

Applied Machine Learning

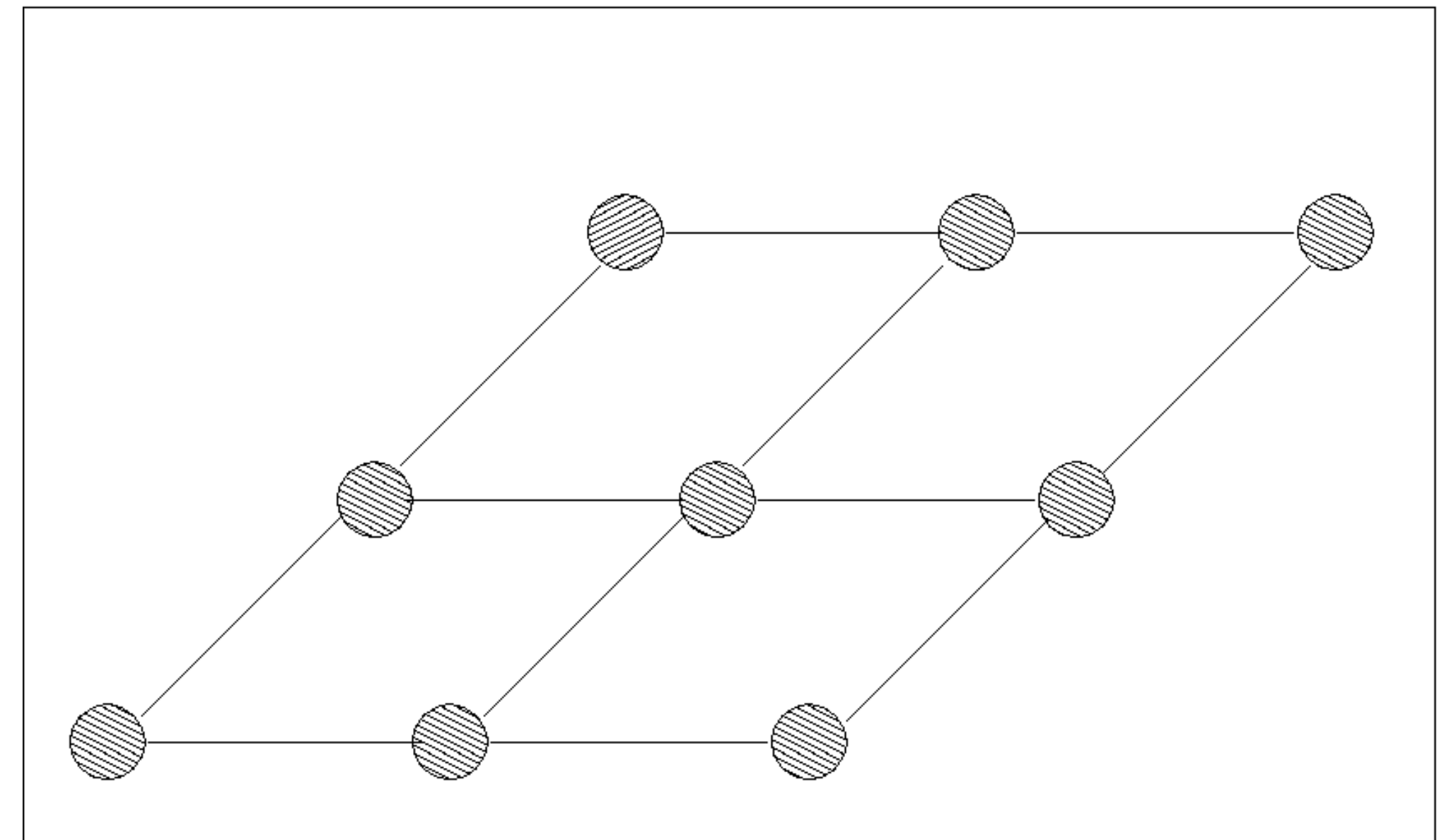
Mean Field Inference for Image Denoising

Mean Field Inference for Image Denoising

- Image Denoising as a Boltzmann Machine
- Optimization Problem
- Mean Field Inference

