Question 8.1

Using the same crime data set uscrime.txt as in Question 8.2, apply Principal Component Analysis and then create a regression model using the first few principal components. Specify your new model in terms of the original variables (not the principal components), and compare its quality to that of your solution to Question 8.2. You can use the R function prcomp for PCA. (Note that to first scale the data, you can include scale. = TRUE to scale as part of the PCA function. Don't forget that, to make a prediction for the new city, you'll need to unscale the coefficients (i.e., do the scaling calculation in reverse)!)

Approach Followed

- 1) After data is ingested, I followed series of steps to see the co-correlation of the predictors among themself and its impact on the Crime responses using Corelation plots, Corelation matrix and the Correlation tables. In my opinion Corelation tables were the best approach to quickly see the correlated fields. 2) This step is followed by creating eigen values and eigen vectors for X(Tranpose)*X; X being the matrix of the Crime data set.I used Caret transformation to get predictors, box cox tranformation and find princial componentswhich suggested that "PCA needed 9 components to capture 95 percent of the variance" 3) Based on this recommendationm I set my "n" values for PCA as 9.Using prcomp, I created PCA and obtained first "9" components using which I created linear regression model(using lm). The model gave the adjusted R squared :0.69;R squared :0.61
- 4) Now, per the requirement, we have to express themodel in terms of originalvairbales and NOT principal components. To achieve this I obtain my components PC1 through PC(and then used pca rotation adn transform function to get the original Coefficients. One item to notice is since the co-efficients are scaled earlier, I need to unscale it to get the exact co-efficients of the model. 5) To scale, it subtracts the mean and divides by the standard deviation, for each variable. I used this logic to unscale the co-efficients 6) Compare the PCA model accuracy vs. the original model' accuracy. The Original model R2 squared is 0.80 and Adju R2 .70 which is more than the PCA model accuracy. Then i ran a loop from 1 through all PCA components to see any of its accuracy beat the original model.. But none. As noted in the lectures, overfitting could have occured with less data points with original linear regression model 7) Finally the prediction for the new City using predict function. I used both the model to predict the outcome of Crime. Predicted value of new data using PCR is 1112 vs. 155 crime data value using original regression with all the predictors

#PCA generated crime ratio 7 times more than the linear regression model with all predictors for the new data value

In [203]: # Clear environment rm(list = ls())# Setting the random number generator seed so that our results are reproducible set.seed(1)

In [204]:

#########LIBRARY##########

library("ggplot2")

#install.packages("devtools") #install.packages("corrplot")

library("devtools")

#library("corrplot") # to use Correlation plot

library(tidyr) # to use "as tibble,

library(plyr) #to use arrange function

library(car) #Scatterplotmatrix

library(GGally) #for gapairs graph

library(pls)# to get pca fit

INGESTION OF DATA AND STUDY THE RELATIONSHIPS

In [205]: ########INGEST FILE#########

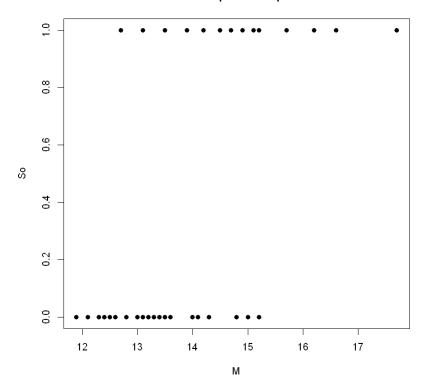
#crime<- read.table("C:\\Preethi\\R\\USCrimes.txt",header=TRUE,stringsAsFactors = FALSE,sep="\t")</pre> data = read.table("C:\\Preethi\\R\\USCrimes.txt",header=TRUE,stringsAsFactors = FALSE,sep="\t") %>% as.data.frame head(data,2)

