Applied Machine Learning

Discrete Markov Random Fields

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- Discrete Markov Random Fields
- Discrete Markov Random Fields for Denoising Grey-Level Images
- Discrete Markov Random Fields for Segmentation of Grey-Level Images

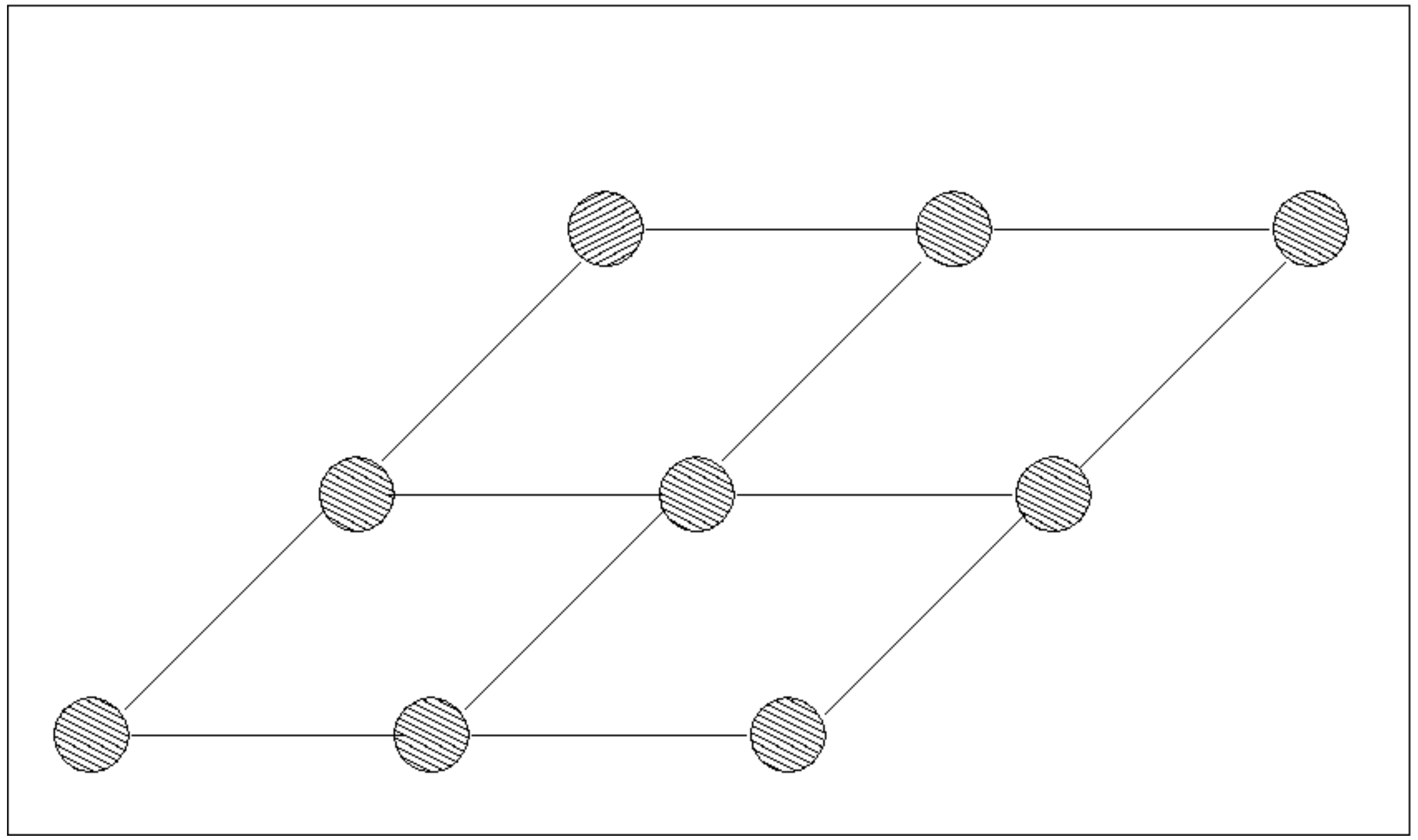
Discrete Markov Random Fields

- Markov Random Fields
  - particular case: Boltzmann Machines
- U_i: discrete random variable
  - k possible values
  - One-hot representation: $\mathbf{u}_i$
    - for values $\in \{0,1,2\}$
      - [1 0 0], [0 1 0], [0 0 1]
- Coupling function: $\theta(U_i, U_j) = \mathbf{u}_i^\top \Theta^{(i,j)} \mathbf{u}_j$
  - Coupling matrix: ${\Theta^{(i,j)}}_{[k \times l]}$
  - ${\Theta^{(i,j)}}_{m,n}$ is coupling between $U_i = m$ and $U_j = n$

- log of joint probability:
- $$\log P(U \mid \theta) = \left(\sum_i \sum_{j \in N(i)} \mathbf{u}_i^\top \Theta^{(i,j)} \mathbf{u}_j \right) - \log Z(\theta)$$
- $$Z(\theta) = \sum_{\text{values of } \mathbf{u}} e^{\sum_i \sum_{j \in N(i)} \mathbf{u}_i^\top \Theta^{i,j} \mathbf{u}_j}$$

Discrete Markov Random Field for Denoising Grey-Level Images

- U_i: discrete: 256 possible values, one-hot vectors with 256 components
  - H: hidden and true pixel values $\mathbf{h}_i$
  - X: observed pixel values affected by noise $\mathbf{x}_i$
- Coupling function: $\theta(U_i, U_j) = \mathbf{u}_i^\top \Theta^{(i,j)} \mathbf{u}_j$
- Coupling matrix: $\Theta^{(i,j)}{}_{[k \times l]}$
  - Coupling between hidden nodes
    - $\theta(H_i, H_j) = \mathbf{h}_i^\top \Theta_h^{(i,j)} \mathbf{h}_j$ $\qquad \Theta_h^{(i,j)} = cI$
  - Coupling between hidden and observed nodes
    - $\theta(H_i, X_i) = \mathbf{h}_i^\top \Theta_x^{(i,i)} \mathbf{x}_i = \mathbf{h}_i^\top \beta_i$ $\qquad \beta_i = (H_i - X_j)^2$



