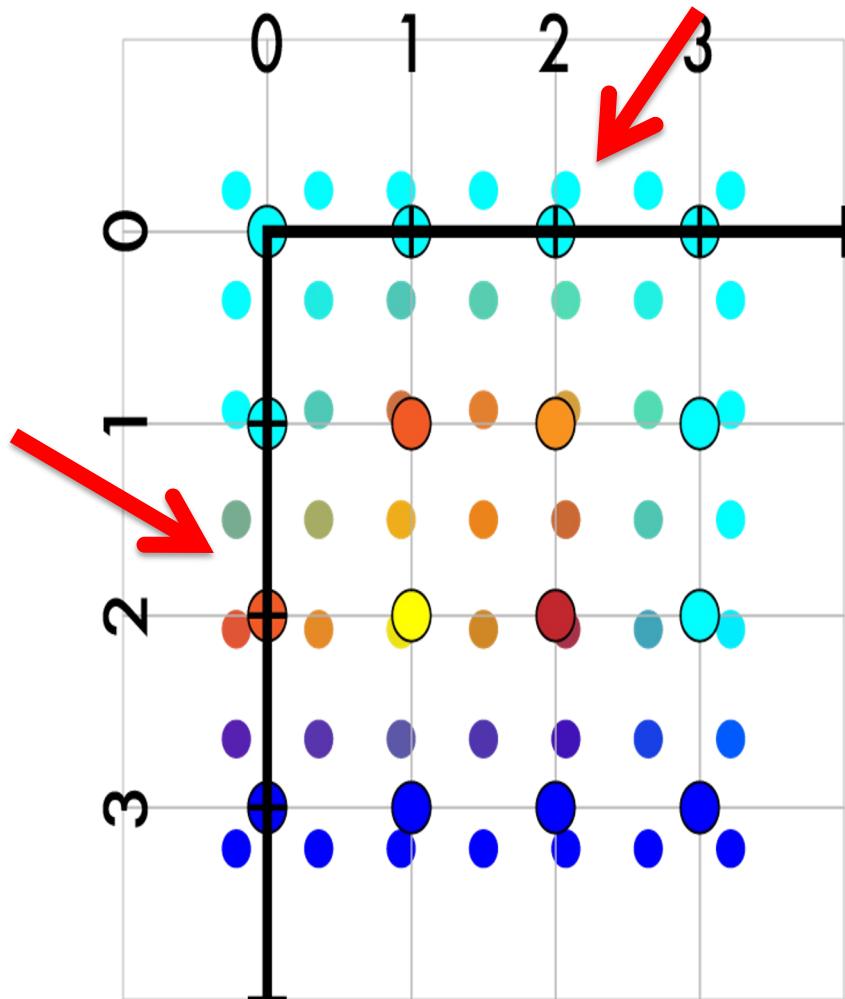
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values
    - $q = (166, 172.5, 100)$
- Fill in the rest



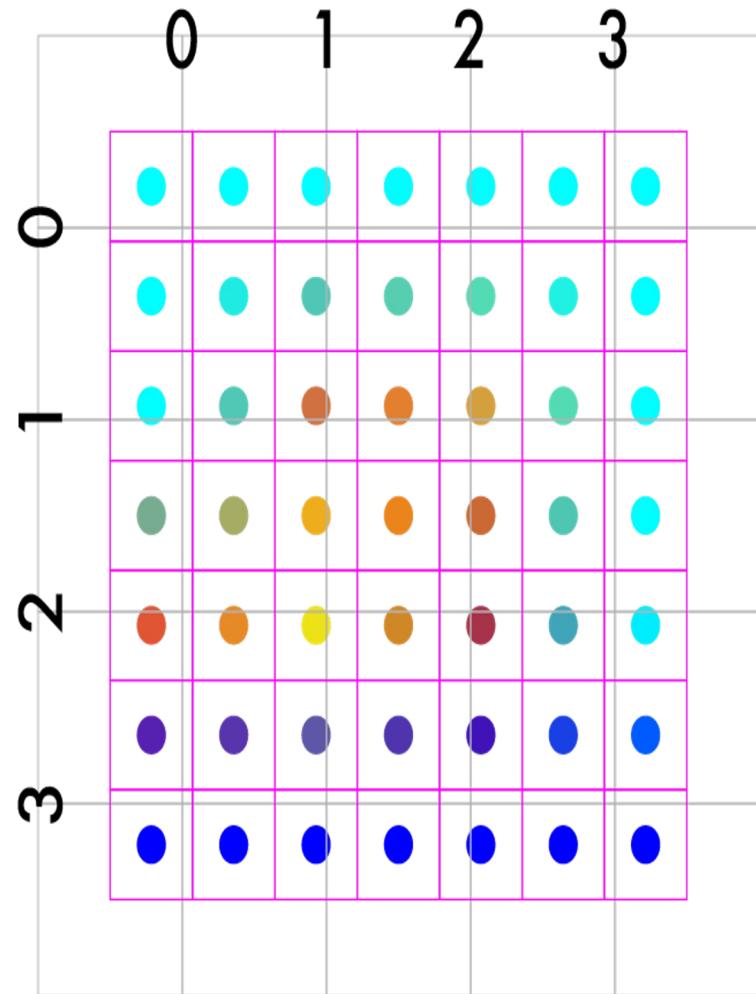
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values
    - $q = (166, 172.5, 100)$
- Fill in the rest
  - On outer edges use padding



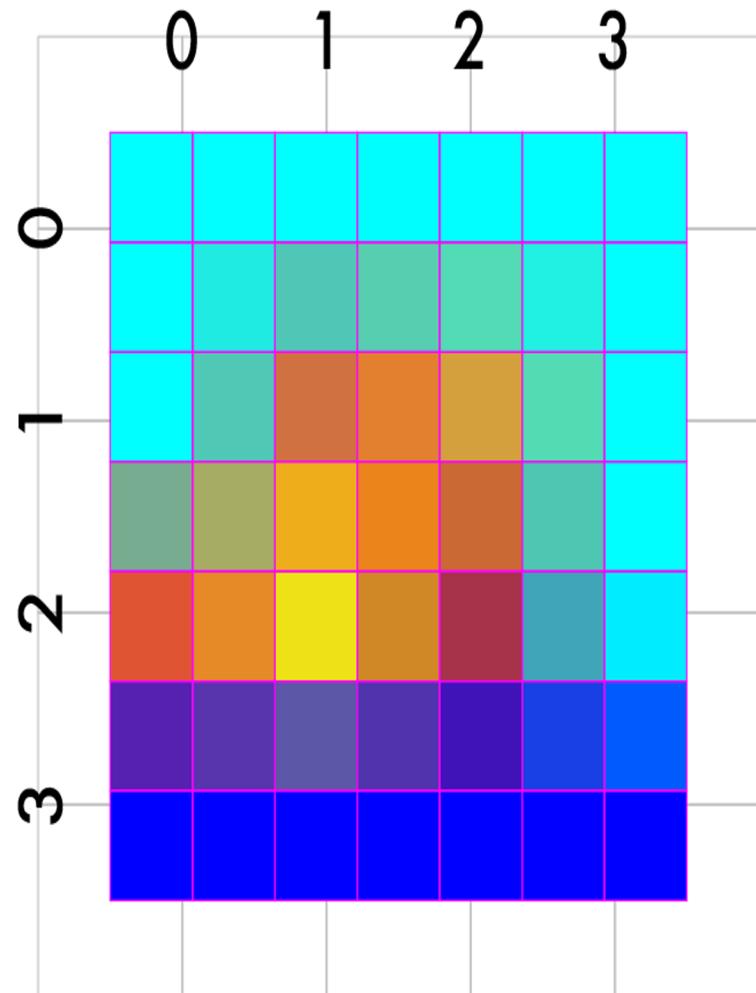
# Resize $4 \times 4$ to $7 \times 7$

- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values
    - $q = (166, 172.5, 100)$
- Fill in the rest

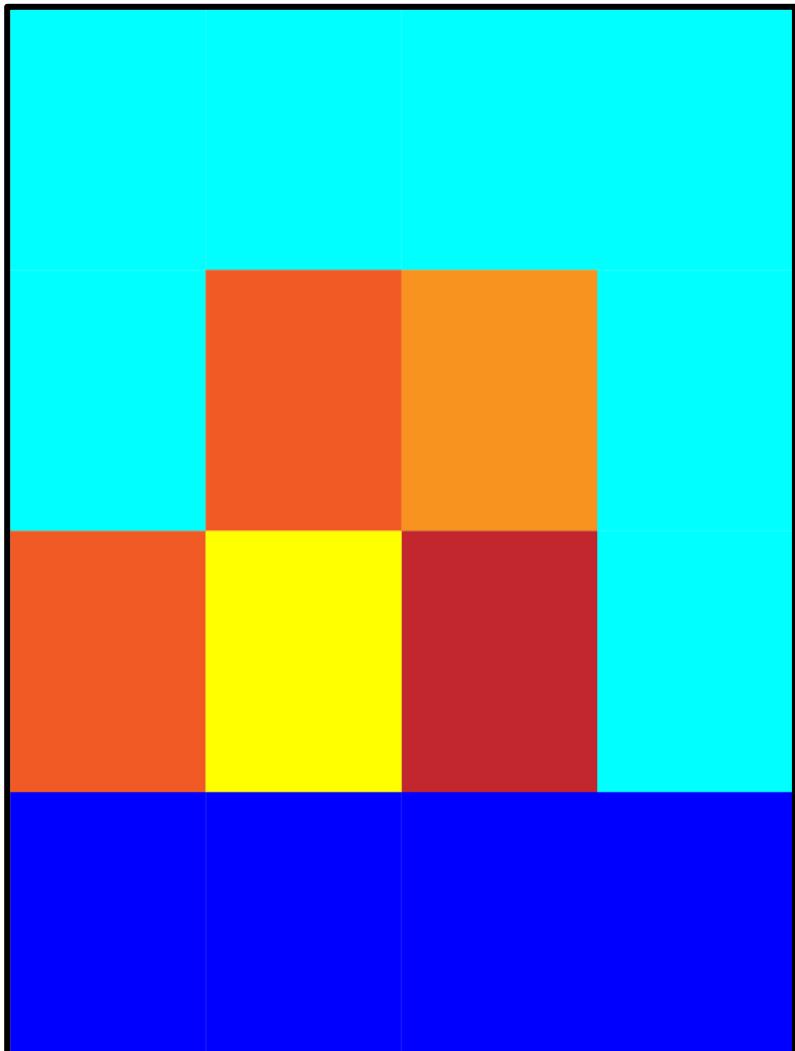


# Resize $4 \times 4$ to $7 \times 7$

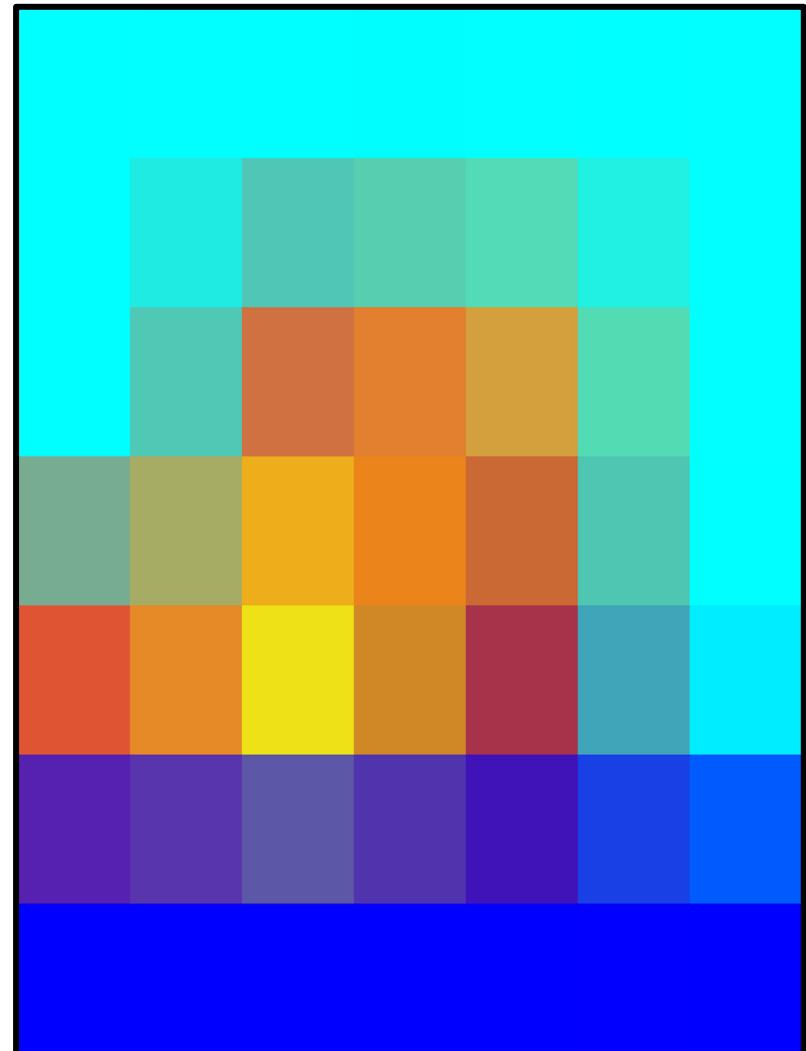
- Create our new image
- Match up coordinates
- Iterate over new points
  - Map to old coordinates
  - Take point  $(1,3)$
  - Map it to  $(5/14, 21/14)$
  - Interpolate old values
    - $q = (166, 172.5, 100)$
- Fill in the rest



# Final result!

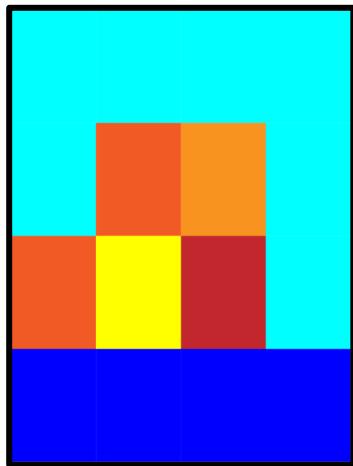


Original image  $4 \times 4$

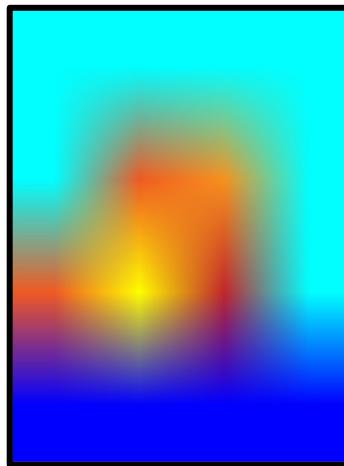


Resized image  $7 \times 7$  using  
bilinear interpolation

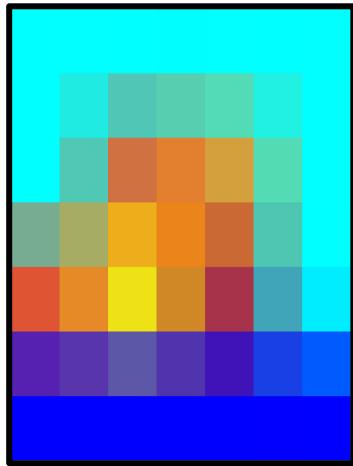
# Different scales



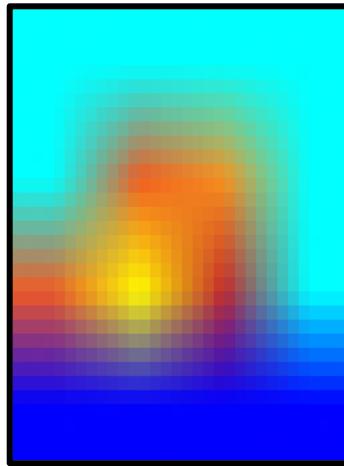
Original image  $4 \times 4$



Resized image  $256 \times 256$   
using bilinear interpolation



Resized image  $7 \times 7$  using  
bilinear interpolation

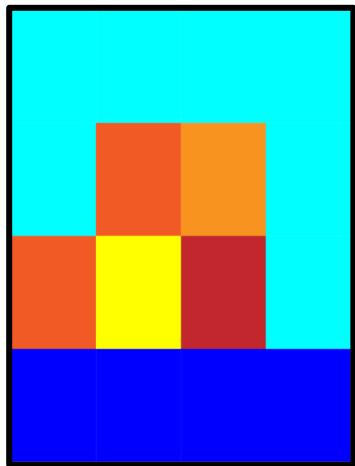


Resized image  $32 \times 32$   
using bilinear interpolation

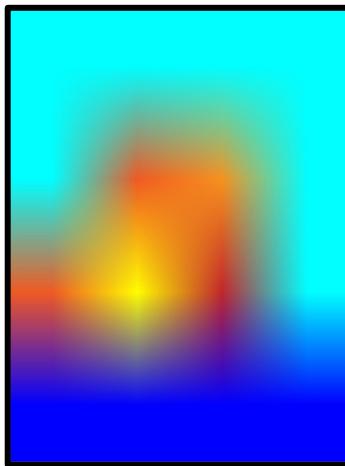


Star like pattern

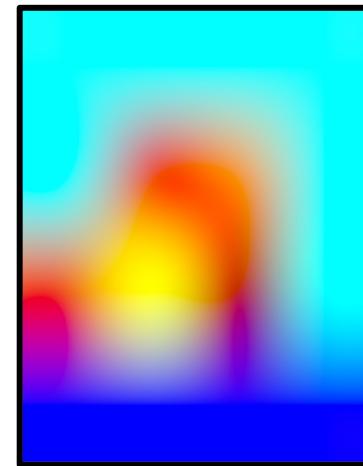
# Different methods



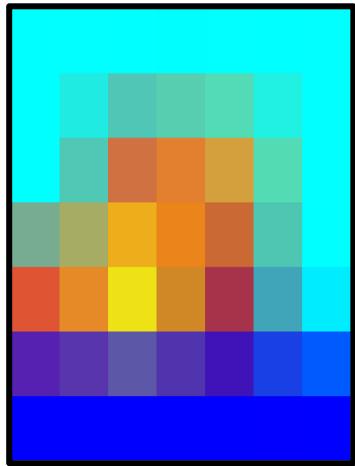
Original image  $4 \times 4$



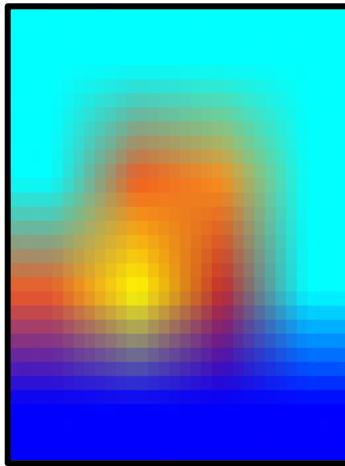
Resized image  $256 \times 256$   
using bilinear interpolation



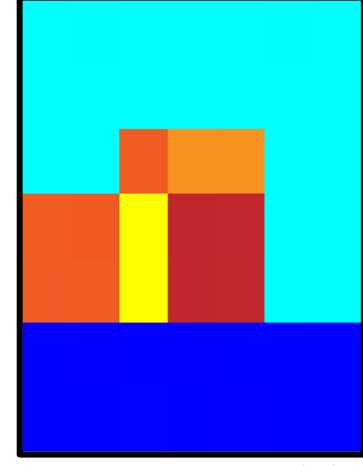
Resized image  $256 \times 256$   
using bicubic interpolation



Resized image  $7 \times 7$  using  
bilinear interpolation



Resized image  $32 \times 32$   
using bilinear interpolation



Resized image  $7 \times 7$  using  
nearest neighbor interpolation

# Want to make image smaller



# From $448 \times 448$ to $64 \times 64$



nearest neighbor  
interpolation



bilinear interpolation

# From $448 \times 448$ to $64 \times 64$



nearest neighbor  
interpolation



bilinear interpolation

# From $448 \times 448$ to $64 \times 64$



nearest neighbor  
interpolation



bilinear interpolation

# From $448 \times 448$ to $64 \times 64$

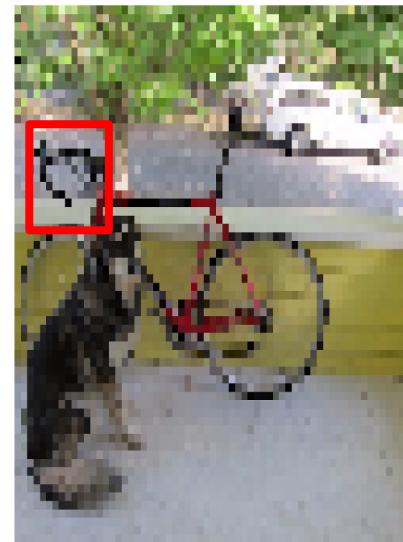
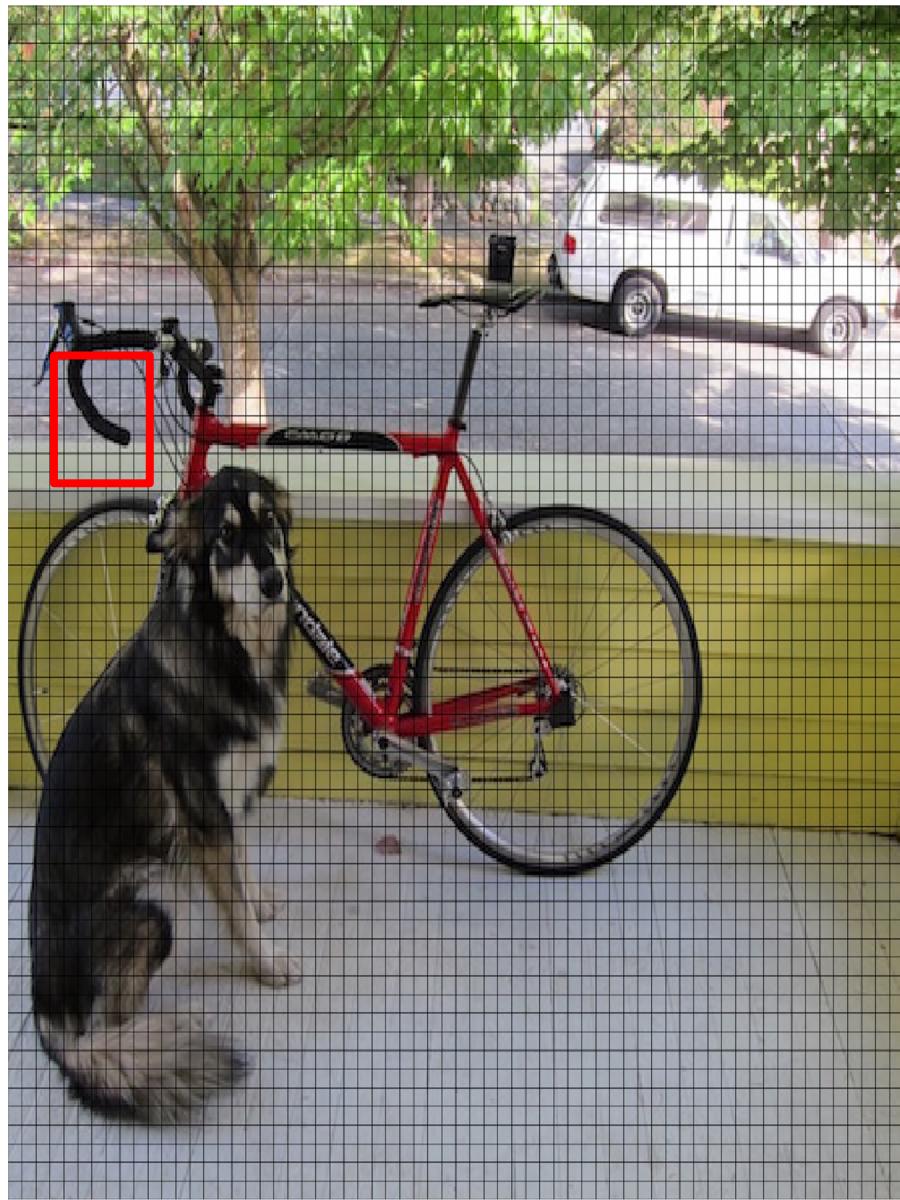


nearest neighbor  
interpolation

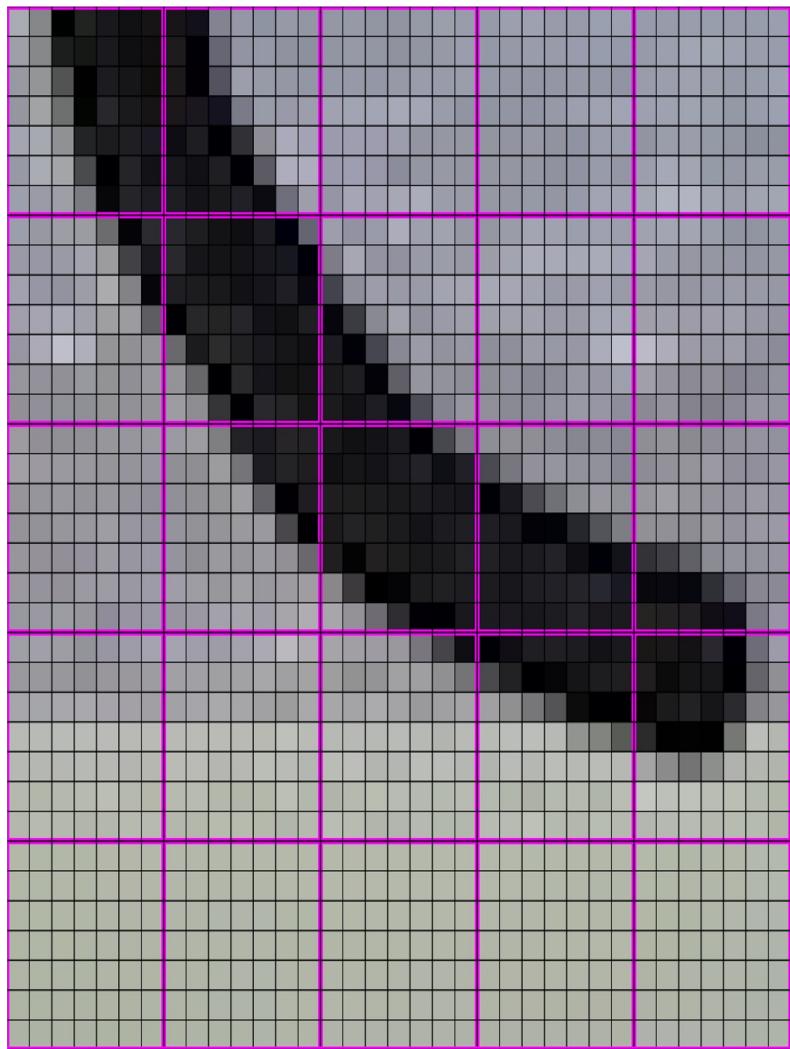
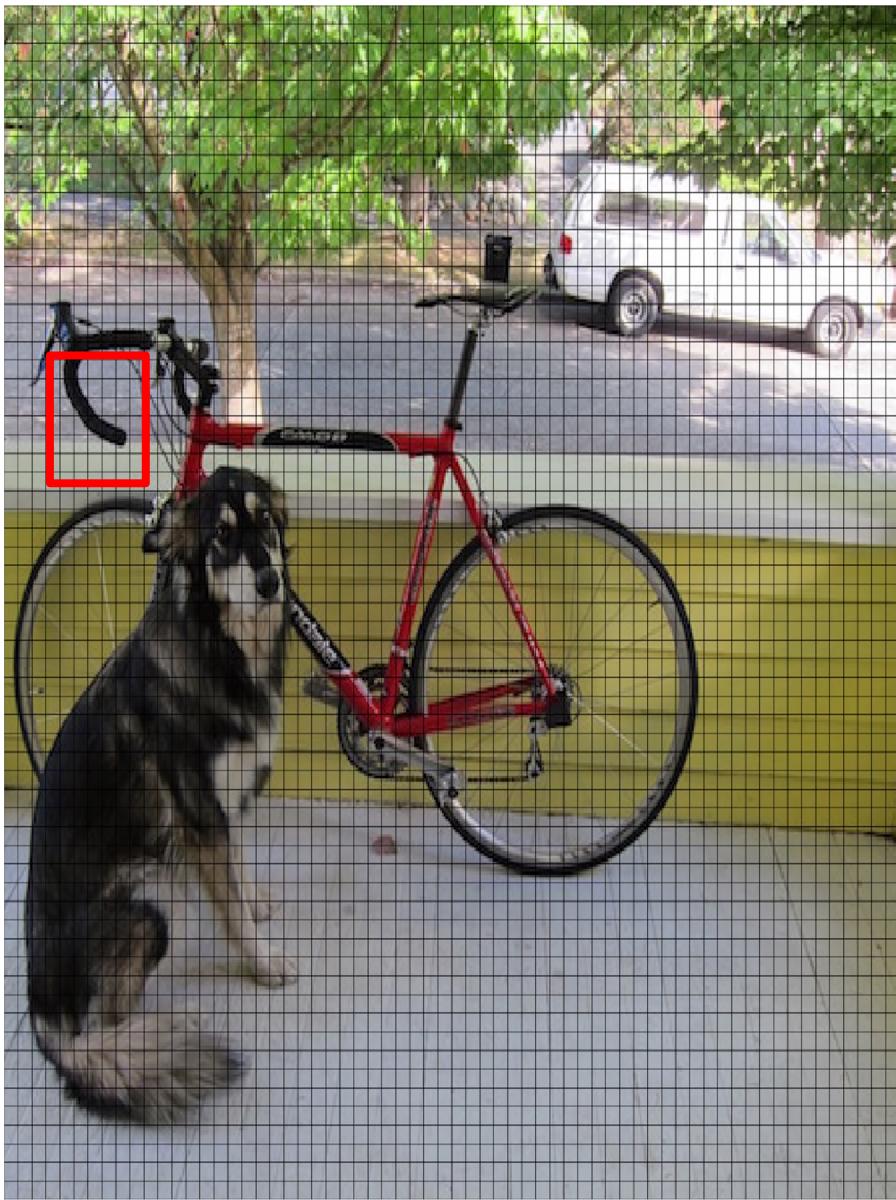


bilinear interpolation

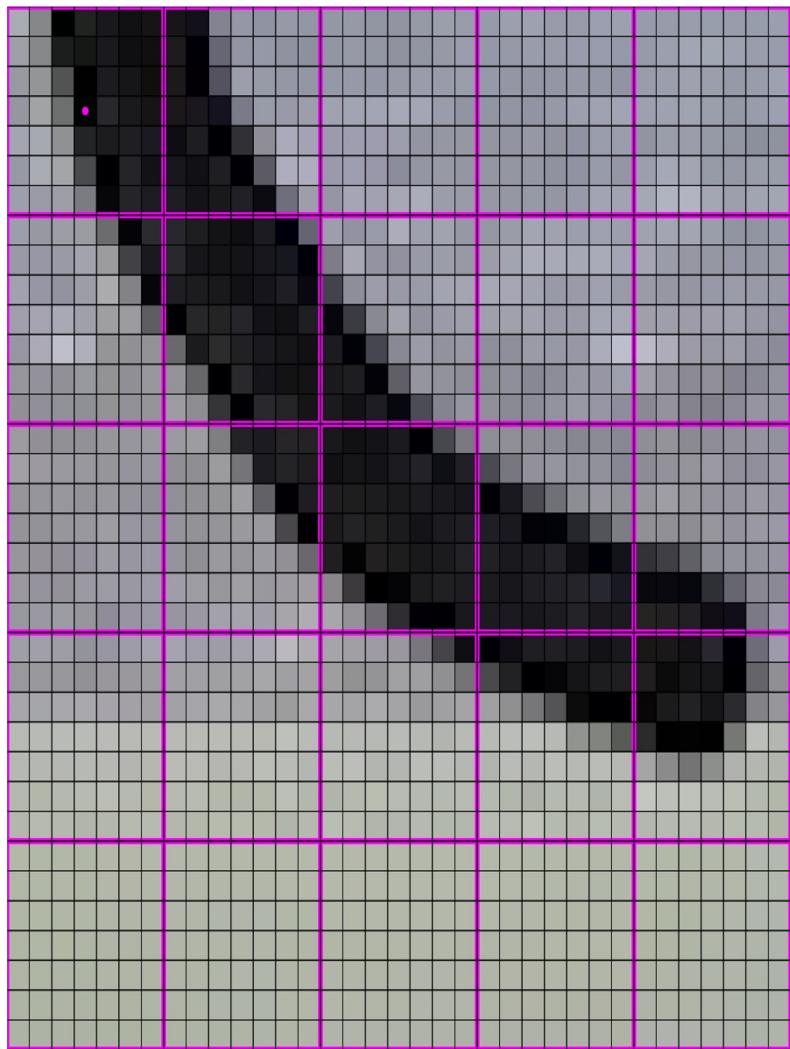
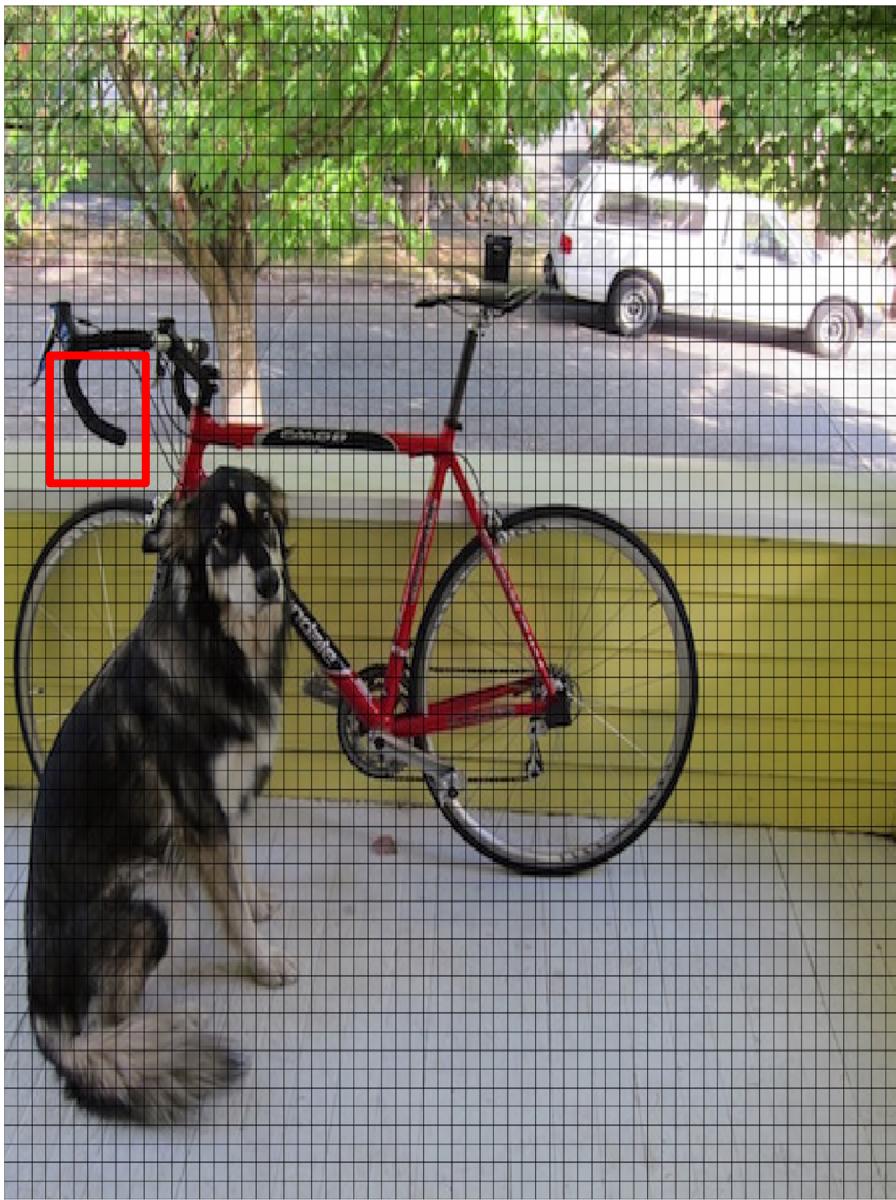
# From $448 \times 448$ to $64 \times 64$



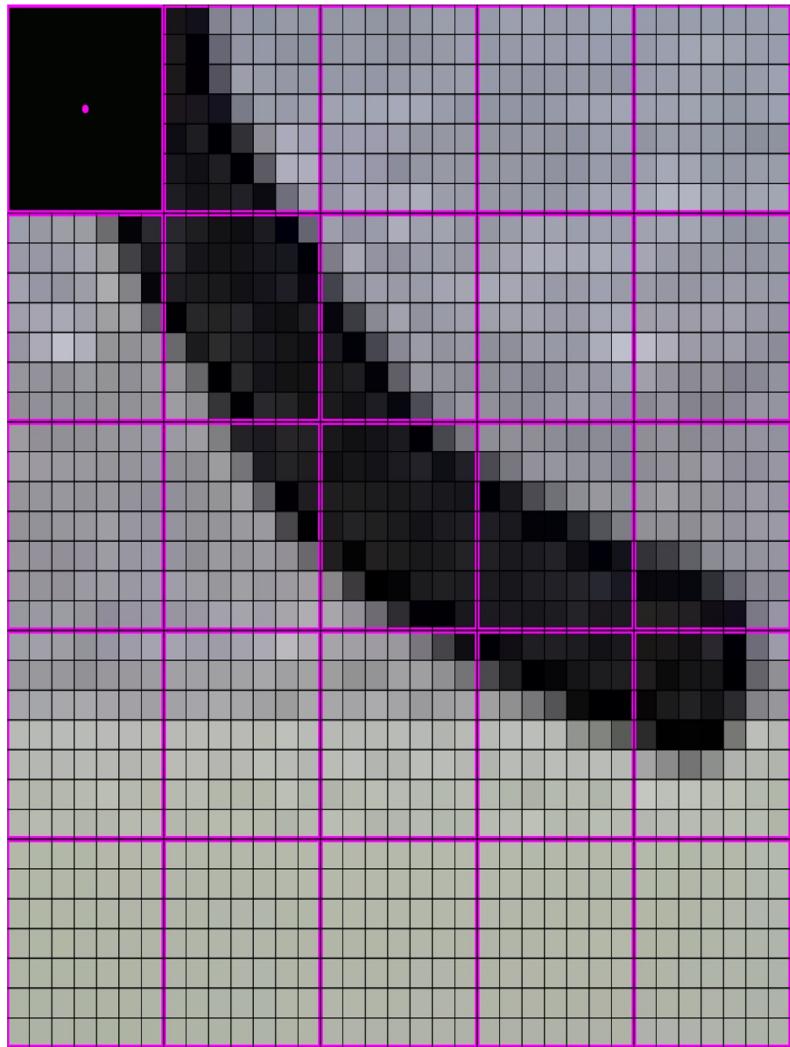
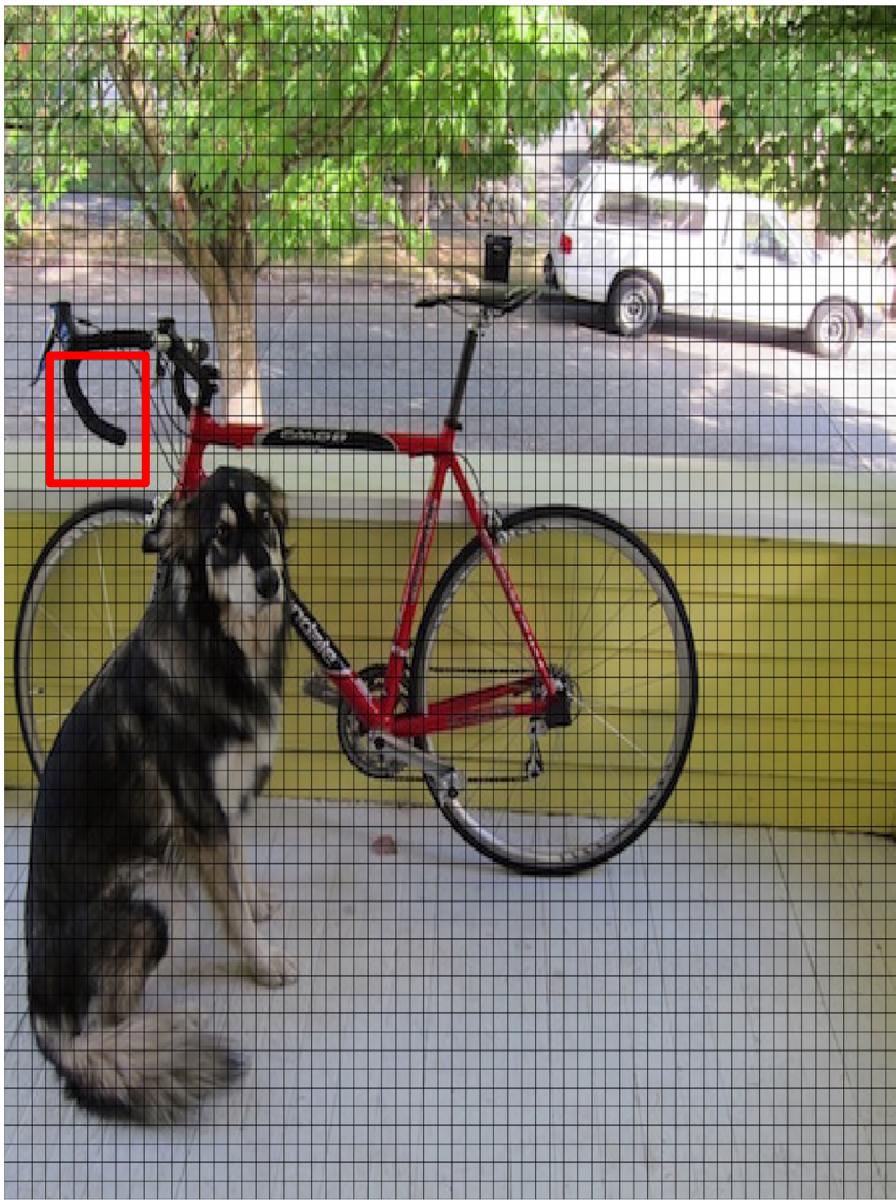
# From $448 \times 448$ to $64 \times 64$



