

## SIRS Equations

We will be solving the stochastic version of the three equations for  $S_i$ ,  $I_i$  and  $R_i$

$$S_{i,t+1} = S_{i,t} - N_{i,S,t}^{inf} + N_{i,t}^{lst}$$

$$I_{i,t+1} = I_{i,t} + N_{i,I,t}^{inf} - N_{i,I,t}^{rec}$$

$$R_{i,t+1} = R_{i,t} + N_{i,t}^{rec} - N_{i,t}^{lst}$$

where we will calculate the probability of population  $i$  to be infected as:

$$p_i^{inf} = 1 - \exp(-\lambda_i \cdot \Delta t)$$

(see above for the definition of  $\lambda_i$ ), the probability of this population to recover as:

$$p_i^{rec} = 1 - \exp(-\Delta t \cdot \gamma_i)$$

and the probability to lose immunity as:

$$p_i^{lst} = 1 - \exp(-\Delta t \cdot w_i)$$

with  $\Delta t$  being the time step. The force of infection,  $\lambda_i$  is given by:

$$\lambda_i = \beta_i \sum_j m_{ij} \frac{I_j}{N_i}$$

Using these three probabilities we can calculate the number of infected/recovered

for each country as:

$$N_{i,S,t}^{inf} = rbinom(1, S_i, p_i^{inf})$$

$$N_{i,I,t}^{rec} = rbinom(1, I_i, p_i^{rec})$$

$$N_{i,I,t}^{lst} = rbinom(1, R_i, p_i^{lst})$$