A data.frame: 2 × 16

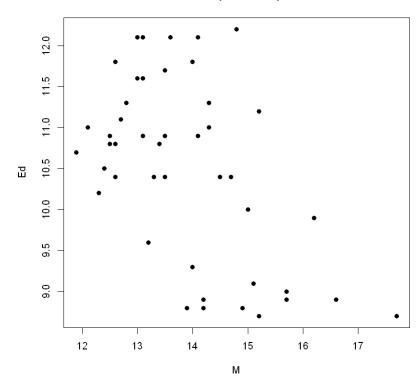
| M | | So | Ed | Po1 | Po2 | LF | M.F | Pop | NW | U1 | U2 | Wealth | Ineq | Prob | Time | Crime |
|--|-----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| <d< th=""><th>bl></th><th><int></int></th><th><dbl></dbl></th><th><dbl></dbl></th><th><dbl></dbl></th><th><dbl></dbl></th><th><dbl></dbl></th><th><int></int></th><th><dbl></dbl></th><th><dbl></dbl></th><th><dbl></dbl></th><th><int></int></th><th><dbl></dbl></th><th><dbl></dbl></th><th><dbl></dbl></th><th><int></int></th></d<> | bl> | <int></int> | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> | <int></int> | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> | <int></int> | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> | <int></int> |
| 1 15. | .1 | 1 | 9.1 | 5.8 | 5.6 | 0.510 | 95.0 | 33 | 30.1 | 0.108 | 4.1 | 3940 | 26.1 | 0.084602 | 26.2011 | 791 |
| 2 14. | .3 | 0 | 11.3 | 10.3 | 9.5 | 0.583 | 101.2 | 13 | 10.2 | 0.096 | 3.6 | 5570 | 19.4 | 0.029599 | 25.2999 | 1635 |

```
In [225]: ##EXPLORING DATA#####
#2-D graphs for each predictors to understand relationship and correlation of predictors
#Just reduced to 2 charts to avoid overfill of assignment with scatterplot
for (i in 1:3){
    for (j in 1:3){
        if (i<j){
            plot(data[,i],data[,j], main="Scatterplot Example",xlab=colnames(data)[i],ylab=colnames(data)[j], pch=19)
        }
    }
}</pre>
```

Scatterplot Example



Scatterplot Example

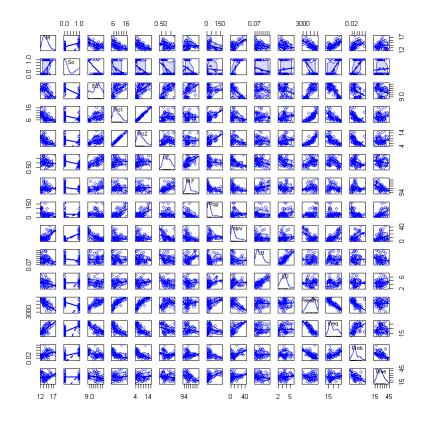


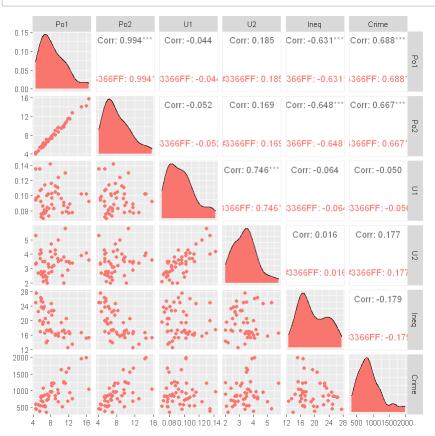
Scatterplot Example



```
In [144]: #library(car)
#All predictors scatterplot
scatterplotMatrix(~ M+So+Ed+Po1+Po2+LF+M.F+Pop+NW+U1+U2+Wealth+Ineq+Prob+Time, data = data)
#too tough to read
```

Warning message in smoother(x[subs], y[subs], col = smoother.args\$col[i], log.x = FALSE, : "could not fit smooth"





