CSC 2510 HW#2

Monday, October 5, 2020 2:00 PM

HOMEWORK #2

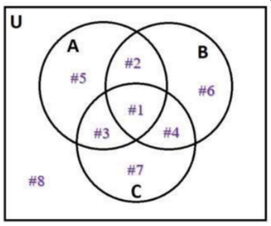
PROBLEM 1.

Given set expressions, indicate which area(s) should be shaded, if any, on given Venn diagrams.

a) $(A \cup B) \cap (A' \cup C)'$

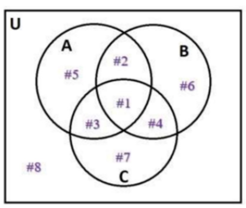
$(A \cup B) \cap (A' \cup C)'$

$(A \cup B) \cap (A \cap C')$



Solution: area(s) 2, 5, 6

b) $C - B - A'$



Solution: area(s) 3

PROBLEM 2.

Write down the power sets of the following sets and find cardinalities:

- A={{a}, a} p(A)= { ∅, a, {a}, {{a}, a} }

  What is the cardinality of set A? 2 What is the cardinality of set p(A)? 4

- B={{b}} p(B)= { ∅, {b} }

  What is the cardinality of set B? 1
  What is the cardinality of set p(B)? 2

- C={{ }} p(C)= { ∅, {∅} }

  What is the cardinality of set C? 1

  What is the cardinality of set p(C)? 2

PROBLEM 3.

Is this a power set of some set A? If so, find that set A.
Explain your answer (possible or impossible; and if it is possible – how you "restored" set A and how to check whether it is correct).

p(A) = { {}, {{a}, {a,b}}, {{a}}, {{a,b},{a}} ,{{a,b}} }

A = This is not a powerset of some set A, since this has 5 elements. Every power set will a total number of elements that can be expressed as a power of 2. It is impossible to restore the original set, as I see a duplicate of the second element.

PROBLEM 4.

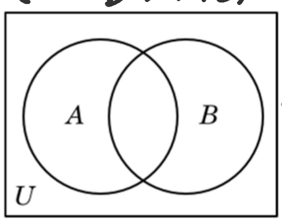
Using set identity laws, simplify. For your convenience, table with rules is copied below.

Show the process step-by-step (and write down which *set identities laws* you used).

$((AAUBB') \cap B)' \cap (AAUBB') \cap A \cap B$ = …

$((A \cup B') \cap B)' \cap (A \cup B') \cap A \cap B$

1. $((A \cup B') \cap B)'$ → Hypothesis



2. $(A' \cap B) \cup B'$ → De Morgan's law
3. $(A' \cup B') \cap (B \cup B')$ 2, Distributive property
4. $(A' \cup B') \cap U$ 3, Complement law
5. $(A' \cup B')$ 4, identity law
6. $(A \cup B') \cap A$ Hypothesis
7. A 6, Absorption
8. B Hypothesis
9. $(A \cap B)'$ 5, De Morgan
10. $(A \cap B) \cap (A \cap B)'$ 7,8,9 Hypothesis
11. $\emptyset$ 10, Complement Law.

Hint: *To check your answer you can draw Venn diagram of the original expression.*

SET IDENTITIES:

Commutative Laws: A ∪ B = B ∪ A A ∩ B = B ∩ A	Identity Laws: A ∪ ∅ = A A ∩ U = A
Associative Laws: (A ∪ B) ∪ C = A ∪ (B ∪ C) (A ∩ B) ∩ C = A ∩ (B ∩ C)	Complement Laws: A ∪ A' = U A ∩ A' = ∅
Distributive Laws: A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)	Double Complement Law: (A')' = A
De Morgan's Laws: (A ∪ B)' = A' ∩ B' (A ∩ B)' = A' ∪ B'	Idempotent Laws: A ∪ A = A A ∩ A = A
Absorption Laws: A ∪ (A ∩ B) = A A ∩ (A ∪ B) = A	Universal Bound Laws: A ∪ U = U A ∩ ∅ = ∅
Set Difference Law: A − B = A ∩ B'	Complements of U and ∅: U' = ∅ ∅' = U

$A \cap (A \cup B) = A$ Set Difference Law: $A - B = A \cap B'$	Complements of U and $\emptyset$: $U' = \emptyset$ $\emptyset' = U$

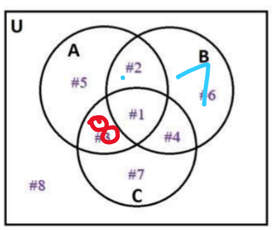
PROBLEM 5.

36 students go to a hot dog stand and order hot dogs. Every student orders at least one topping out of three choices (mustard, onion, relish). You have the following information about their topping choices:

(a) 18 ask for mustard.
(b) 21 ask for onions.
(c) 18 ask for relish.
(d) 8 ask for mustard but not onions.
(e) 31 ask for onions or relish (or both).
(f) 17 ask for exactly two toppings.
(g) 2 ask for all three toppings.

How many students order exactly one topping? 17

Let set A stand for student who ordered mustard topping, B – for relish, C for onion. Find cardinalities of as many of areas as possible (there are 8 areas total – see the picture below).



(note: Knowing 18 students asked for mustard does NOT mean that area 5 cardinality is 18, but that cardinality for combined areas 1,2,3,5 is. Now you need to find cardinalities of as many separate areas as you can!)

Explain your answers.

Area #	Cardinality	Explanation
1	2	This is the section where the children asked for all 3 toppings.

2		
3		DOWN
4		
5	5	
6		
7		
8	0	Every student had at least 1 topping.

6		
7		
8	0	Every student had at least 1 topping.

#1: 2 students asked for all 3 toppings.

#2: We know that 8 students asked for mustard, but not onions. We also know that 31 students out of 36 asked for Onions or Relish (or possibly both) . We know that 5 students asked only for mustard, as they didn't ask for onions or relish. That leaves 3 students out as to who asked for only mustard and relish.

#3: Since we know that 18 students asked for mustard, we also know that #1 + #2 + #3 + #5 = 18. We found out that #1 = 2, #2 = 3, and #5 = 5. So this leaves out 8 students for area 3.

#4: We know that 17 students asked for exactly 2 toppings, which means #2 + #3 + #4 = 17. Based on #2 and #3, we have 6 students for #4.

#5: This is where the students asked only for mustard. We also know that 31 students out of 36 asked for Onions or Relish (or possibly both) . We know that 5 students asked only for mustard, as they didn't ask for onions or relish.

#6: 18 students asked for Relish, we know that #1 + #2 + #4 + #6 = 18. Based on #1, #2, and #4, we have only 7 students remaining.

#7: Since 21 students asked for Onions, so #1 + #3 + #4 + #7 = 21. We know the values of #1, #3, and #4, we have 5 students who **only** asked for Onions.

#8: It is 0, since all students had at least one topping.