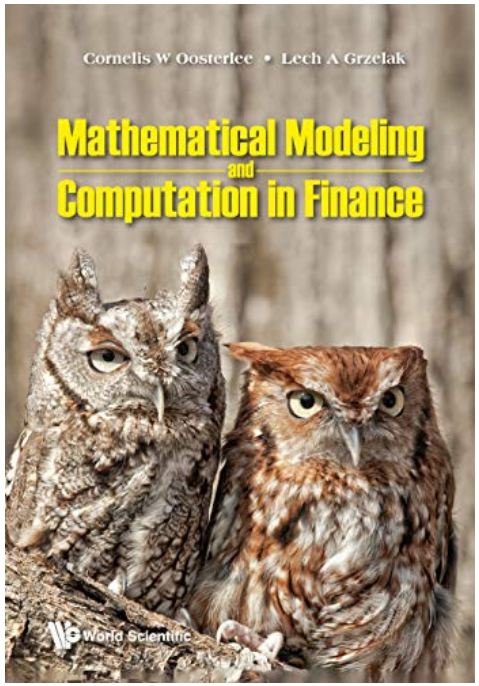
Materials for the course

The course is based on book "*Mathematical Modeling and Computation in Finance: With Exercises and Python and MATLAB Computer Codes*", by C.W. Oosterlee and L.A. Grzelak, World Scientific Publishing Europe Ltd, 2019. For more details go here.



- YouTube Channel with courses can be found here.
- Slides and the codes can be found here.

List of content

Computational Finance

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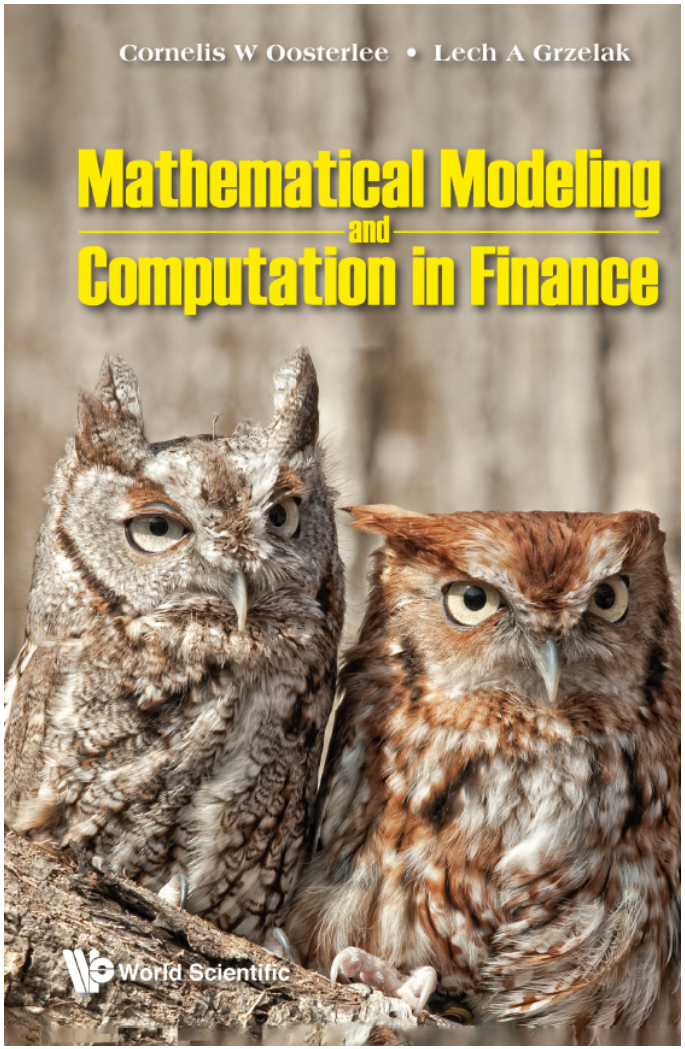
Computational Finance



⇒ Background knowledge ...