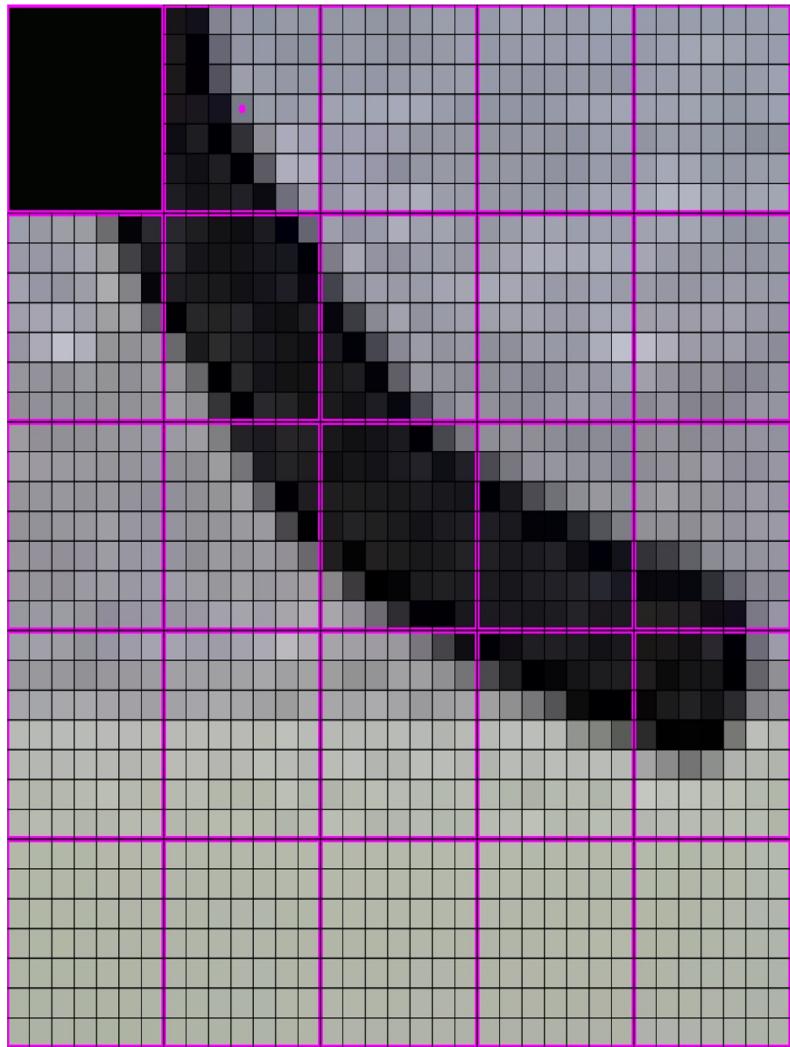
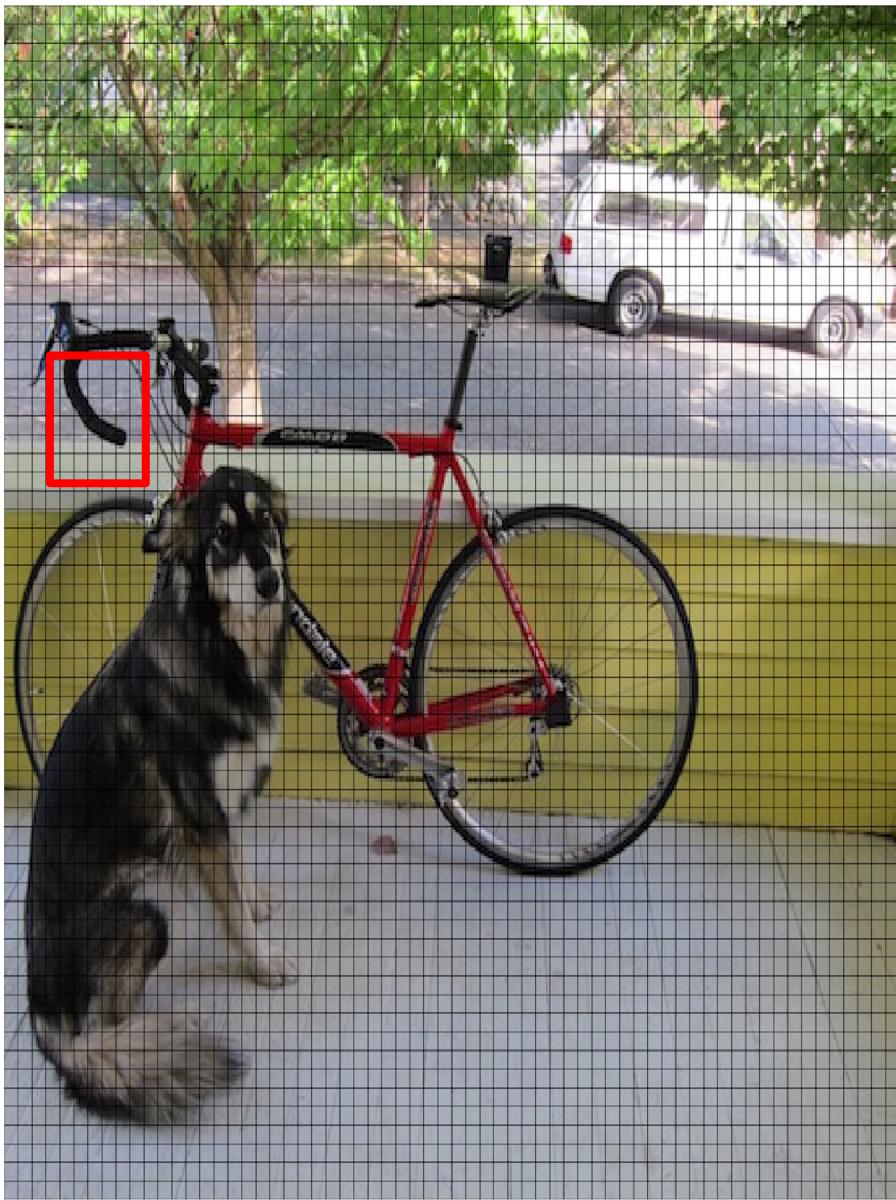
# From $448 \times 448$ to $64 \times 64$



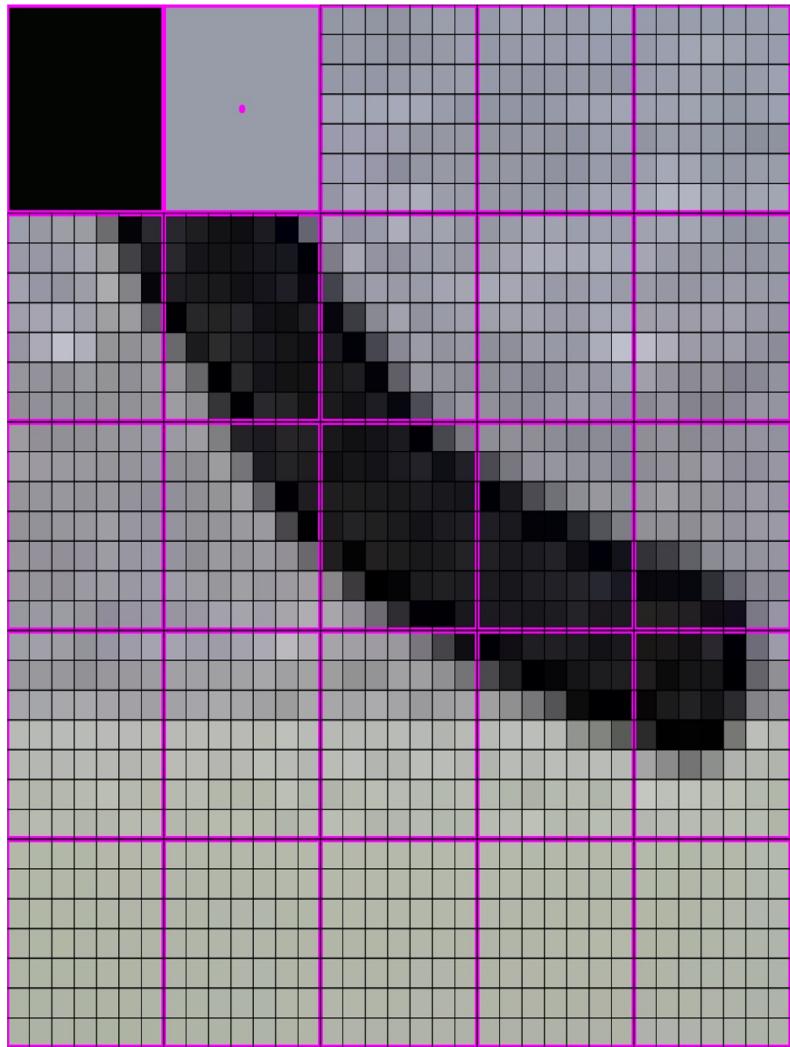
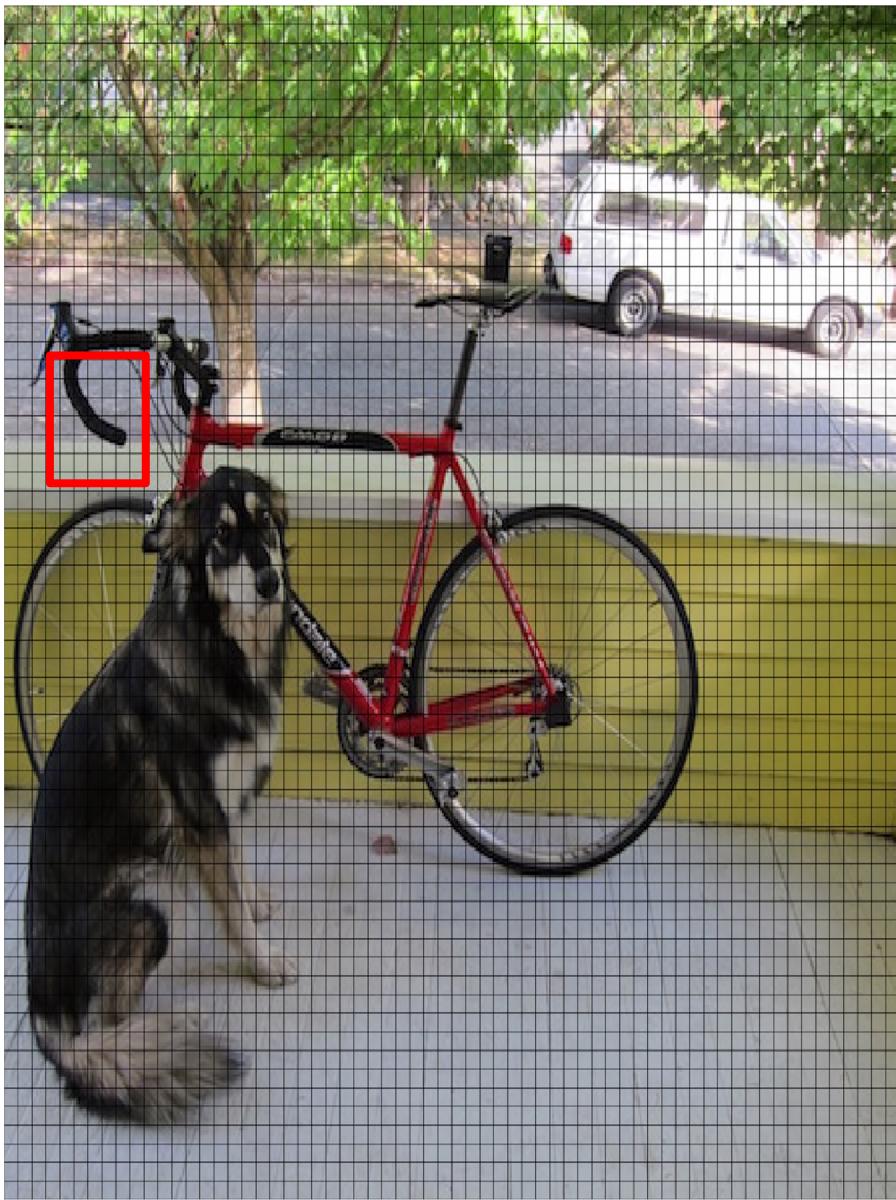
# From $448 \times 448$ to $64 \times 64$



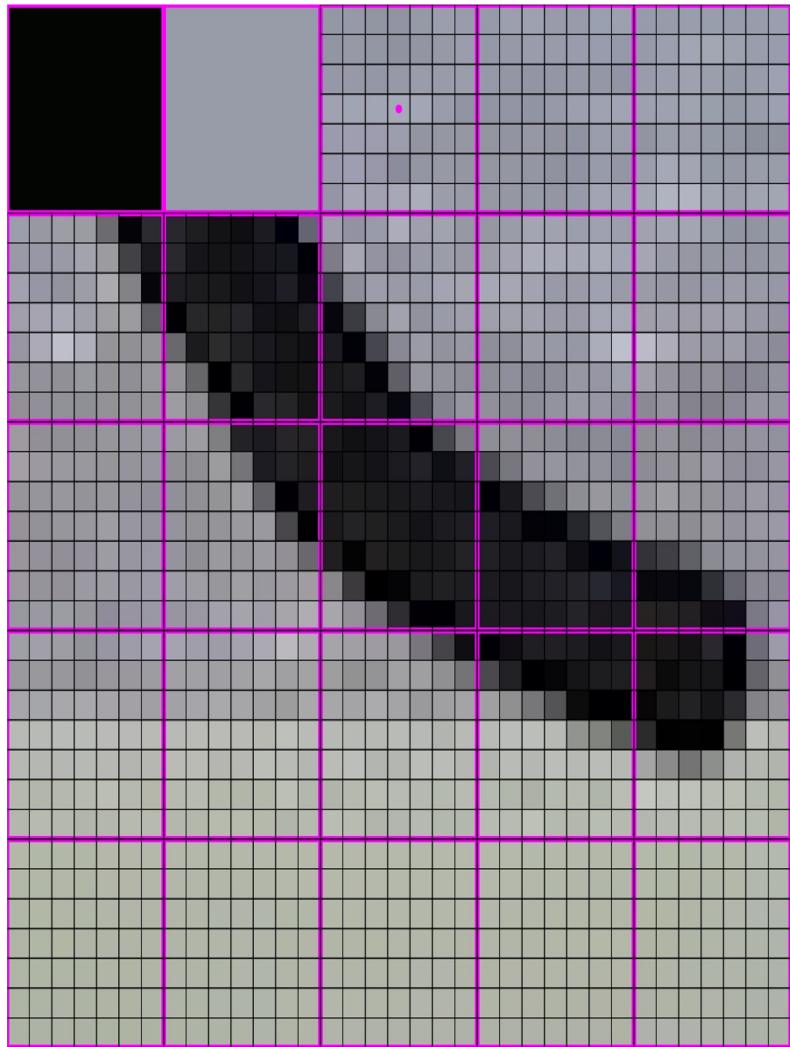
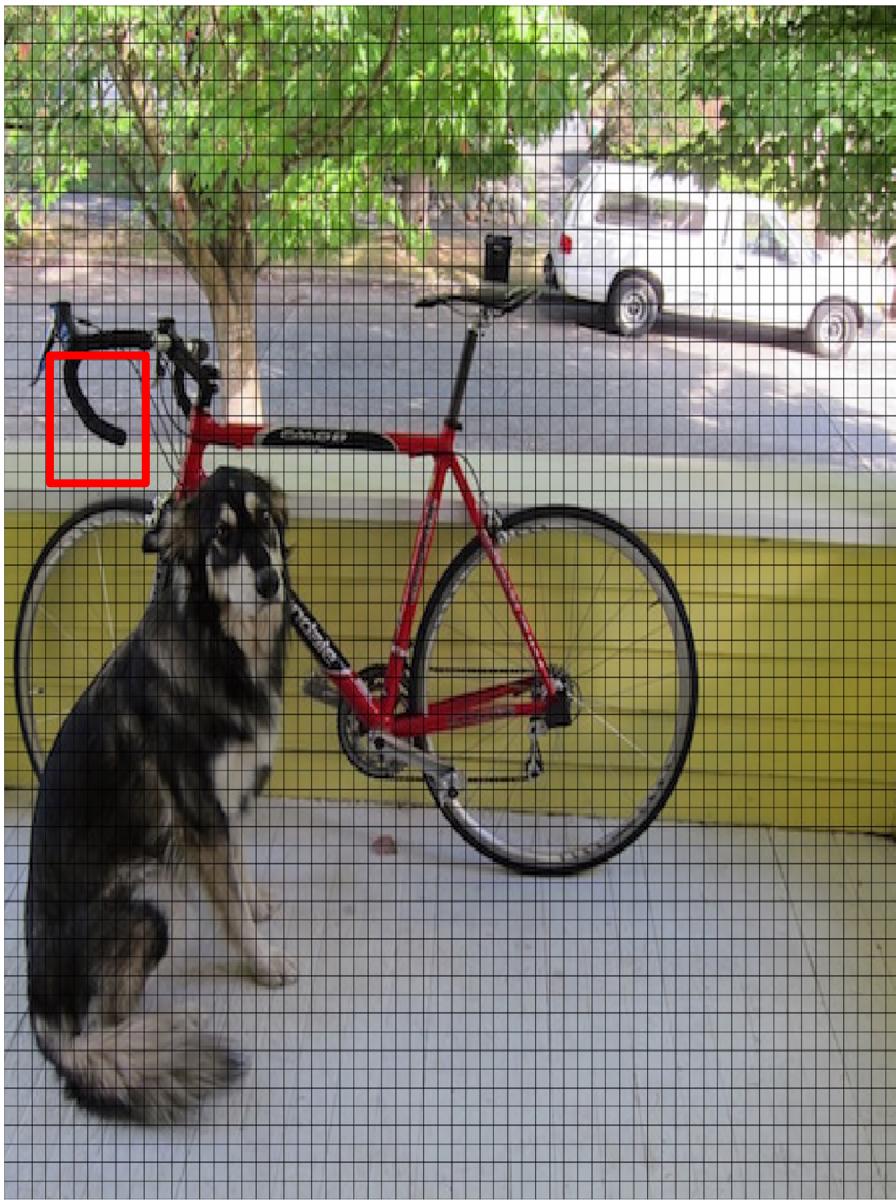
# From $448 \times 448$ to $64 \times 64$



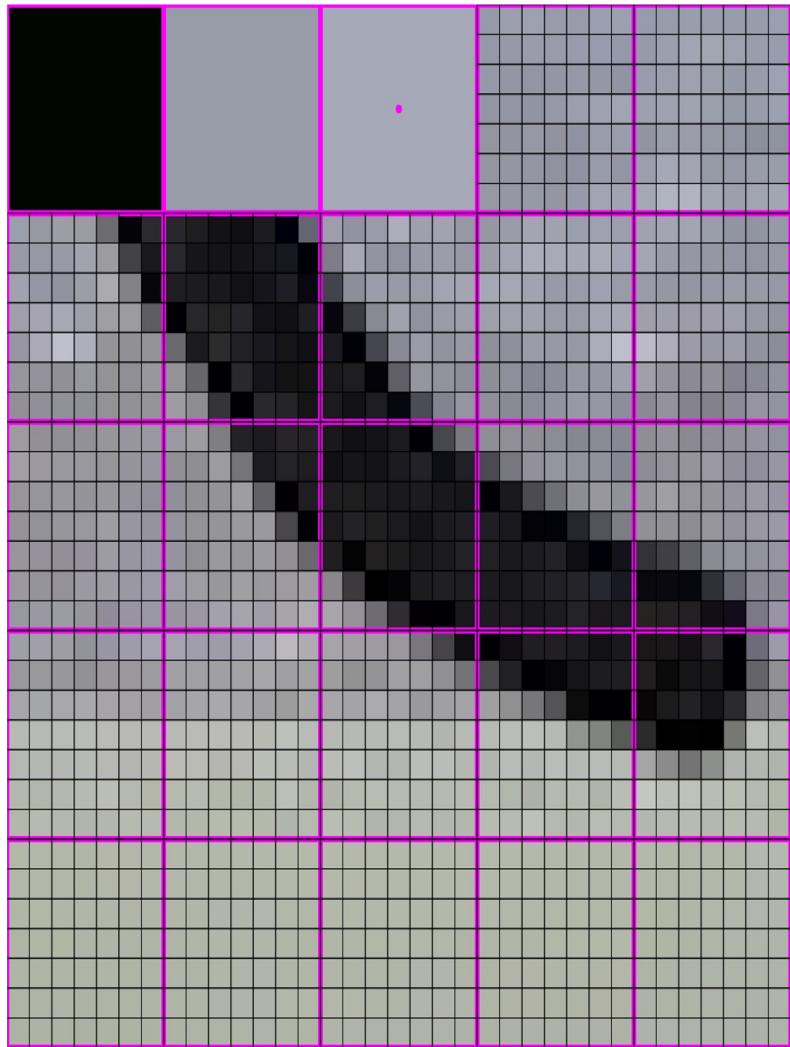
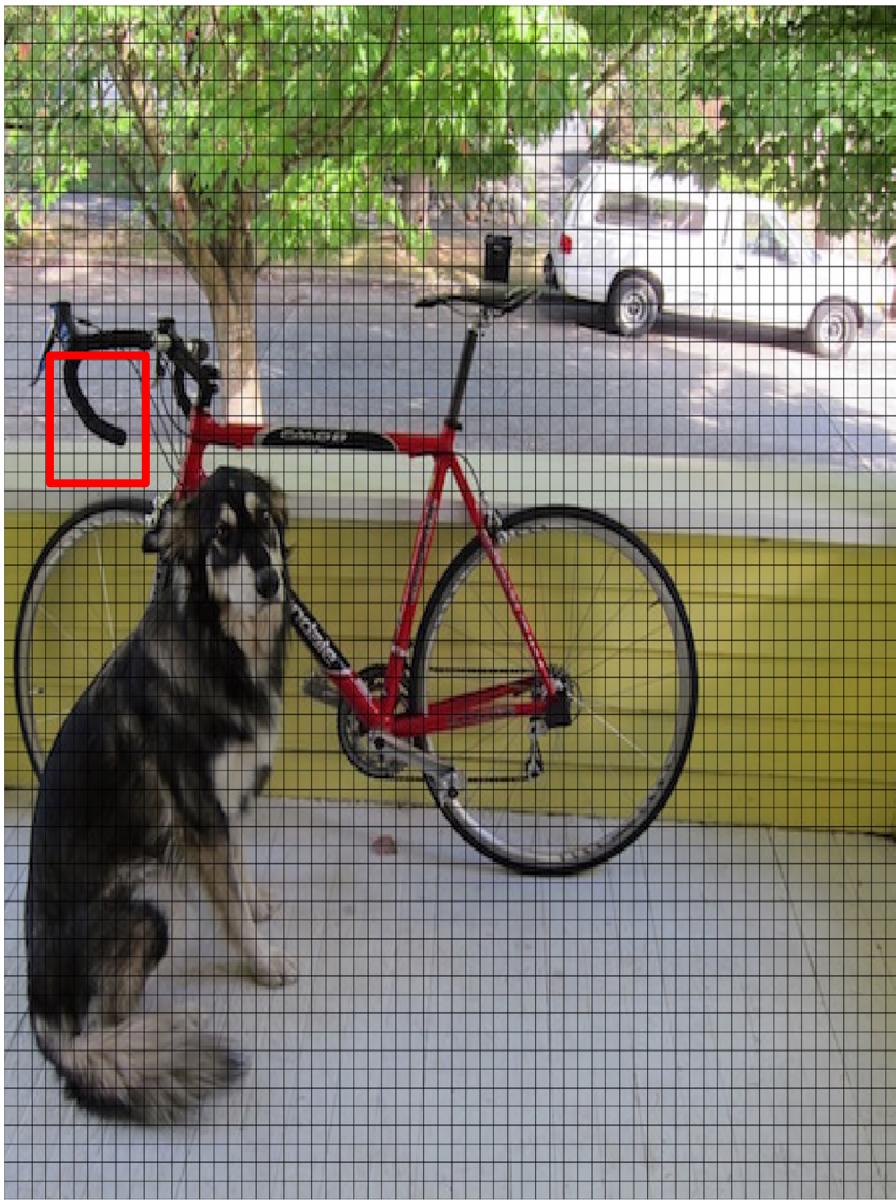
# From $448 \times 448$ to $64 \times 64$



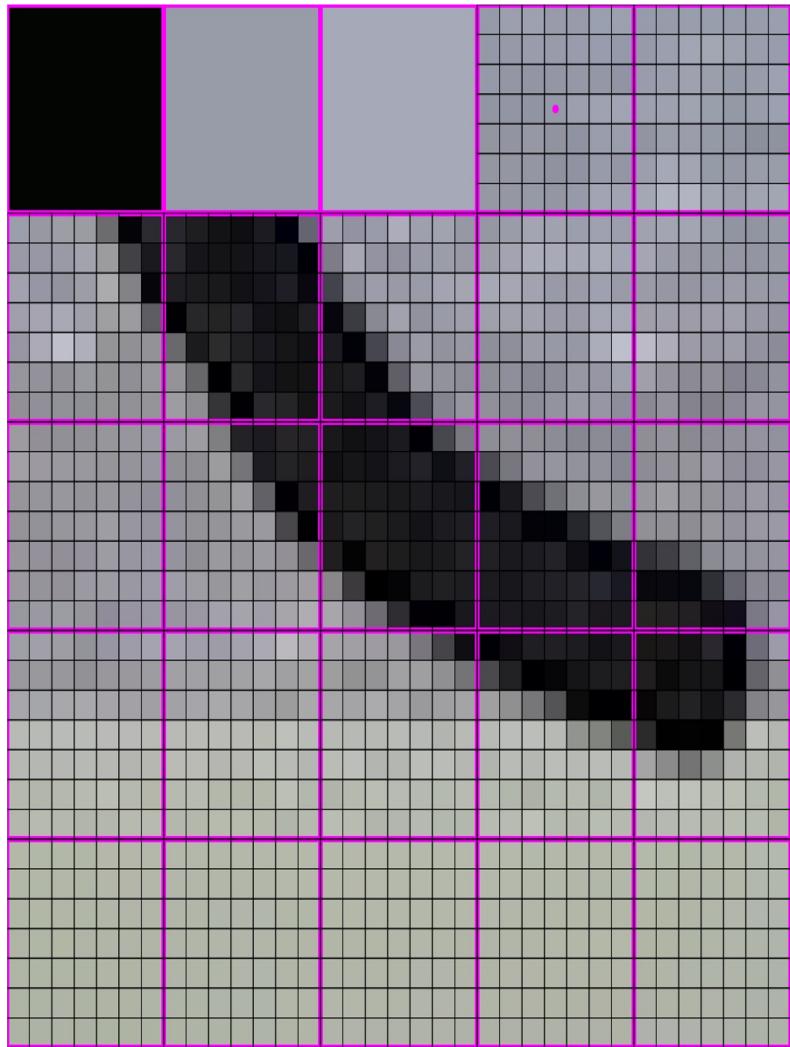
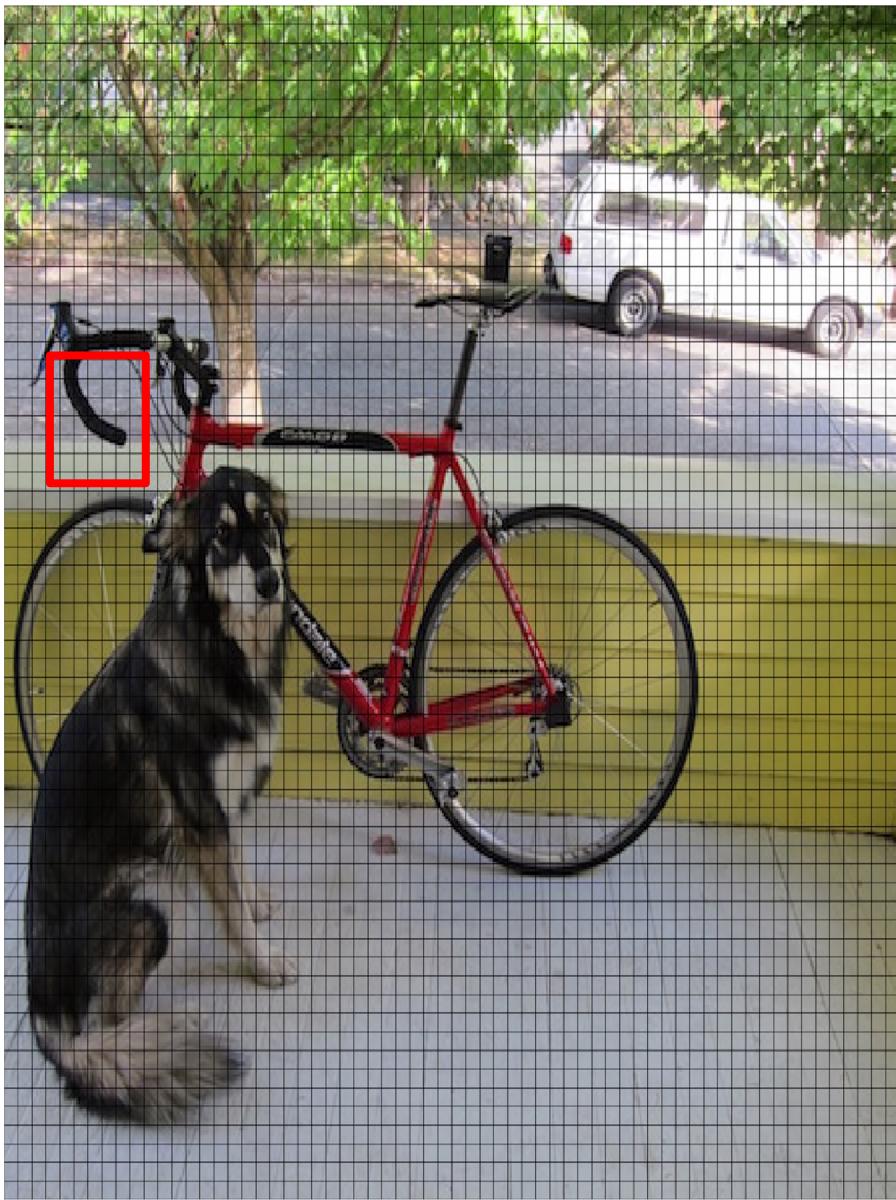
# From $448 \times 448$ to $64 \times 64$



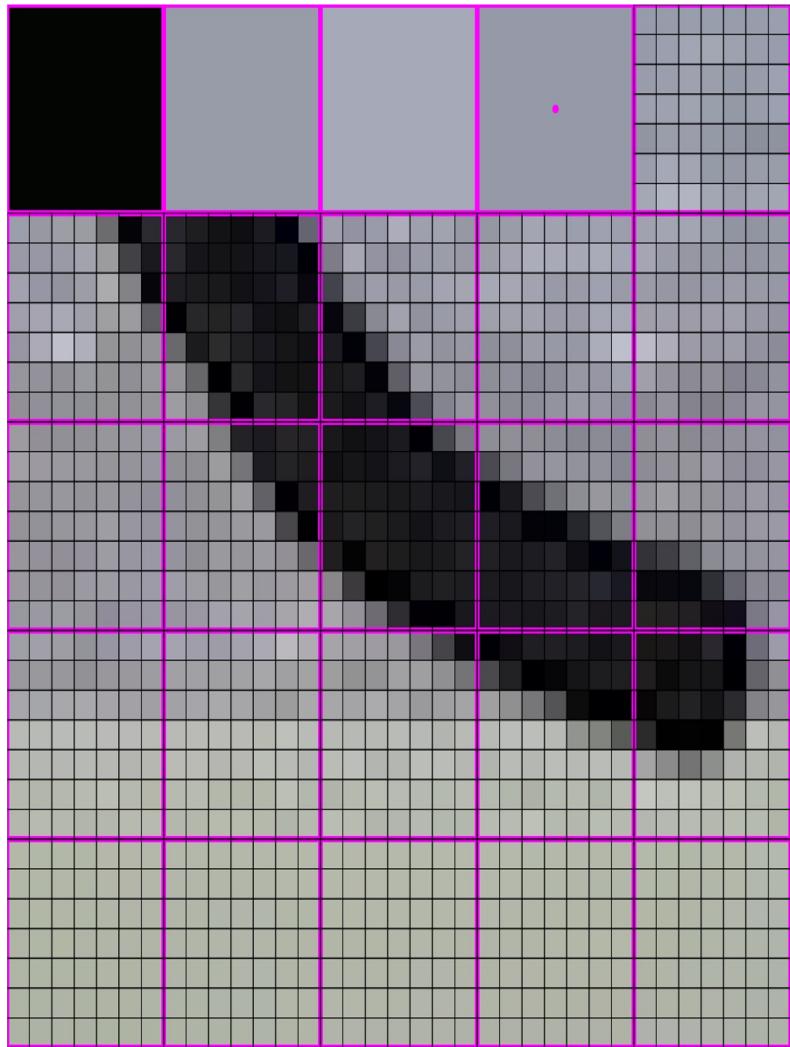
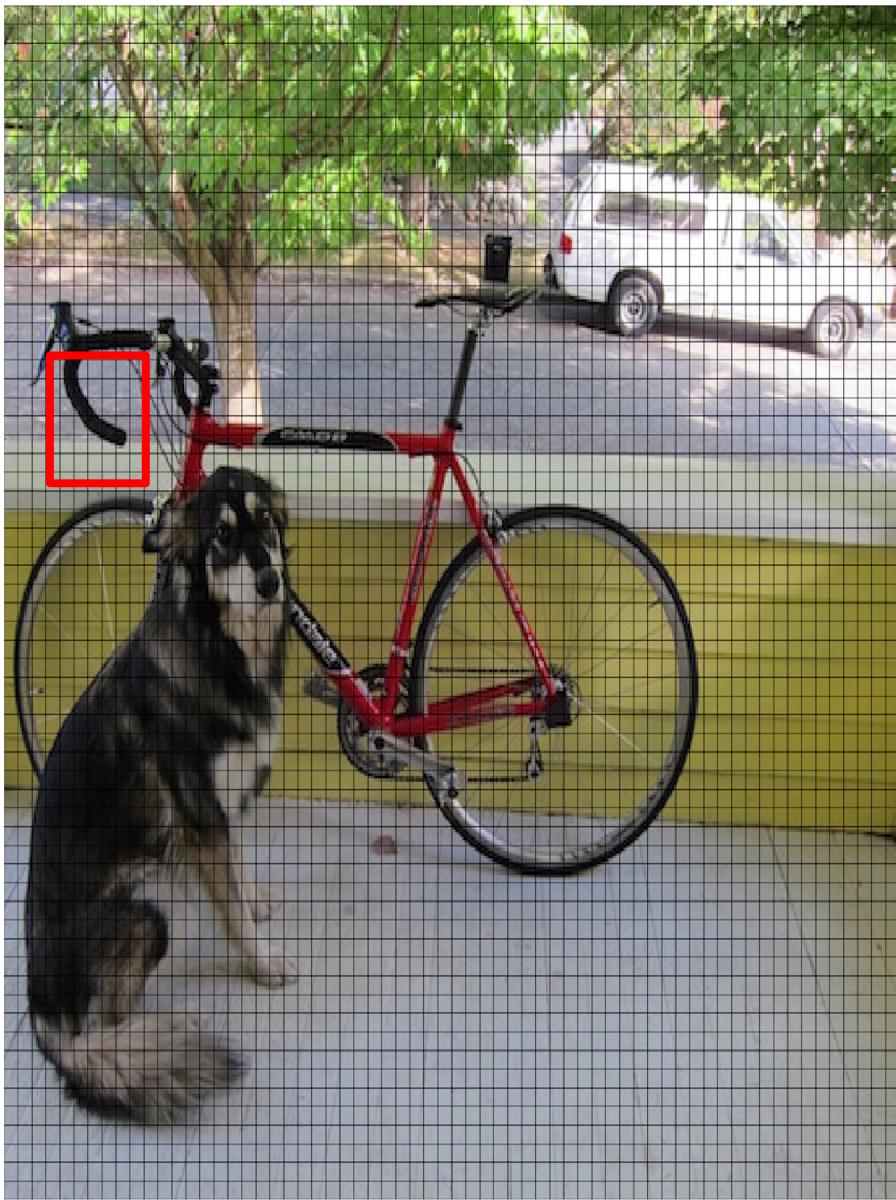
# From $448 \times 448$ to $64 \times 64$



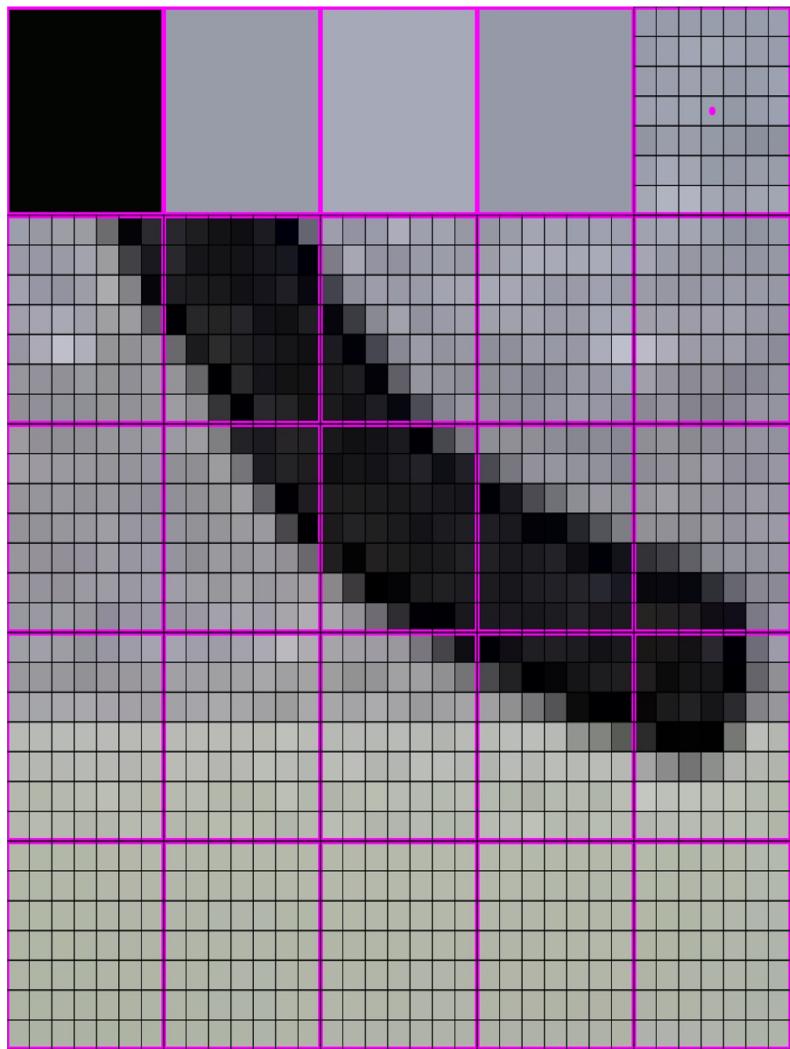
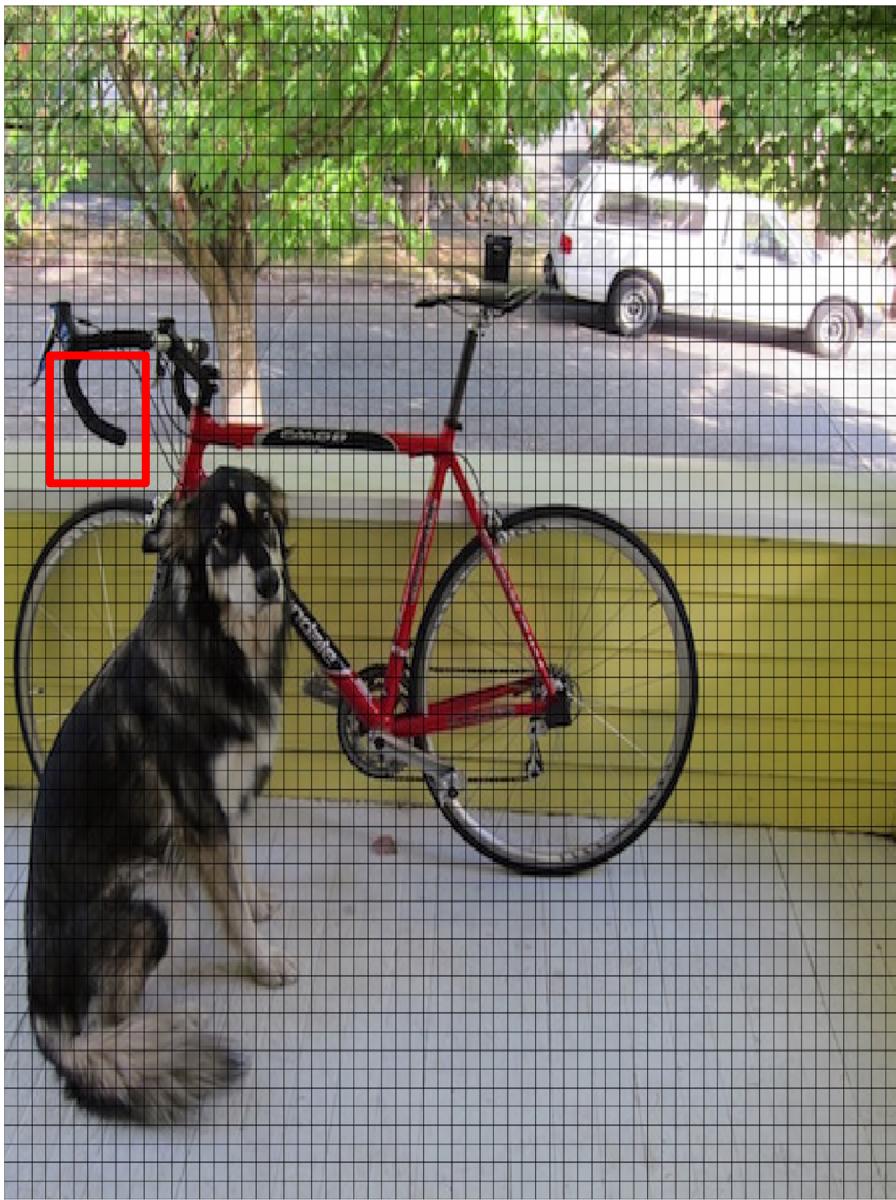
# From $448 \times 448$ to $64 \times 64$