A matrix: 16 × 16 of type dbl

So-Ed;Ed-Ineq;P

| | M | So | Ed | Po1 | Po2 | LF | M.F | Pop | NW | U1 | U2 | Wealth | Ineq | Prob | Time | Crime |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|-------|-------|-------|-------|
| М | 1.00 | 0.58 | -0.53 | -0.51 | -0.51 | -0.16 | -0.03 | -0.28 | 0.59 | -0.22 | -0.24 | -0.67 | 0.64 | 0.36 | 0.11 | -0.09 |
| So | 0.58 | 1.00 | -0.70 | -0.37 | -0.38 | -0.51 | -0.31 | -0.05 | 0.77 | -0.17 | 0.07 | -0.64 | 0.74 | 0.53 | 0.07 | -0.09 |
| Ed | -0.53 | -0.70 | 1.00 | 0.48 | 0.50 | 0.56 | 0.44 | -0.02 | -0.66 | 0.02 | -0.22 | 0.74 | -0.77 | -0.39 | -0.25 | 0.32 |
| Po1 | -0.51 | -0.37 | 0.48 | 1.00 | 0.99 | 0.12 | 0.03 | 0.53 | -0.21 | -0.04 | 0.19 | 0.79 | -0.63 | -0.47 | 0.10 | 0.69 |
| Po2 | -0.51 | -0.38 | 0.50 | 0.99 | 1.00 | 0.11 | 0.02 | 0.51 | -0.22 | -0.05 | 0.17 | 0.79 | -0.65 | -0.47 | 0.08 | 0.67 |
| LF | -0.16 | -0.51 | 0.56 | 0.12 | 0.11 | 1.00 | 0.51 | -0.12 | -0.34 | -0.23 | -0.42 | 0.29 | -0.27 | -0.25 | -0.12 | 0.19 |
| M.F | -0.03 | -0.31 | 0.44 | 0.03 | 0.02 | 0.51 | 1.00 | -0.41 | -0.33 | 0.35 | -0.02 | 0.18 | -0.17 | -0.05 | -0.43 | 0.21 |
| Pop | -0.28 | -0.05 | -0.02 | 0.53 | 0.51 | -0.12 | -0.41 | 1.00 | 0.10 | -0.04 | 0.27 | 0.31 | -0.13 | -0.35 | 0.46 | 0.34 |
| NW | 0.59 | 0.77 | -0.66 | -0.21 | -0.22 | -0.34 | -0.33 | 0.10 | 1.00 | -0.16 | 0.08 | -0.59 | 0.68 | 0.43 | 0.23 | 0.03 |
| U1 | -0.22 | -0.17 | 0.02 | -0.04 | -0.05 | -0.23 | 0.35 | -0.04 | -0.16 | 1.00 | 0.75 | 0.04 | -0.06 | -0.01 | -0.17 | -0.05 |
| U2 | -0.24 | 0.07 | -0.22 | 0.19 | 0.17 | -0.42 | -0.02 | 0.27 | 0.08 | 0.75 | 1.00 | 0.09 | 0.02 | -0.06 | 0.10 | 0.18 |
| Wealth | -0.67 | -0.64 | 0.74 | 0.79 | 0.79 | 0.29 | 0.18 | 0.31 | -0.59 | 0.04 | 0.09 | 1.00 | -0.88 | -0.56 | 0.00 | 0.44 |
| Ineq | 0.64 | 0.74 | -0.77 | -0.63 | -0.65 | -0.27 | -0.17 | -0.13 | 0.68 | -0.06 | 0.02 | -0.88 | 1.00 | 0.47 | 0.10 | -0.18 |
| Prob | 0.36 | 0.53 | -0.39 | -0.47 | -0.47 | -0.25 | -0.05 | -0.35 | 0.43 | -0.01 | -0.06 | -0.56 | 0.47 | 1.00 | -0.44 | -0.43 |
| Time | 0.11 | 0.07 | -0.25 | 0.10 | 0.08 | -0.12 | -0.43 | 0.46 | 0.23 | -0.17 | 0.10 | 0.00 | 0.10 | -0.44 | 1.00 | 0.15 |
| Crime | -0.09 | -0.09 | 0.32 | 0.69 | 0.67 | 0.19 | 0.21 | 0.34 | 0.03 | -0.05 | 0.18 | 0.44 | -0.18 | -0.43 | 0.15 | 1.00 |

CHECKING THEORY: EIGEN VALUES AND VECTORS

```
In [23]: #Eigen values & vectors
X <- as.matrix(data)
X_Trans_X <- t(X)%*%X
ev <- eigen(X_Trans_X)
#Eigenvalues
ev$values
#Eigenvectors
#ev$vectors</pre>
```

```
1380740012.90473 · 5384801.42652306 · 59330.4704502898 · 20626.7567816144 · 2187.41595483789 · 1677.27232653165 · 151.07239653922 · 88.021838531293 · 41.095517797618 · 24.8002928942946 · 9.65667653420426 · 2.80155722968799 · 1.96793750526589 · 0.0256320722571422 · 0.00769541551602604 · 0.00252687018876677
```

```
In [136]: #Find Determinant of X(Transpose)*X -lamba(identity matrix) for 1st Eigen value and vector and test the theory
#diag(ncol(data))
identity_mat <-diag(ncol(data))
det(X_Trans_X-ev$values[2]*identity_mat)</pre>
```