Course Outline

⇒ Book: Mathematical Modeling and Computation in Finance, by Kees Oosterlee and myself, World Scientific Press, 2019

- Start: Asset prices, option basics
  1. Background stochastic calculus (Ch. 1)
  2. Stock prices and options (Ch. 2, Ch. 3)
  3. Black-Scholes PDE (Ch. 3)

⇒ Beyond Black & Scholes equation (Ch. 5, Ch. 7, Ch. 8)

  1. Models with jumps
  2. Stochastic volatility

- Numerical methods:
  1. Fourier option pricing techniques (Ch.6)
  2. Monte Carlo methods (Ch.9)
- Pricing of exotic option (Ch.10)
- Focus on Modeling and Numerical Pricing Techniques.
- The book and the slides are important reading material!

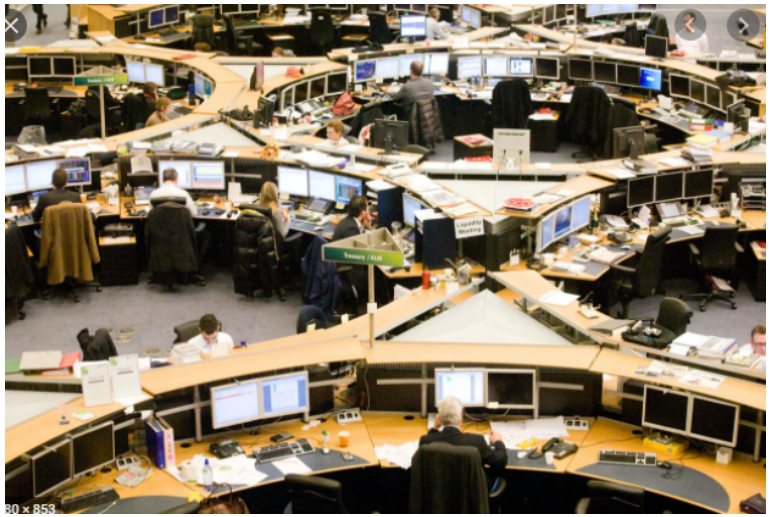
Course Outline

- Lecture 1- Introduction and Overview of Asset Classes
- Lecture 2- Stock, Options and Stochastics
- Lecture 3- Option Pricing and Simulation in Python
- Lecture 4- Implied Volatility
- Lecture 5- Jump Processes
- Lecture 6- Affine Jump Diffusion Processes
- Lecture 7- Stochastic Volatility Models
- Lecture 8- Fourier Transformation for Option Pricing
- Lecture 9- Monte Carlo Simulation
- Lecture 10- Monte Carlo Simulation of the Heston Model
- Lecture 11- Hedging and Monte Carlo Greeks
- Lecture 12- Forward Start Options and Model of Bates
- Lecture 13- Exotic Derivatives
- Lecture 14- Summary

Grading

- Grading: Successful completion of
  - Two sets of exercises related to the material taught, including computer exercises, by means of a written report (50% of the grade);
  - A written examination about all material (50% of grade).
- Credit: 6 ECTS points
- (Online) Excursion to financial industry