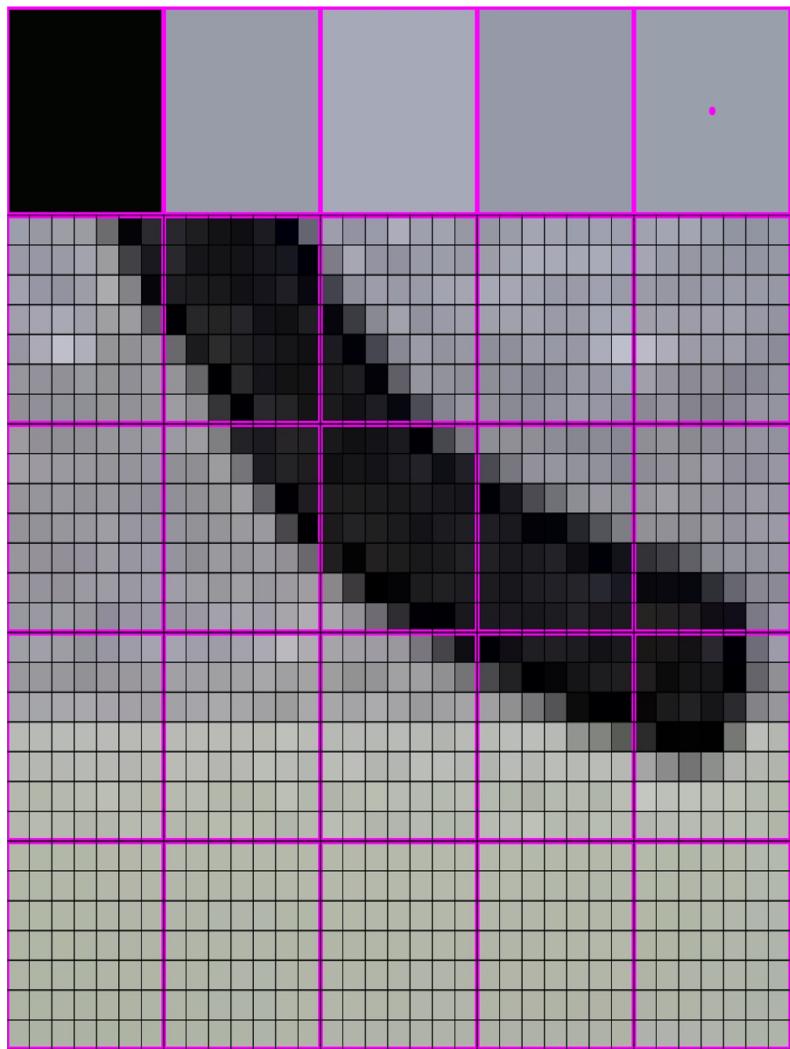
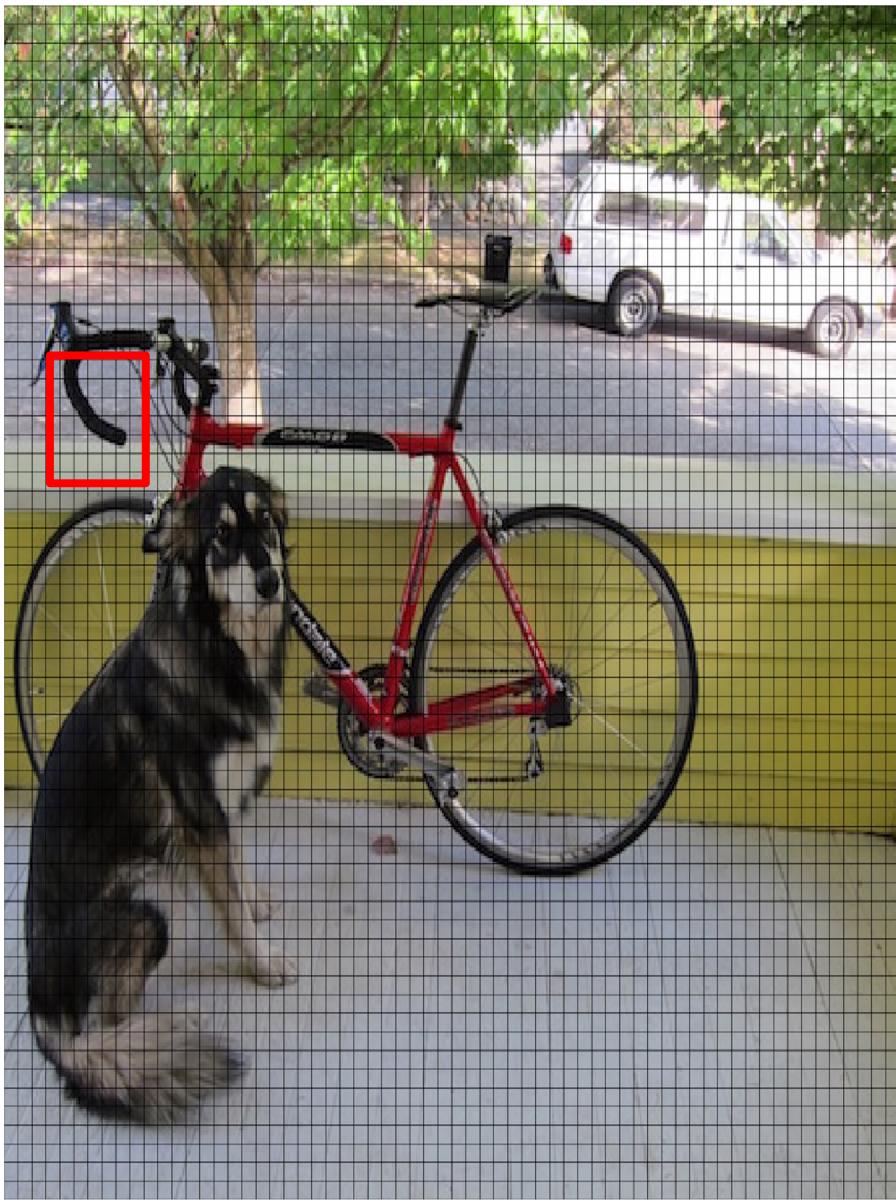
# From $448 \times 448$ to $64 \times 64$



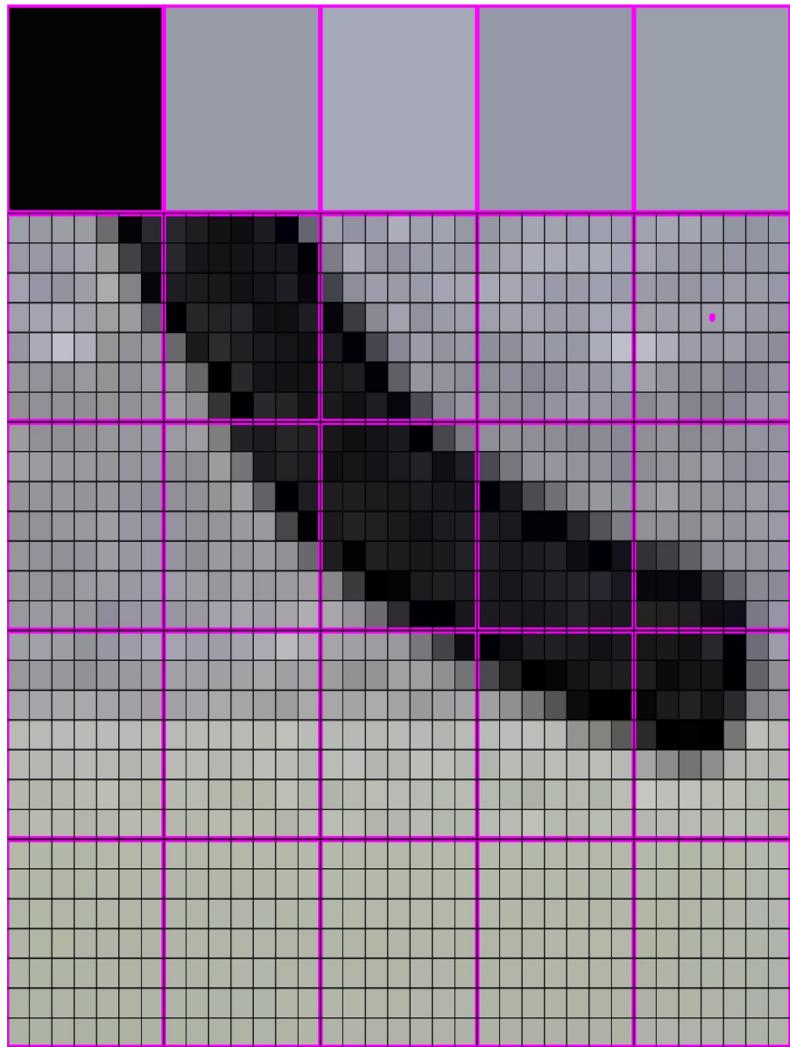
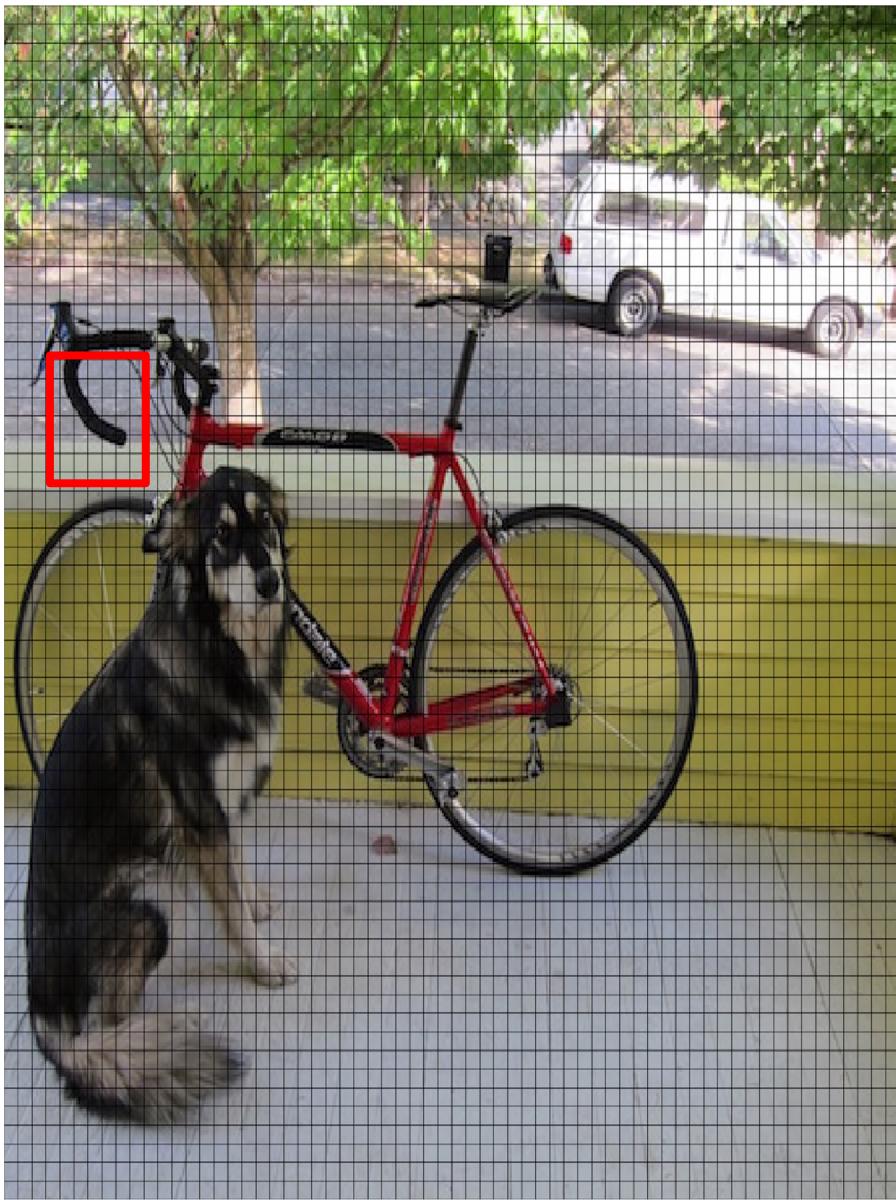
# From $448 \times 448$ to $64 \times 64$



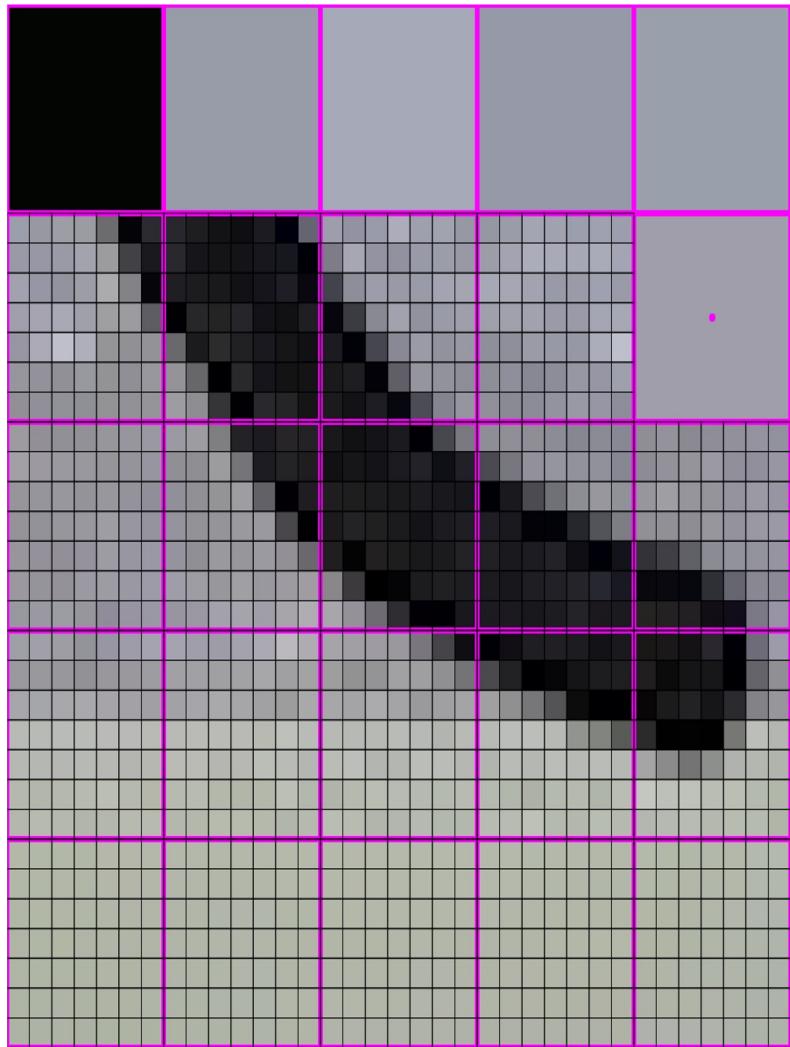
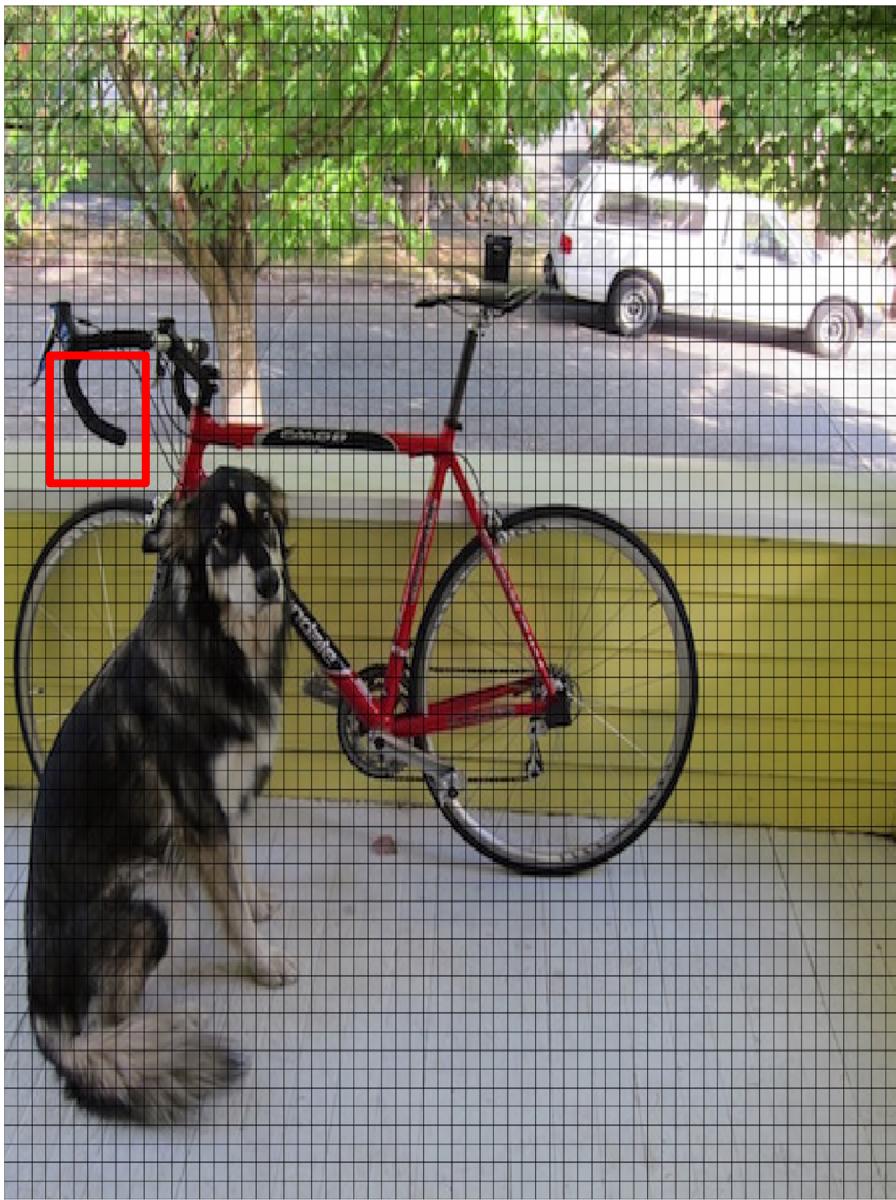
# From $448 \times 448$ to $64 \times 64$



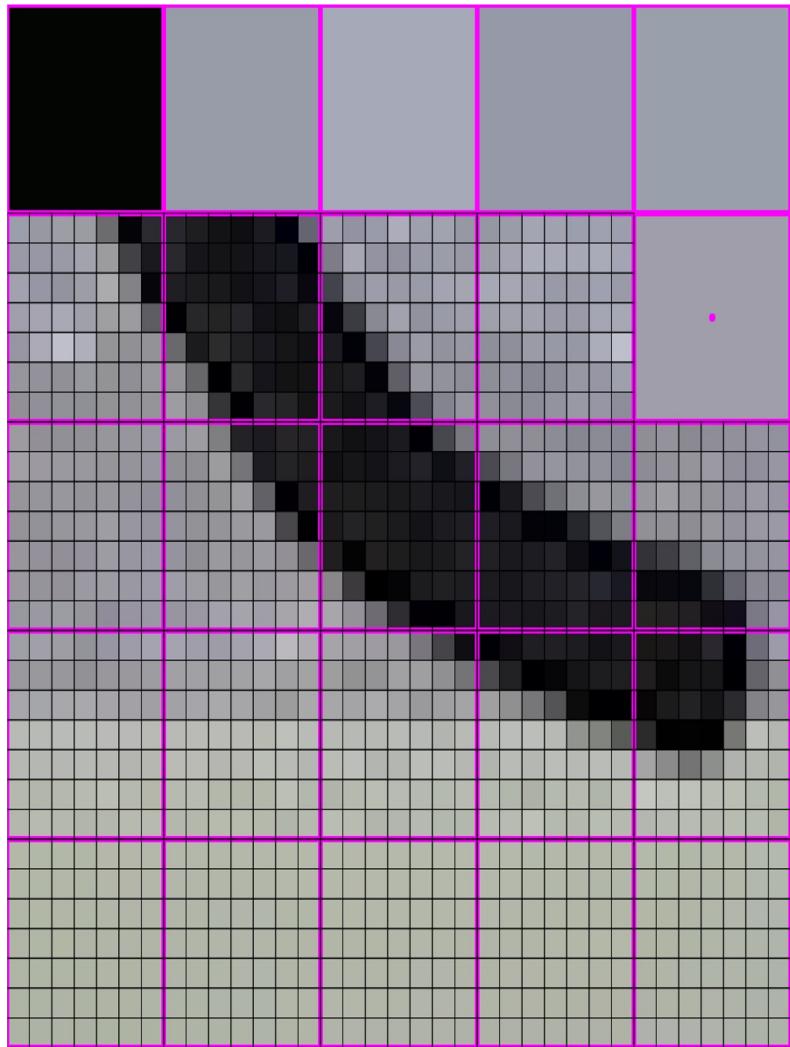
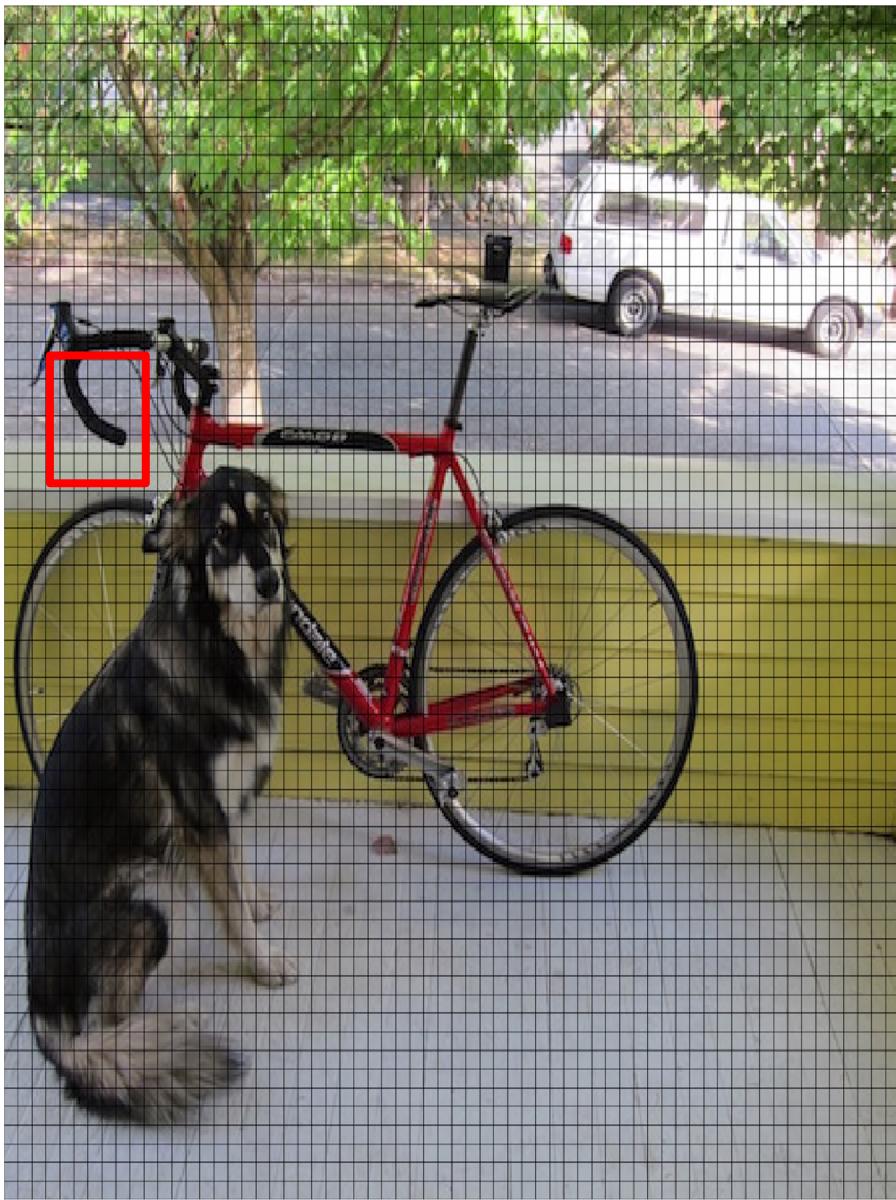
# From $448 \times 448$ to $64 \times 64$



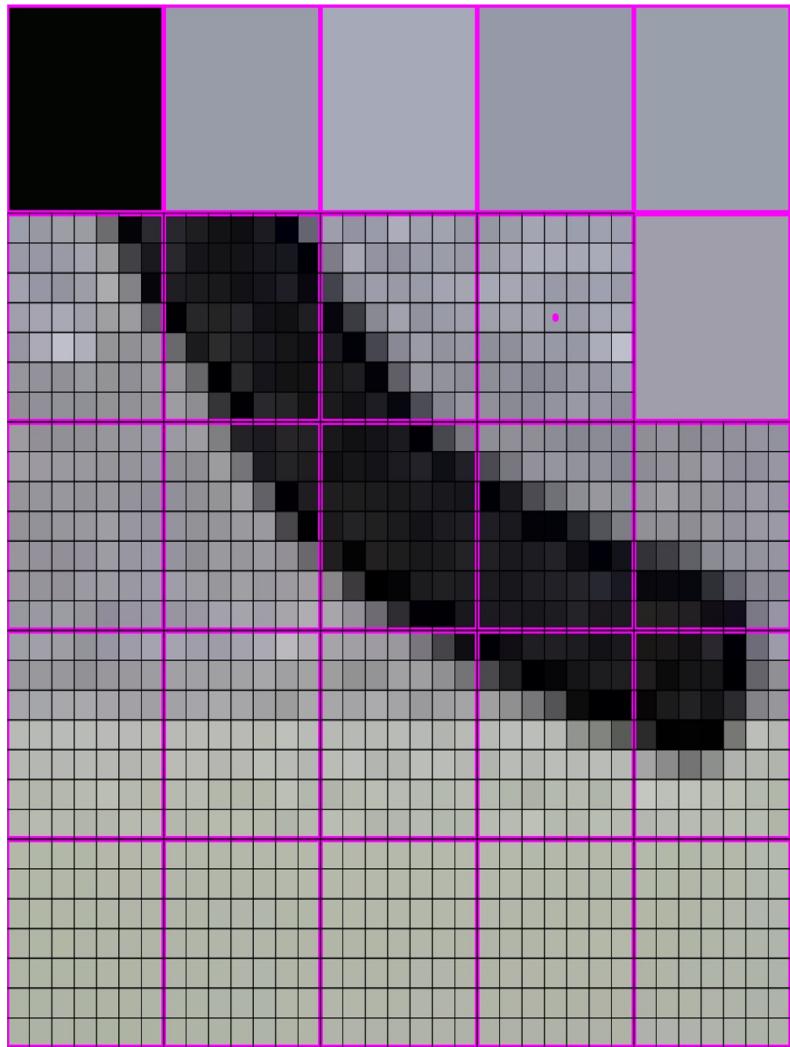
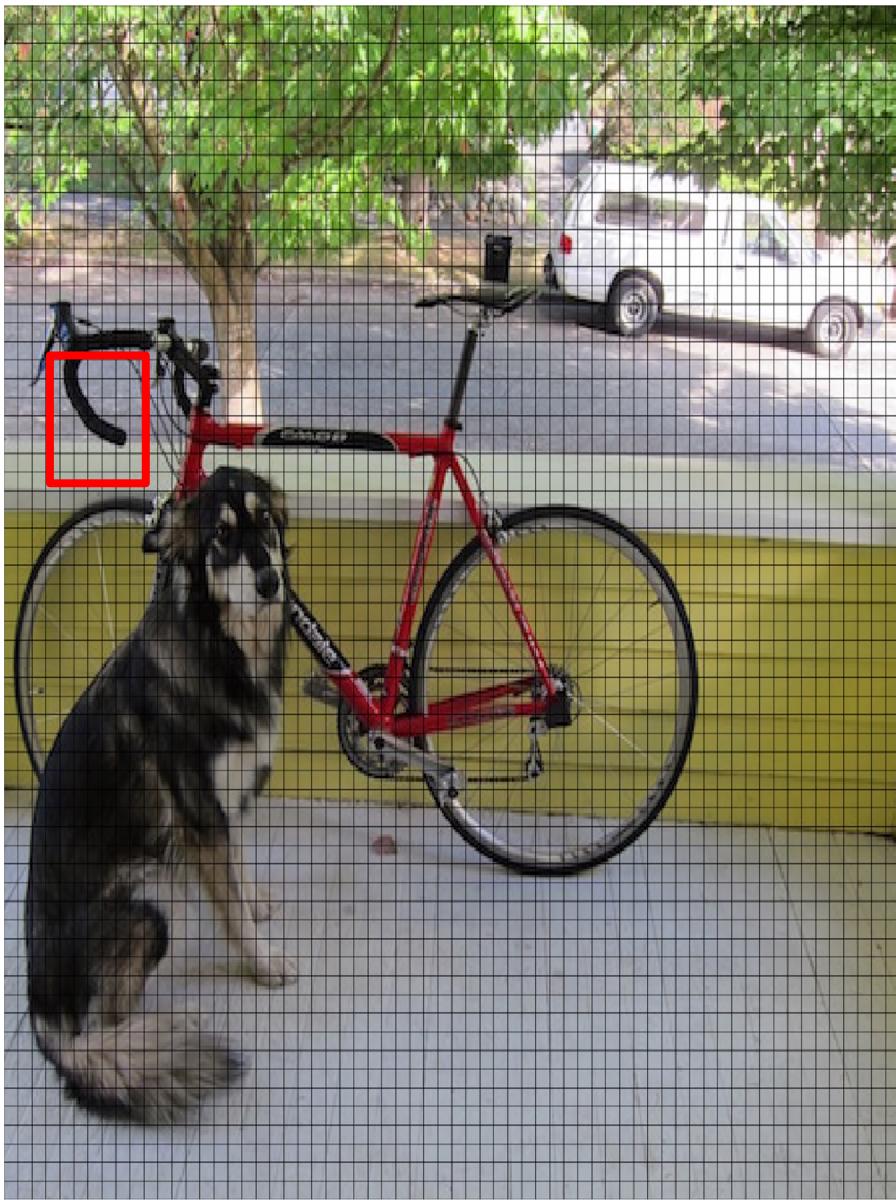
# From $448 \times 448$ to $64 \times 64$



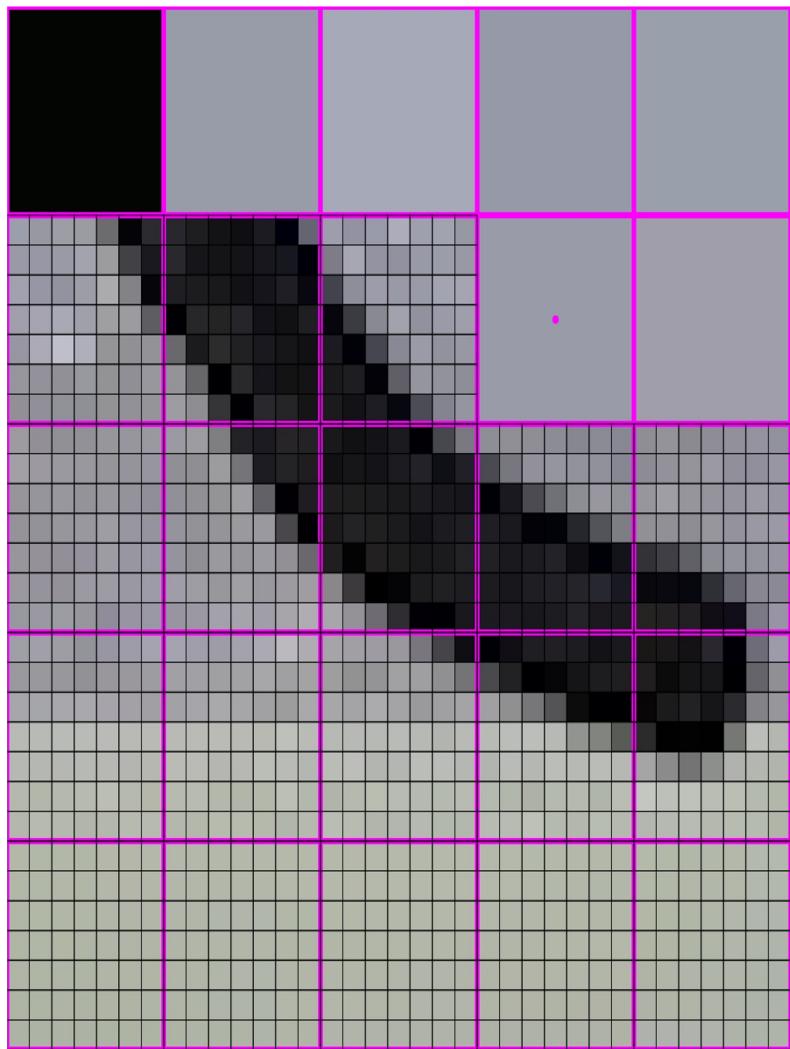
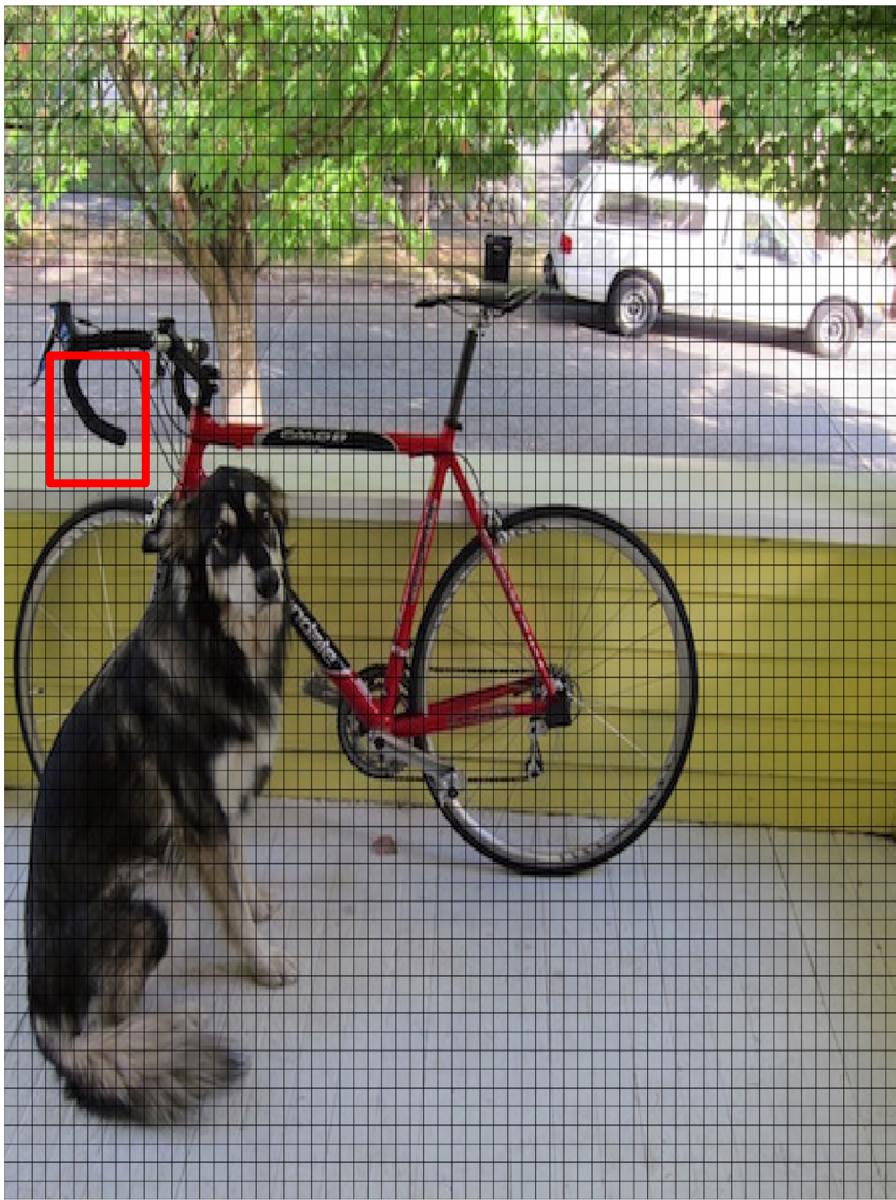
# From $448 \times 448$ to $64 \times 64$



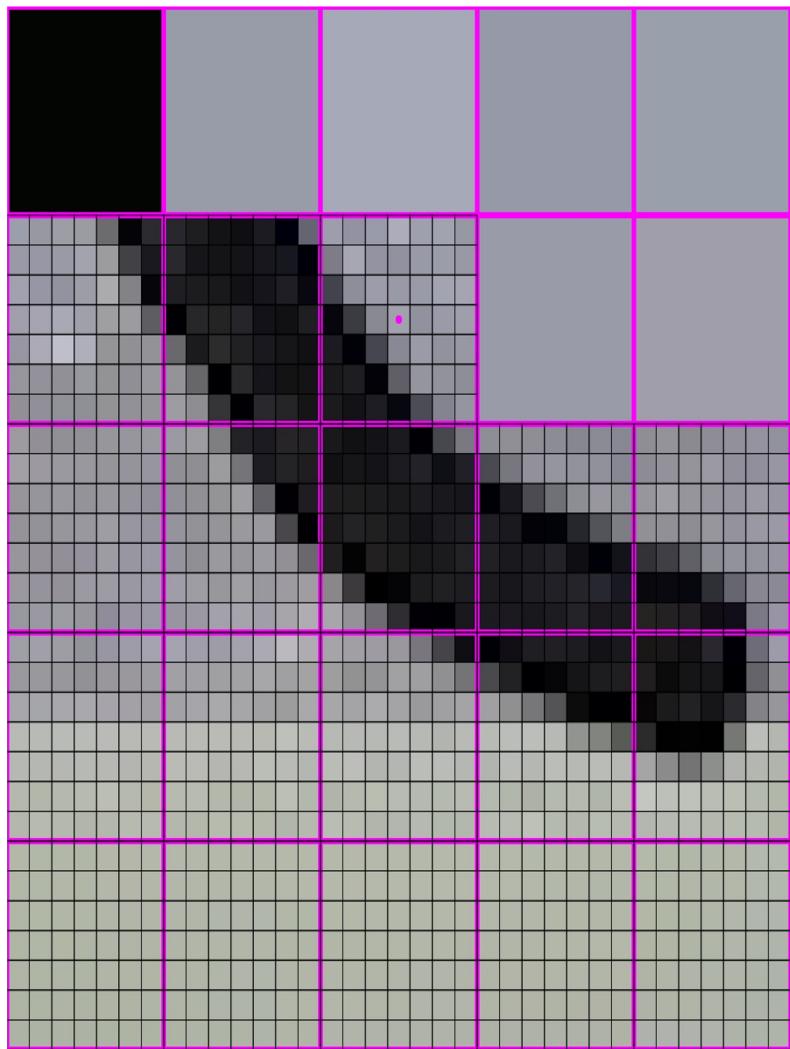
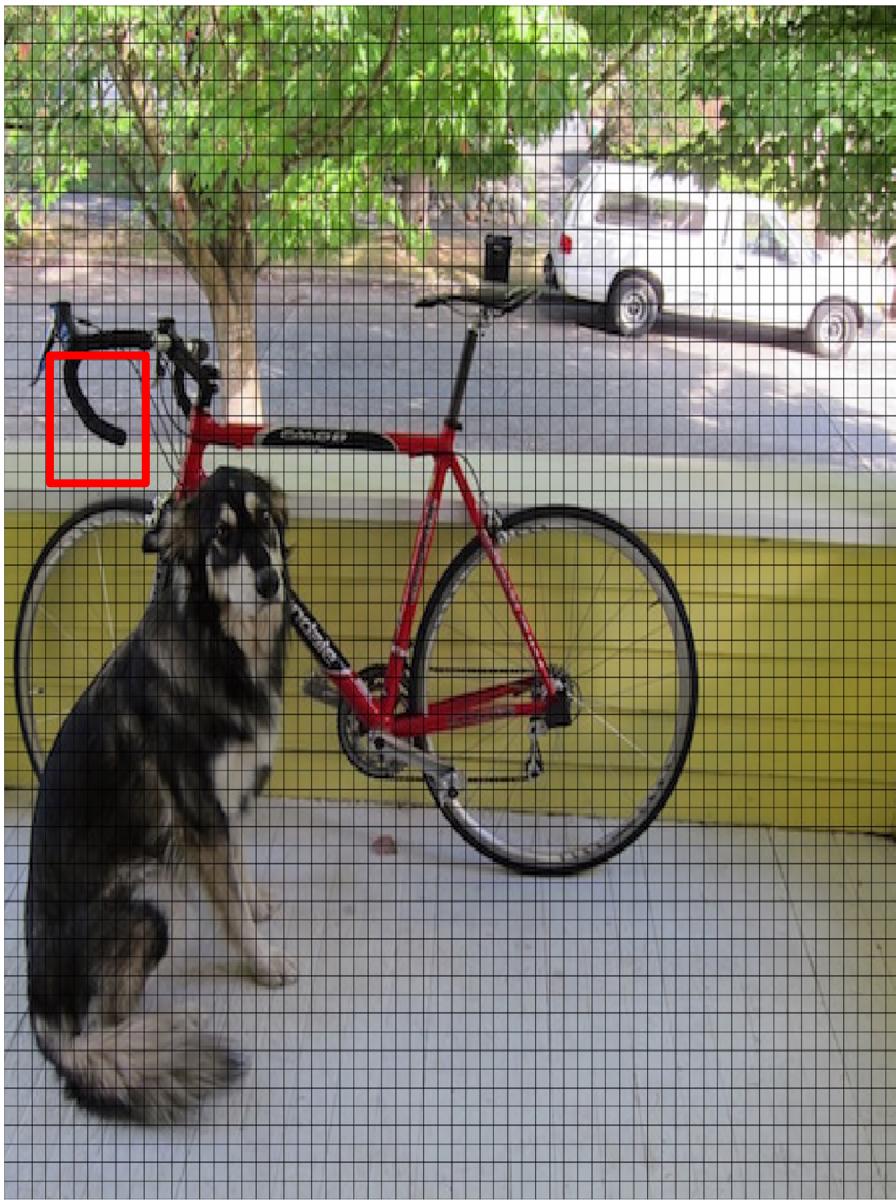
# From $448 \times 448$ to $64 \times 64$