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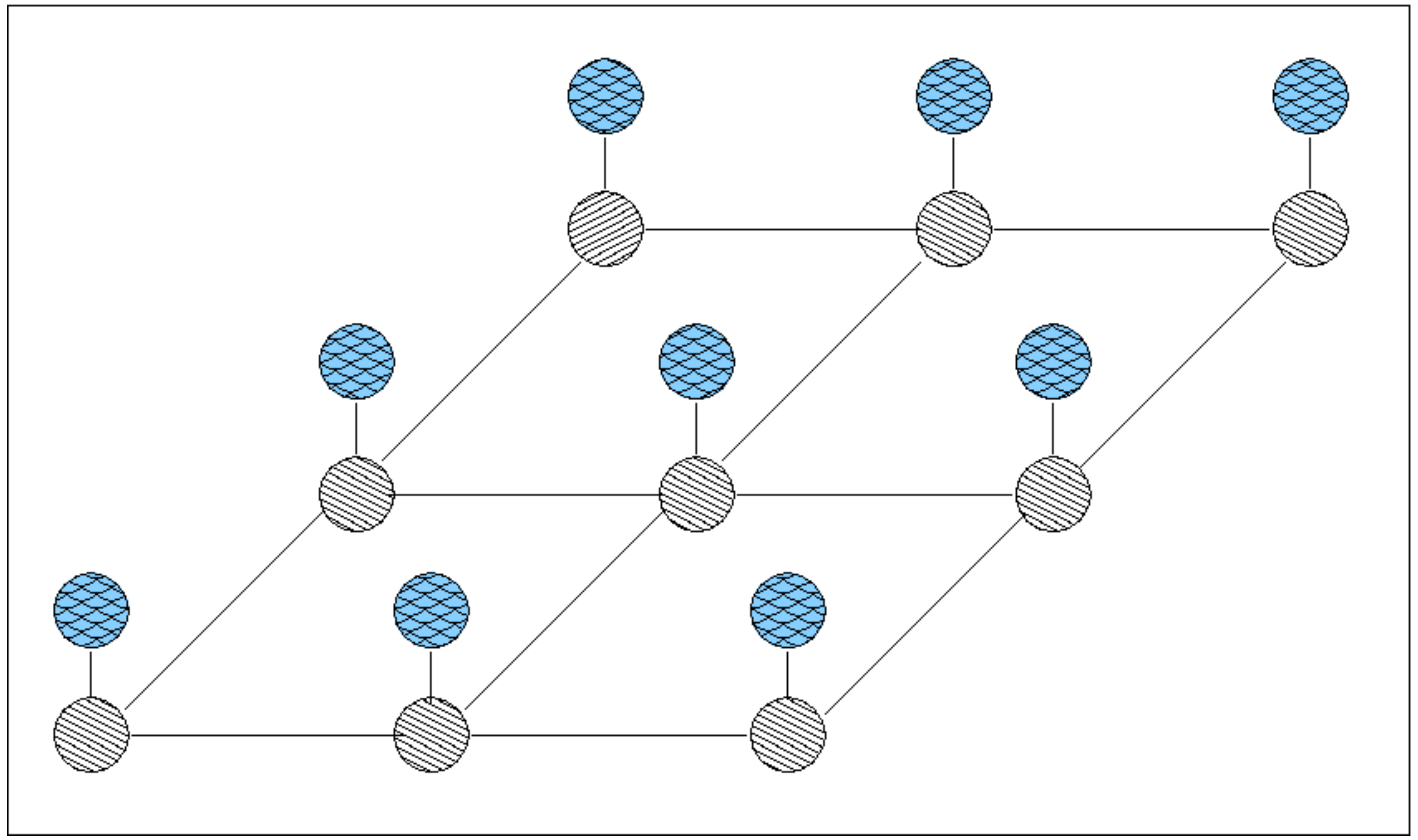
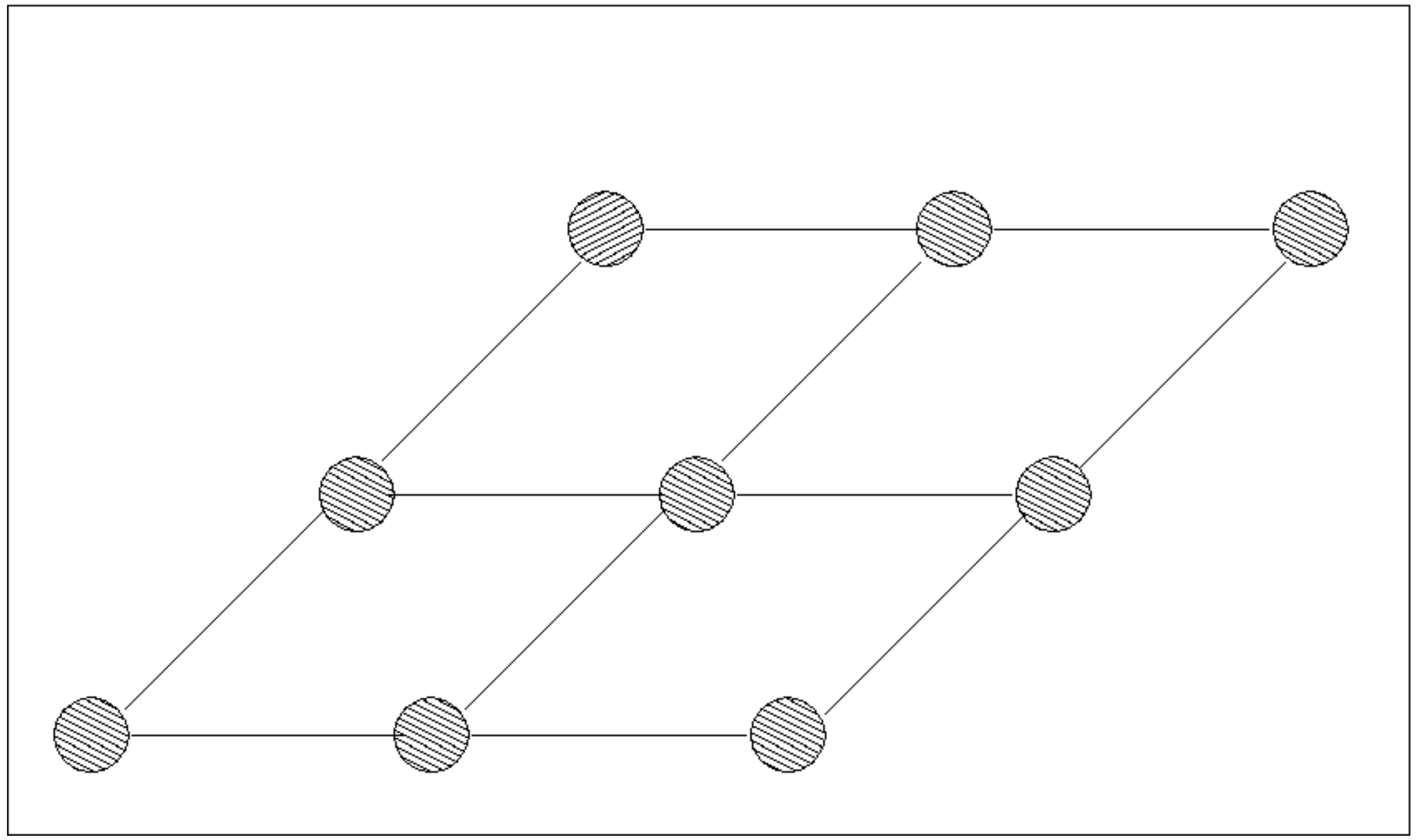
- Log of joint probability
  - $$logP(H|\theta) = \left(\sum_{i,j} \mathbf{h}_i^\top \Theta_h^{(i,j)} \mathbf{h}_j \right) + \sum_i \mathbf{h}_i^\top \beta_i - \log Z(\theta)$$



Discrete Markov Random Field for Image Segmentation

- U_i: discrete: one-hot vectors with 256 components
  - H: hidden pixel labels $\mathbf{h}_i$: as many values as labels
    - ["red", "blue", "magenta",…]
  - X: observed pixel values $\mathbf{x}_i$: 256 possible values,
- Coupling function: $\theta(U_i, U_j) = \mathbf{u}_i^\top \Theta^{(i,j)} \mathbf{u}_j$
- Coupling matrix: $\Theta^{(i,j)}{}_{[k \times l]}$
  - Coupling between hidden nodes: $\theta(H_i, H_j) = \mathbf{h}_i^\top \Theta_h^{(i,j)} \mathbf{h}_j$
  - Coupling between hidden and observed nodes: $\theta(H_i, X_i) = \mathbf{h}_i^\top \Theta_x^{(i,i)} \mathbf{x}_i$
- Log of joint probability
  - $$logP(H|\theta) = \left(\sum_{i,j} \mathbf{h}_i^\top \Theta_h^{(i,j)} \mathbf{h}_j \right) + \sum_i \mathbf{h}_i^\top \Theta_x^{(i,i)} \mathbf{x}_i - \log Z(\theta)$$



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