Financial engineering



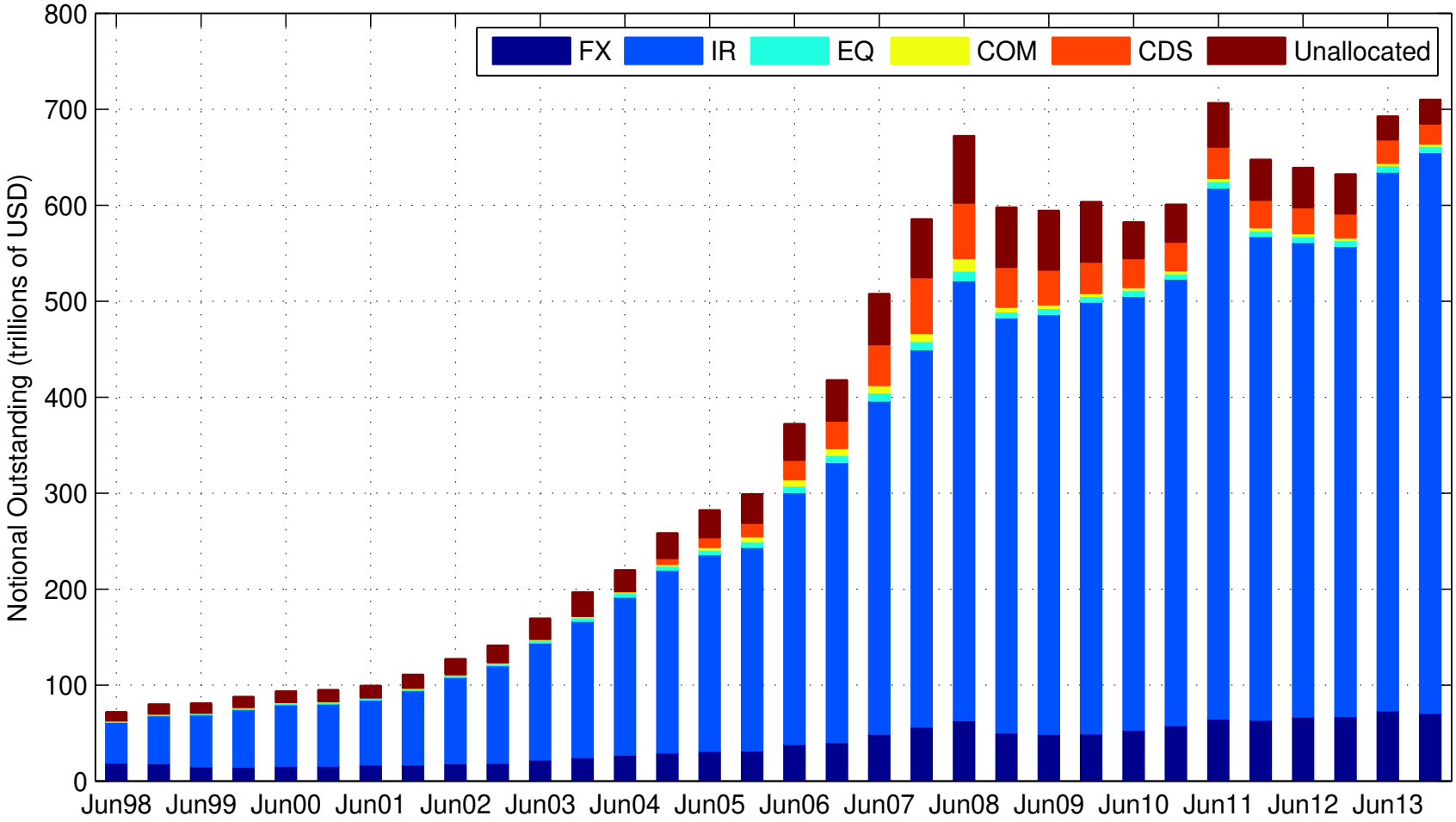
- Financial Engineers develop and maintain models used for pricing of derivatives.
- Financial engineering is a field involving financial theory, methods of engineering, tools of mathematics and the practice of programming.

Financial Markets, different markets

- Stock market (selling and buying stocks)
- Option exchanges (trading financial options)
- Interest rates market
- FX market
- Credit market
- Commodity market (gold, metals, corn, meat, futures, . . .)
- Energy market (oil, gas, . . .)
- OTC market (over the counter, not traded on a regulated exchange)
- Crypto currencies

$\Rightarrow$ Each market gives us mathematical questions

Market share



Stock options represent a tiny piece of the financial world!

- $\Rightarrow$ . . . but a good basis for education and research!

Financial instruments–Equities/Stocks

Another basic financial instrument is the equity, stock or share.

- This is the ownership of a small piece of a company. The price is determined by the value of the company and by the expectations of the performance of the company.
- The expectations are in the bid and ask behavior in the stock market.
- They give an uncertainty to the future price development of the stock. The exact profit is known at the date of selling.
- The true value of a stock is sometimes a bit higher, sometimes a bit lower than the expected value. The amount in which the stock price development can differ from the expected value is determined by the so-called volatility.