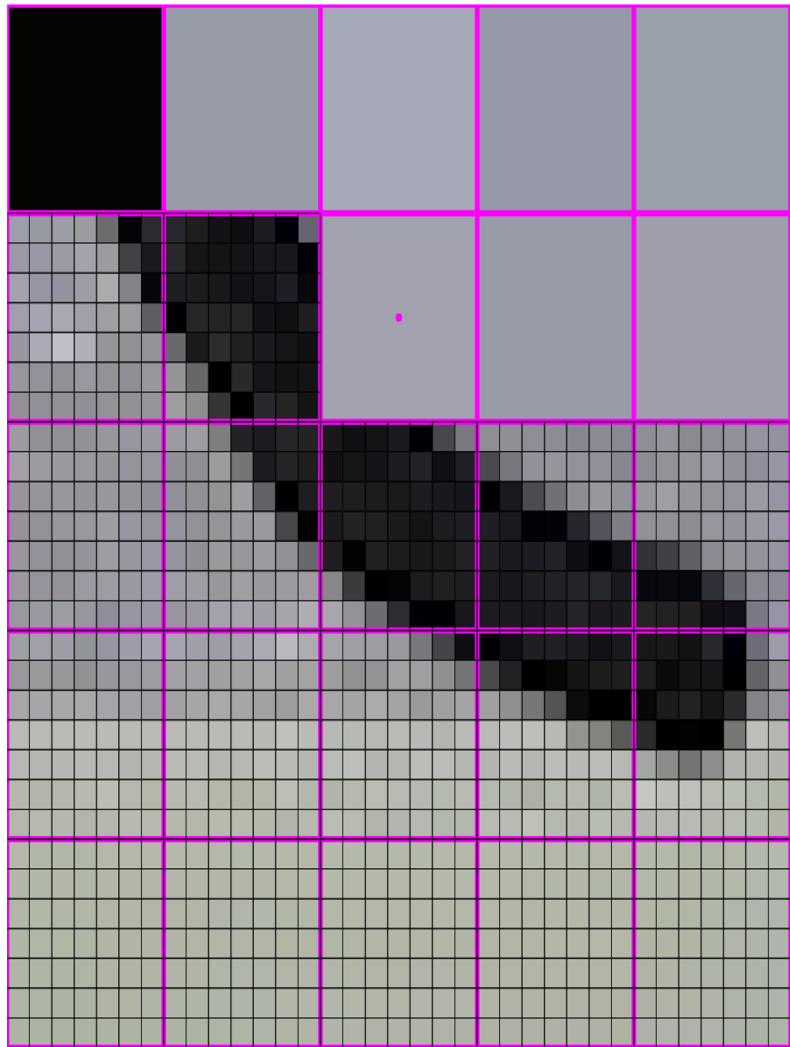
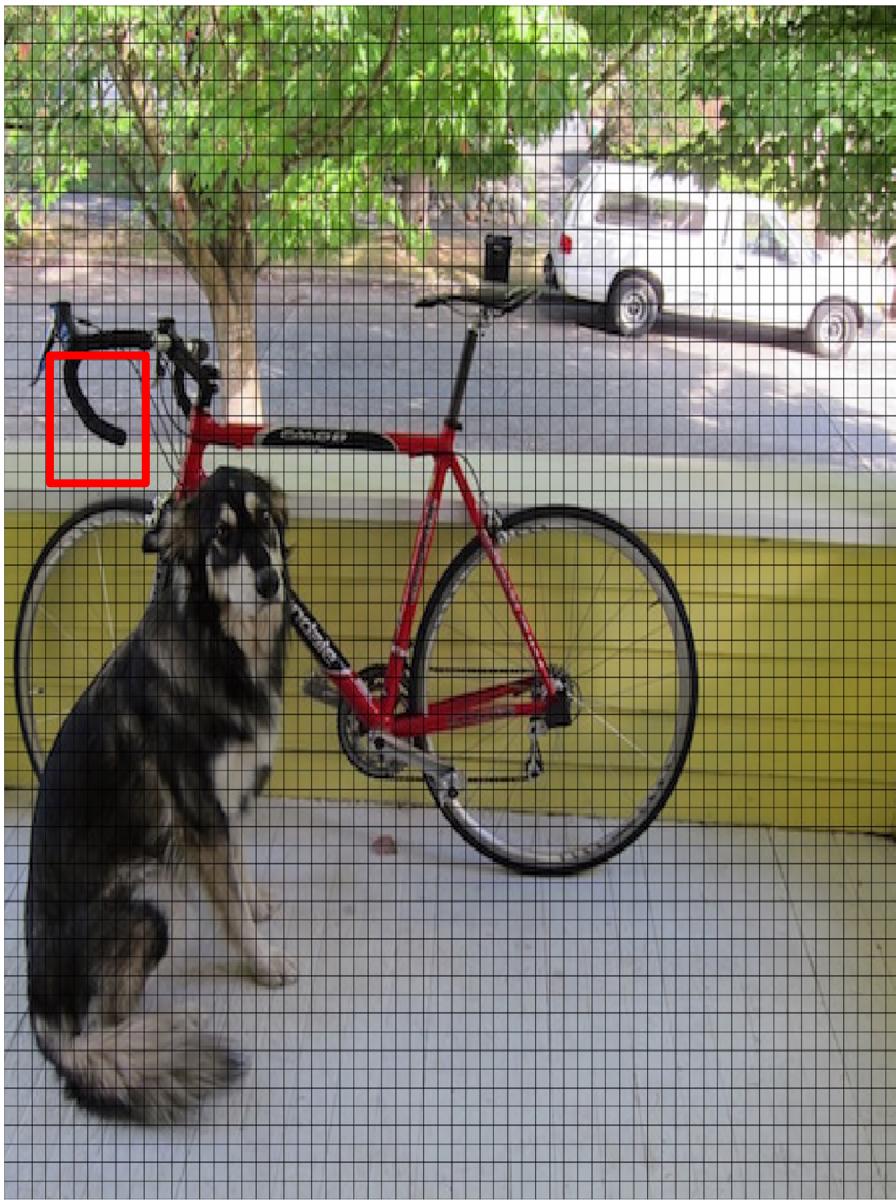
# From $448 \times 448$ to $64 \times 64$



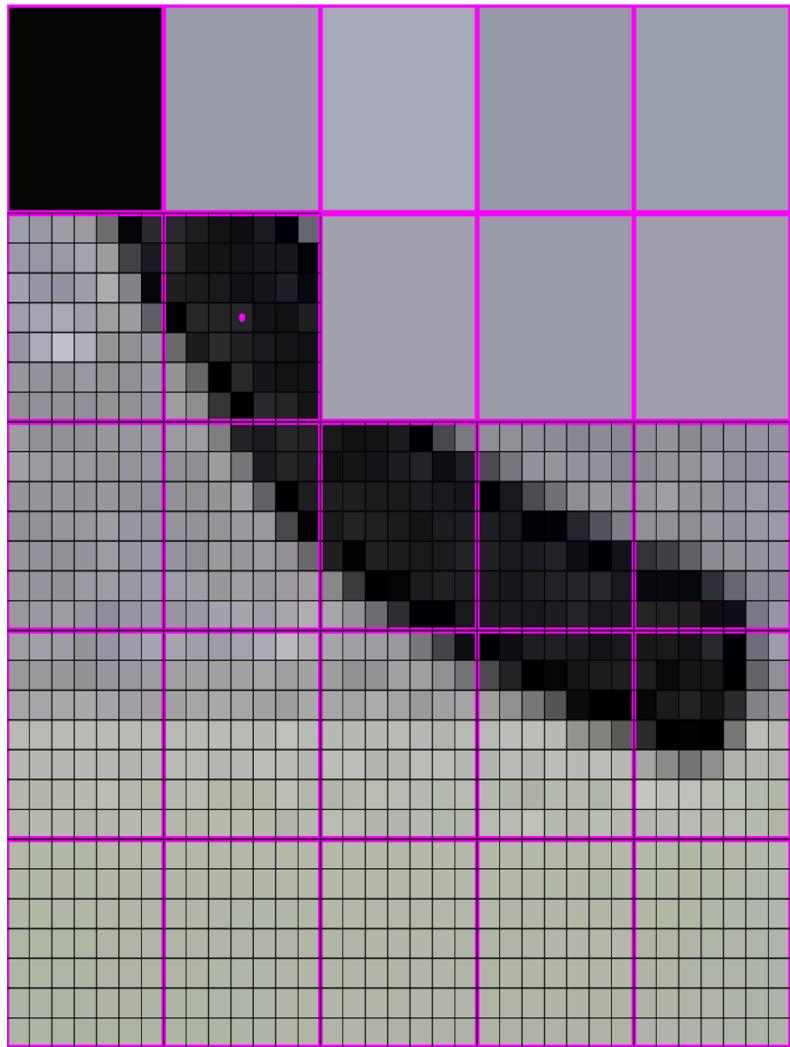
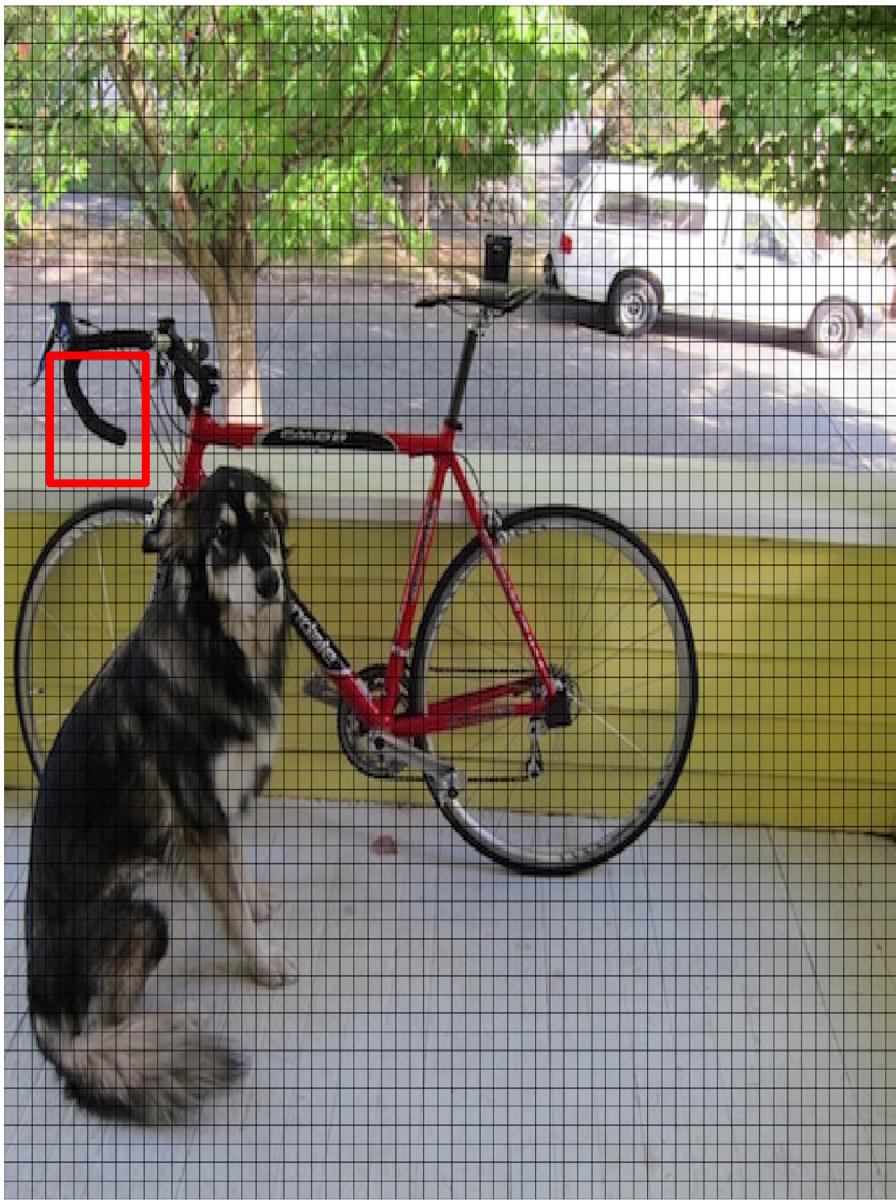
# From $448 \times 448$ to $64 \times 64$



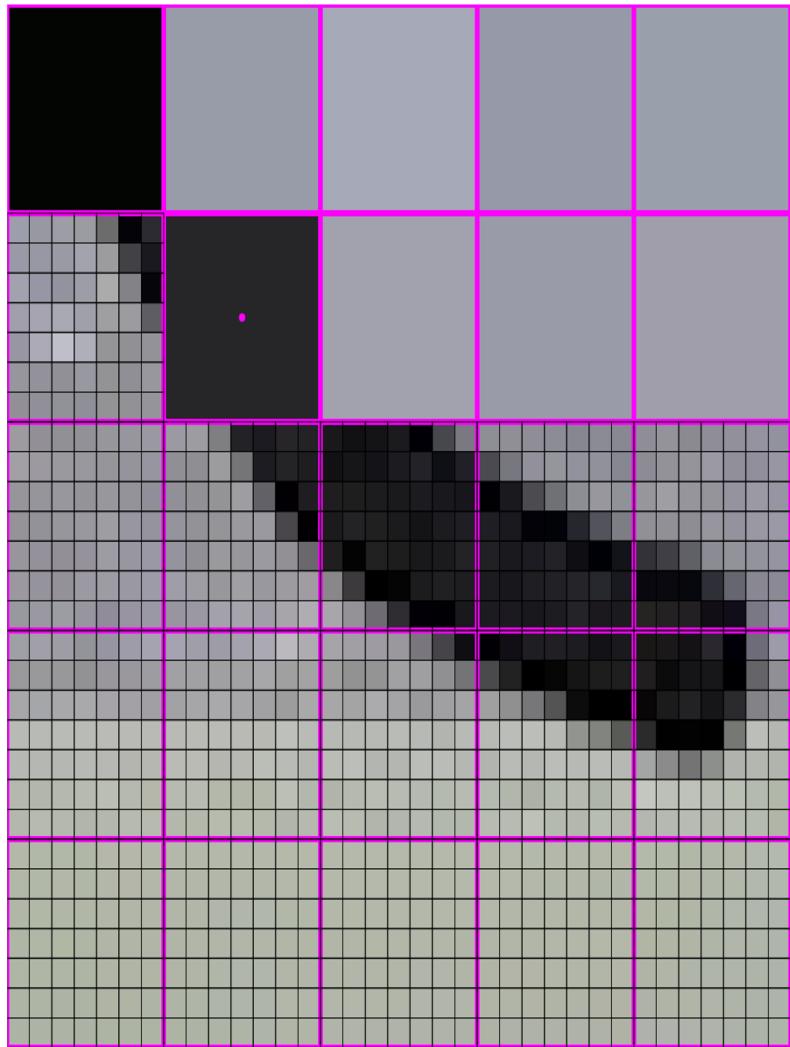
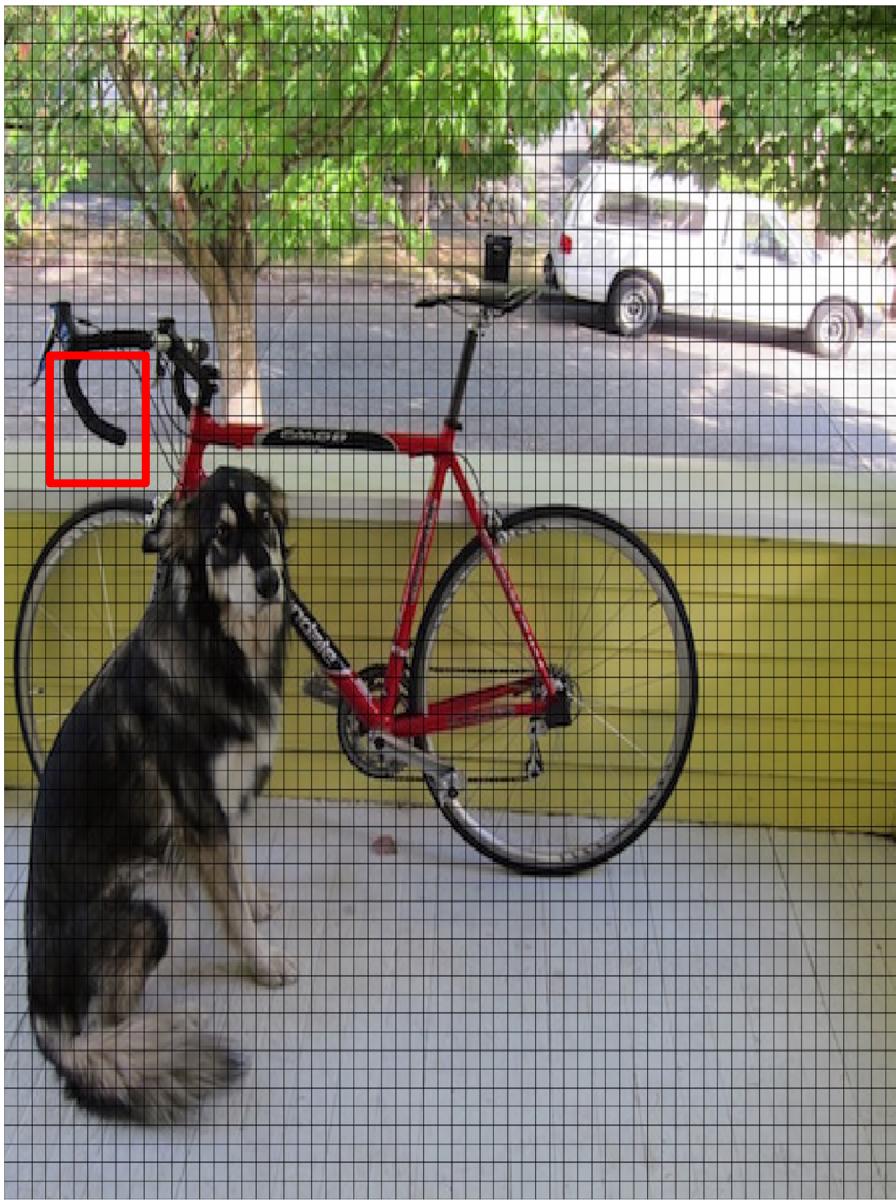
# From $448 \times 448$ to $64 \times 64$



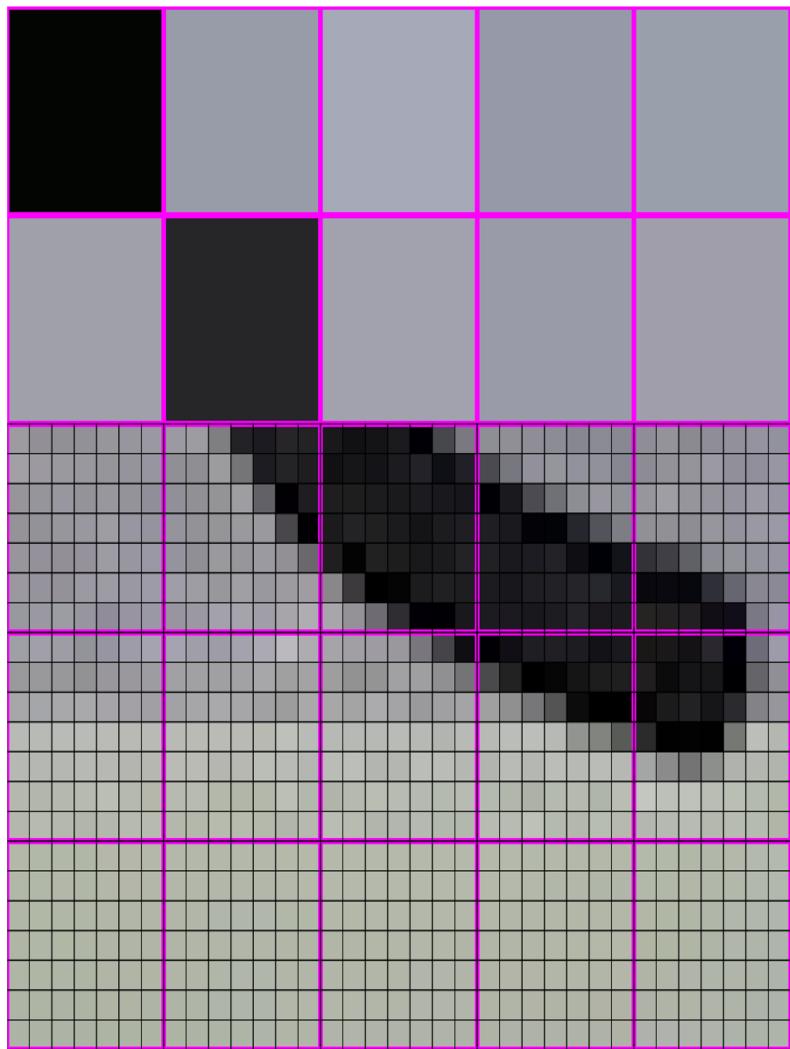
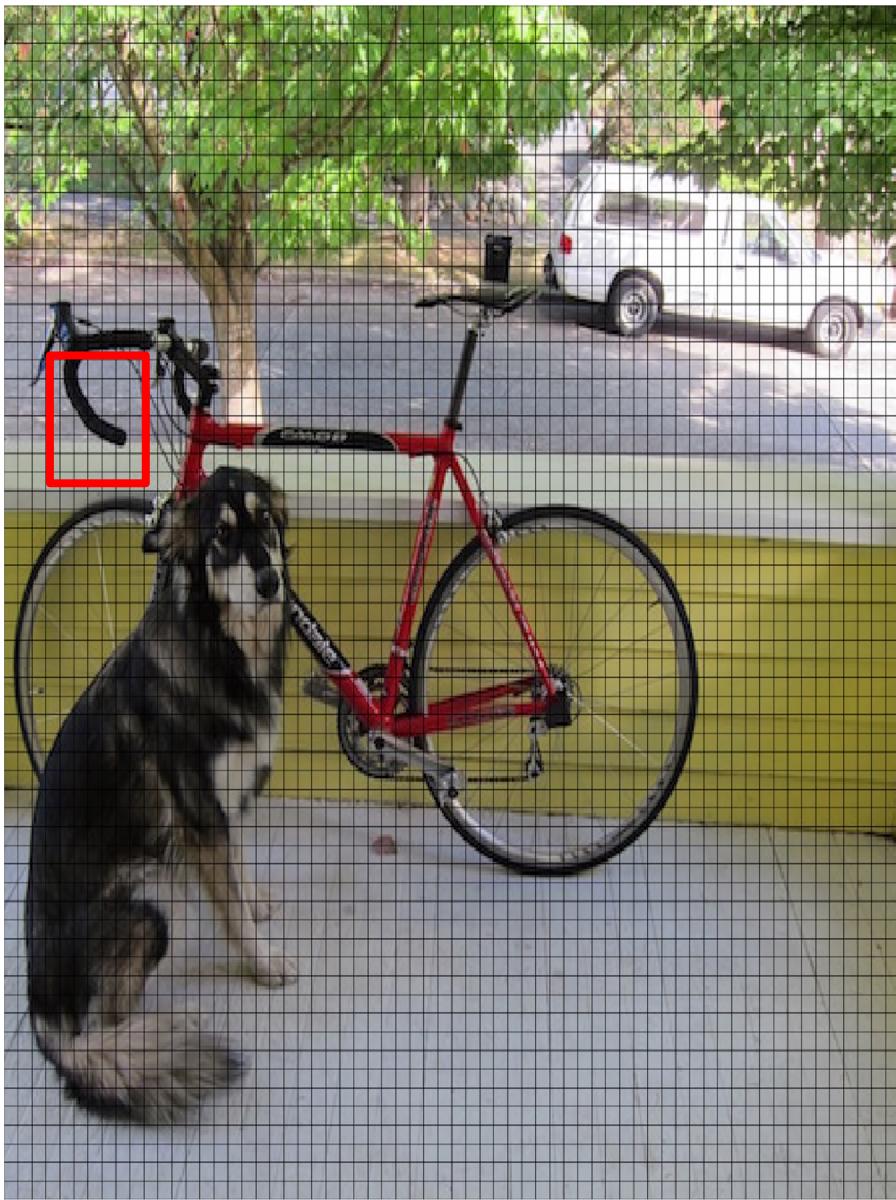
# From $448 \times 448$ to $64 \times 64$



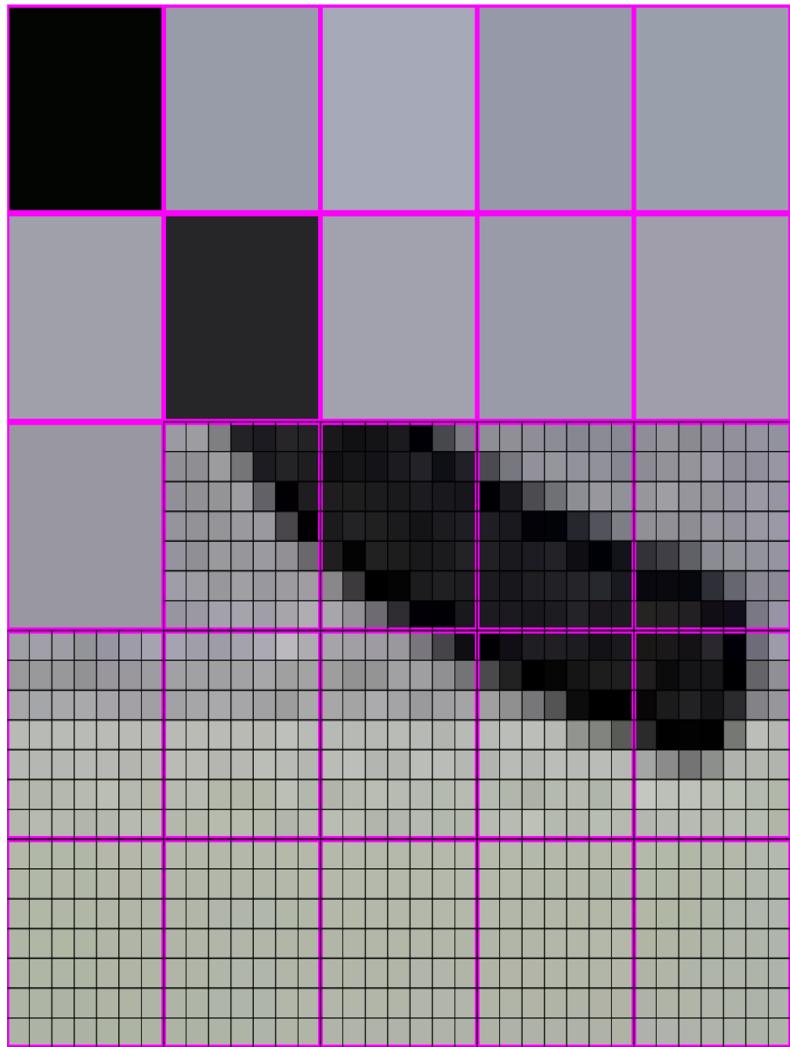
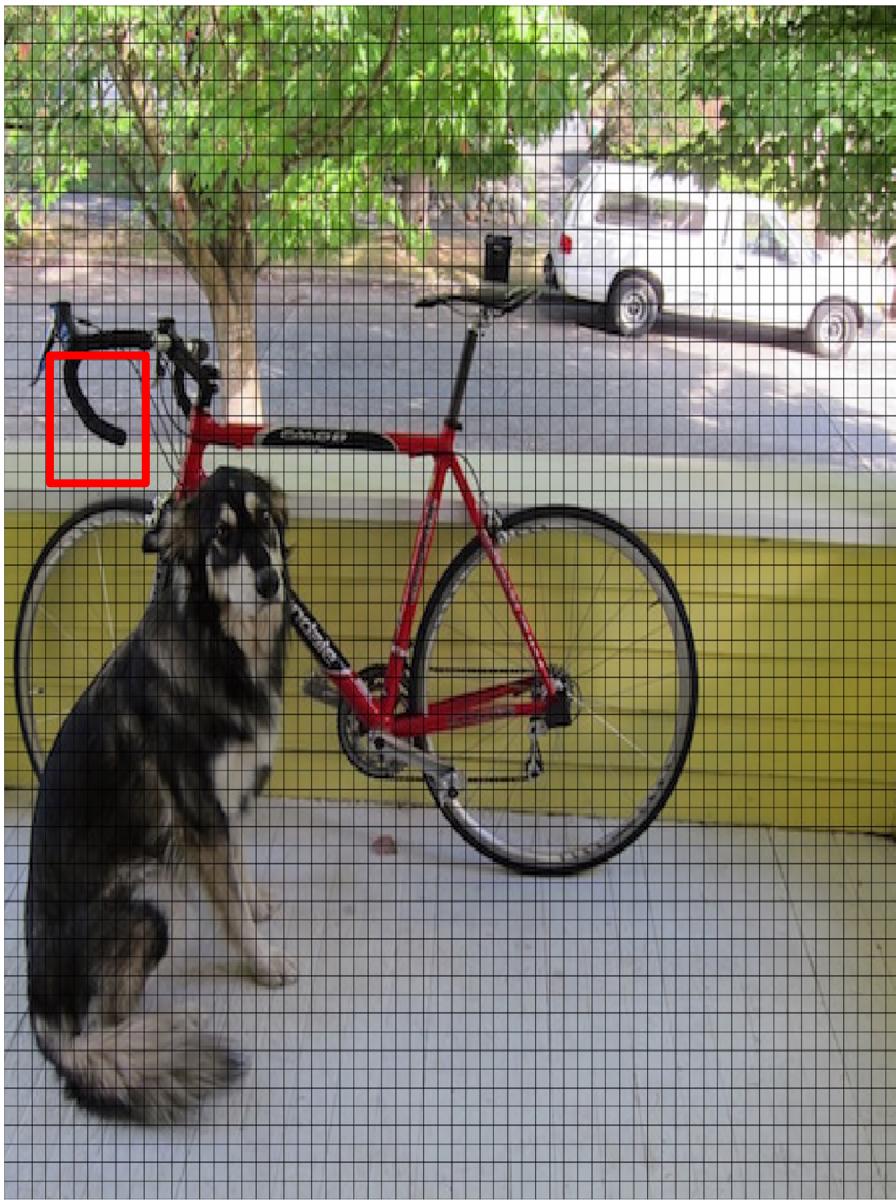
# From $448 \times 448$ to $64 \times 64$



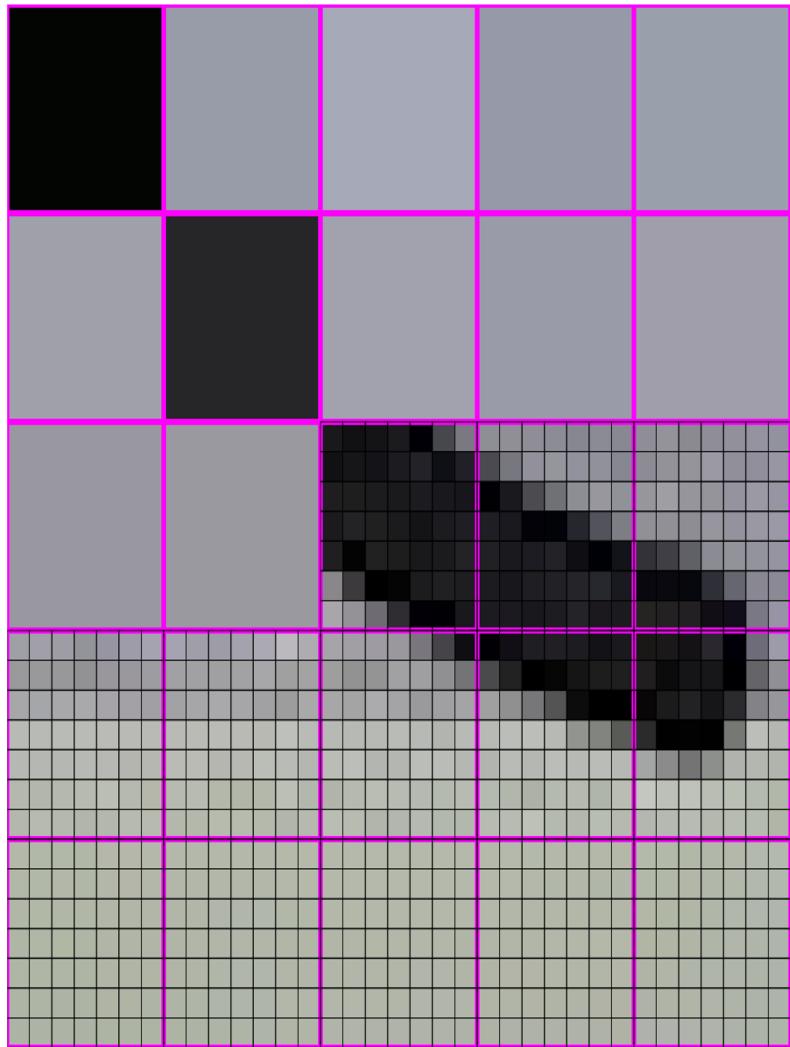
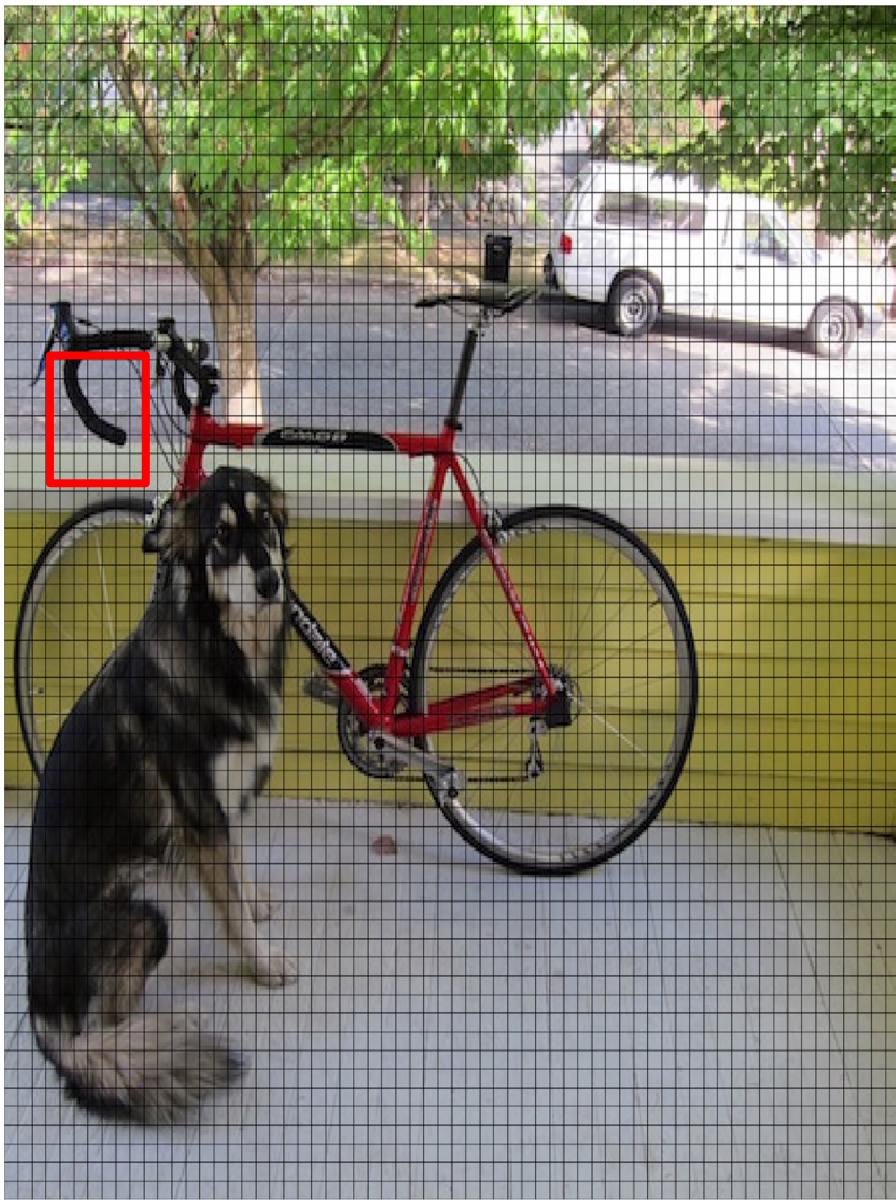
# From $448 \times 448$ to $64 \times 64$



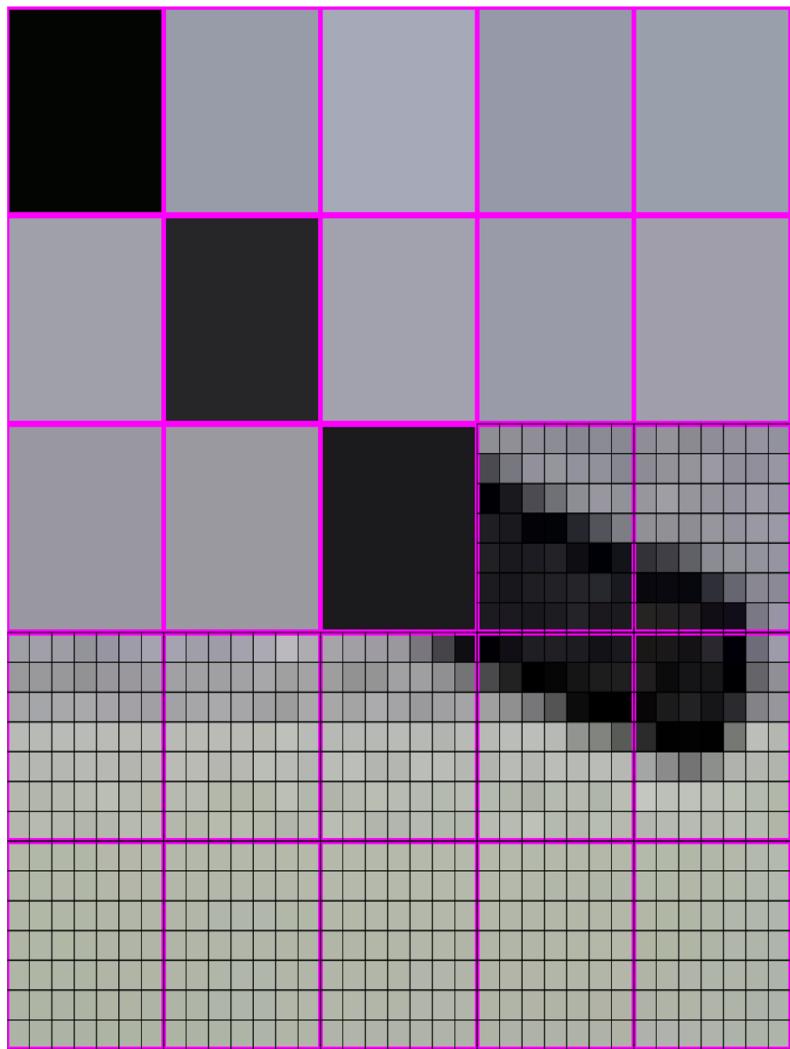
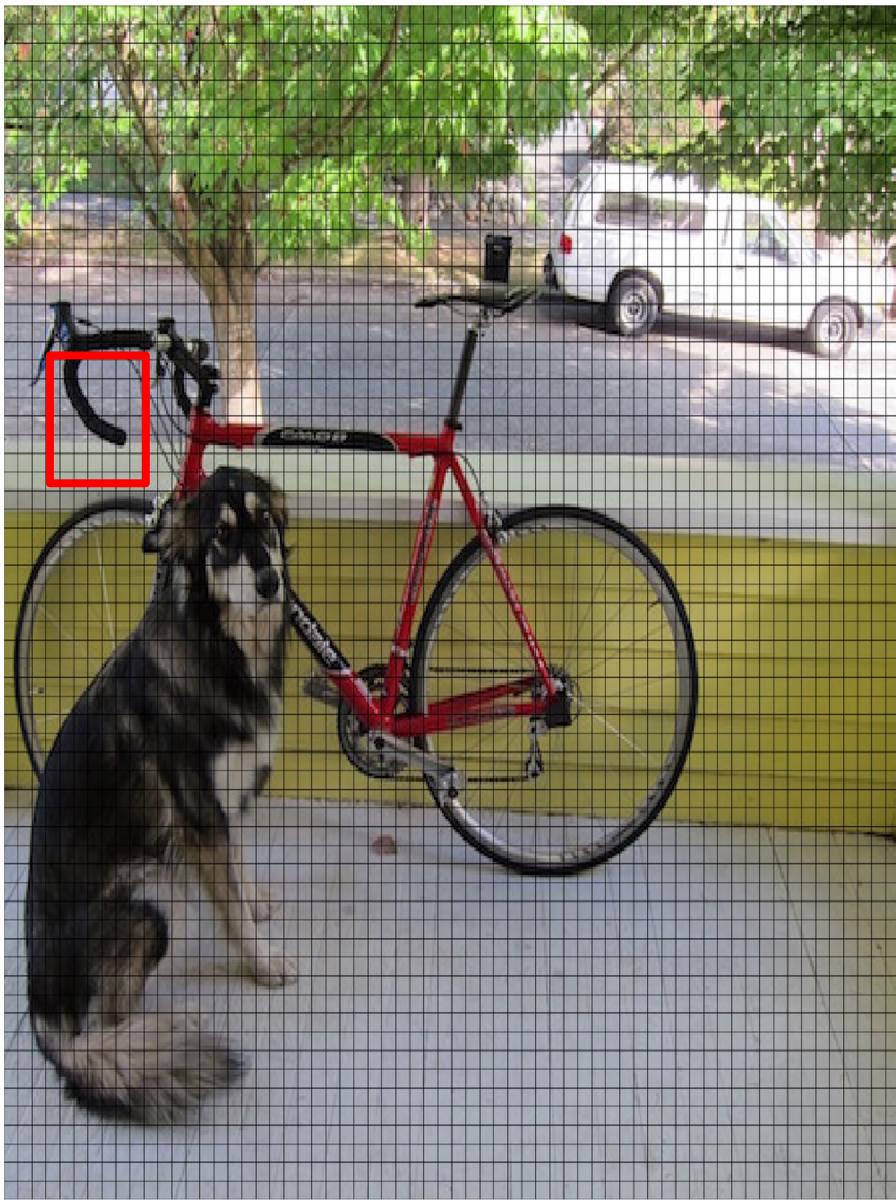
# From $448 \times 448$ to $64 \times 64$