4.25568663425159e+94

```
In [206]: #Caret to transform predictors, box cox tranformation and find princial components
           pca data <-caret::preProcess(</pre>
                                   data %>% dplyr::select(-Crime),
                                   method=c('center','scale','nzv','pca'))
           pca data # suggestion is to use PCA for 9 components out of 15 components
           #linear combination of predictors for each principal component
           pca data$rotation
```

Created from 47 samples and 15 variables

Pre-processing:

- centered (15)
- ignored (0)
- principal component signal extraction (15)
- scaled (15)

PCA needed 9 components to capture 95 percent of the variance

A matrix: 15 × 9 of type dbl

| | PC1 | PC2 | PC3 | PC4 | PC5 | PC6 | PC7 | PC8 | PC9 |
|--------|-------------|-------------|---------------|-------------|-------------|--------------|-------------|-------------|----------|
| M | -0.30371194 | 0.06280357 | 0.1724199946 | -0.02035537 | -0.35832737 | -0.449132706 | -0.15707378 | -0.55367691 | 0.154747 |
| So | -0.33088129 | -0.15837219 | 0.0155433104 | 0.29247181 | -0.12061130 | -0.100500743 | 0.19649727 | 0.22734157 | -0.65599 |
| Ed | 0.33962148 | 0.21461152 | 0.0677396249 | 0.07974375 | -0.02442839 | -0.008571367 | -0.23943629 | -0.14644678 | -0.44326 |
| Po1 | 0.30863412 | -0.26981761 | 0.0506458161 | 0.33325059 | -0.23527680 | -0.095776709 | 0.08011735 | 0.04613156 | 0.194254 |
| Po2 | 0.31099285 | -0.26396300 | 0.0530651173 | 0.35192809 | -0.20473383 | -0.119524780 | 0.09518288 | 0.03168720 | 0.195120 |
| LF | 0.17617757 | 0.31943042 | 0.2715301768 | -0.14326529 | -0.39407588 | 0.504234275 | -0.15931612 | 0.25513777 | 0.143934 |
| M.F | 0.11638221 | 0.39434428 | -0.2031621598 | 0.01048029 | -0.57877443 | -0.074501901 | 0.15548197 | -0.05507254 | -0.24378 |
| Pop | 0.11307836 | -0.46723456 | 0.0770210971 | -0.03210513 | -0.08317034 | 0.547098563 | 0.09046187 | -0.59078221 | -0.20244 |
| NW | -0.29358647 | -0.22801119 | 0.0788156621 | 0.23925971 | -0.36079387 | 0.051219538 | -0.31154195 | 0.20432828 | 0.189841 |
| U1 | 0.04050137 | 0.00807439 | -0.6590290980 | -0.18279096 | -0.13136873 | 0.017385981 | -0.17354115 | -0.20206312 | 0.020693 |
| U2 | 0.01812228 | -0.27971336 | -0.5785006293 | -0.06889312 | -0.13499487 | 0.048155286 | -0.07526787 | 0.24369650 | 0.055760 |
| Wealth | 0.37970331 | -0.07718862 | 0.0100647664 | 0.11781752 | 0.01167683 | -0.154683104 | -0.14859424 | 0.08630649 | -0.23196 |

| | PC1 | PC2 | PC3 | PC4 | PC5 | PC6 | PC7 | PC8 | PC9 |
|------|-------------|-------------|---------------|-------------|-------------|--------------|-------------|-------------|----------|
| Ineq | -0.36579778 | -0.02752240 | -0.0002944563 | -0.08066612 | -0.21672823 | 0.272027031 | 0.37483032 | 0.07184018 | -0.02494 |
| Prob | -0.25888661 | 0.15831708 | -0.1176726436 | 0.49303389 | 0.16562829 | 0.283535996 | -0.56159383 | -0.08598908 | -0.05306 |
| Time | -0.02062867 | -0.38014836 | 0.2235664632 | -0.54059002 | -0.14764767 | -0.148203050 | -0.44199877 | 0.19507812 | -0.23551 |