Dividends

- The owner of a stock theoretically owns a piece of the company.
- To the average investor the value in holding the stock comes from dividends and any growth in the stock's value. Dividends are lump payments, paid out every six months, to the stock owner.
- The amount of dividend varies from year to year depending on the profitability of the company.
- The amount of dividend is decided by the board of directors and is usually set a month before the dividend is actually paid.
- When the stock is bought it either comes with its entitlement to the next dividend (cum) or not (ex). There is a date at around the time of dividend payment when the stock goes from cum to ex.

The money market

- Interest is money paid for a credit or similar liability. Examples are bond yields, interest paid for bank loans, and returns on savings.
- Interest is typically calculated as a percentage of the principal, the amount owed to the lender. The percentage that is paid over a certain time period is the interest rate. Interest rates are market prices which are determined by supply and demand.
- As money became a commodity, the money market became a component of financial markets for assets involved in short-term borrowing, lending, buying and selling.
- Money markets, which provide liquidity for the global financial system, are part of the financial market.
- Trading in money markets is done directly with a financial counterparty (over the counter, OTC).
- Interest rates, derivatives and their models are part of the follow up course.

Reasons for interest rate changes

- Banks may change the interest rate to either slow down or speed up economy growth.
- Lowering interest rates can give the economy a short-run boost.
- Generally, if the economy is strong then the interest rates will be high, if the economy is weak the interest rates will be low.
- Economies typically exhibit inflation, a given amount of money buys fewer goods in the future than it will now.
- Risks of investment: The borrower may go bankrupt, or default on the loan. A lender generally charges a risk premium to ensure that she/he is compensated for those that fail.
- Taxes: Because some of the gains from interest may be subject to taxes, the lender may insist on a higher rate to make up for this loss.

Interest Rate–Safe Money

The simplest concept in finance is the time value of money.

- €1 today is worth more than €1 in a year's time.
- Invest €1 a discrete interest rate of r (assumed to be constant), paid once per year. After one year your bank account contains $1 \times (1 + r)$.
- There are several types of interest:
  - There is simple and compound interest. Simple interest is when the interest you receive is based only on the amount you initially invest, compound interest is when you get interest on your interest.
  - Interest typically comes in two forms, discretely compounded and continuously compounded.

Interest Rates

- With $M(t)$ in the bank at time t, how does this increase with time?

  If you check your account a short period later, time $t + \mathrm{d}t$, the amount will have increased by

$$M(t + \mathrm{d}t) - M(t) \approx \frac{\mathrm{d}M}{\mathrm{d}t}\mathrm{d}t + \ldots, \quad \text{(Taylor series expansion)}.$$

  The interest you receive must be proportional to the amount you have, M, the interest rate r and the time-step, dt. Thus,

$$\frac{\mathrm{d}M}{\mathrm{d}t}\mathrm{d}t = rM(t)\mathrm{d}t \text{ giving } \frac{\mathrm{d}M}{\mathrm{d}t} = rM(t) \ .$$

