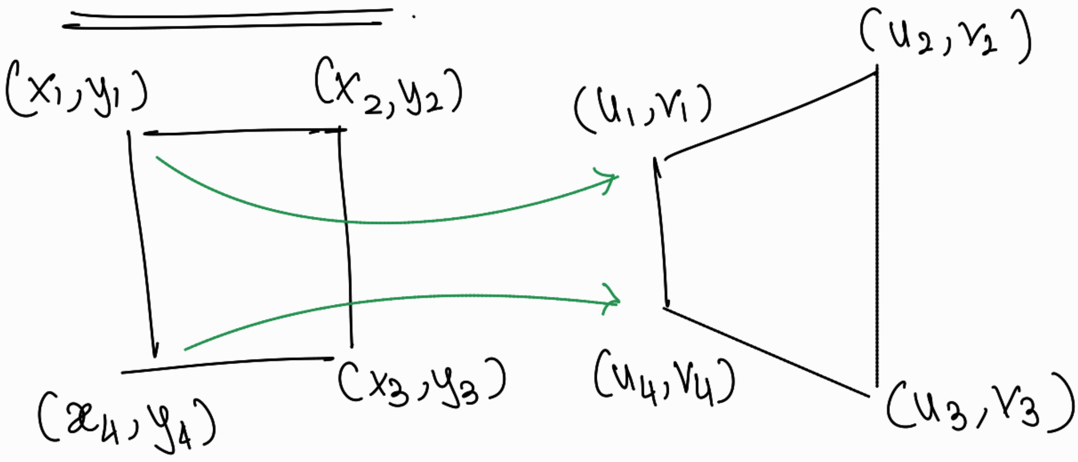
CV Homework 2

Q 1.1.3 – Derive A_i



Given input image with pixels (x, y) we want homography H to get (u, v)

Thus,

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$\left\{ \begin{array}{l} \text{from question,} \\ x_2 = (x, y) \\ x_1 = (u, v) \end{array} \right\}$

$$\Rightarrow \lambda u = h_{11} x + h_{12} y + h_{13} \quad \text{—— (1)}$$

$$\lambda v = h_{21} x + h_{22} y + h_{23} \quad \text{—— (2)}$$

$$\lambda . 1 = h_{31} x + h_{32} y + h_{33}$$

] Due to '$\equiv$', the eqn is valid upto known scale factor λ.

We can substitute λ from last eqn in (1) & (2).

$$\Rightarrow (h_{31}x + h_{32}y + h_{33})u = h_{11}x + h_{12}y + h_{13}$$

$$\& \ (h_{31}x + h_{32}y + h_{33})v = h_{21}x + h_{22}y + h_{23}$$

$$\Rightarrow xh_{11} + yh_{12} + h_{13} + 0h_{21} + 0h_{22} + 0h_{23} - xuh_{31} - yuh_{32} - uh_{33} = 0$$

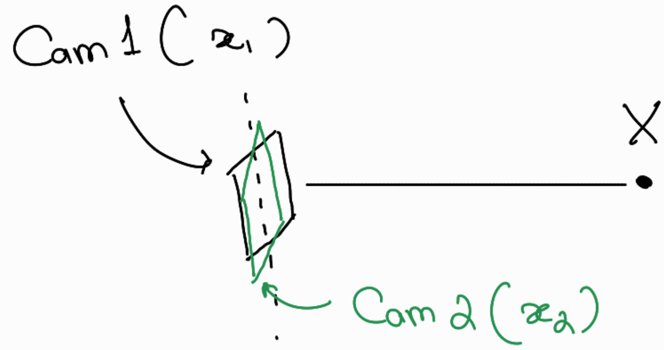
$$\& \ 0h_{11} + 0h_{12} + 0h_{13} + xh_{21} + yh_{22} + h_{23} - xvh_{31} - yvh_{32} - vh_{33} = 0$$

This can be written in matrix form as:

$$\underbrace{\begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -xu & -yu & -u \\ 0 & 0 & 0 & x & y & 1 & -xv & -yv & -v \end{bmatrix}}_{A_i} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

$$\therefore A_i = \begin{bmatrix} x_2^i & y_2^i & 1 & 0 & 0 & 0 & -x_2^i x_1^i & -y_2^i x_1^i & -x_1^i \\ 0 & 0 & 0 & x_2^i & y_2^i & 1 & -x_2^i y_1^i & -y_2^i y_1^i & -y_1^i \end{bmatrix}$$

Q 1.2.1 To prove a Homography always exists that satisfies $x_1 \equiv H x_2$, given two cameras are separated by pure rotation.



→ Cam 1 & Cam 2 are separated only by rotation R.

→ Without loss of generality, we can assume that Cam 1 is @ origin.

Thus,

projection matrix

$$x_1 \equiv P_1 X$$

$$x_1 \equiv K [I_1 | 0] X$$

& Similarly,

$$x_2 \equiv K [R | 0] X$$

$$\Rightarrow X_3 \equiv R^{-1} K^{-1} x_2$$

(first 3 elements of X)

$$\therefore x_1 \equiv K [I][R^{-1}] K^{-1} x_2$$

Thus homography $\boxed{H = K R^{-1} K}$

Q. 1.2.2 To show that H^2 is homography corresponding to 2θ.

$\Rightarrow$ We just proved that,

$$H = KRK^{-1}$$

let $R = R(\theta)$

$$\therefore H^2 = (KR(\theta)K^{-1})(KR(\theta)K^{-1})$$

$$= K\,R(\theta)^2\,K^{-1}$$

$\longrightarrow$ By rotation composition property.

$$R(\theta)^2 = R(2\theta)$$

Intuitively, this is 2 successive rotations by θ about a fixed axis which is equivalent to 1 rotation by 2θ about the fixed axis

$$\therefore \boxed{H^2 = KR(2\theta)K^{-1}}$$