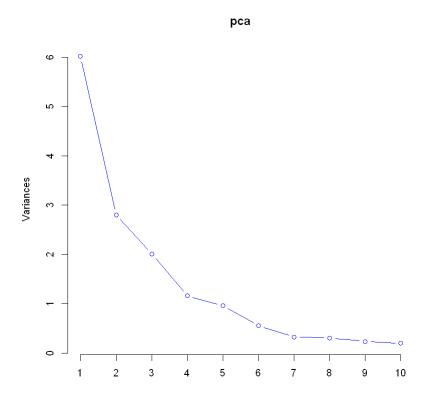
PCA COMPUTATION USING PRCOMP

In [207]: pca <- prcomp(data[,1:15], scale. = TRUE)
 summary(pca)</pre>

Importance of components:

PC1 PC5 PC6 PC2 PC3 PC4 PC7 2.4534 1.6739 1.4160 1.07806 0.97893 0.74377 0.56729 Standard deviation Proportion of Variance 0.4013 0.1868 0.1337 0.07748 0.06389 0.03688 0.02145 Cumulative Proportion 0.4013 0.5880 0.7217 0.79920 0.86308 0.89996 0.92142 PC8 PC9 PC10 PC11 PC12 PC13 PC14 Standard deviation 0.55444 0.48493 0.44708 0.41915 0.35804 0.26333 0.2418 Proportion of Variance 0.02049 0.01568 0.01333 0.01171 0.00855 0.00462 0.0039 Cumulative Proportion 0.94191 0.95759 0.97091 0.98263 0.99117 0.99579 0.9997 PC15

Standard deviation 0.06793 Proportion of Variance 0.00031 Cumulative Proportion 1.00000 In [150]: plot(pca, type="lines",col="blue")



In [208]: #get first 9 Principal componet PCs <- pca\$x[,1:9]</pre> head(PCs,5)

A matrix: 5 × 9 of type dbl

| PC1 | PC2 | PC3 | PC4 | PC5 | PC6 | PC7 | PC8 | PC9 |
|-----------|------------|-------------|-------------|-------------|------------|-------------|--------------|-------------|
| -4.199284 | -1.0938312 | -1.11907395 | 0.67178115 | 0.05528338 | 0.3073383 | -0.56640816 | -0.007801727 | 0.22350995 |
| 1.172663 | 0.6770136 | -0.05244634 | -0.08350709 | -1.17319982 | -0.5832373 | 0.19561119 | 0.154566472 | 0.43677720 |
| -4.173725 | 0.2767750 | -0.37107658 | 0.37793995 | 0.54134525 | 0.7187223 | 0.10330693 | 0.351138883 | 0.06299232 |
| 3.834962 | -2.5769060 | 0.22793998 | 0.38262331 | -1.64474650 | 0.7294884 | 0.26699499 | -1.547460841 | -0.37954181 |
| 1.839300 | 1.3309856 | 1.27882805 | 0.71814305 | 0.04159032 | -0.3940902 | 0.07050766 | -0.543237437 | 0.22463245 |

Build linear regression model with the first 9 principal components

```
In [209]: PCcrime <- cbind(PCs, data$Crime) #Create new data matrix with first 5 PCs and crime rate
          #PCcrime
          model <- lm(V10~., data = as.data.frame(PCcrime)) #Create regression model based on 5 principal components
          #modeL
          summary(model) #PCA: Adjusted R squared :0.69; R squared :0.61
          Call:
          lm(formula = V10 ~ ., data = as.data.frame(PCcrime))
          Residuals:
             Min
                    1Q Median
                                        Max
          -455.9 -132.5 21.5 139.9 393.0
          Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
                       905.09
                                   34.91 25.928 < 2e-16 ***
          (Intercept)
          PC1
                        65.22
                                   14.38 4.535 5.88e-05 ***
          PC2
                        -70.08
                                   21.08 -3.325 0.00201 **
          PC3
                        25.19
                                   24.92 1.011 0.31857
          PC4
                        69.45
                                   32.73 2.122 0.04061 *
          PC5
                       -229.04
                                   36.04 -6.355 2.08e-07 ***
          PC6
                       -60.21
                                   47.44 -1.269 0.21228
                       117.26
          PC7
                                   62.20
                                         1.885 0.06728 .
                        28.72
                                   63.64 0.451 0.65446
          PC8
          PC9
                       -37.18
                                   72.76 -0.511 0.61244
          Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
          Residual standard error: 239.3 on 37 degrees of freedom
          Multiple R-squared: 0.692,
                                         Adjusted R-squared: 0.6171
          F-statistic: 9.239 on 9 and 37 DF, p-value: 3.588e-07
```