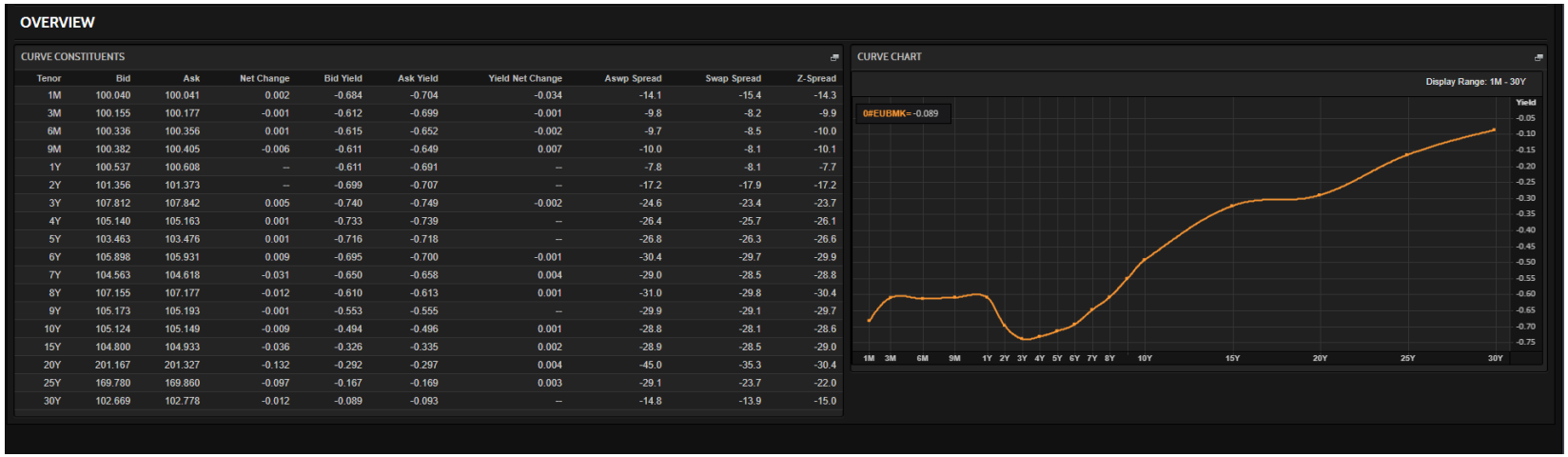
  If you have €M(0) initially, then the solution is $M(t) = M(0)\mathrm{e}^{rt}$.

  Conversely, if you know you will get 1€ at time T in the future, its value at an earlier time t is simply

$$\mathrm{e}^{-r(T-t)} \ .$$

Interest Rates

- In practice the rate r is not a constant but it is time-dependent.