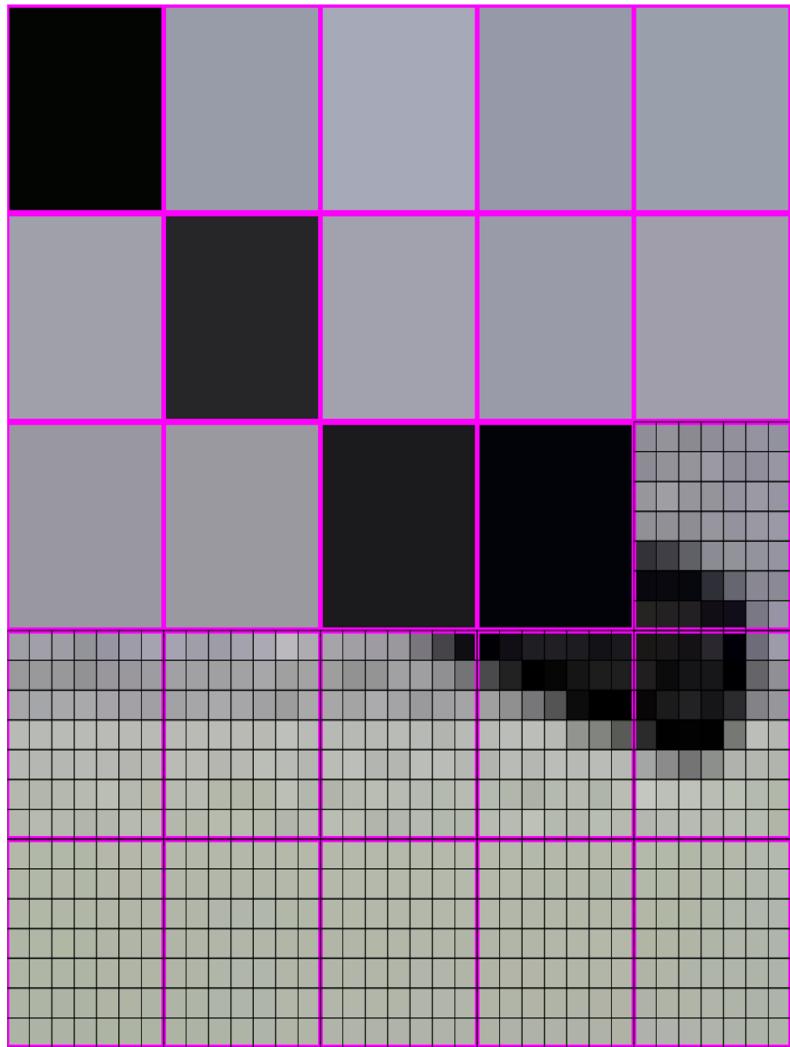
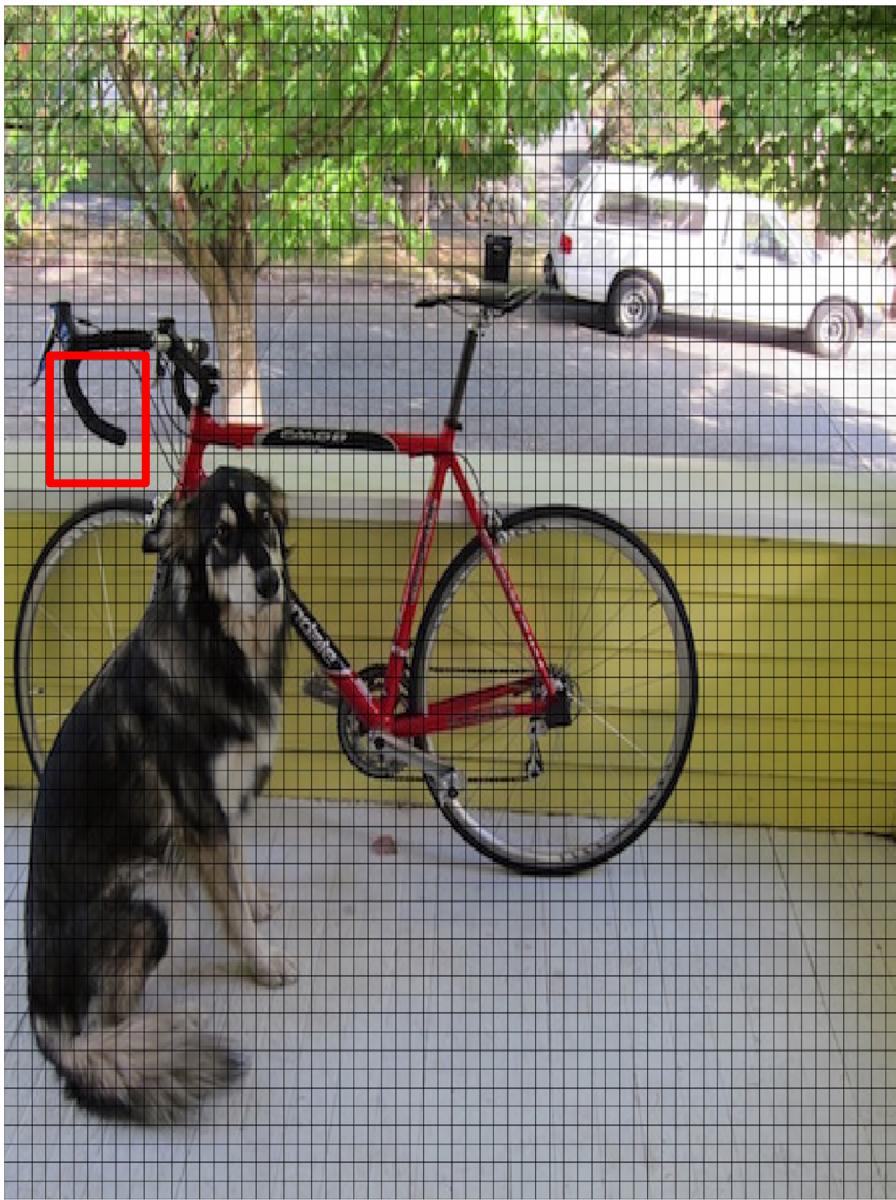
# From $448 \times 448$ to $64 \times 64$



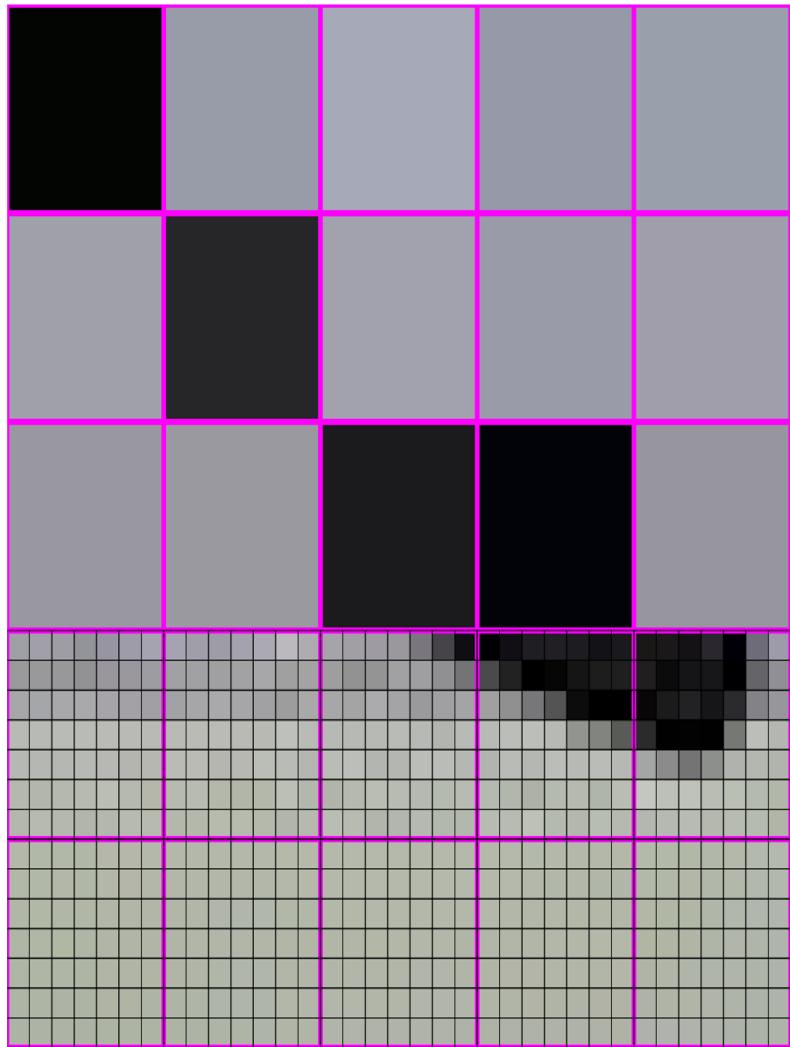
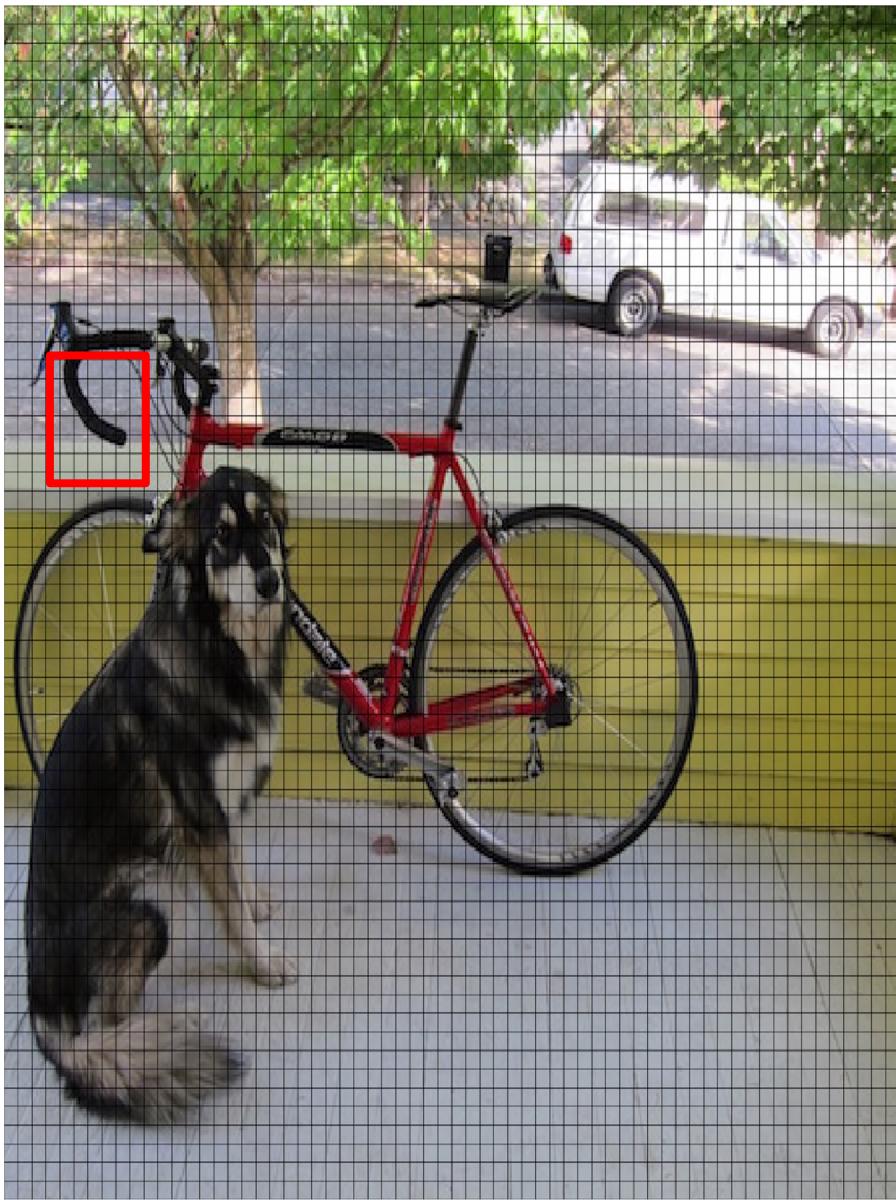
# From $448 \times 448$ to $64 \times 64$



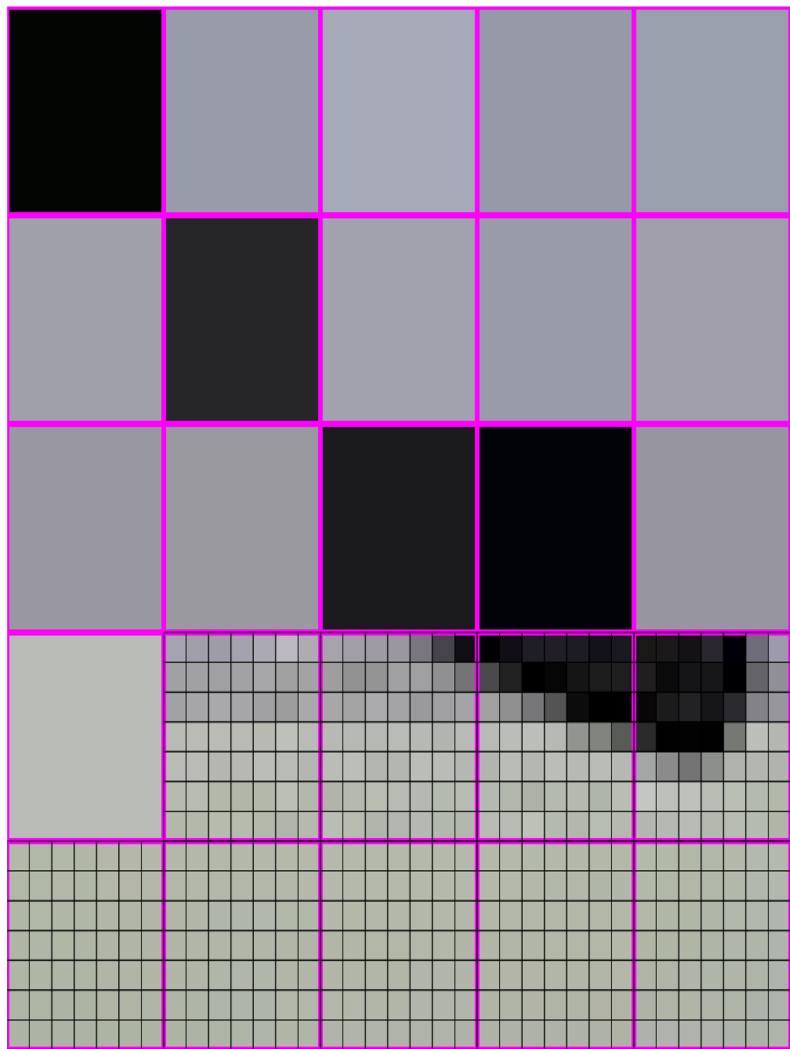
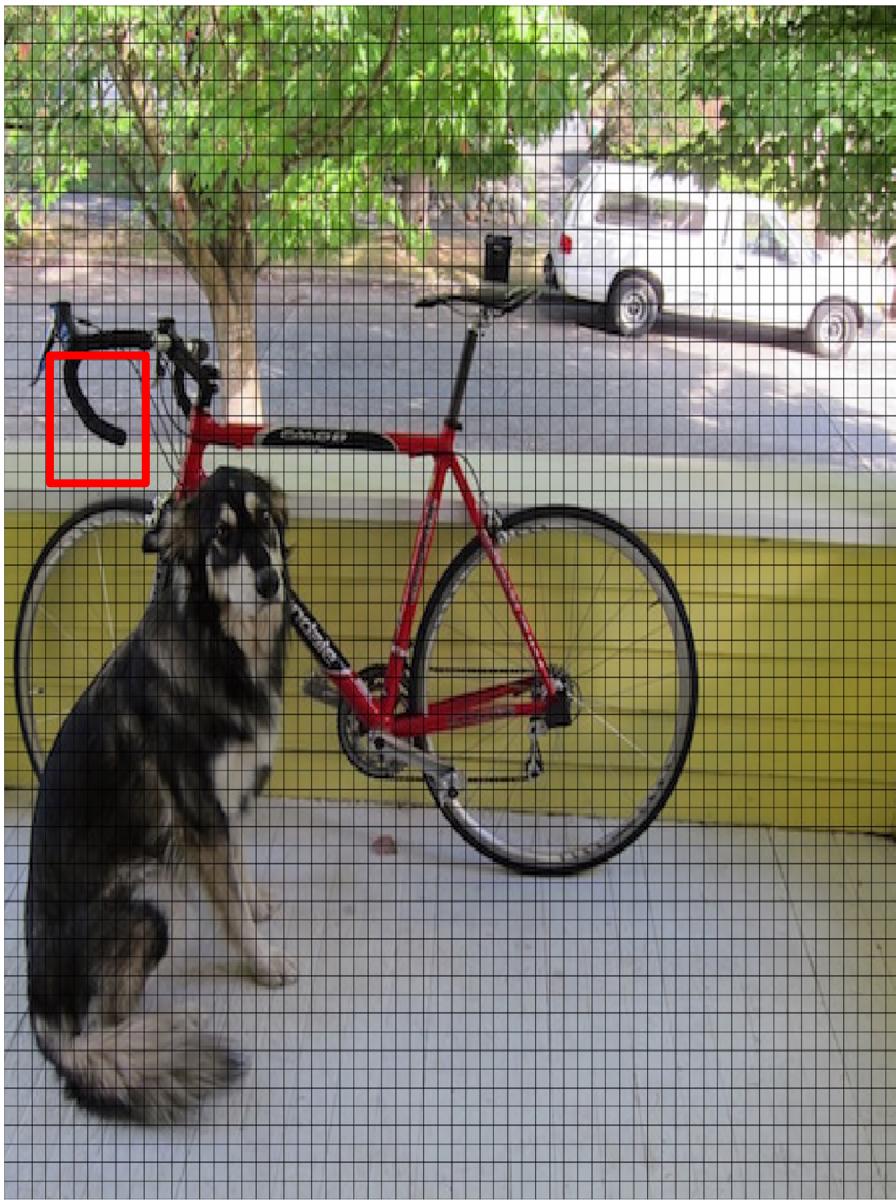
# From $448 \times 448$ to $64 \times 64$



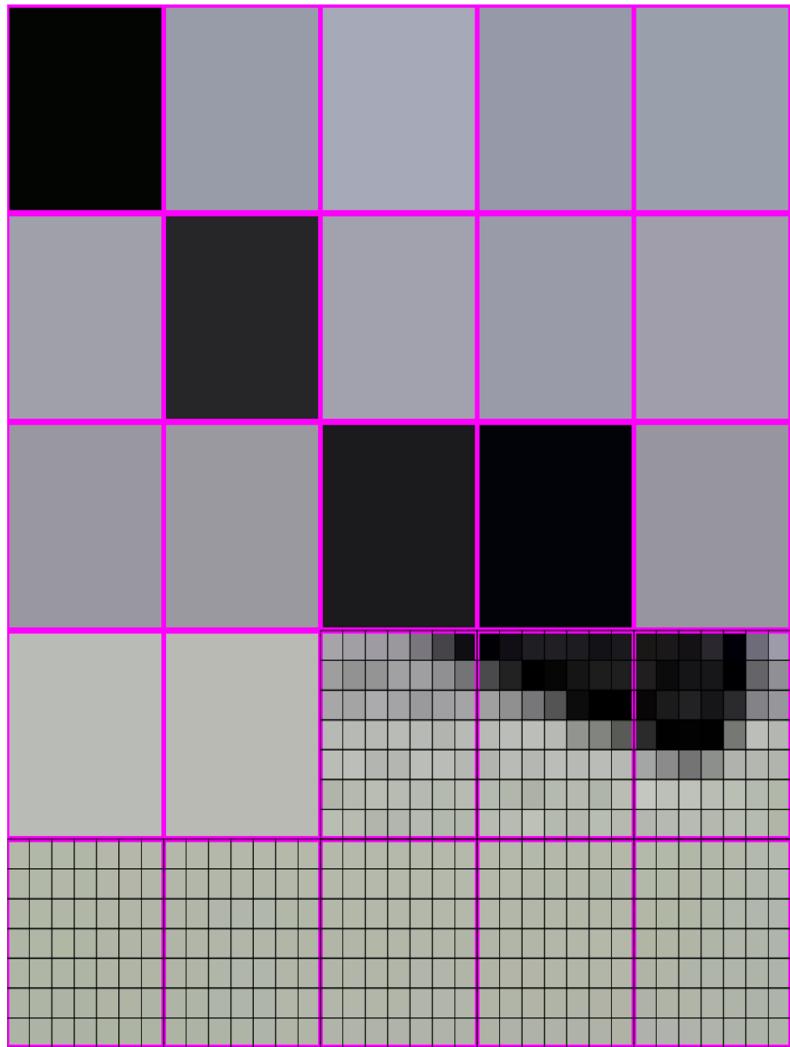
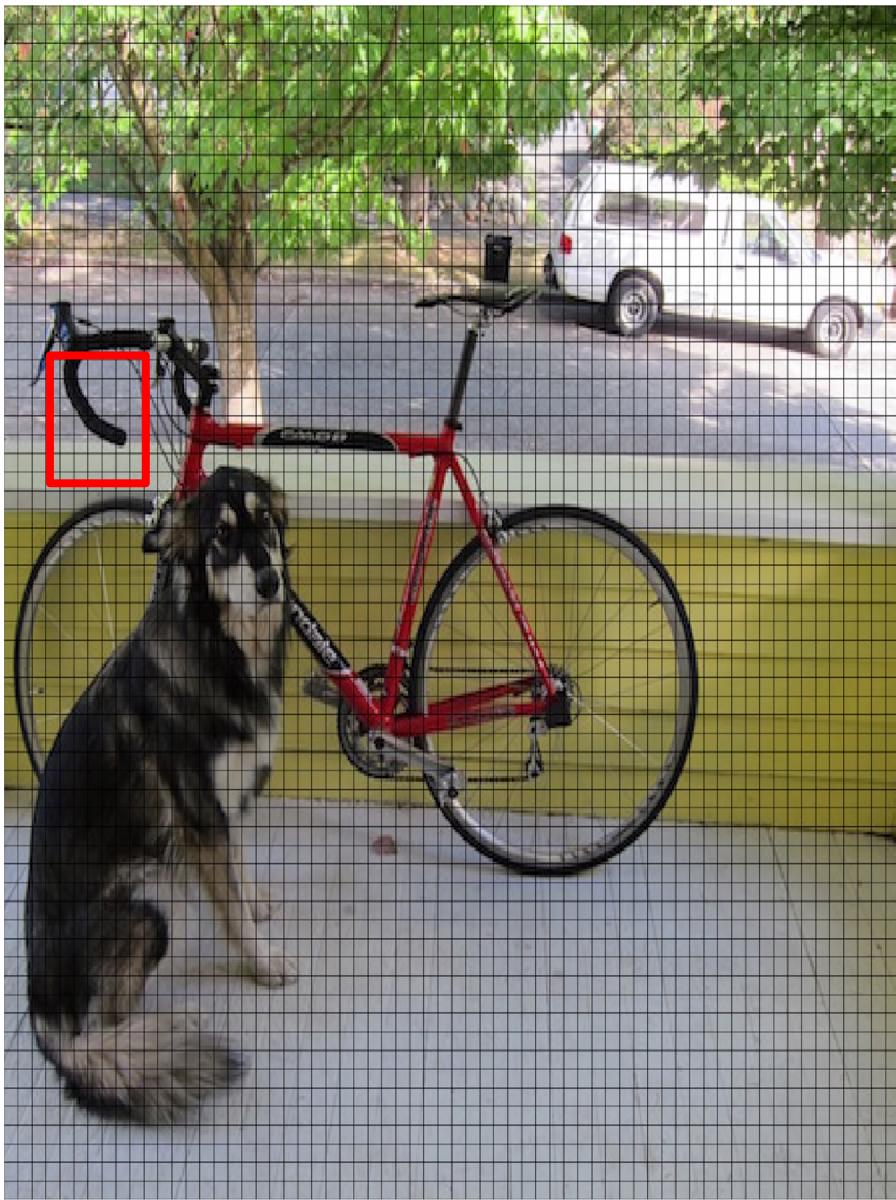
# From $448 \times 448$ to $64 \times 64$



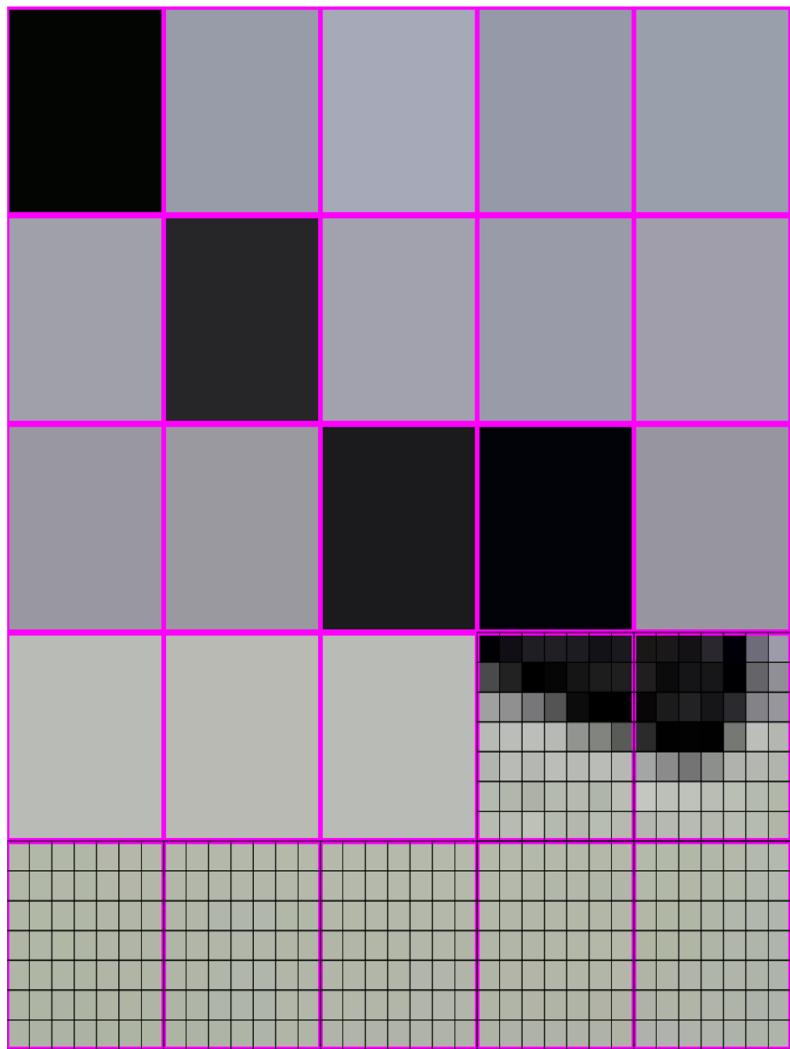
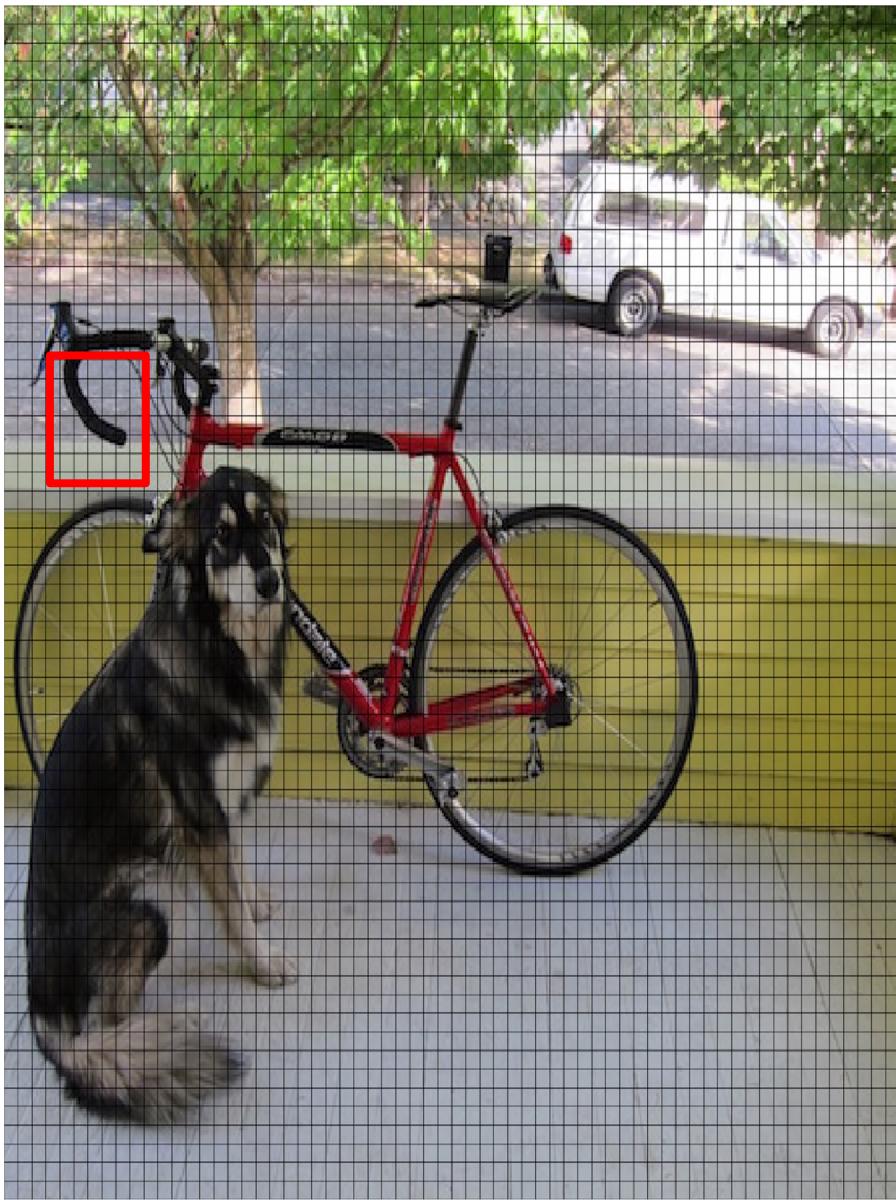
# From $448 \times 448$ to $64 \times 64$



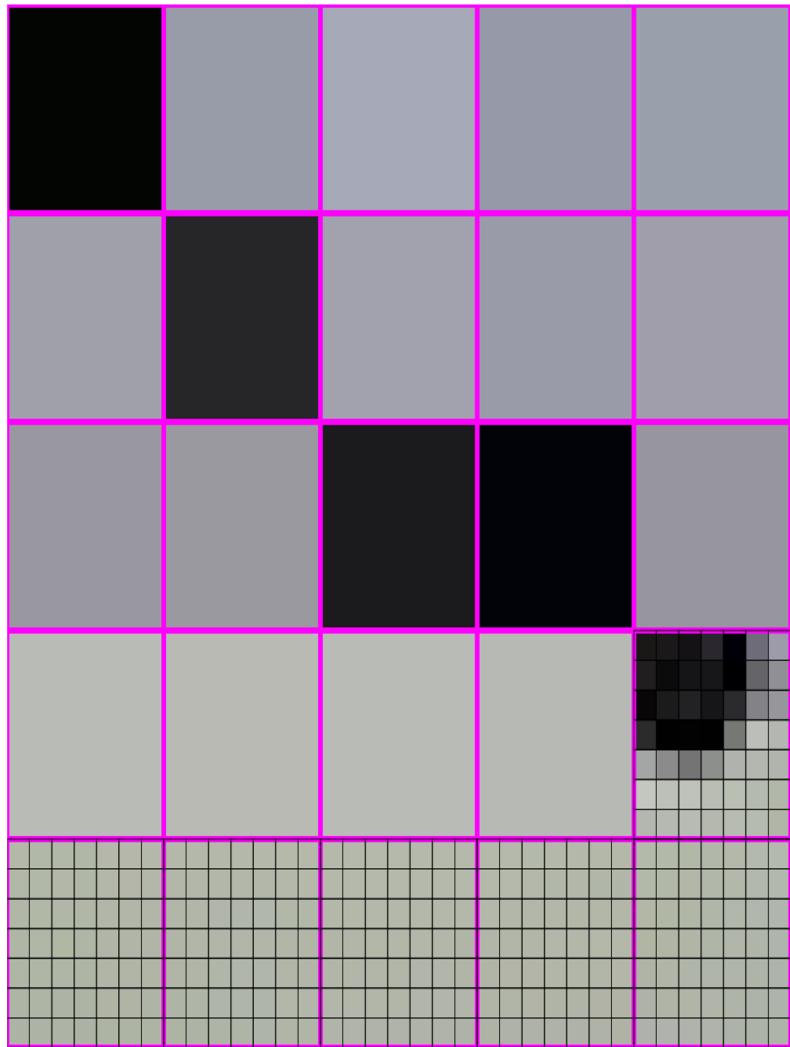
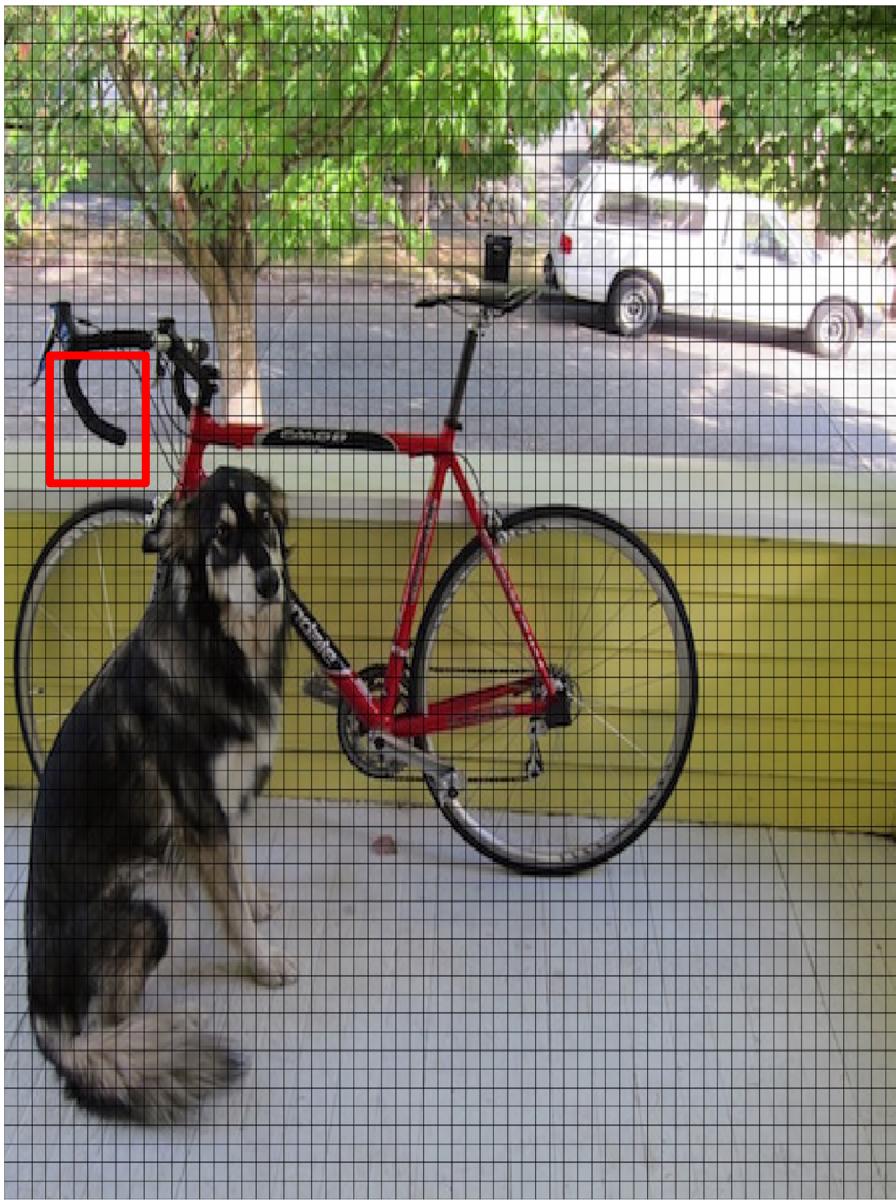
# From $448 \times 448$ to $64 \times 64$



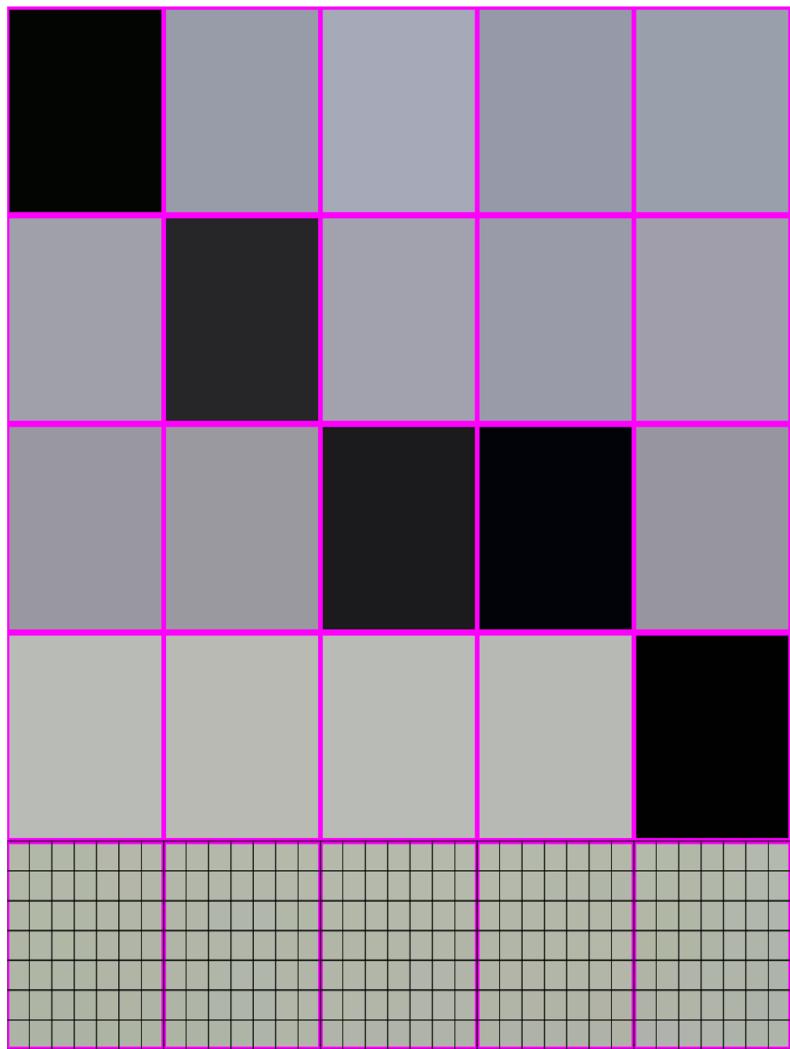
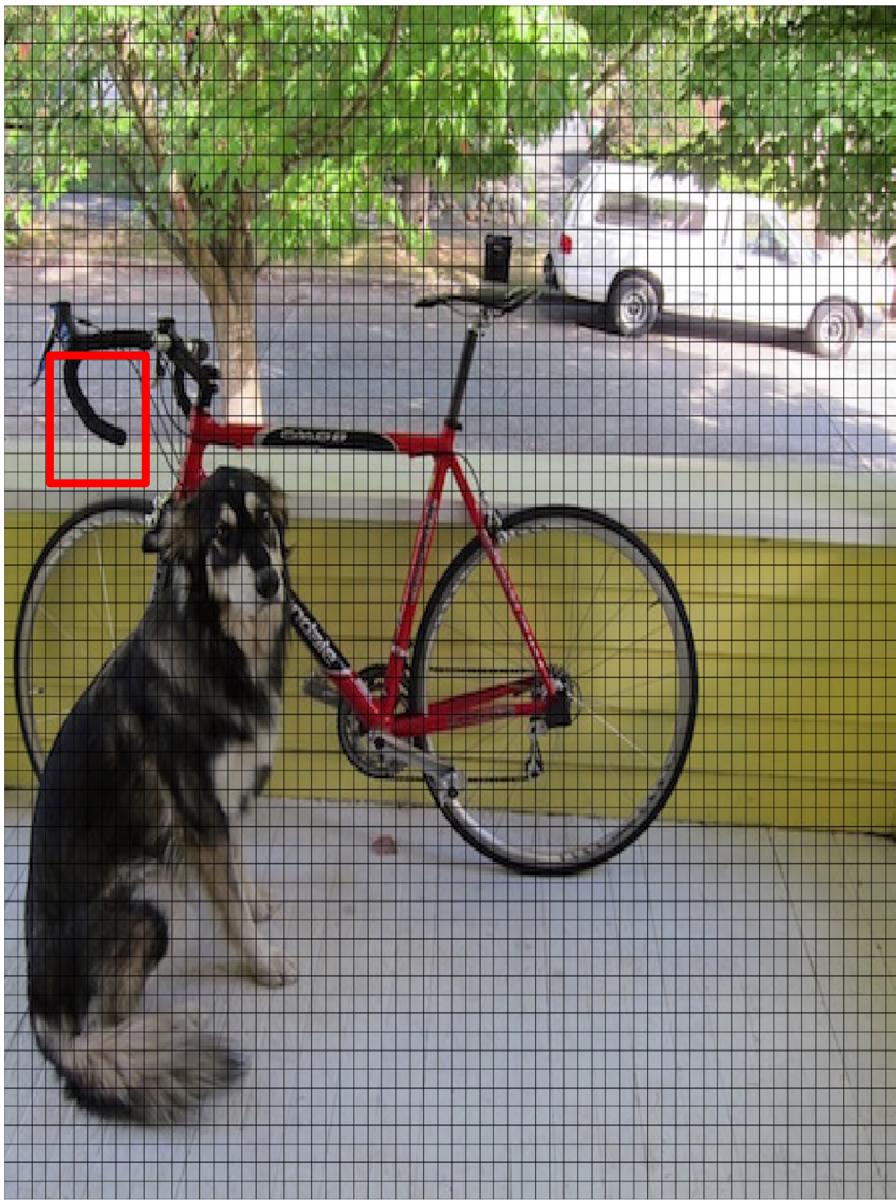
# From $448 \times 448$ to $64 \times 64$