Specify the new model in terms of the original variables (not the principal components), UNSCALE data in revrse

In [212]: # Get coefficients in terms of original data from PCA coefficients model\$coefficient beta0 <- model\$coefficients[1] #intercept</pre> betas <- model\$coefficients[2:10] #PC</pre> print(paste("Intercept Co-efficient: ",beta0)) betas

> (Intercept): 905.085106382979 PC1: 65.215930138666 PC2: -70.0831185497858 PC3: 25.1940780425772 PC4: 69.446030796839 PC5: -229.042822001686 PC6: -60.2132861756709 PC7: 117.255897957602 PC8: 28.7165560702805 PC9: -37.1756419454256

[1] "Intercept Co-efficient: 905.085106382979"

PC1: 65.215930138666 PC2: -70.0831185497858 PC3: 25.1940780425772 PC4: 69.446030796839 PC5: -229.042822001686 PC6: -60.2132861756709 PC7: 117.255897957602 PC8: 28.7165560702805 PC9: -37.1756419454256

In [213]: # Transform the PC coefficients into coefficients for the original variables

#pca\$rotation[,1:4] alphas <- pca\$rotation[,1:9] %*% betas t(alphas) #tranform PC co-efficients into original variable co-efficients

A matrix: 1 × 15 of type dbl

| М | So | Ed | Po1 | Po2 | LF | M.F | Pop | NW | U1 | U2 | Wealth | Ineq |
|----------|----------|----------|----------|----------|----------|---------|----------|----------|-----------|----------|----------|--------|
| 47.76773 | 97.85583 | 4.661941 | 126.6093 | 123.4647 | 29.18835 | 138.325 | 27.10574 | 57.26768 | -25.10066 | 25.54553 | 38.92574 | 52.665 |

```
In [214]: # Above is SCALED coefficients
          #To convert into original
          # When scaling, this function subtracts the mean and divides by the standard deviation, for each variable.
          # So, alpha * (x - mean)/sd = originalAlpha * x.
           # That means:
          # (1) originalAlpha = alpha/sd
          # (2) we have to modify the constant term a0 by alpha*mean/sd
          originalAlpha <- alphas/sapply(data[,1:15],sd)</pre>
          originalBeta0 <- beta0 - sum(alphas*sapply(data[,1:15],mean)/sapply(data[,1:15],sd))</pre>
           originalBeta0
```

(Intercept): -5742.13576339317

In [216]: # Here are the coefficients for unscaled data: t(originalAlpha)

A matrix: 1 × 15 of type dbl

| M | So | Ed | Po1 | Po2 | LF | M.F | Рор | NW | U1 | U2 | Wealth | In |
|----------|----------|----------|----------|----------|----------|----------|-----------|----------|-----------|----------|------------|----|
| 38.00853 | 204.3025 | 4.167285 | 42.60219 | 44.15555 | 722.2726 | 46.94177 | 0.7119751 | 5.569224 | -1392.255 | 30.24768 | 0.04034134 | 13 |