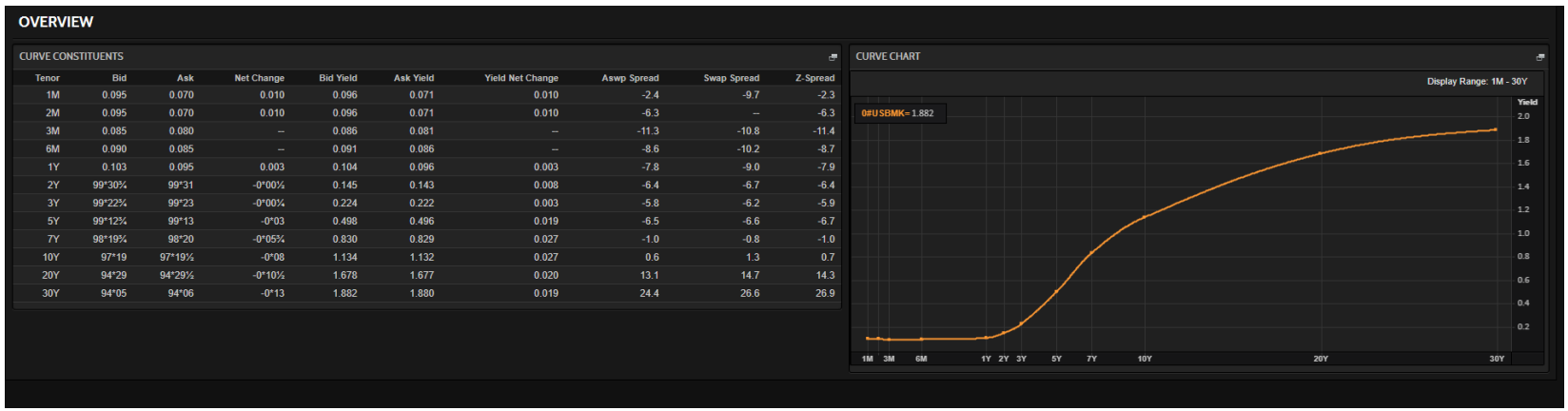
OVERVIEW

CURVE CONSTITUENTS

Tenor	Bid	Ask	Net Change	Bid Yield	Ask Yield	Yield Net Change	Aswp Spread	Swap Spread	Z-Spread
1M	100.040	100.041	0.002	-0.684	-0.704	-0.034	-14.1	-15.4	-14.3
3M	100.155	100.177	-0.001	-0.612	-0.699	-0.001	-9.8	-8.2	-9.9
6M	100.336	100.356	0.001	-0.615	-0.652	-0.002	-9.7	-8.5	-10.0
9M	100.382	100.405	-0.006	-0.611	-0.649	0.007	-10.0	-8.1	-10.1
1Y	100.537	100.608	--	-0.611	-0.691	--	-7.8	-8.1	-7.7
2Y	101.356	101.373	--	-0.699	-0.707	--	-17.2	-17.9	-17.2
3Y	107.812	107.842	0.005	-0.740	-0.749	-0.002	-24.6	-23.4	-23.7
4Y	105.140	105.163	0.001	-0.733	-0.739	--	-26.4	-25.7	-26.1
5Y	103.463	103.476	0.001	-0.716	-0.718	--	-26.8	-26.3	-26.6
6Y	105.898	105.931	0.009	-0.695	-0.700	-0.001	-30.4	-29.7	-29.9
7Y	104.563	104.618	-0.031	-0.650	-0.658	0.004	-29.0	-28.5	-28.8
8Y	107.155	107.177	-0.012	-0.610	-0.613	0.001	-31.0	-29.8	-30.4
9Y	105.173	105.193	-0.001	-0.553	-0.555	--	-29.9	-29.1	-29.7
10Y	105.124	105.149	-0.009	-0.494	-0.496	0.001	-28.8	-28.1	-28.6
15Y	104.800	104.933	-0.036	-0.326	-0.335	0.002	-28.9	-28.5	-29.0
20Y	201.167	201.327	-0.132	-0.292	-0.297	0.004	-45.0	-35.3	-30.4
25Y	169.780	169.860	-0.097	-0.167	-0.169	0.003	-29.1	-23.7	-22.0
30Y	102.669	102.778	-0.012	-0.089	-0.093	--	-14.8	-13.9	-15.0

CURVE CHART

Figure: Interest Rates (Yield Curve) for EUR (sounce: Reuters)



OVERVIEW

CURVE CONSTITUENTS

Tenor	Bid	Ask	Net Change	Bid Yield	Ask Yield	Yield Net Change	Aswp Spread	Swap Spread	Z-Spread
1M	0.095	0.070	0.010	0.096	0.071	0.010	-2.4	-9.7	-2.3
2M	0.095	0.070	0.010	0.096	0.071	0.010	-6.3	--	-6.3
3M	0.085	0.080	--	0.086	0.081	--	-11.3	-10.8	-11.4
6M	0.090	0.085	--	0.091	0.086	--	-8.6	-10.2	-8.7
1Y	0.103	0.095	0.003	0.104	0.096	0.003	-7.8	-9.0	-7.9
2Y	99*30¾	99*31	-0*00¼	0.145	0.143	0.008	-6.4	-6.7	-6.4
3Y	99*22¾	99*23	-0*00¼	0.224	0.222	0.003	-5.8	-6.2	-5.9
5Y	99*12¾	99*13	-0*03	0.498	0.496	0.019	-6.5	-6.6	-6.7
7Y	98*19¾	98*20	-0*05¾	0.830	0.829	0.027	-1.0	-0.8	-1.0
10Y	97*19	97*19¼	-0*08	1.134	1.132	0.027	0.6	1.3	0.7
20Y	94*29	94*29¼	-0*10¼	1.678	1.677	0.020	13.1	14.7	14.3
30Y	94*05	94*06	-0*13	1.882	1.880	0.019	24.4	26.6	26.9

CURVE CHART

Figure: Interest Rates (Yield Curve) for USD (sounce: Reuters)

Volatility

- Volatility is a measure for variation of financial prices over time. The symbol σ is used for volatility, and corresponds to standard deviation, not to be confused with variance, σ^2.
- A statistical measure of the tendency of a market or security to rise or fall sharply within a period of time.
- Volatility is typically calculated by using variance of the price or return. A highly volatile market means that prices have huge swings in very short periods of time.
- Security: An instrument representing ownership (stocks), a debt agreement (bonds), or the rights to ownership (derivatives).
- Return: The gain or loss of a security in a particular period. The return consists of the income and the capital gains relative on an investment. It is usually quoted as a percentage.