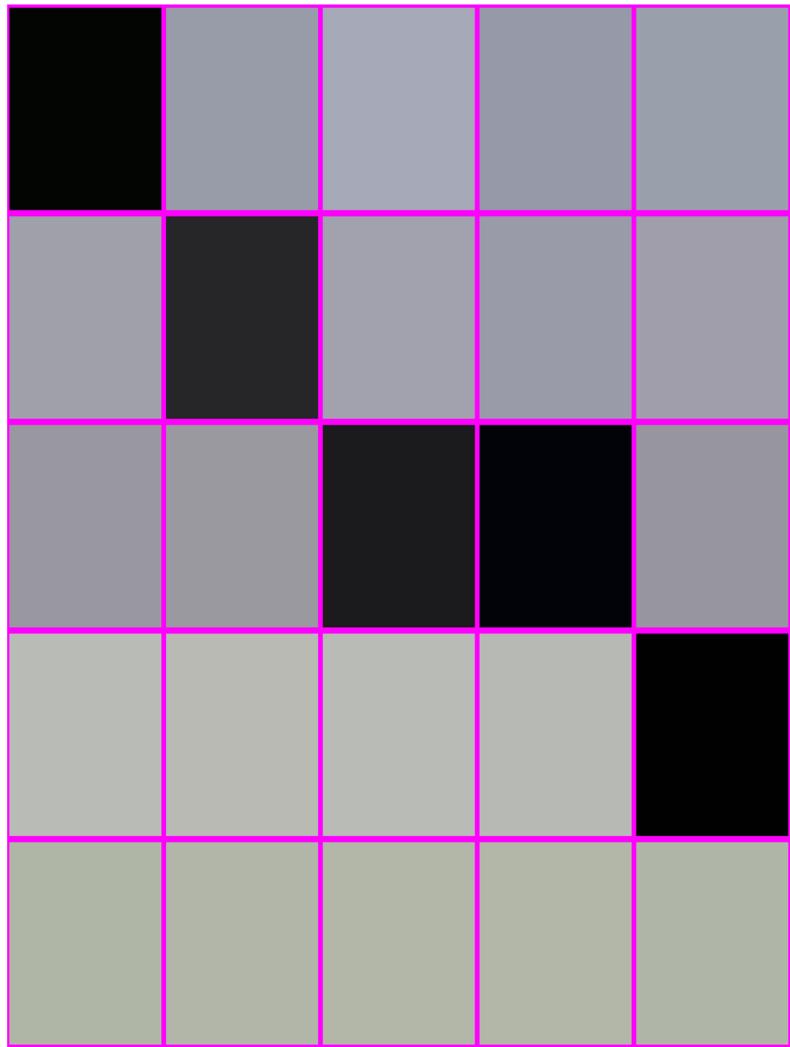
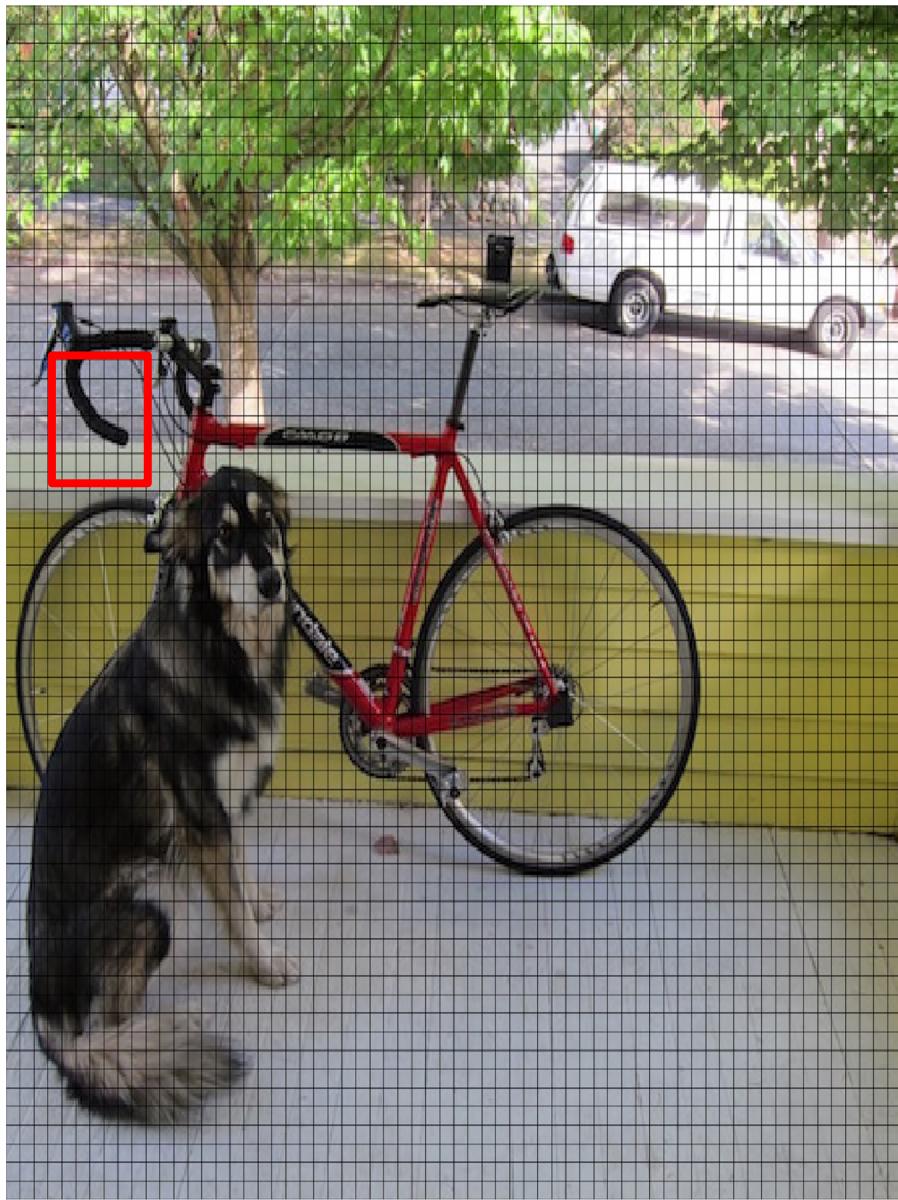
# From $448 \times 448$ to $64 \times 64$



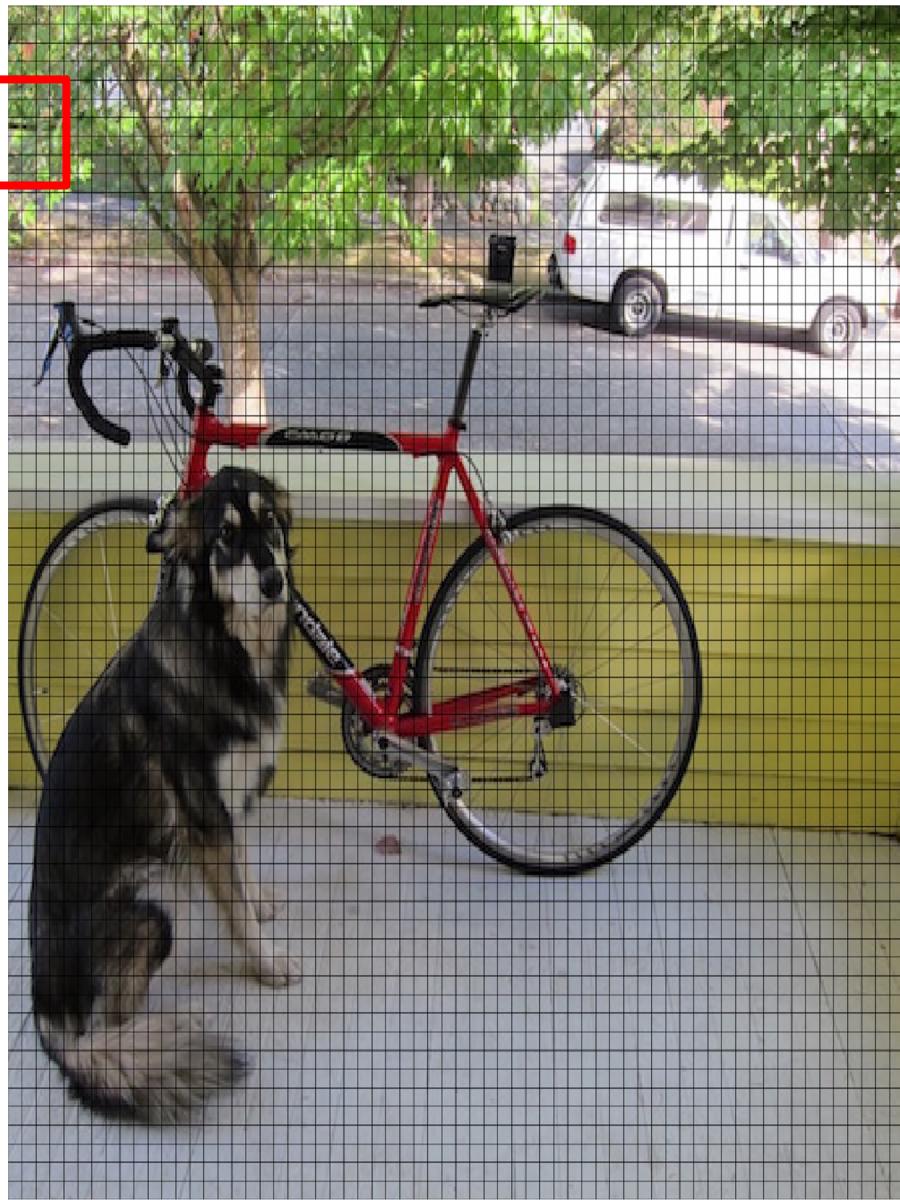
# From $448 \times 448$ to $64 \times 64$



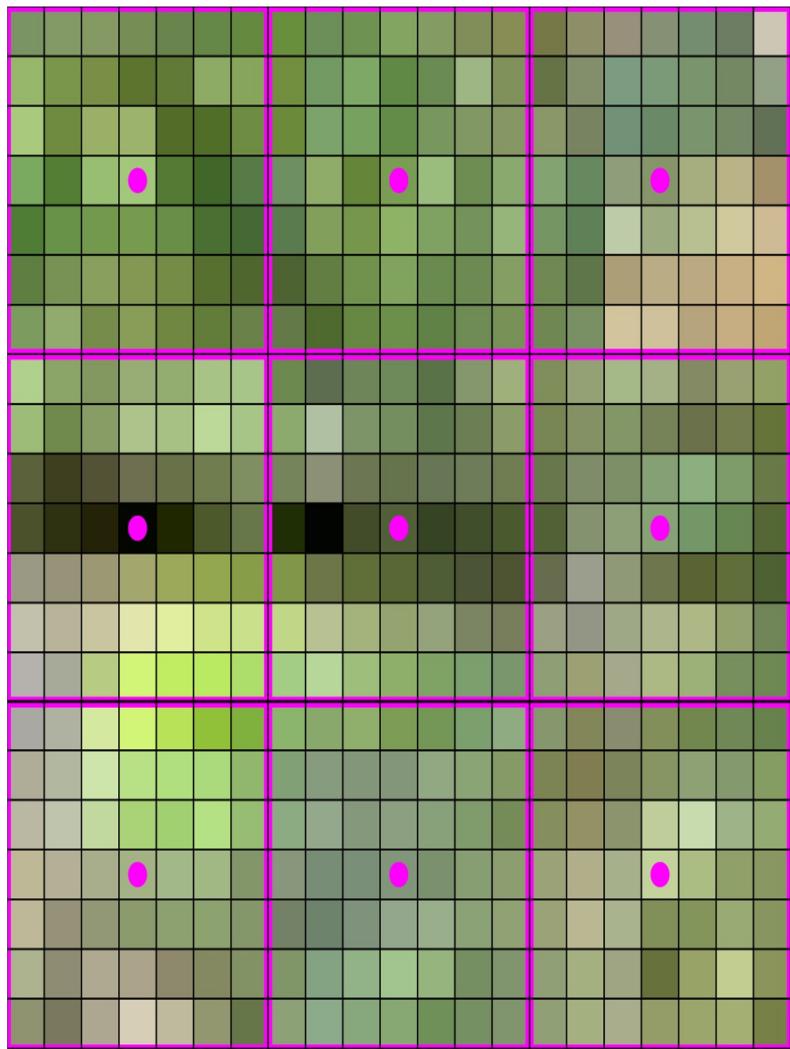
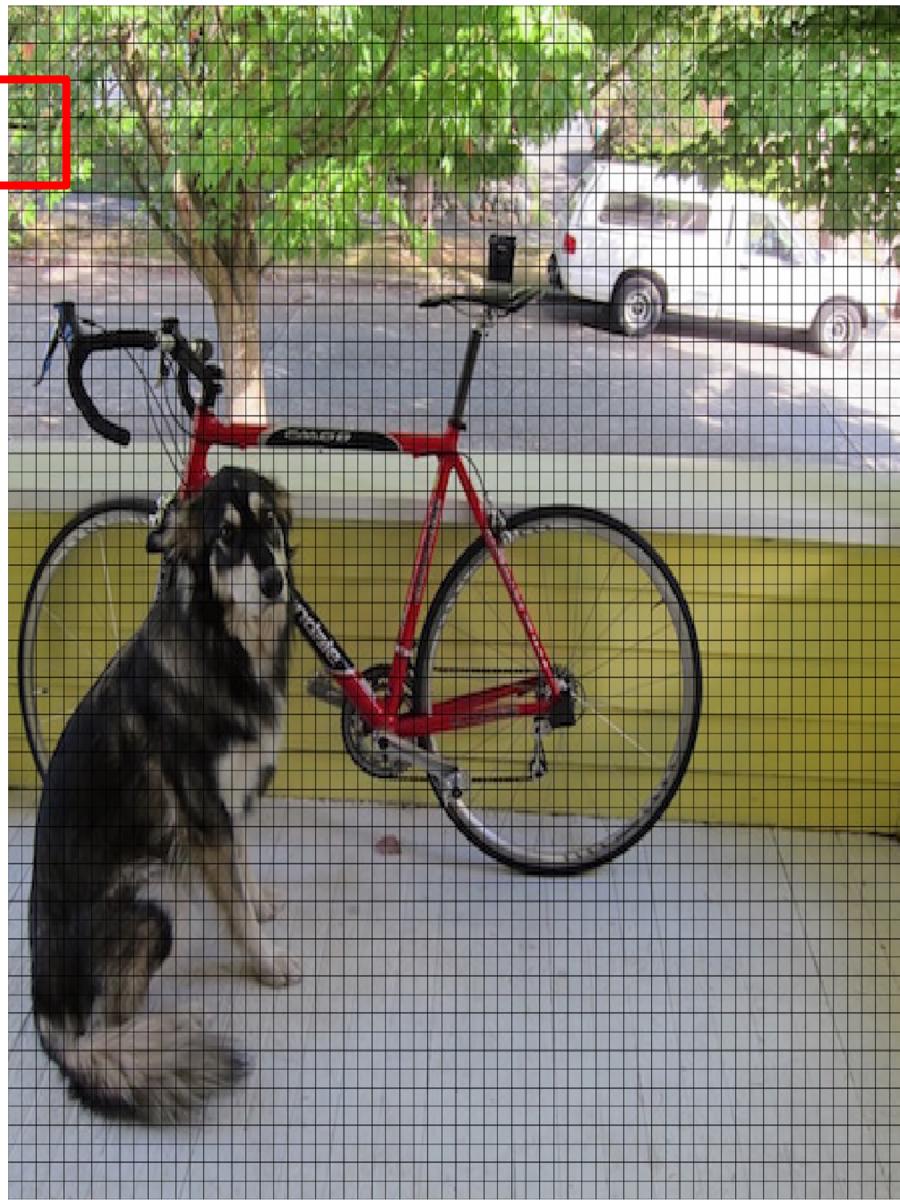
# From $448 \times 448$ to $64 \times 64$



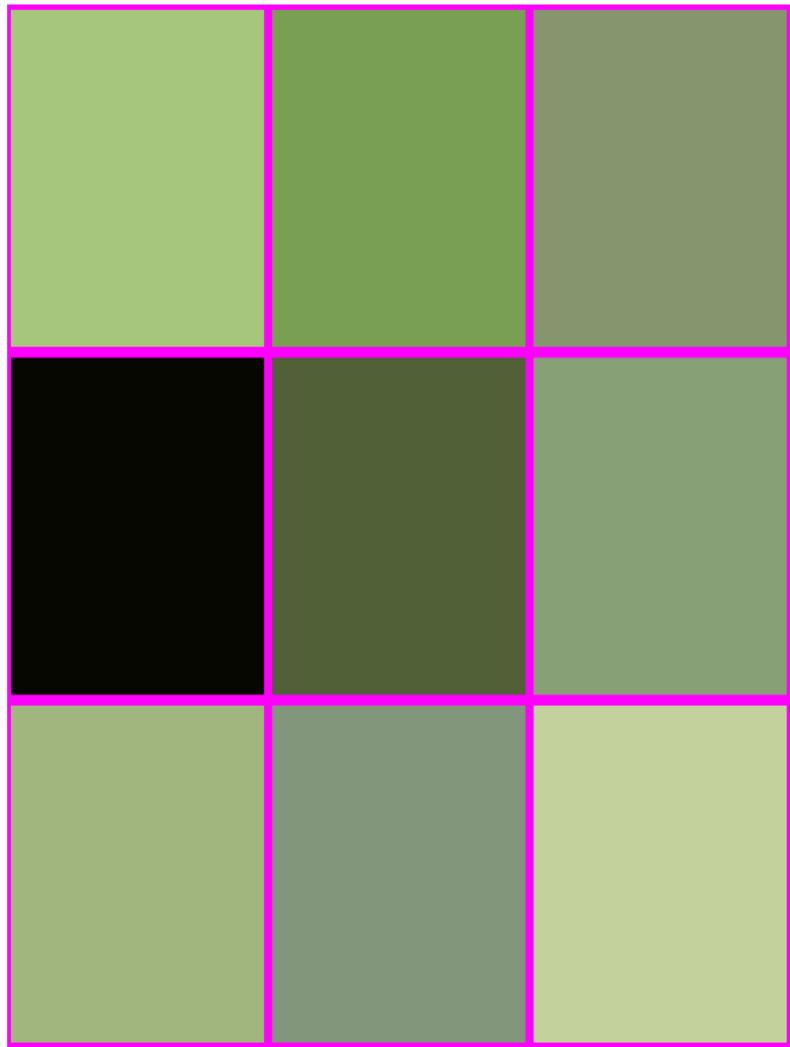
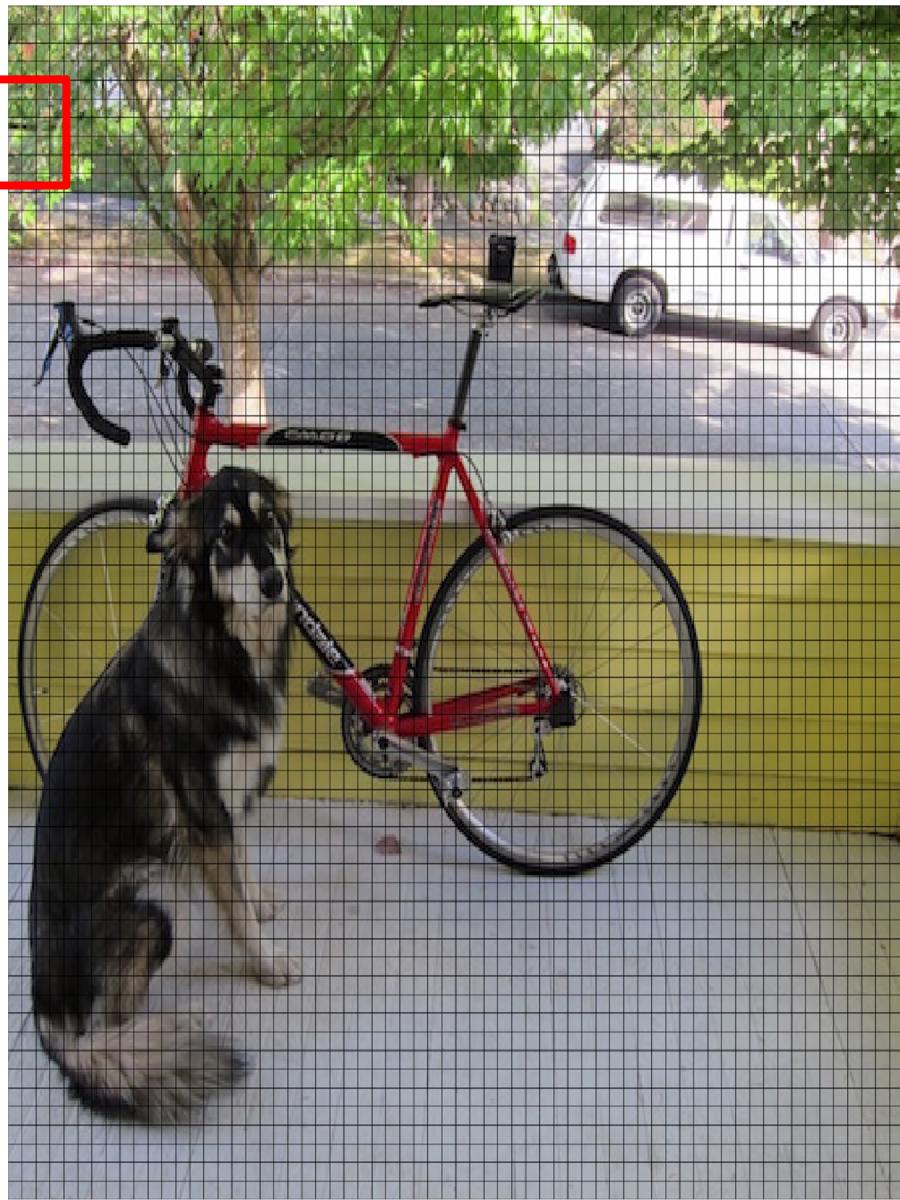
# From $448 \times 448$ to $64 \times 64$



# From $448 \times 448$ to $64 \times 64$



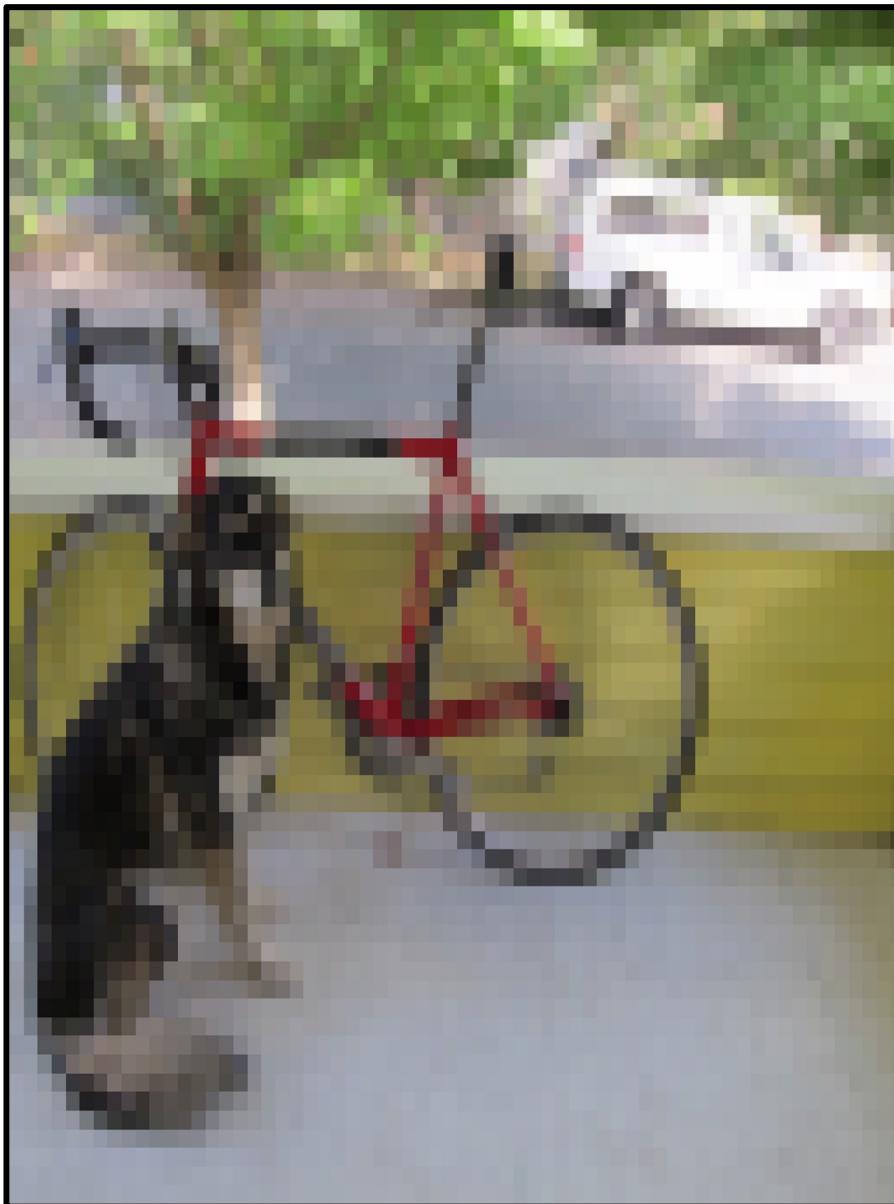
# From $448 \times 448$ to $64 \times 64$



# IS THIS ALL THERE IS??



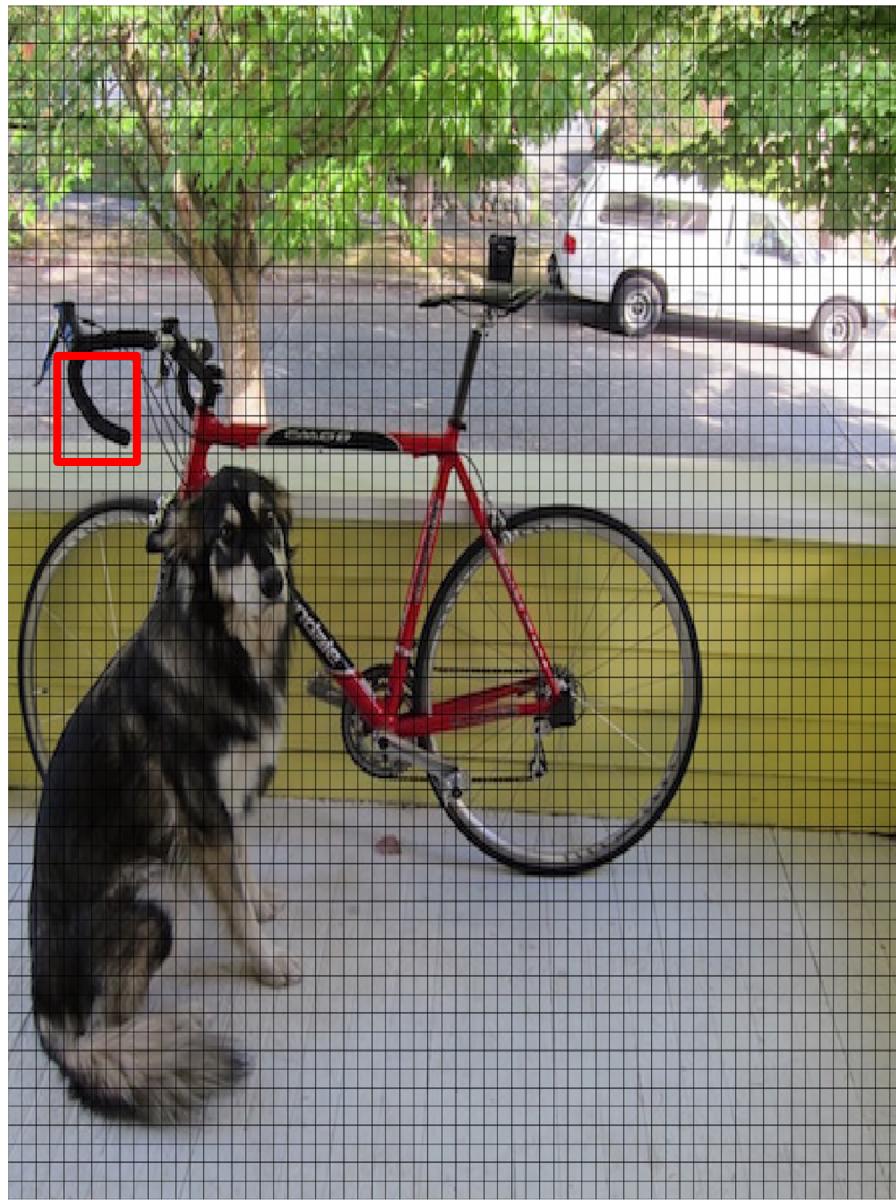
# THERE IS A BETTER WAY!



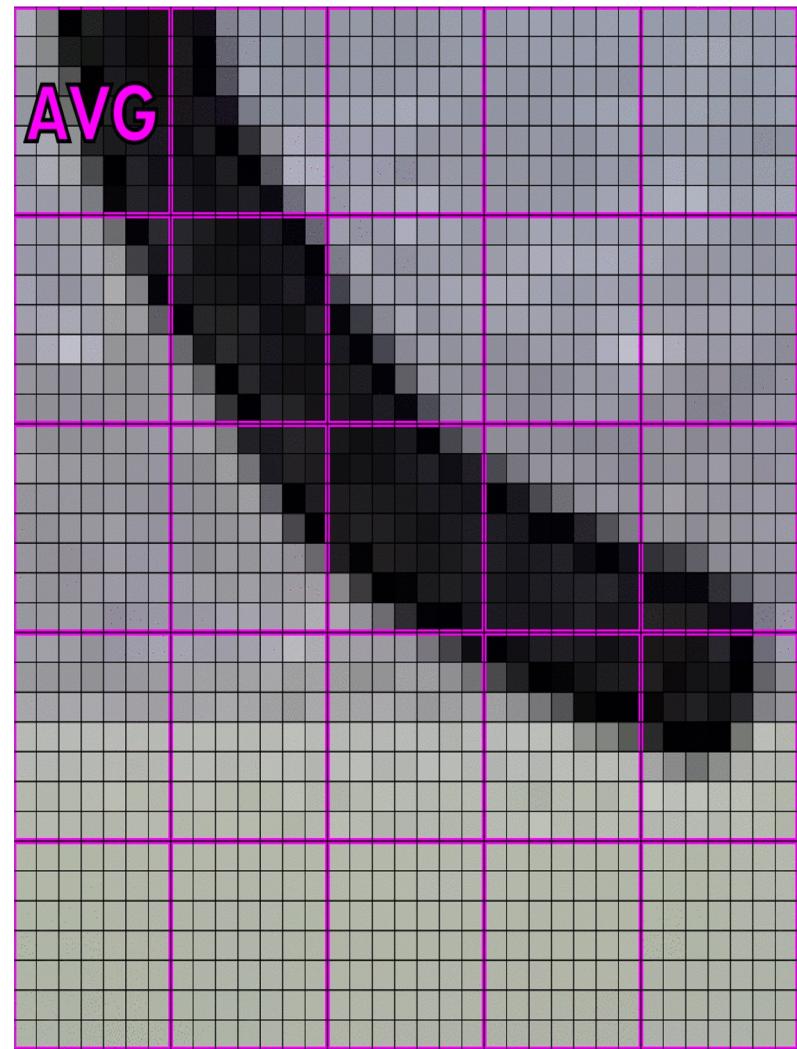
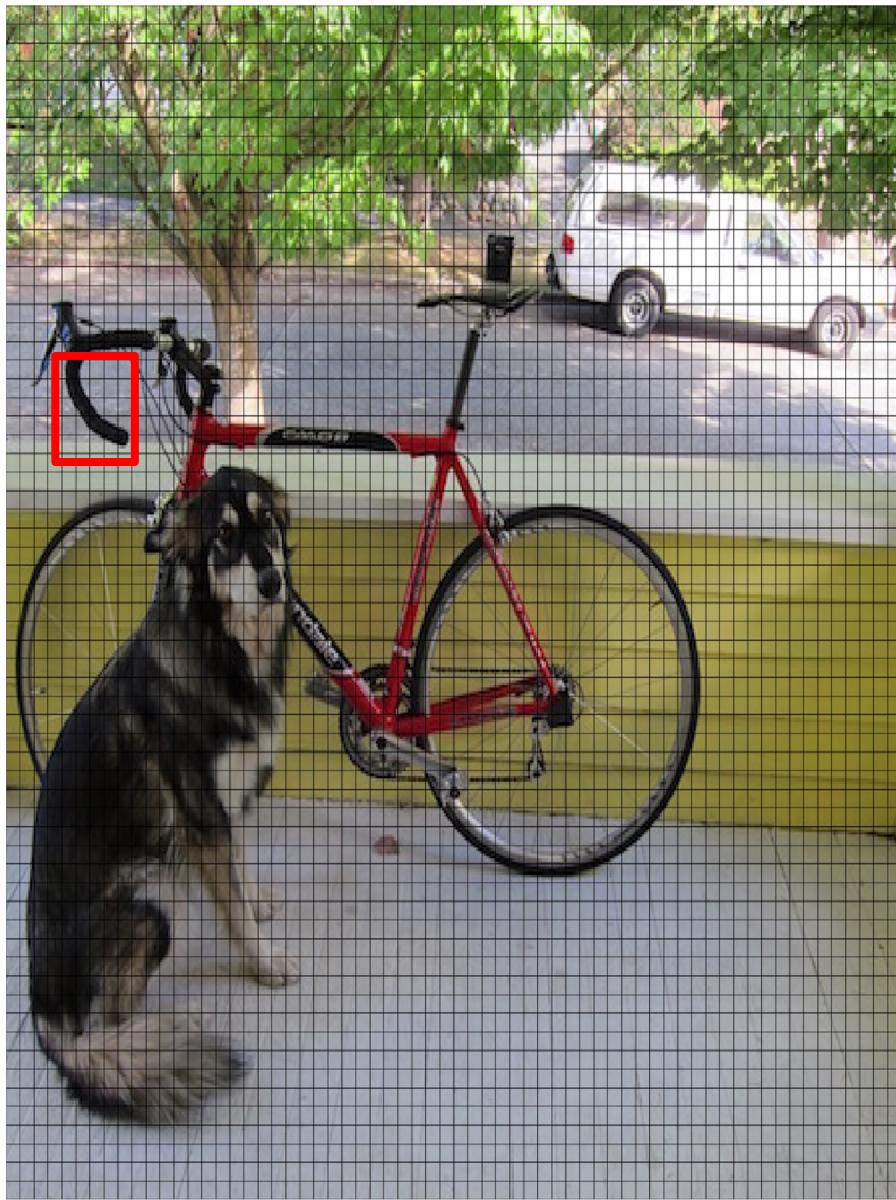
# OBTAİN MUCH BETTER RESULTS!



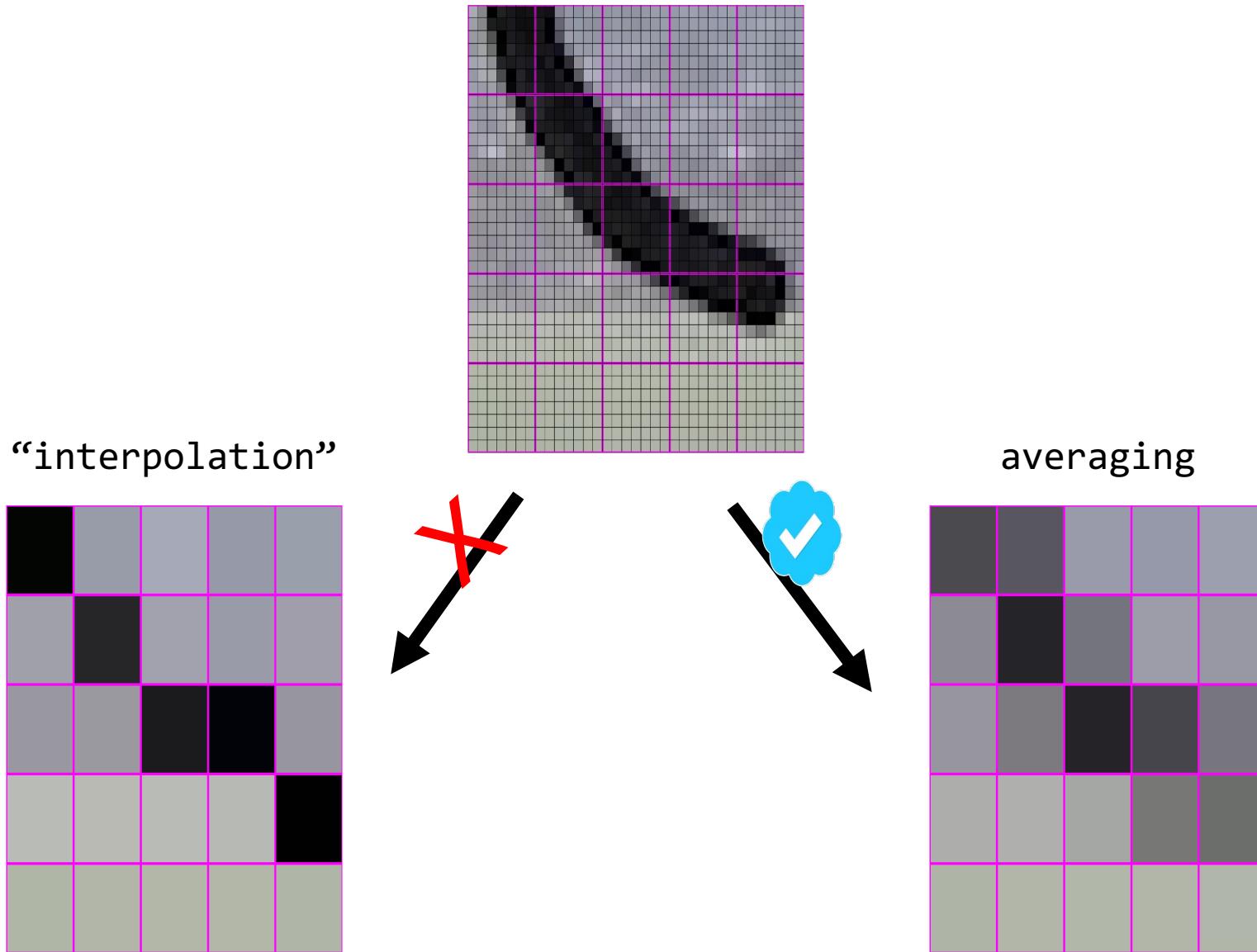
# How to do it?



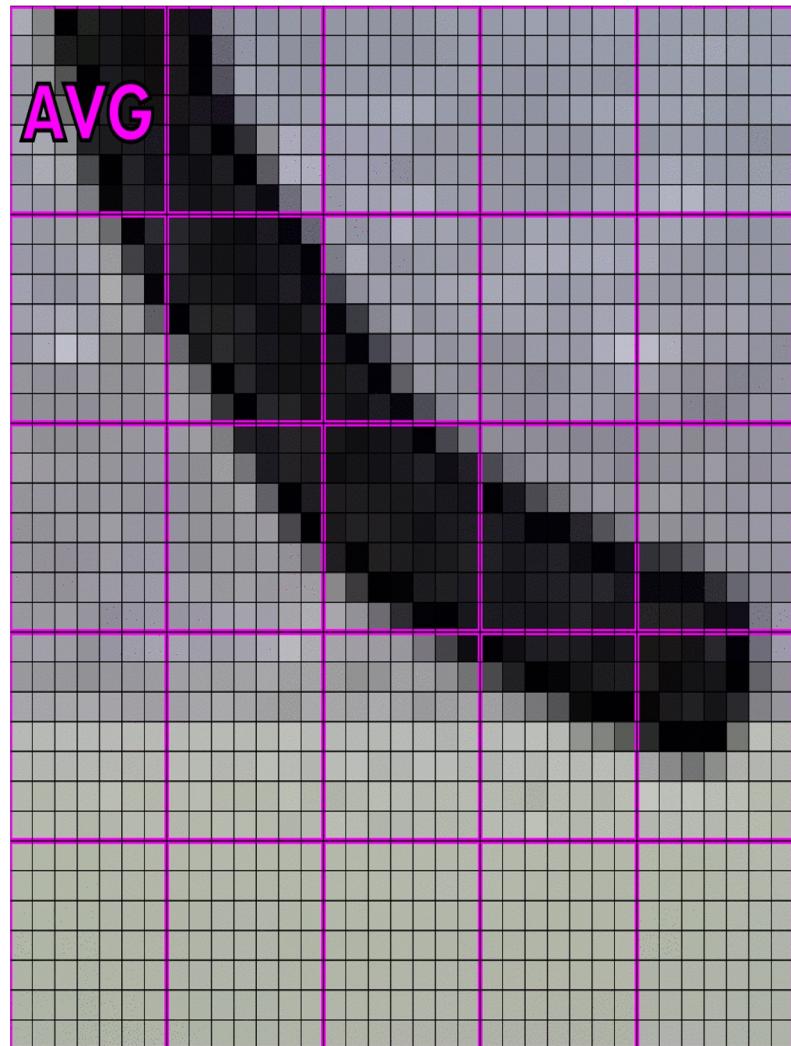
# How to do it? Averaging!



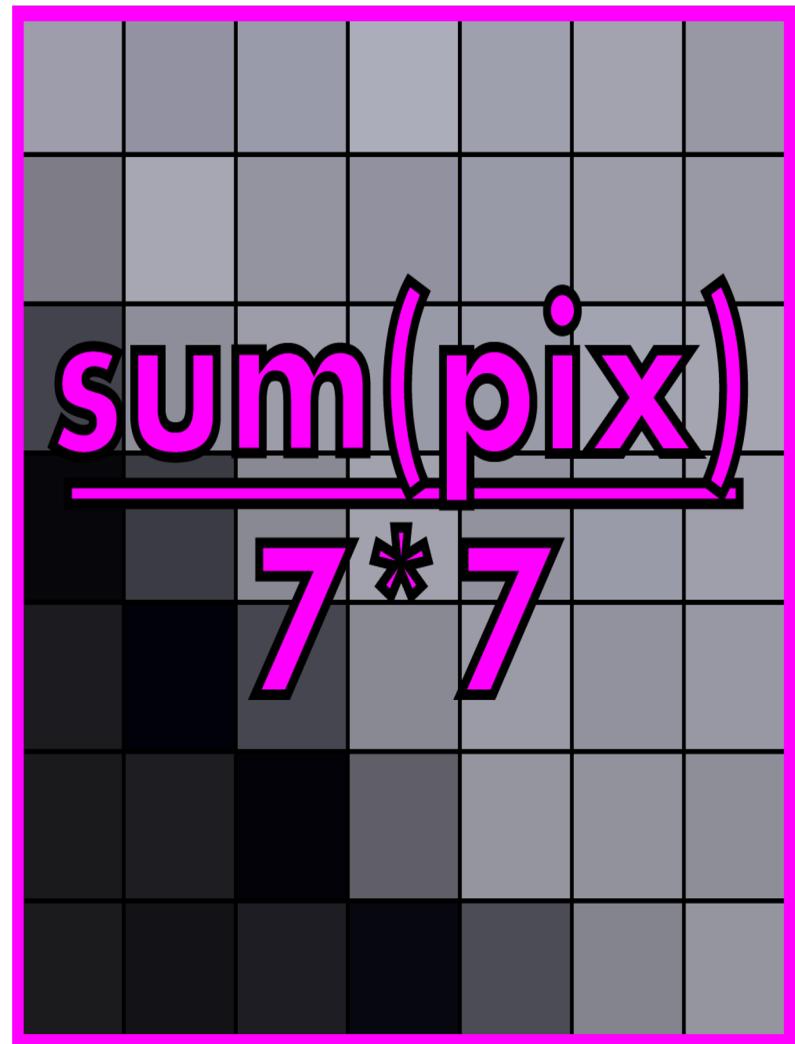
# How to do it ? Averaging!



# What is averaging?

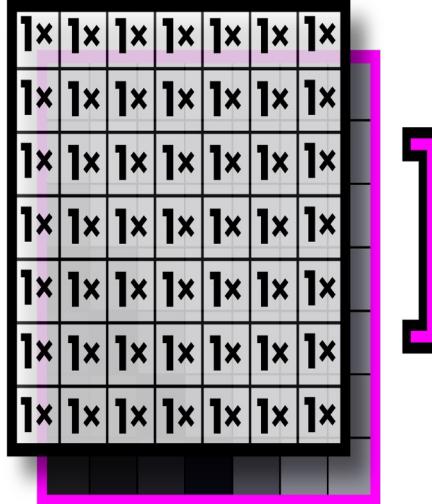


# What is averaging? A weighted sum



# What is averaging? A weighted sum

sum [  $\frac{1}{49}$  ]



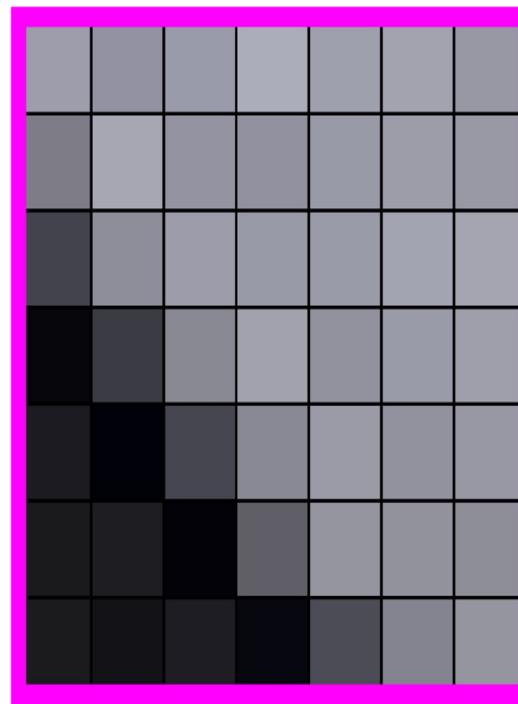
A 7x7 grid of 49 cells, each containing the text "1x". The entire grid is enclosed in a thick black border, which is itself enclosed in a thick magenta border.