localhost:8888/notebooks/Week6-HW-PCA-Regression.ipynb#

```
In [217]: #ORIGINAL MODEL Now let's compare with the regression model from the previous homework
          model2 <- lm( Crime ~ ., data = data)</pre>
          summary(model2) #R2 .80 and Adju R2 .70 (vs. #PCA: Adjusted R squared :0.69; R squared :0.61)
          Call:
          lm(formula = Crime ~ ., data = data)
          Residuals:
             Min
                      1Q Median
                                      30
                                             Max
          -395.74 -98.09 -6.69 112.99 512.67
          Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
          (Intercept) -5.984e+03 1.628e+03 -3.675 0.000893 ***
                      8.783e+01 4.171e+01 2.106 0.043443 *
         Μ
          So
                     -3.803e+00 1.488e+02 -0.026 0.979765
                      1.883e+02 6.209e+01 3.033 0.004861 **
          Ed
                      1.928e+02 1.061e+02 1.817 0.078892 .
          Po1
          Po2
                     -1.094e+02 1.175e+02 -0.931 0.358830
          LF
                     -6.638e+02 1.470e+03 -0.452 0.654654
          M.F
                     1.741e+01 2.035e+01 0.855 0.398995
                     -7.330e-01 1.290e+00 -0.568 0.573845
          Pop
                     4.204e+00 6.481e+00 0.649 0.521279
          NW
                     -5.827e+03 4.210e+03 -1.384 0.176238
          U1
          U2
                      1.678e+02 8.234e+01 2.038 0.050161 .
          Wealth
                      9.617e-02 1.037e-01 0.928 0.360754
                      7.067e+01 2.272e+01 3.111 0.003983 **
          Ineq
                     -4.855e+03 2.272e+03 -2.137 0.040627 *
          Prob
                     -3.479e+00 7.165e+00 -0.486 0.630708
          Time
          Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
          Residual standard error: 209.1 on 31 degrees of freedom
         Multiple R-squared: 0.8031,
                                        Adjusted R-squared: 0.7078
          F-statistic: 8.429 on 15 and 31 DF, p-value: 3.539e-07
```

Compare quality of PCA Model vs. Original Model based on straightforward regression model

```
In [ ]: # These results suggest that we are better off using a more straightforward regression model
                                       # instead of PCA before using regression.
                                       # If we had used all 15 principal components, we would have obtained
                                       # an R-squared value of 0.803, which is the same R-squared value when using all
                                        # 15 regular predictors in a basic linear regression model.
In [166]:
                                          # all possibilities: for i=1..15, run a regression using the first i principal components
                                        r2 <- numeric(15) # create a vector to store the R-squared values
                                        for (i in 1:15) {
                                               pclist <- pca$x[,1:i] # use the first i prinicipal components</pre>
                                               pcc <- cbind(data[,16],pclist) # create data set</pre>
                                              model <- lm(V1~.,data = as.data.frame(pcc)) # fit model</pre>
                                               r2[i] <- 1 - sum(model$residuals^2)/sum((data$Crime - mean(data$Crime))^2) # calculate R-squared
                                        r2
                                                      0.658602327815706 \cdot \quad 0.688181873005123 \cdot \quad 0.689876527649446 \cdot \quad 0.692049141529042 \cdot \quad 0.696287251226172 \cdot \quad 0.69628726172 \cdot \quad 0.696287251226172 \cdot \quad 0.696287251226172 \cdot \quad 0.69628726172 \cdot \quad 0.696287251226172 \cdot \quad 0.696287251226172 \cdot \quad 0.696287251226172 \cdot \quad 0.696287251226172 \cdot \quad 0.69628726172 \cdot \quad 0.696287
```

 $0.697386539400947 \cdot 0.769265609989774 \cdot 0.77236636338524 \cdot 0.791144655274498 \cdot 0.803086758316909$

In [167]: # All PCA first "n" components generate model R-square less than original model R square value of 0.80. #As noted earlier, with smaller subset of data, we are running into overfitting vielding high R square for origin

Predict new observations' Crime fit

crime prediction for new point of observation based on Original vs PCA model

```
In [218]: #datapoint from previous homework
          new_observe = data.frame(
                  M = 14.0,
                  So = 0,
                  Ed = 10.0,
                  Po1 = 12.0,
                  Po2 = 15.5,
                  LF = 0.640,
                  M.F = 94.0,
                  Pop = 150,
                  NW = 1.1,
                  U1 = 0.120,
                  U2 = 3.6,
                  Wealth = 3200,
                  Ineq = 20.1,
                  Prob = 0.04,
                  Time = 39.0
```

In [219]: #ORIGINAL Prediction #model2 <- lm(Crime ~ ., data = data) crime_pred_orig = predict(model2, new_observe) %>% as_tibble() crime_pred_orig

A tibble: 1

× 1

value

<dbl>

155.4349

In [221]: #PCA FITTED PREDICTION library(pls)# to get pcr fit # Run principal component regression function with only the first 4 principal components numcomp <- 9 pcr.fit <- pcr(Crime ~ ., data = data, scale = TRUE, ncomp = numcomp) summary(pcr.fit) head(pcr.fit\$scores,5)</pre>