Volatility

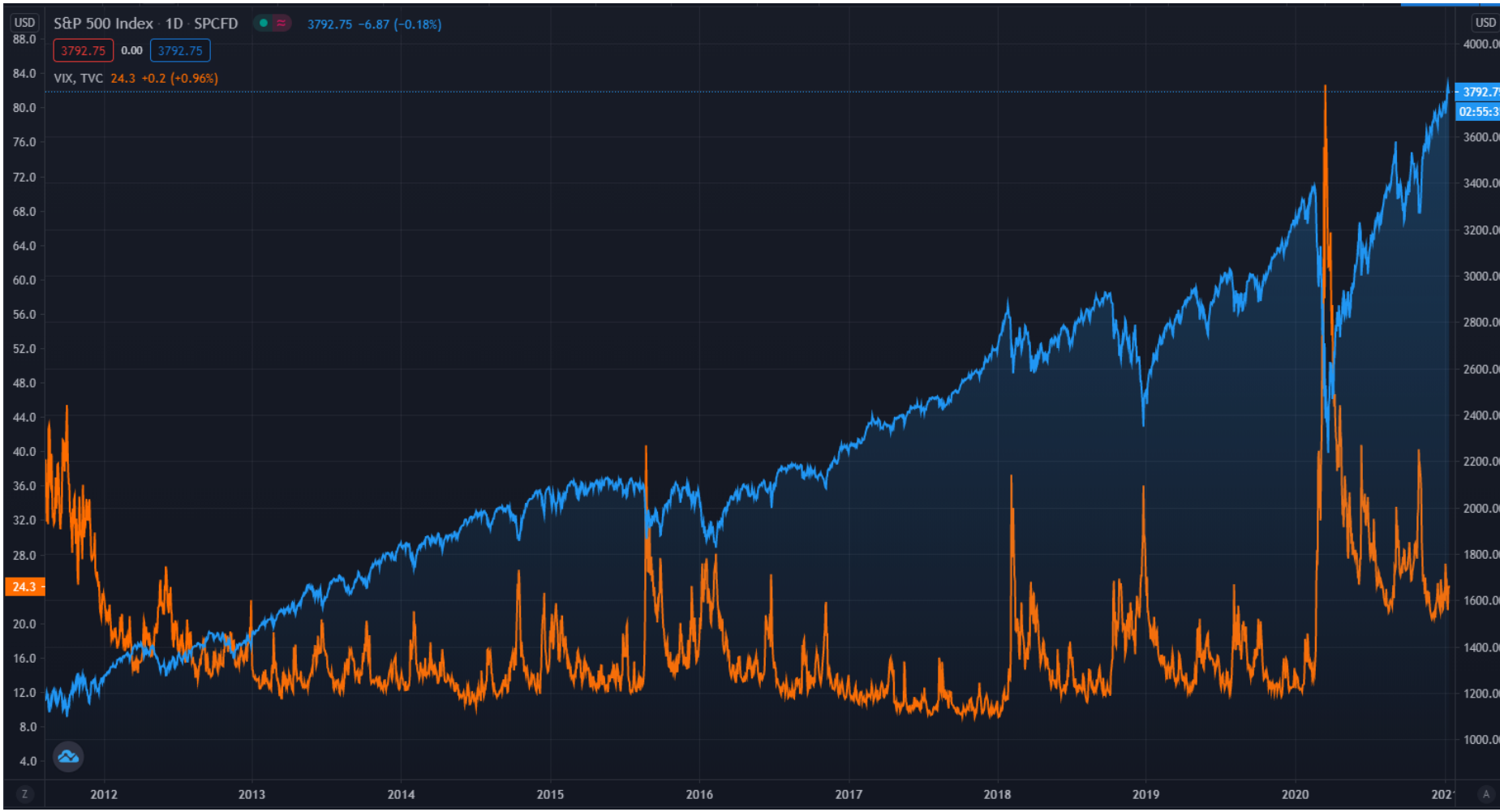


Figure: S&P vs. VIX (source tradingview.com)

Stock indices, S&P500

- Whereas the Dow Jones index is based on 30 leading stocks, the S&P500 is an American stock market index based on 500 large companies having stock listed on the NYSE or NASDAQ.
- Euronext NV is a European stock exchange seated in Amsterdam, Brussels, London, Lisbon and Paris, cash and derivatives markets. The AEX consists of 25 stocks.