# Call this operation “*convolution*”

*Filter or kernel*

$$\frac{1}{49}$$

1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x



# Convolutions on larger images

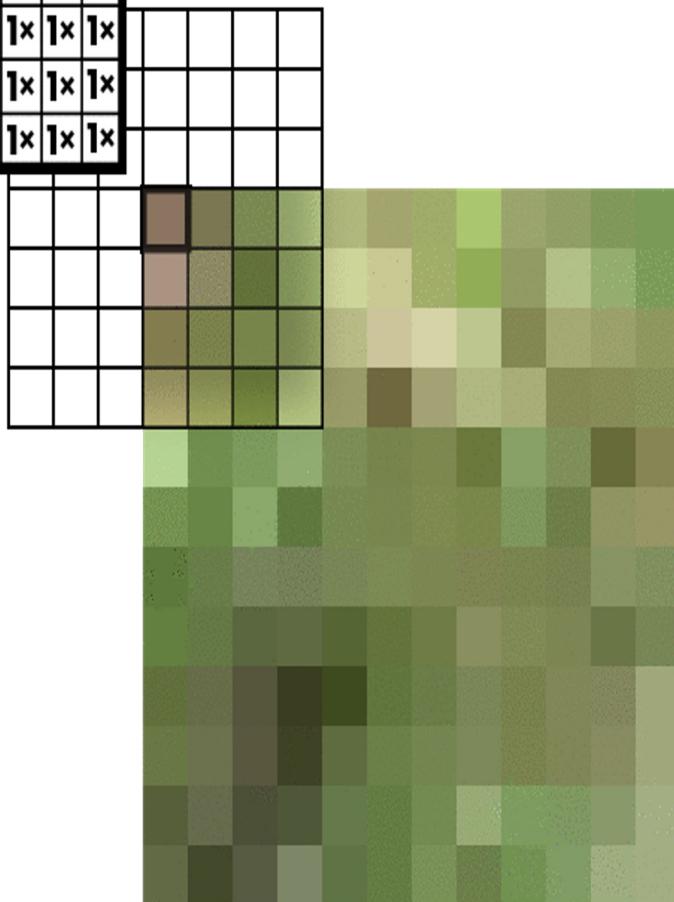
$\frac{1}{49}$

$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$
$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$
$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$
$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$
$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$
$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$
$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$
$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$	$1 \times$



# Kernel slides across image

1  
49

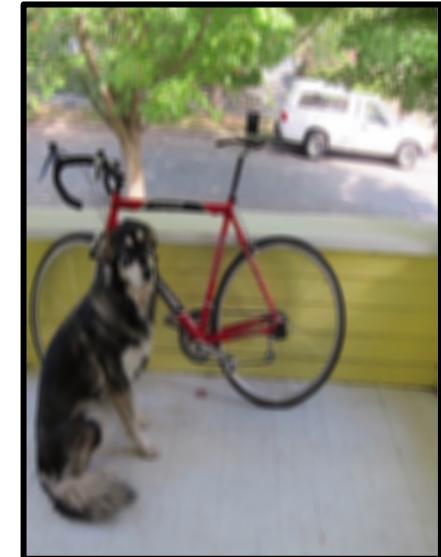
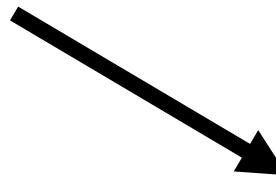


A 10x10 grid of squares. The top-left square is filled with black, while all other squares are white.

# Convolutions on larger images

$\frac{1}{49}$

1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x



# This is called box filter

$$\frac{1}{49}$$

1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x



Box filters

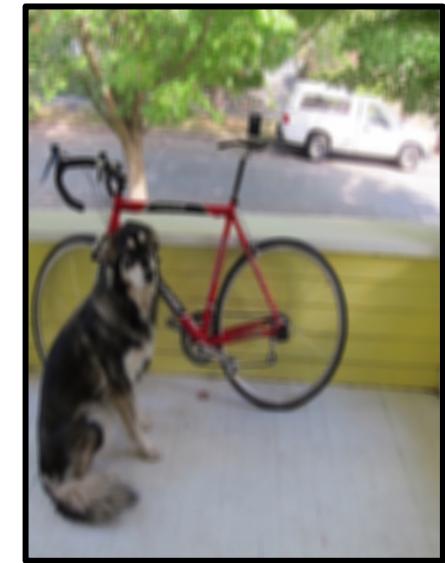
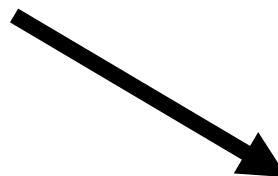
$$\frac{1}{N \times M}$$

1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
					...	
						1x
						1x
1x	1x	1x	1x	1x	1x	1x

N

M

...



# Box filters smooth image

$$\frac{1}{49}$$

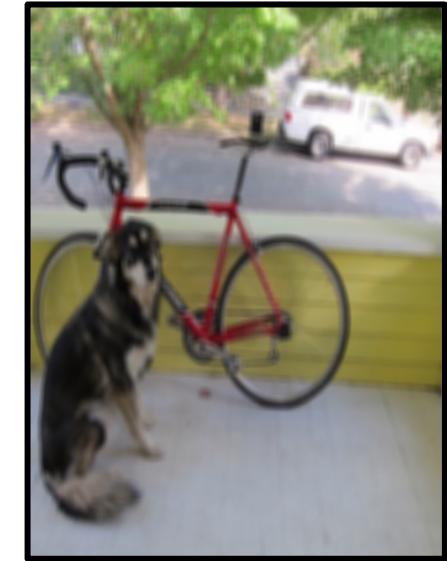
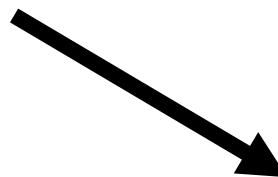
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x



Box filters

$$\frac{1}{N \cdot M}$$

1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
			...			
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x



# Box filters smooth image

$$\frac{1}{49}$$

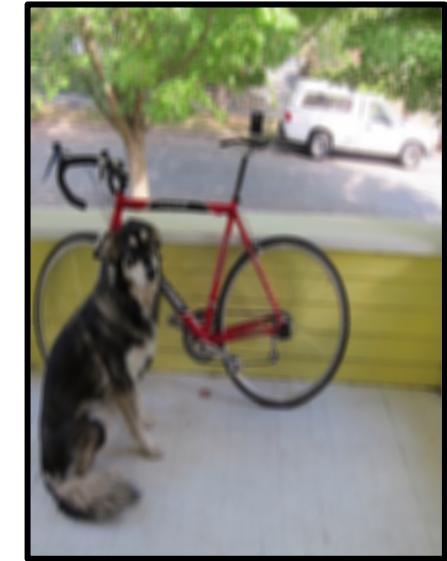
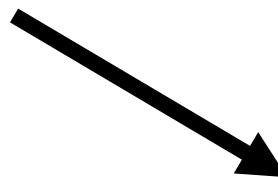
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x	1x



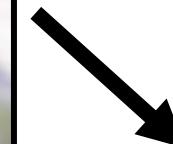
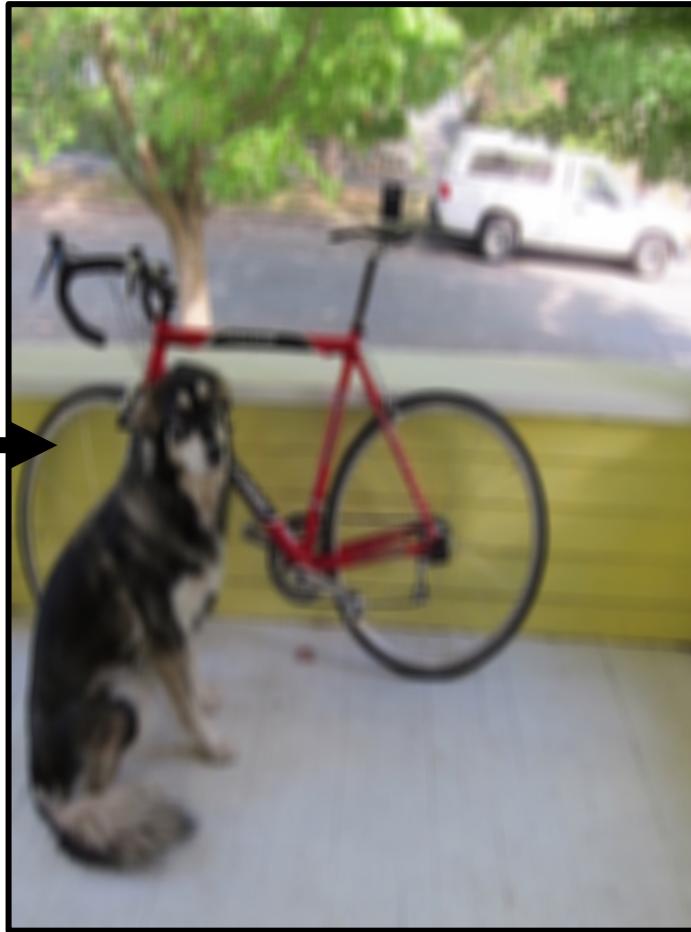
Box filters

$$\frac{1}{N \cdot M}$$

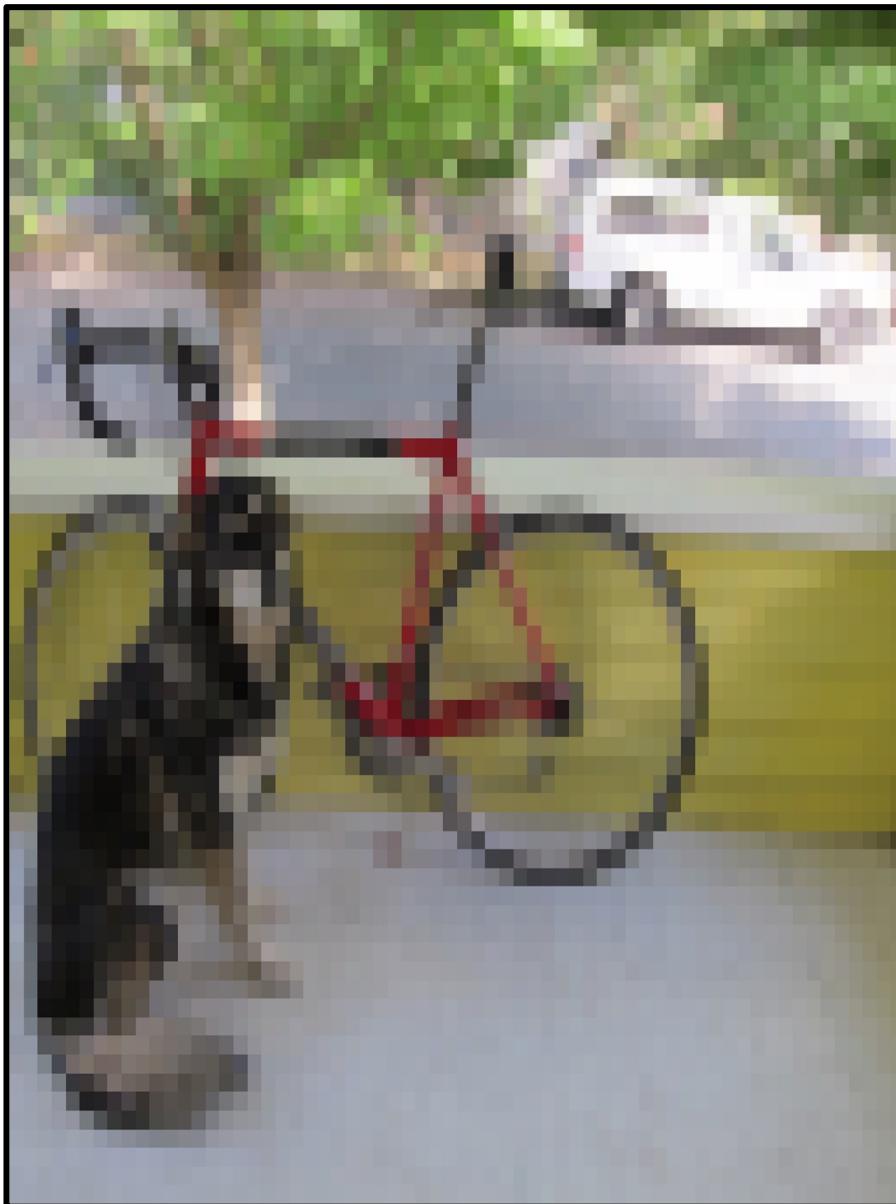
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
			...			
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x
1x	1x	1x	1x	1x		1x



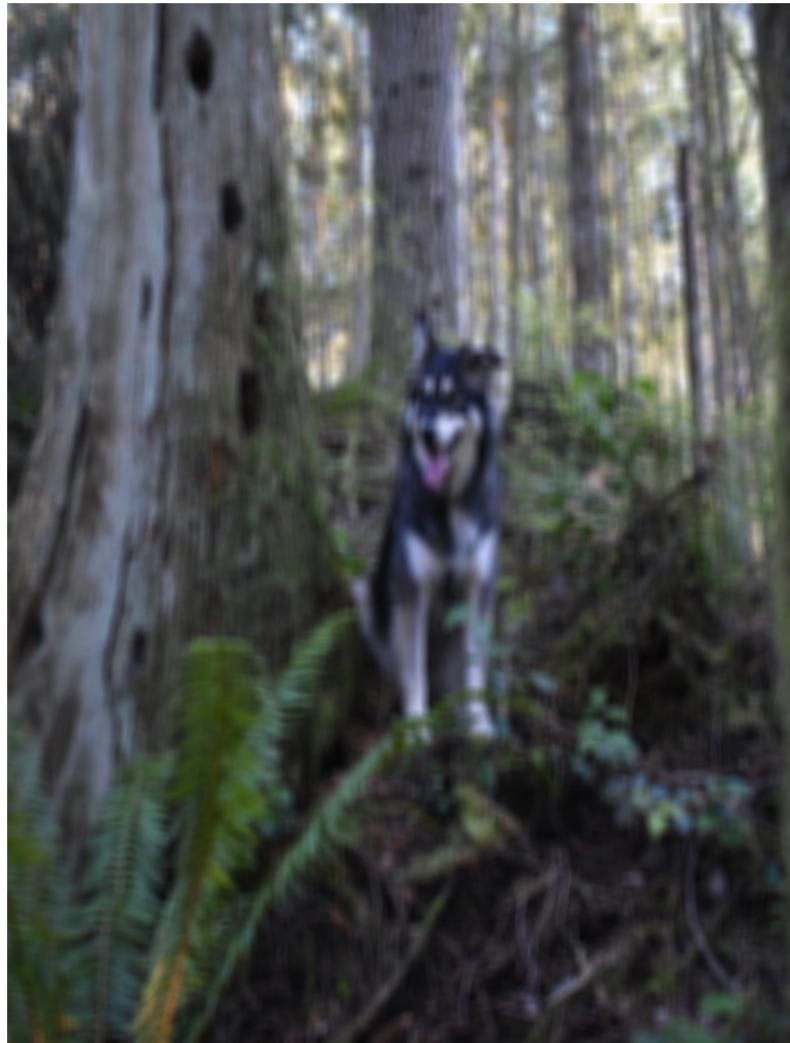
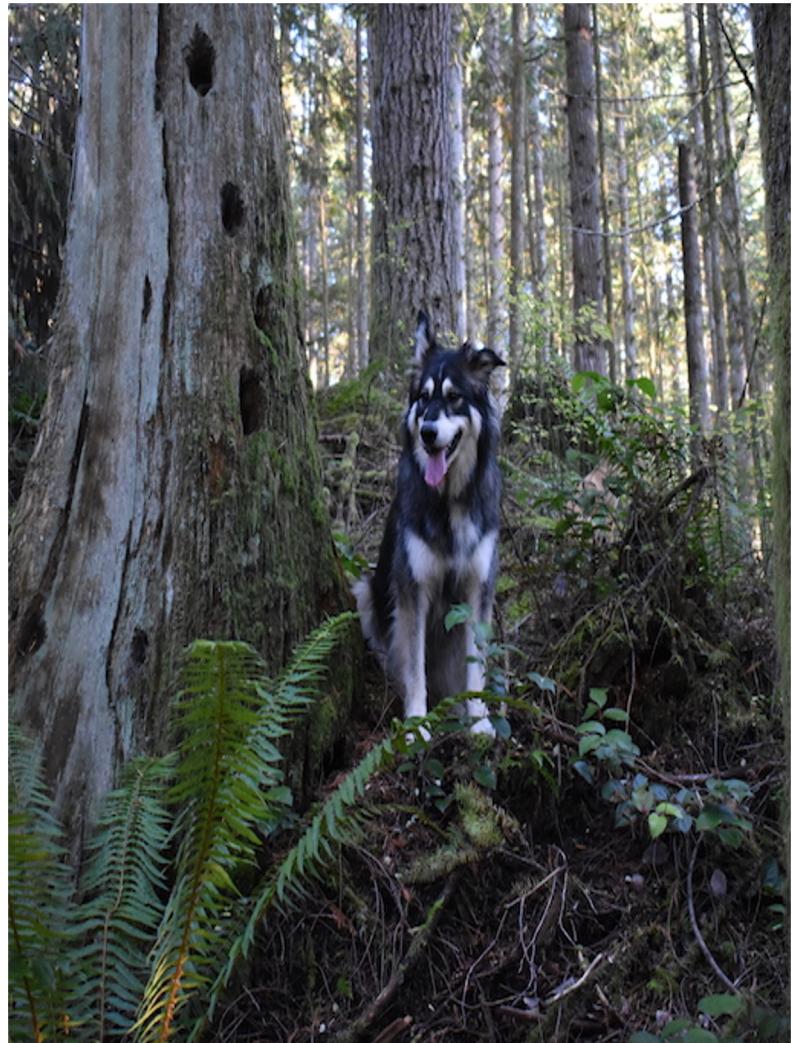
# Now we resize our smoothed image



# So much better!

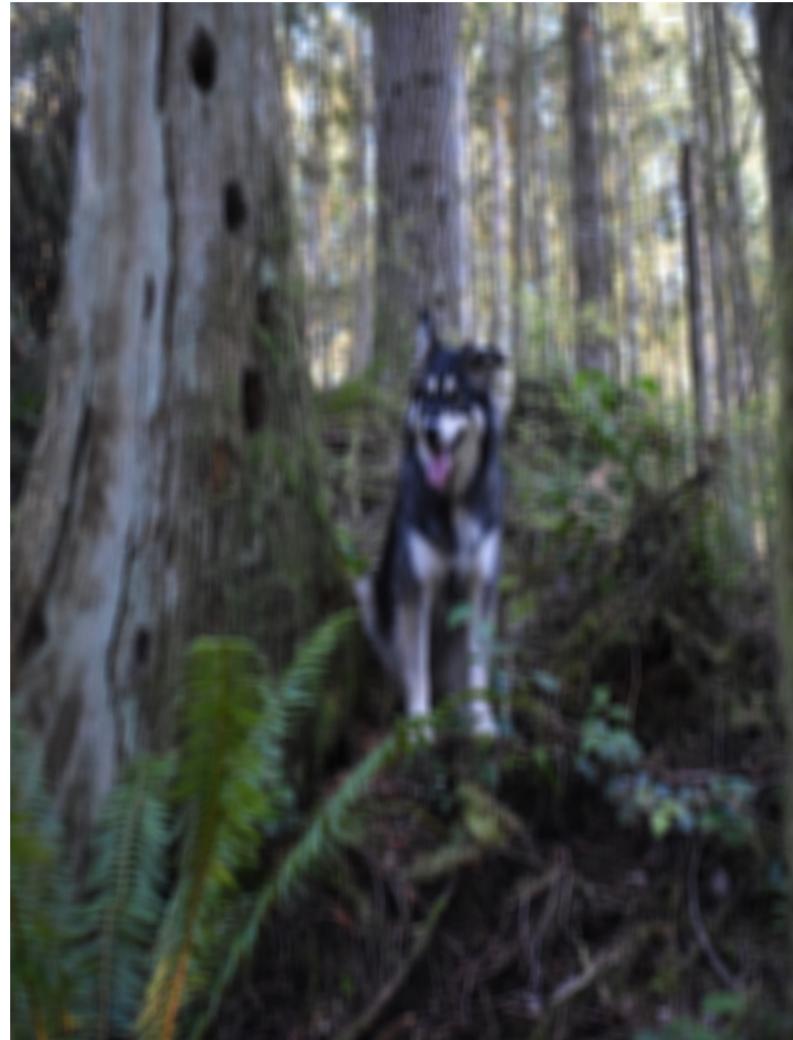
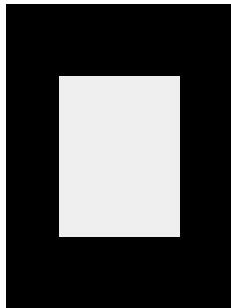
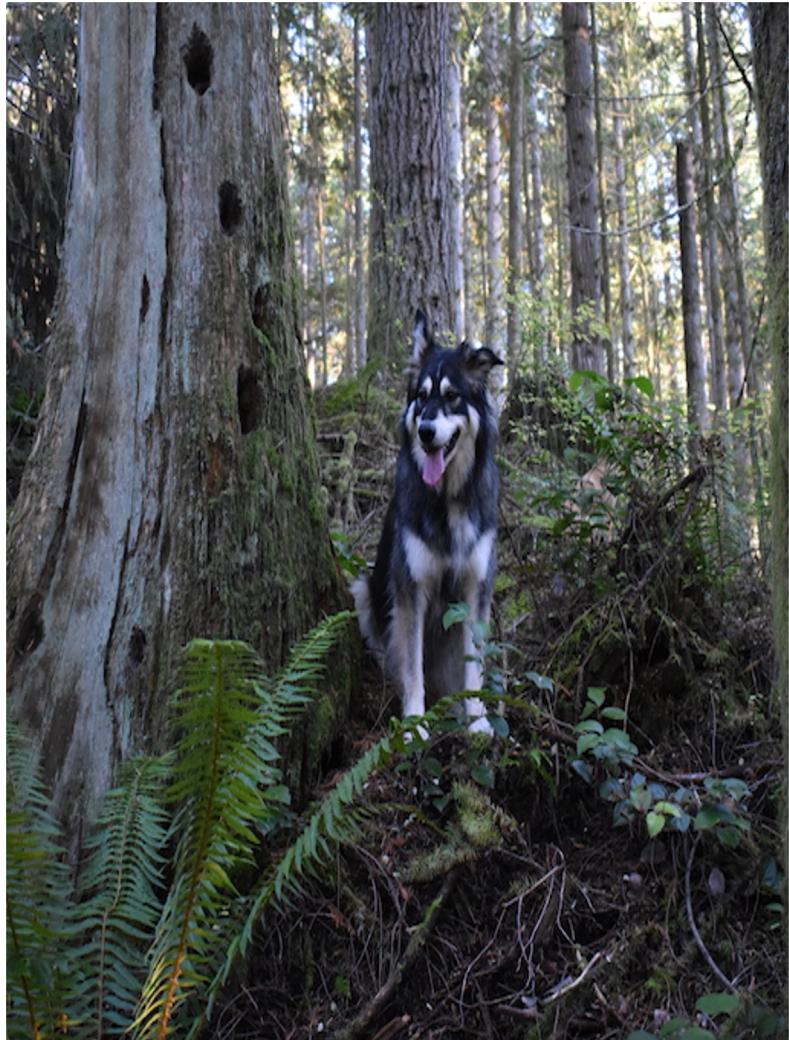


# Box filters have artifacts

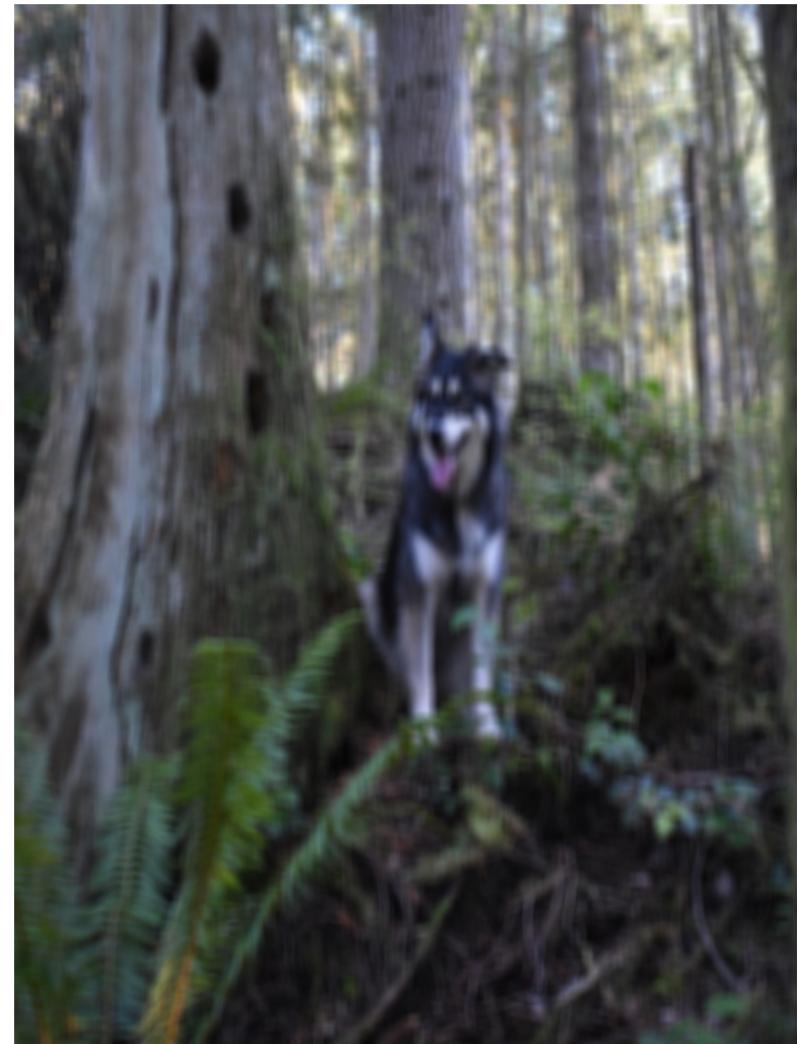
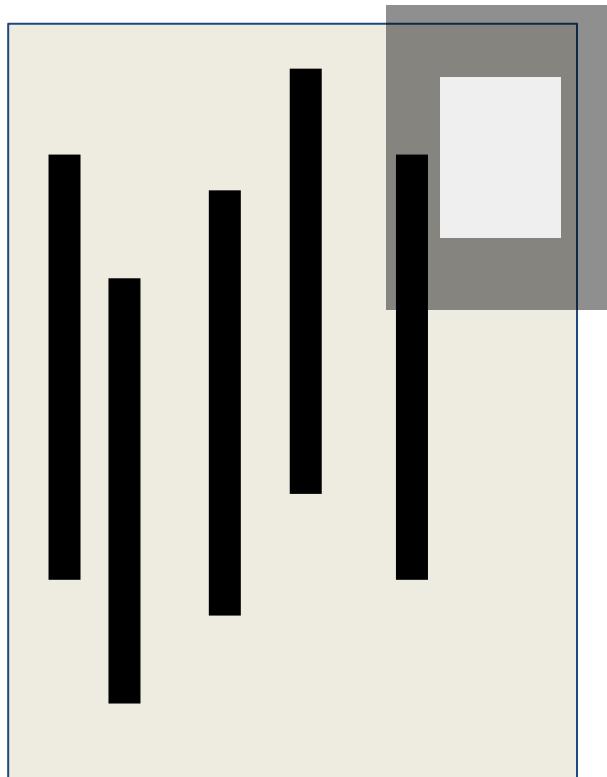




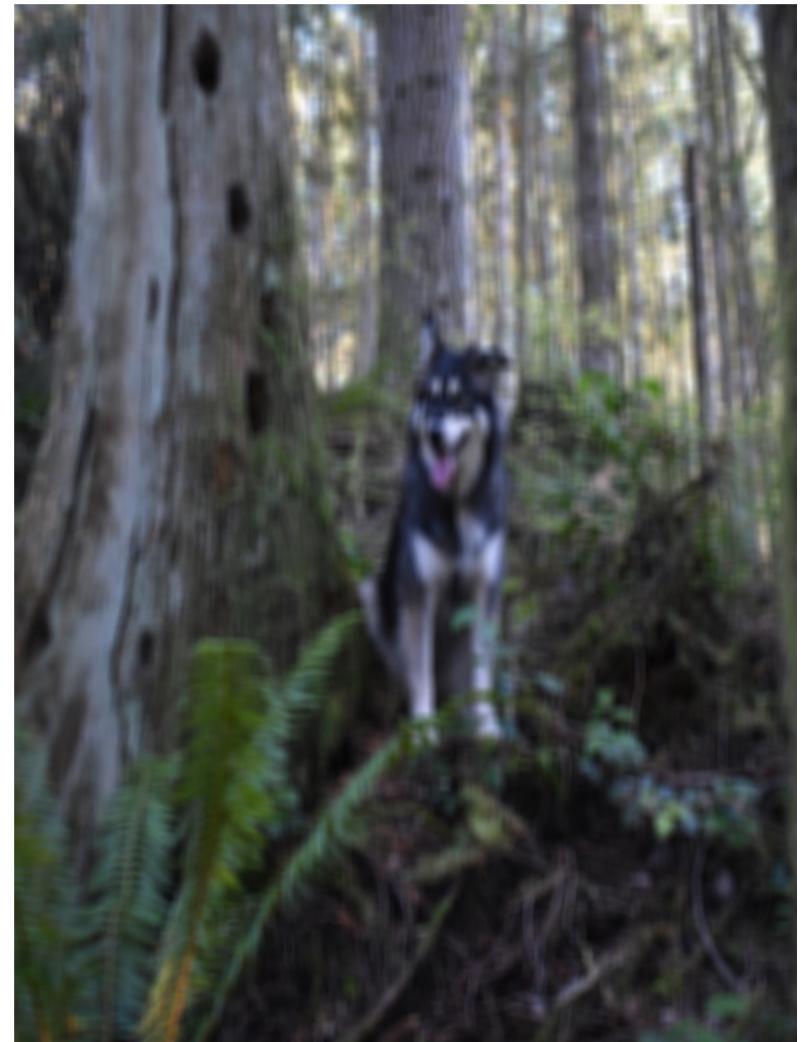
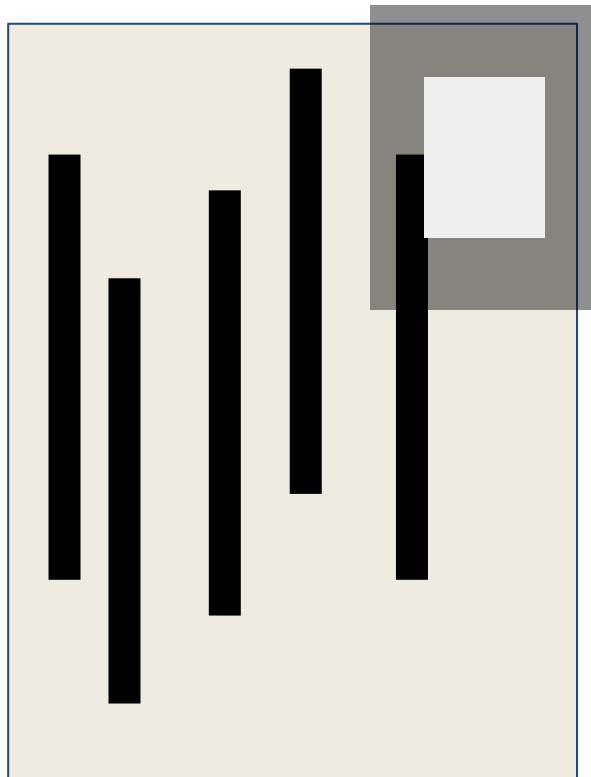
# Box filters: vertical + horizontal streaking



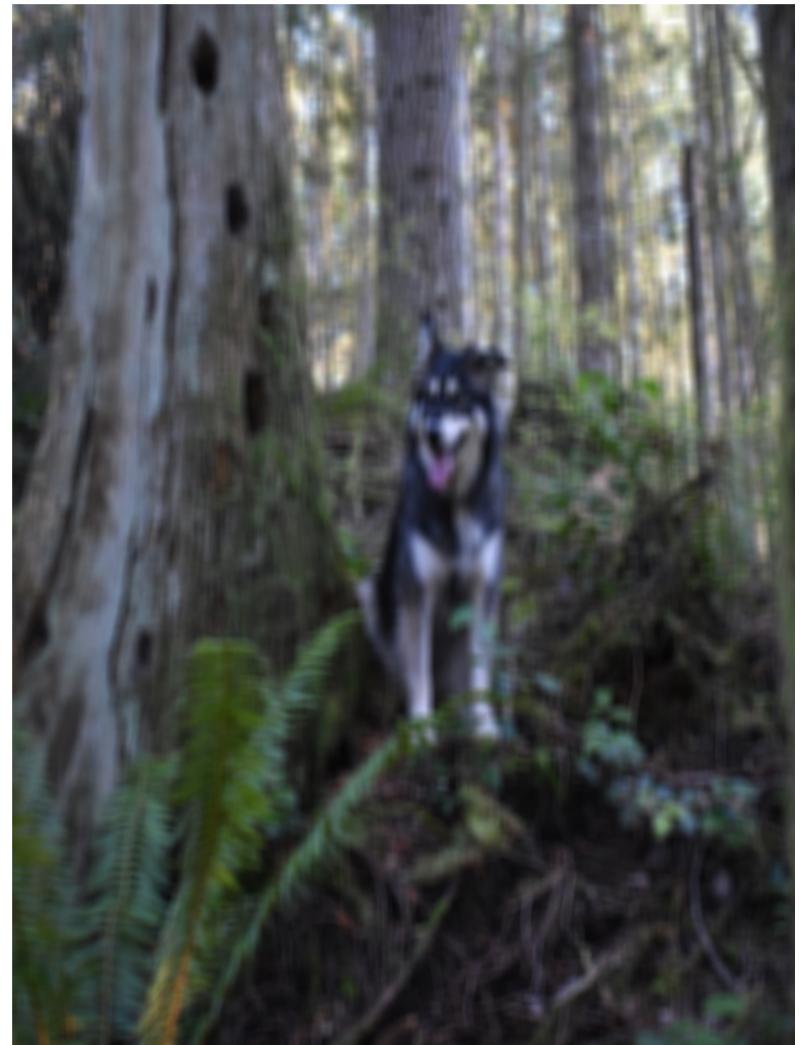
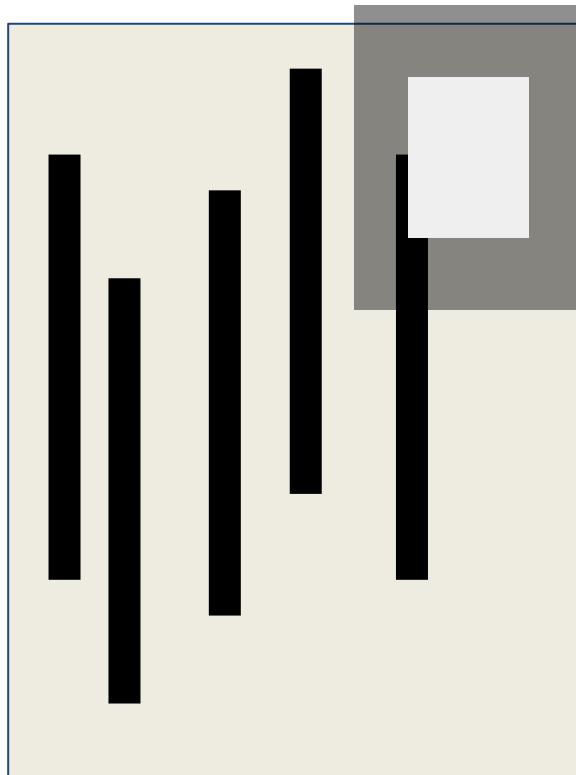
# Box filters: vertical + horizontal streaking



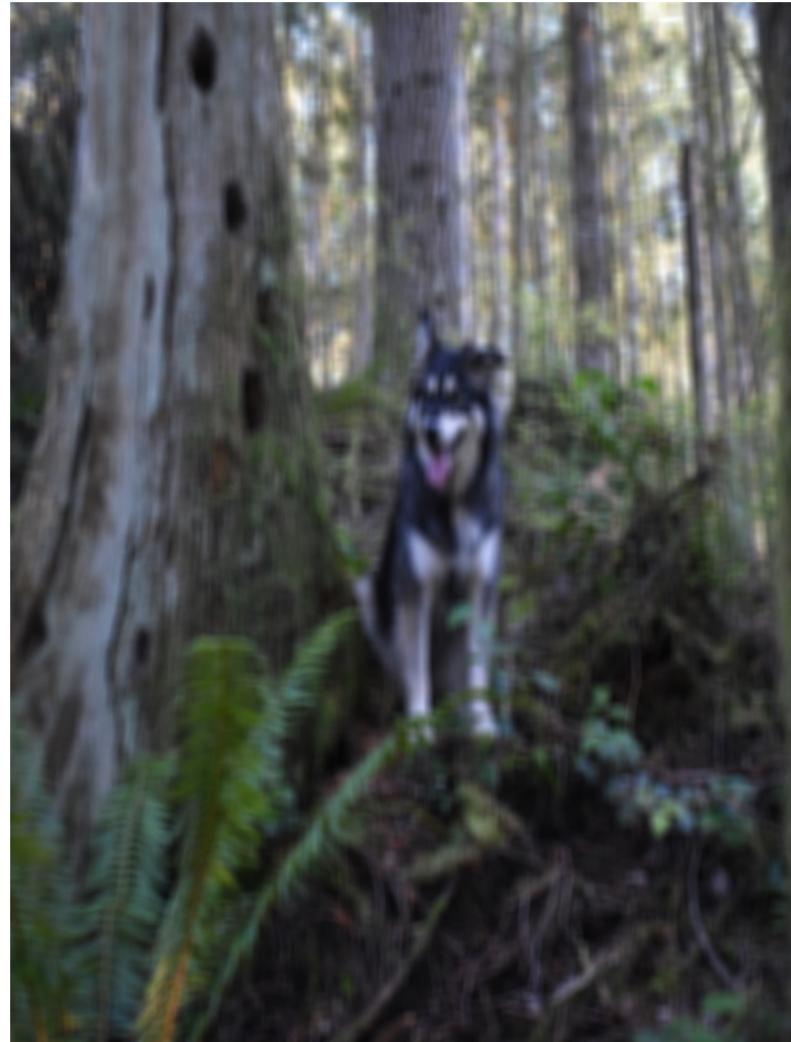
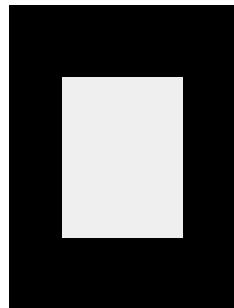
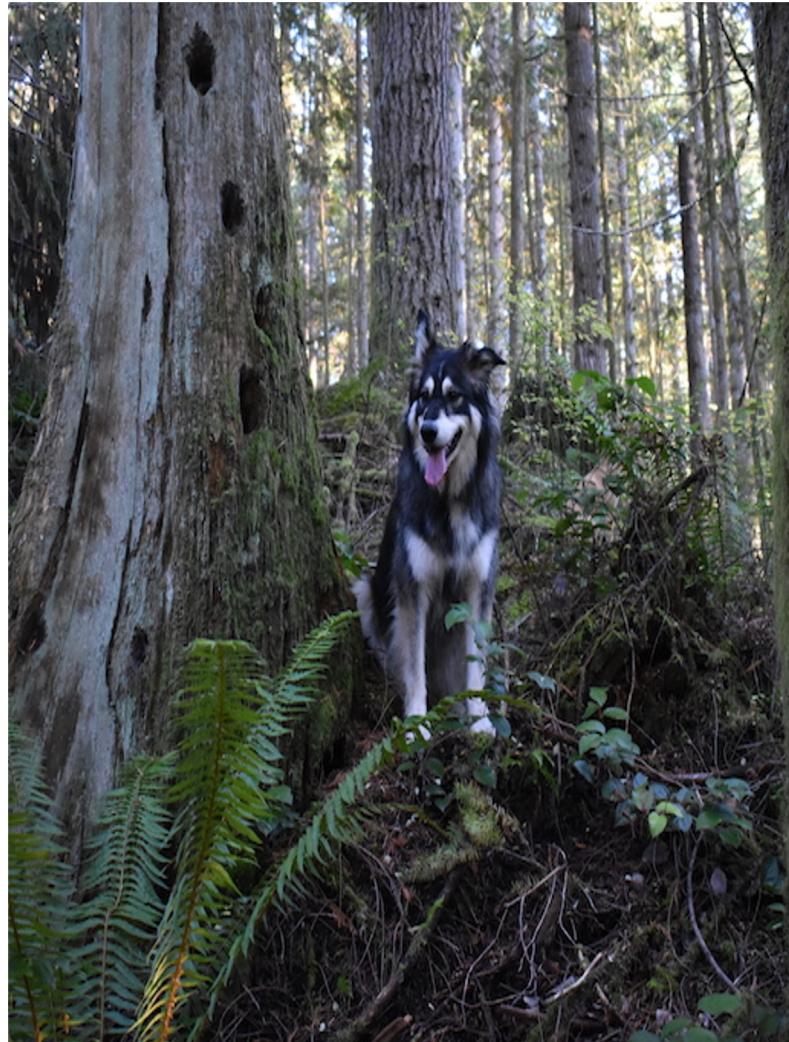
# Box filters: vertical + horizontal streaking