Data: X dimension: 47 15 Y dimension: 47 1

Fit method: svdpc

Number of components considered: 9
TRAINING: % variance explained

1 comps 2 comps 3 comps 4 comps 5 comps 6 comps 7 comps 8 comps 72.17 Χ 40.13 58.81 79.92 86.31 90.00 92.14 94.19 Crime 17.11 27.16 30.91 64.52 65.86 68.82 68.99 26.31

9 comps X 95.76 Crime 69.20

A matrix: 5 × 9 of type dbl

| | Comp 1 | Comp 2 | Comp 3 | Comp 4 | Comp 5 | Comp 6 | Comp 7 | Comp 8 | Comp 9 |
|---|-----------|------------|-------------|-------------|-------------|------------|-------------|--------------|-------------|
| • | -4.199284 | -1.0938312 | -1.11907395 | 0.67178115 | 0.05528338 | 0.3073383 | -0.56640816 | -0.007801727 | 0.22350995 |
| 2 | 1.172663 | 0.6770136 | -0.05244634 | -0.08350709 | -1.17319982 | -0.5832373 | 0.19561119 | 0.154566472 | 0.43677720 |
| ; | -4.173725 | 0.2767750 | -0.37107658 | 0.37793995 | 0.54134525 | 0.7187223 | 0.10330693 | 0.351138883 | 0.06299232 |
| 4 | 3.834962 | -2.5769060 | 0.22793998 | 0.38262331 | -1.64474650 | 0.7294884 | 0.26699499 | -1.547460841 | -0.37954181 |
| | 1.839300 | 1.3309856 | 1.27882805 | 0.71814305 | 0.04159032 | -0.3940902 | 0.07050766 | -0.543237437 | 0.22463245 |

```
In [222]: #use model to make predictions on a test set
          test <- pcr(Crime ~ ., data = data, scale=TRUE, validation="CV") #Cross validation
          summary(test)
          #Root Mean Square error(RMSE)
          #If we use interceptonly , the square error is 390.9
          #If we use first components , the square error is dropped to 364
          #If we add in second components , the square error is dropped to 354
                  X dimension: 47 15
          Data:
                  Y dimension: 47 1
          Fit method: svdpc
          Number of components considered: 15
          VALIDATION: RMSEP
          Cross-validated using 10 random segments.
                 (Intercept) 1 comps 2 comps 3 comps 4 comps 5 comps 6 comps
          CV
                       390.9
                                364.8
                                         354.1
                                                  364.0
                                                           366.9
                                                                    262.5
                                                                             262.8
                                         352.6
                       390.9
                                364.0
                                                  362.2
                                                           367.0
                                                                    260.3
                                                                             260.4
          adjCV
                 7 comps 8 comps 9 comps 10 comps 11 comps 12 comps 13 comps
          CV
                                     270.6
                                                         298.8
                   259.6
                            266.7
                                               276.6
                                                                   264.5
                                                                             276.4
          adjCV
                   253.7
                            263.7
                                     267.7
                                               273.2
                                                         298.6
                                                                   260.1
                                                                             271.9
                 14 comps 15 comps
                    271.2
          CV
                              266.2
          adjCV
                    265.7
                              261.0
          TRAINING: % variance explained
                 1 comps 2 comps 3 comps 4 comps 5 comps 6 comps 7 comps 8 comps
                                                                90.00
          Χ
                   40.13
                            58.81
                                     72.17
                                              79.92
                                                       86.31
                                                                         92.14
                                                                                  94.19
          Crime
                   17.11
                            26.31
                                     27.16
                                              30.91
                                                       64.52
                                                                65.86
                                                                         68.82
                                                                                  68.99
                 9 comps 10 comps 11 comps
                                              12 comps 13 comps 14 comps 15 comps
                   95.76
                                       98.26
                                                 99.12
                                                           99.58
                                                                     99.97
          Χ
                             97.09
                                                                              100.00
                   69.20
          Crime
                             69.63
                                       69.74
                                                 76.93
                                                           77.24
                                                                     79.11
                                                                               80.31
```

```
In [223]: pcr_pred <- predict(test, new_observe, ncomp=4)
    pcr_pred</pre>
```

predicted value of new data using PCR is 1112 vs. 155 crime data value using original regression with all the #PCA generated crime ratio 7 times more than the linear regression model with all predictors for the new data val

1112.67763659883