Figure: S&P vs. VIX (source tradingview.com)

Options

Definition (Option)

Is a contract written by a seller, that gives the right (but not the obligation) to the holder to trade in the future the underlying asset at a previously agreed price.

Most popular options are call and put options: At a prescribed time in the future, (maturity: T):

- European Call option: The holder of the option may purchase a prescribed asset (shares, stocks : S) for a prescribed amount (strike: K) and the writer of the contract must sell the asset, if the holder decides to buy it.
- European Put option: The holder of the option may sell a prescribed asset (shares, stocks : S) for a prescribed amount (strike: K) and the writer of the contract must buy the asset, if the holder decides to sell it.

Options & Payoffs

The value of European call option at the expiry T is given by:

$$V_C(T, S_T) = \max(S_T - K, 0).$$

The value of European put option at the expiry T is given by:

$$V_P(T, S_T) = \max(K - S_T, 0).$$

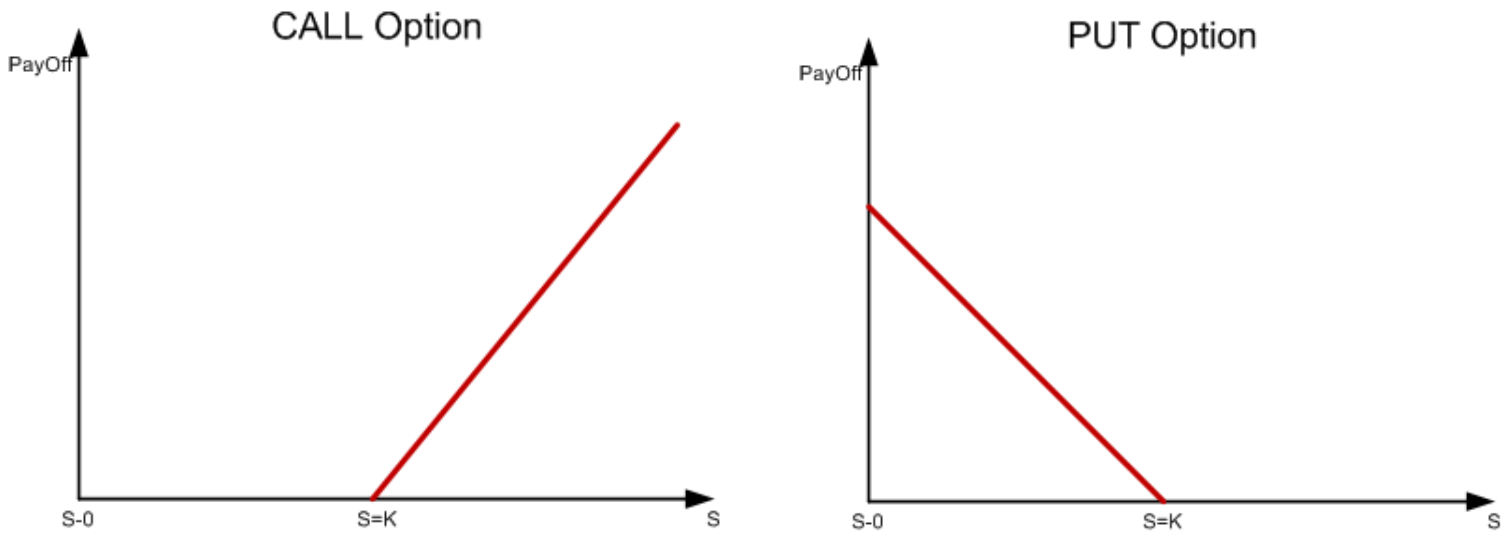


Figure: Payoff diagram for European Call (left), and European Put (right).