# Box filters: vertical + horizontal streaking



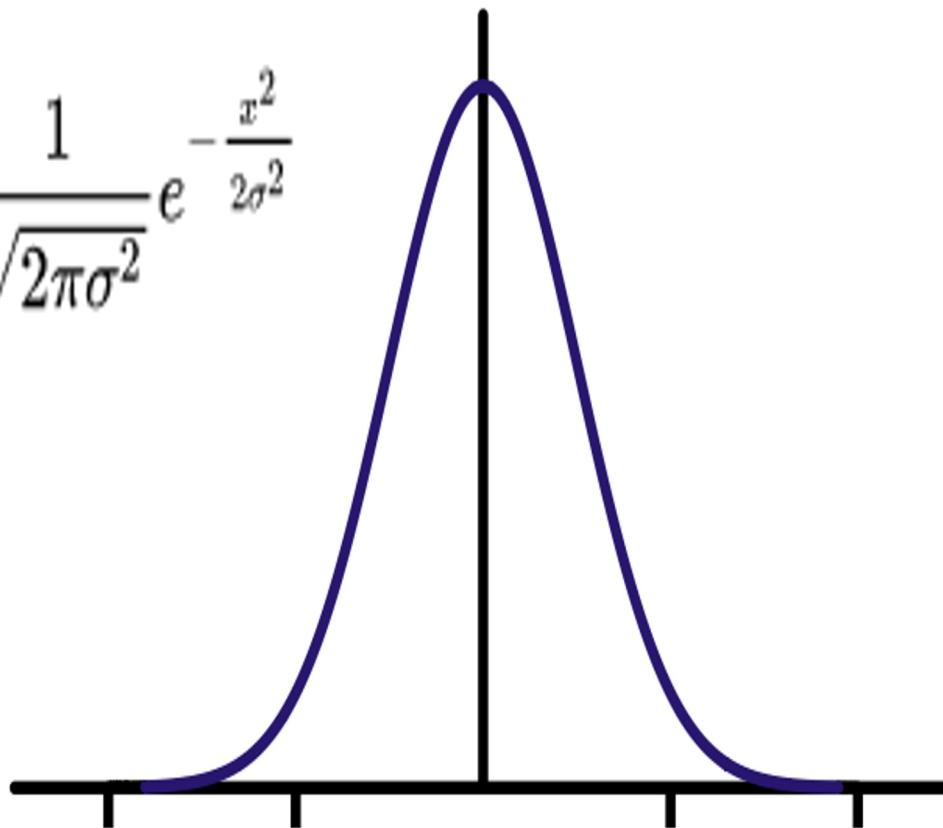
# We want a smoothly weighted kernel



# Gaussians

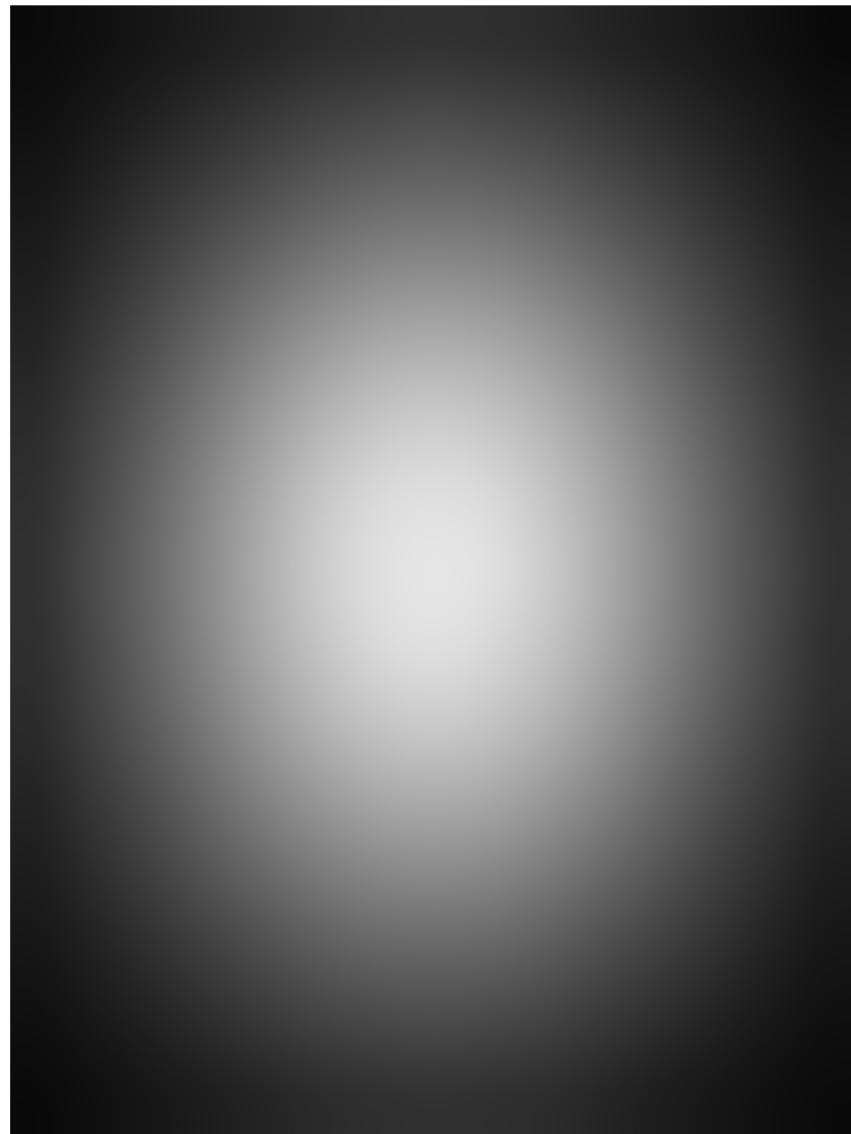
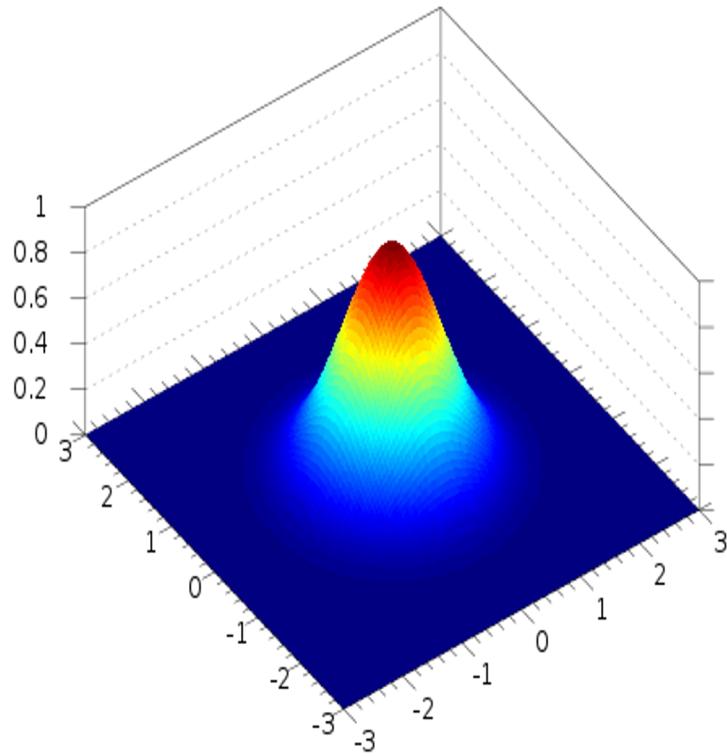
---

$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

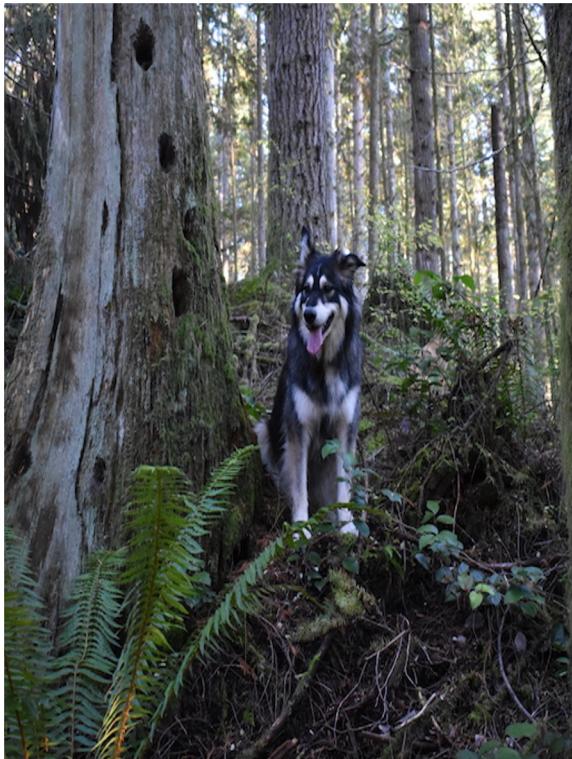


# 2D Gaussian

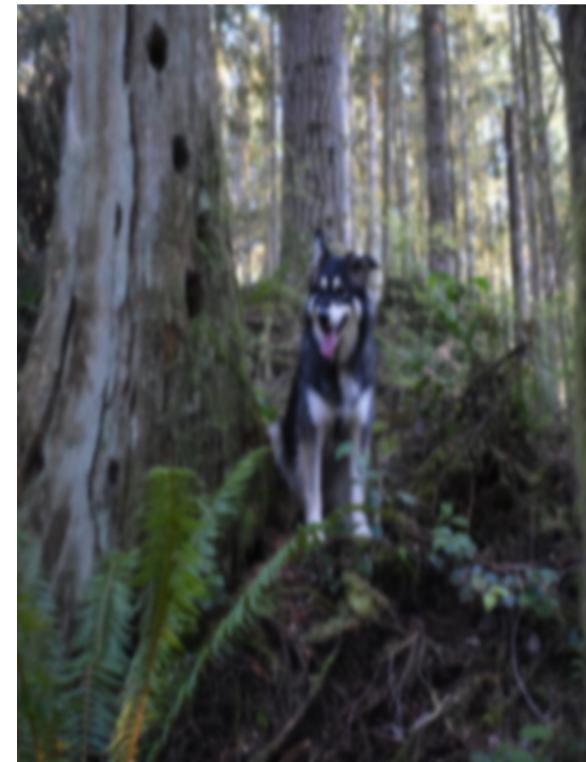
$$g(x, y) = \frac{1}{2\pi\sigma^2} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$$



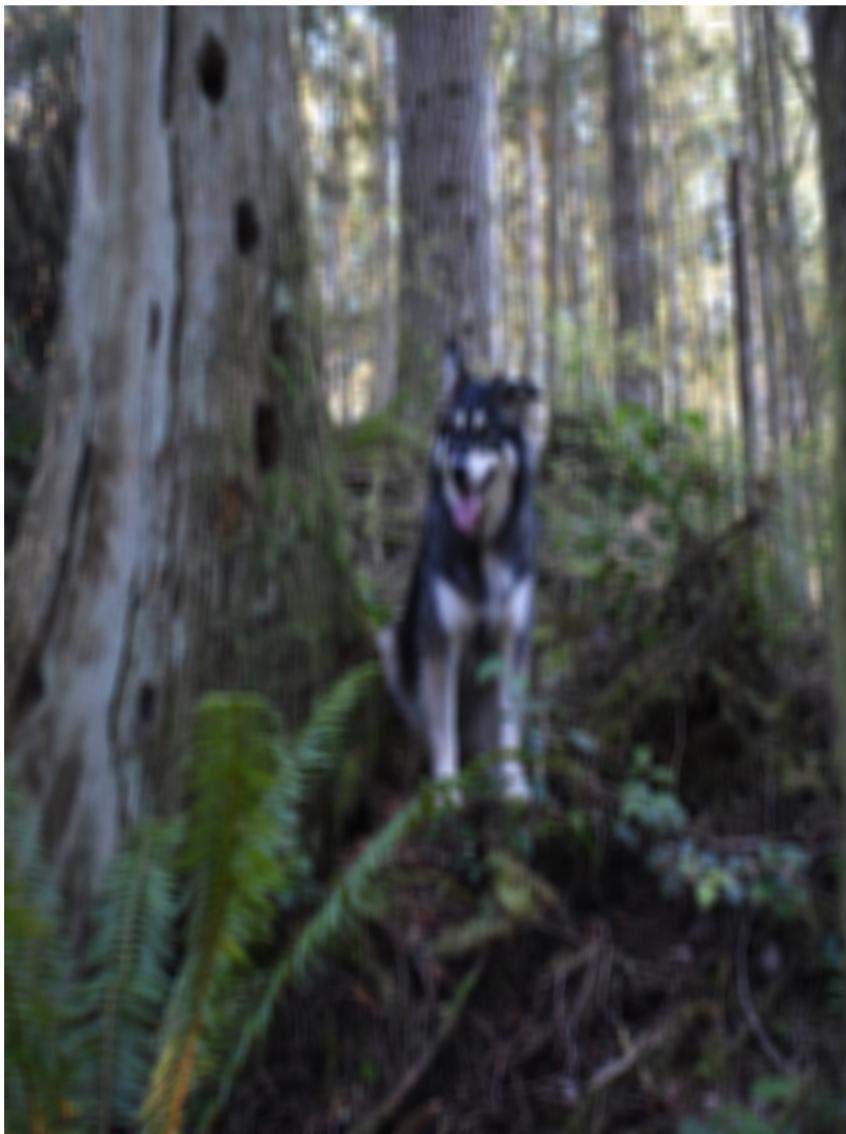
# Better smoothing with Gaussians



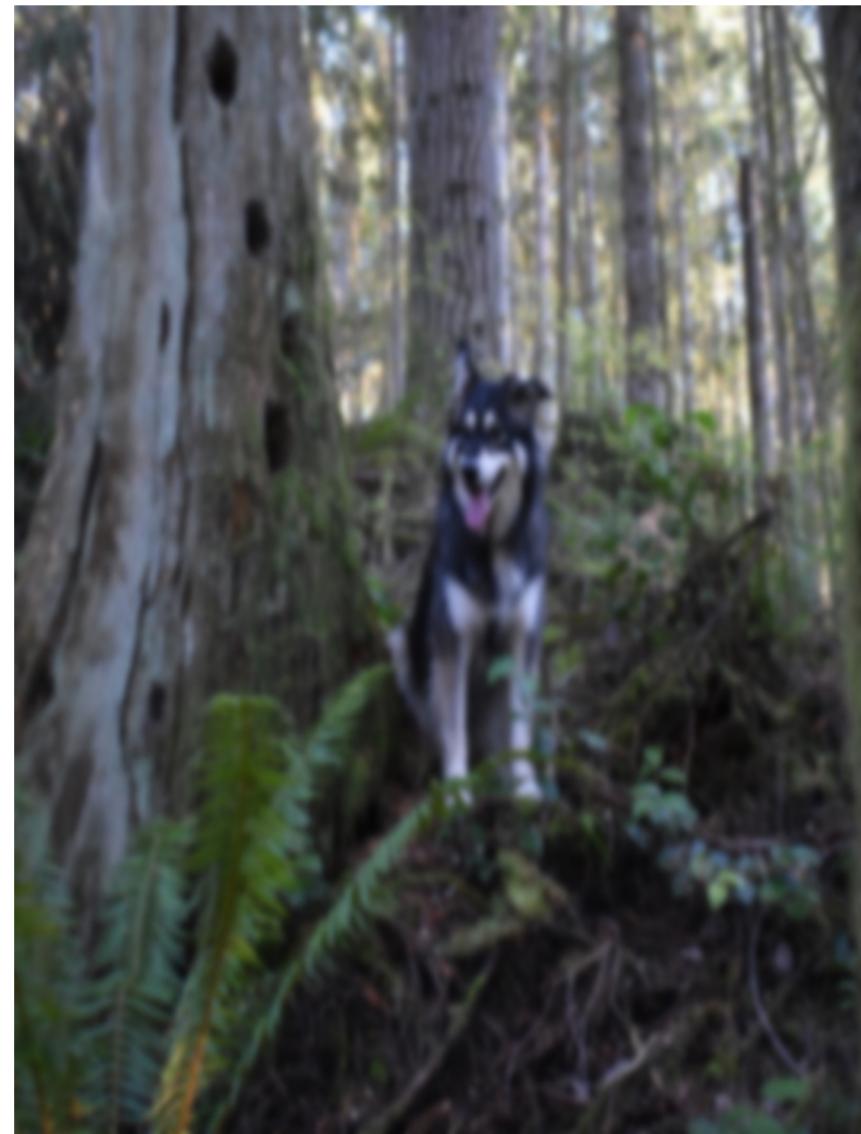
$$\ast \quad \begin{matrix} \text{Blurry image} \\ \text{(Gaussian kernel)} \end{matrix} = \quad \begin{matrix} \text{Smoothed image} \end{matrix}$$



# Better smoothing with Gaussians

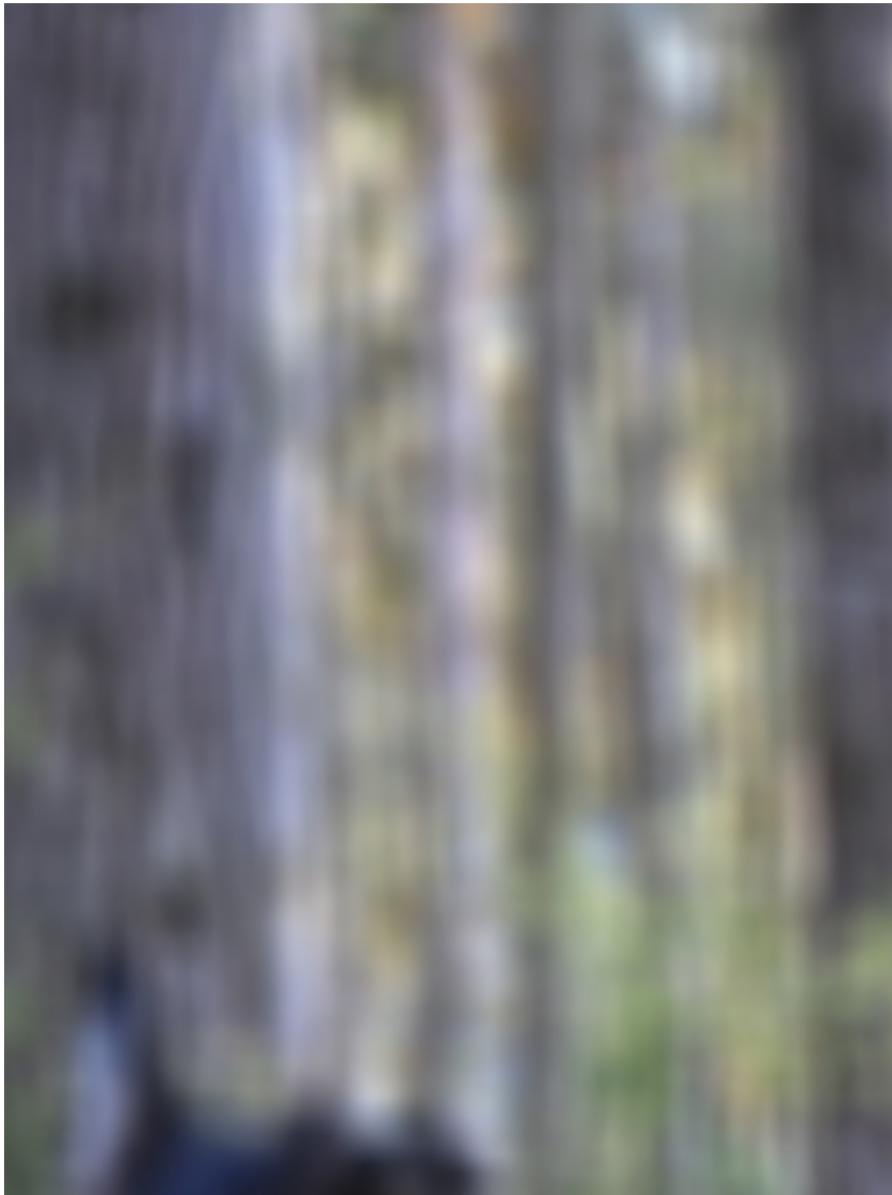


Smoothed with box filter

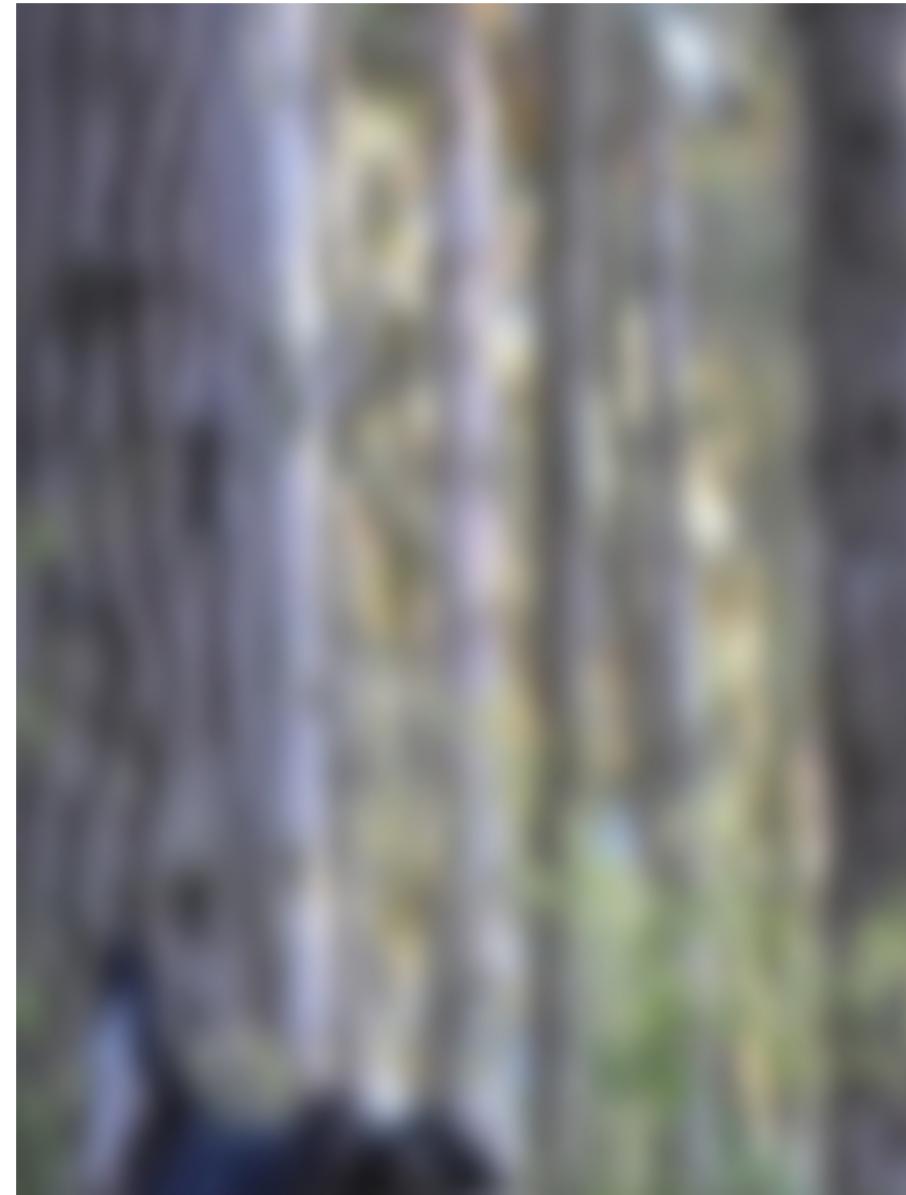


Smoothed with Gaussian filter

# Better smoothing with Gaussians

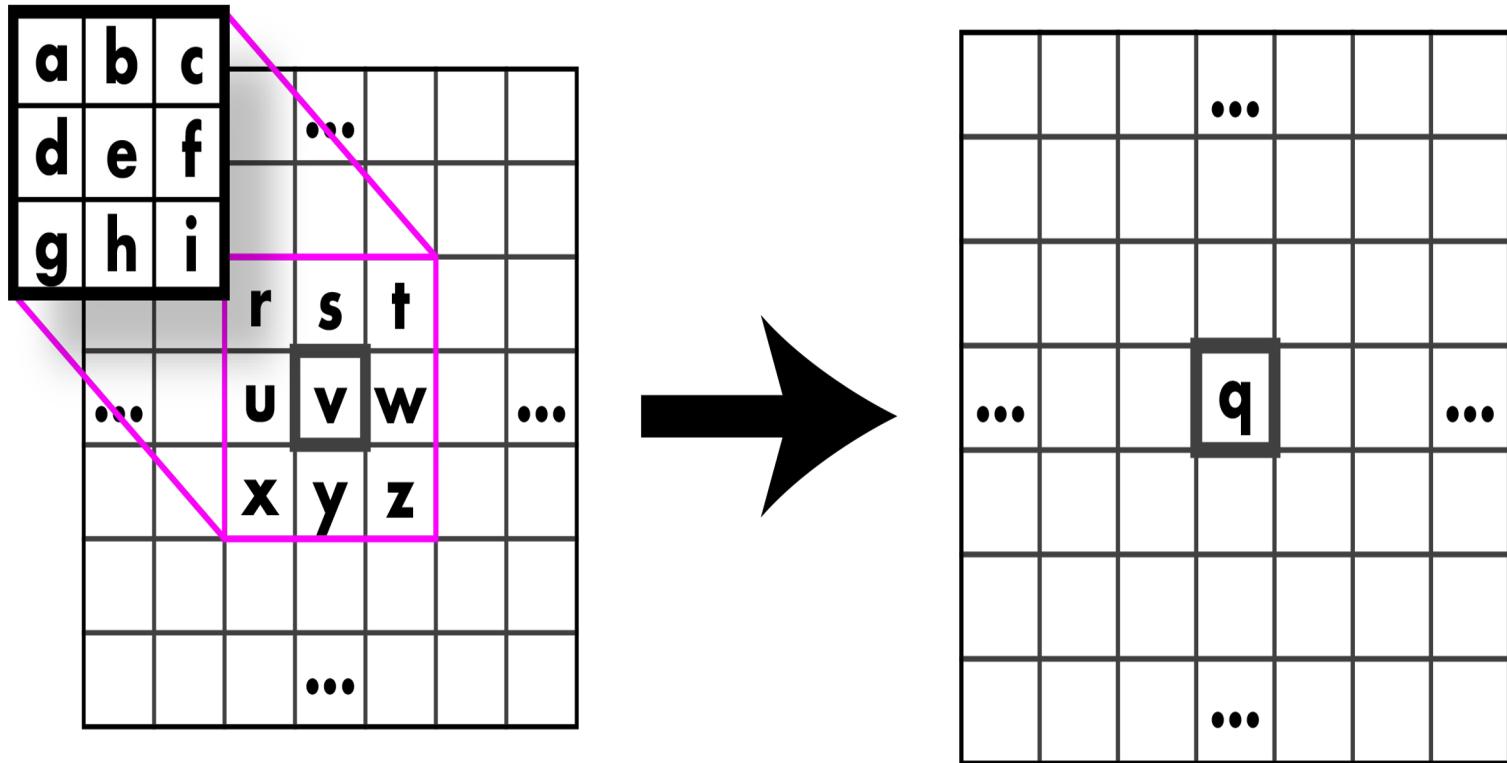


Smoothed with box filter

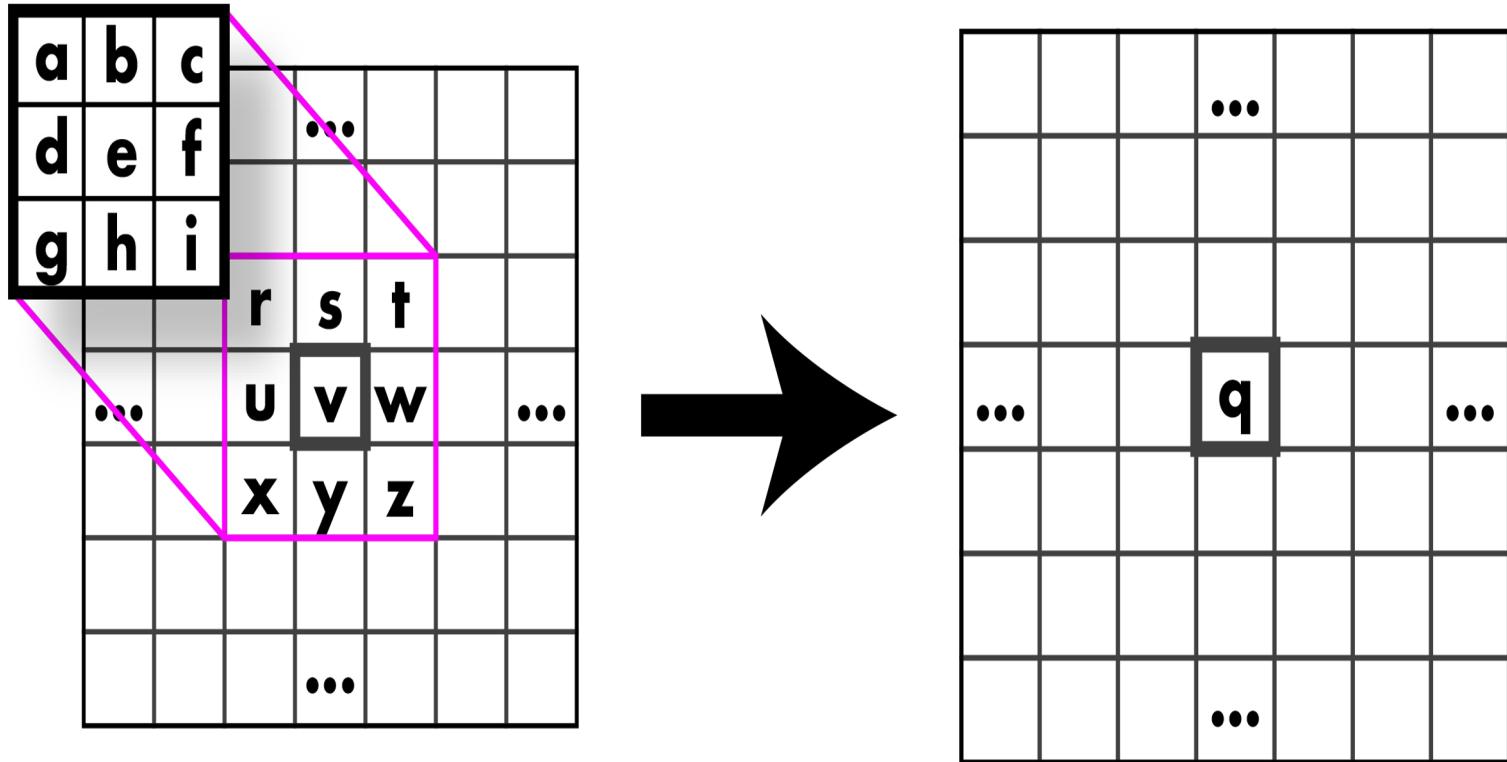


Smoothed with Gaussian filter

# Convolution

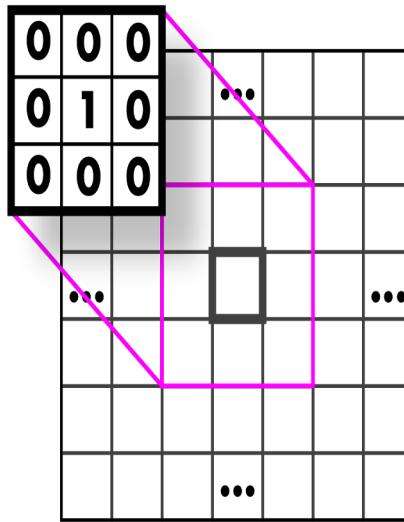


# Convolution

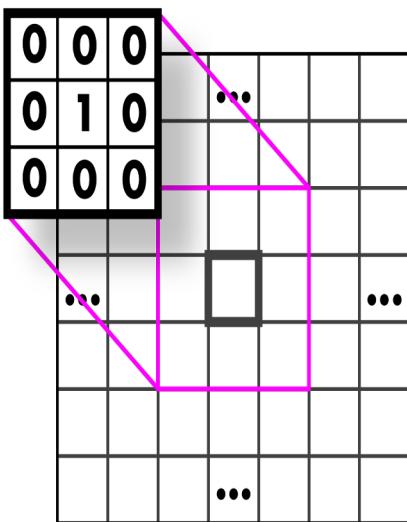


$$q = a \times r + b \times s + c \times t + d \times u + e \times v + f \times w + g \times x + h \times y + i \times z$$

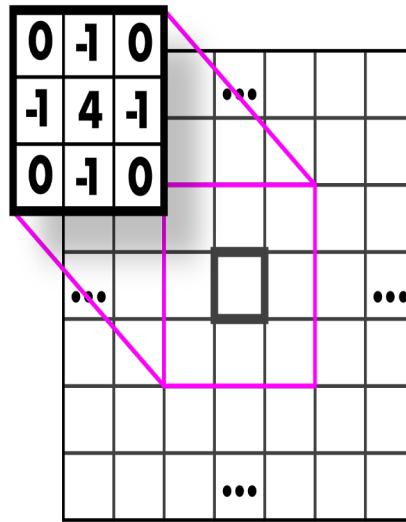
# Guess that kernel!



# Identity Kernel: Does nothing!



# Guess that kernel!

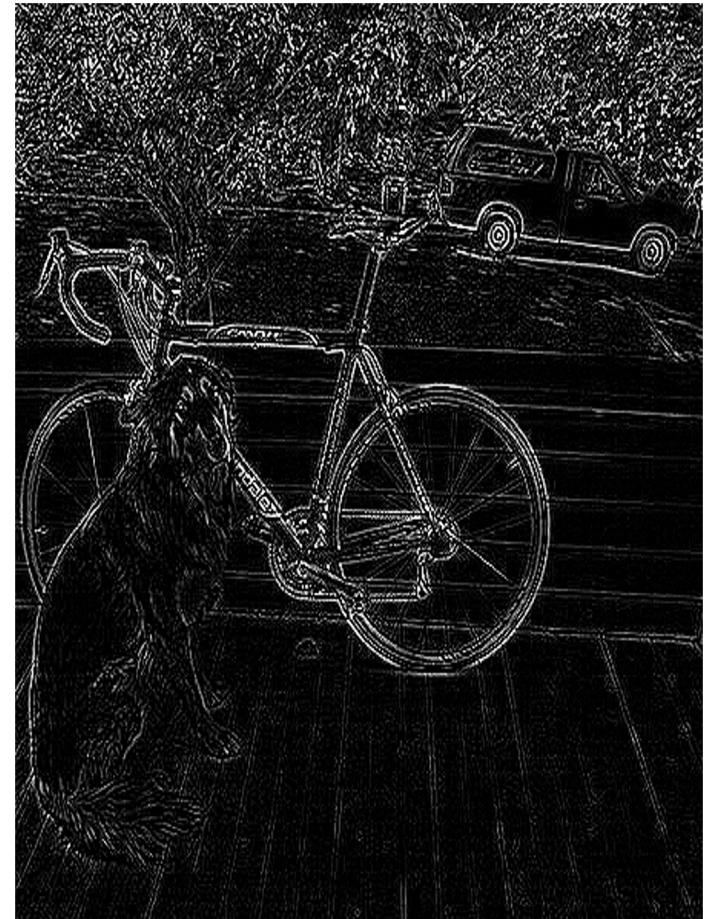


# Highpass Kernel: finds edges

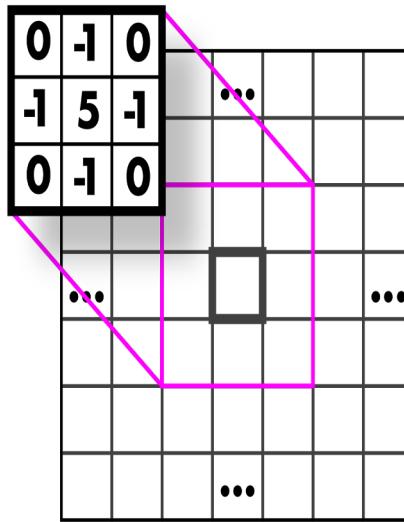


$$\begin{array}{|c|c|c|} \hline 0 & -1 & 0 \\ \hline 1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$

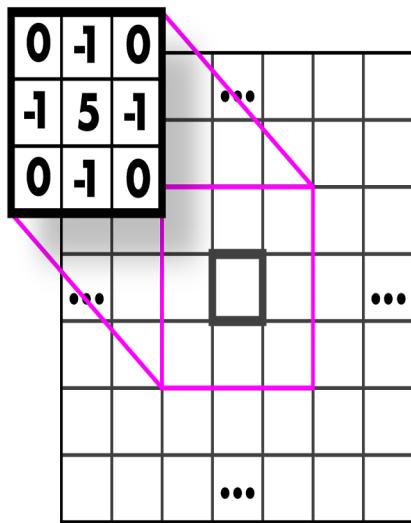
A diagram illustrating the application of a 3x3 highpass kernel to a 7x7 input image patch. The kernel values are shown in the top-left 3x3 grid. The input patch is a 7x7 grid of gray squares. A central square in the input patch is highlighted in black. A pink L-shaped mask indicates the receptive field of the central black square, covering the 3x3 kernel area. Ellipses (..) are used to indicate the continuation of the input patch beyond the 7x7 boundary.



# Guess that kernel!

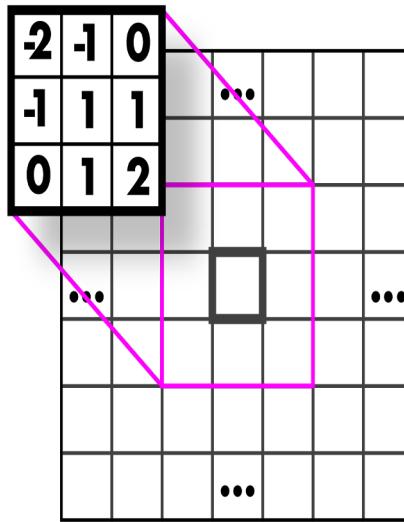


# Sharpen Kernel: sharpens!

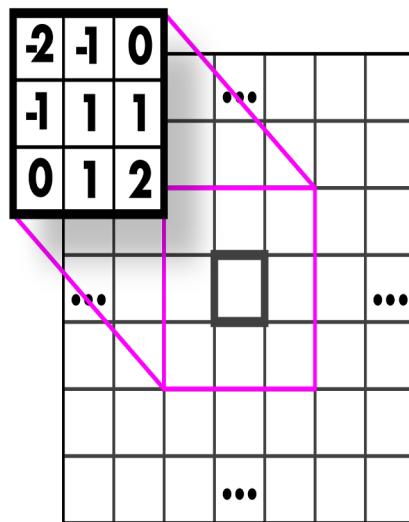


Note: sharpen = highpass + identity!

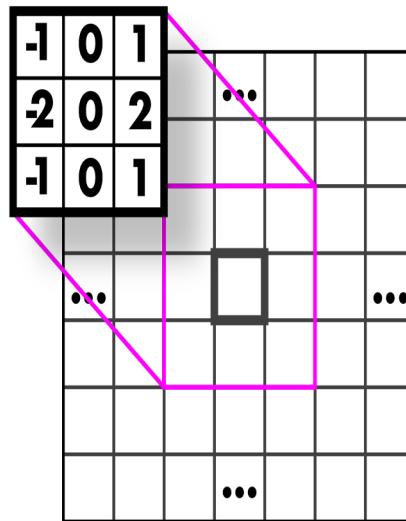
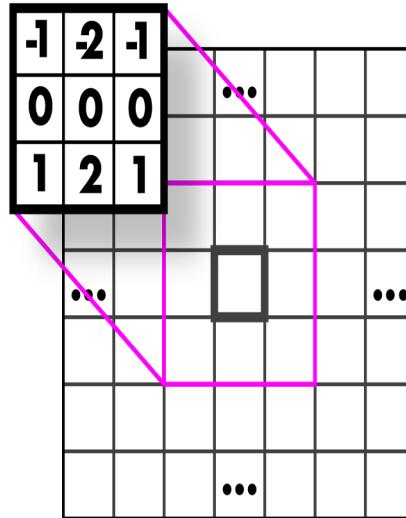
# Guess that kernel!



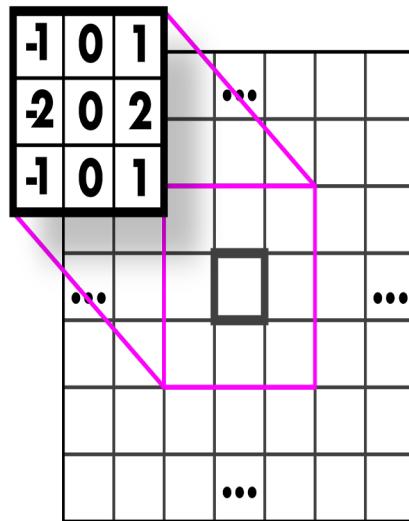
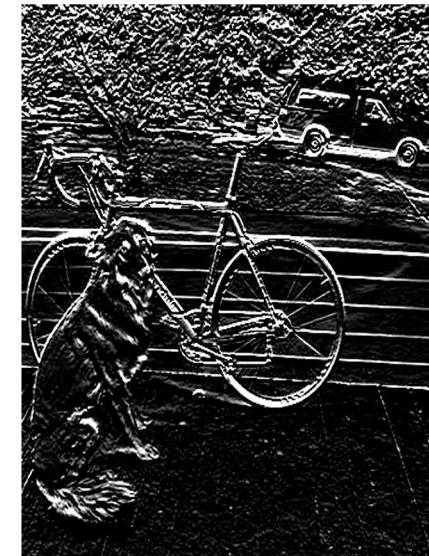
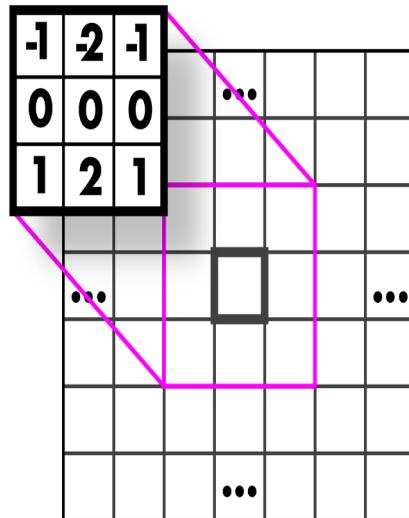
# Emboss Kernel: stylin'



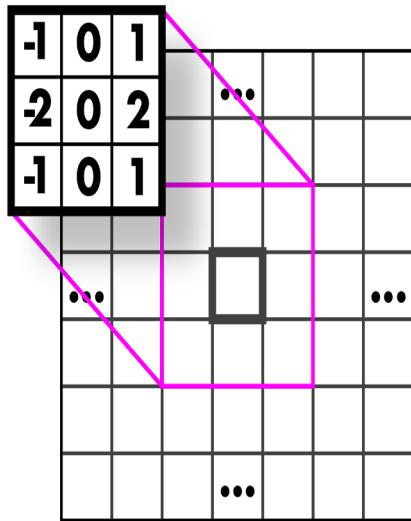
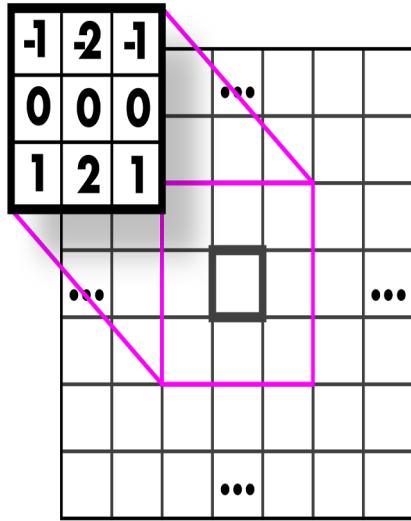
# Guess those kernels!



# Sobel Kernels: edges and...



# Sobel Kernels: edges and gradient!



# Sobel Kernels: edges and gradient!