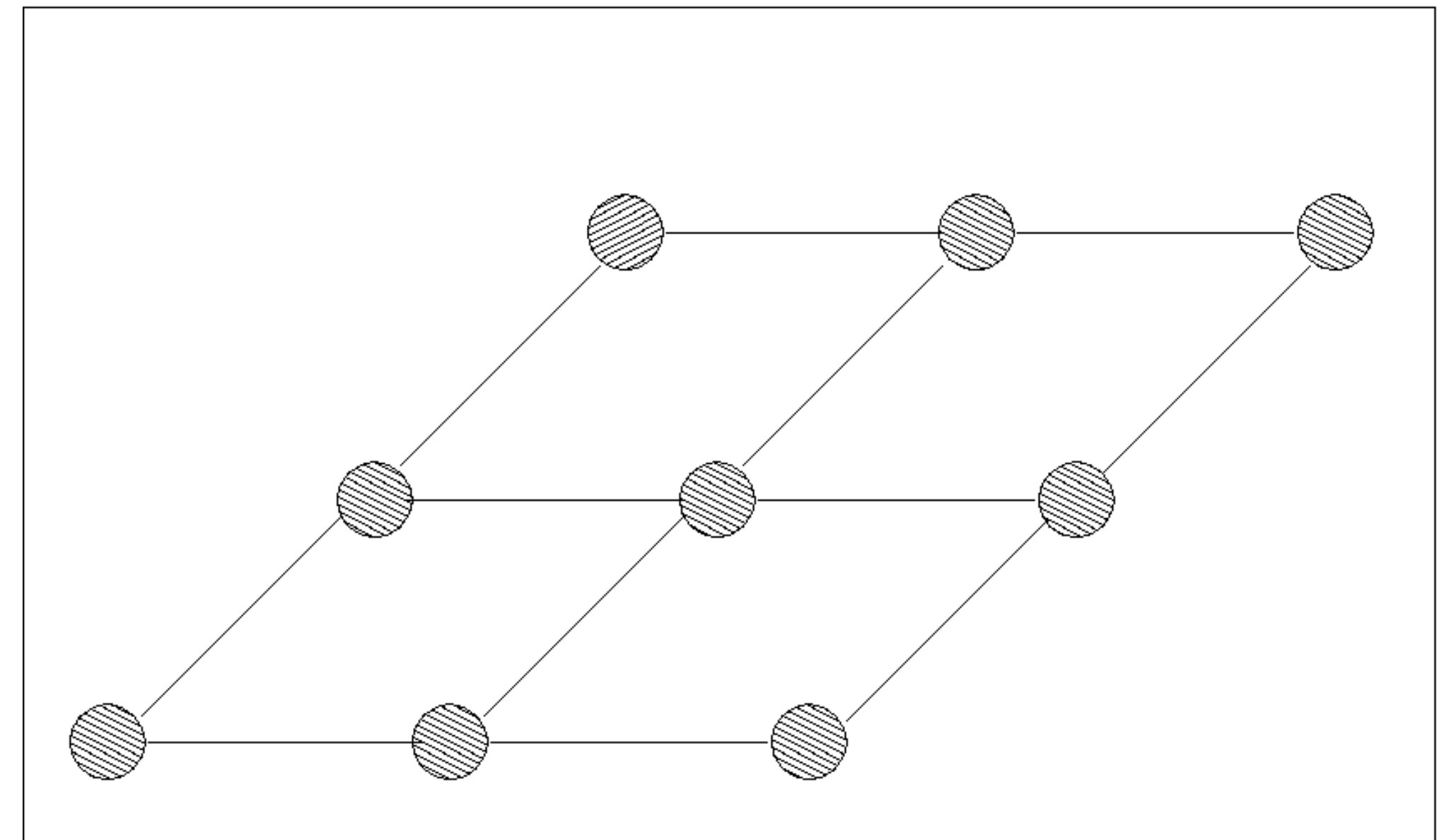
Binary Image

- Binary image
 - True pixel values [on,off]
 - Hidden nodes
 - Connections to adjacent nodes
 - local dependencies around each node
 - Noisy pixel values [on,off]
 - Observed nodes
 - Connections to associated true pixel node
 - Noise may flip the true pixel value



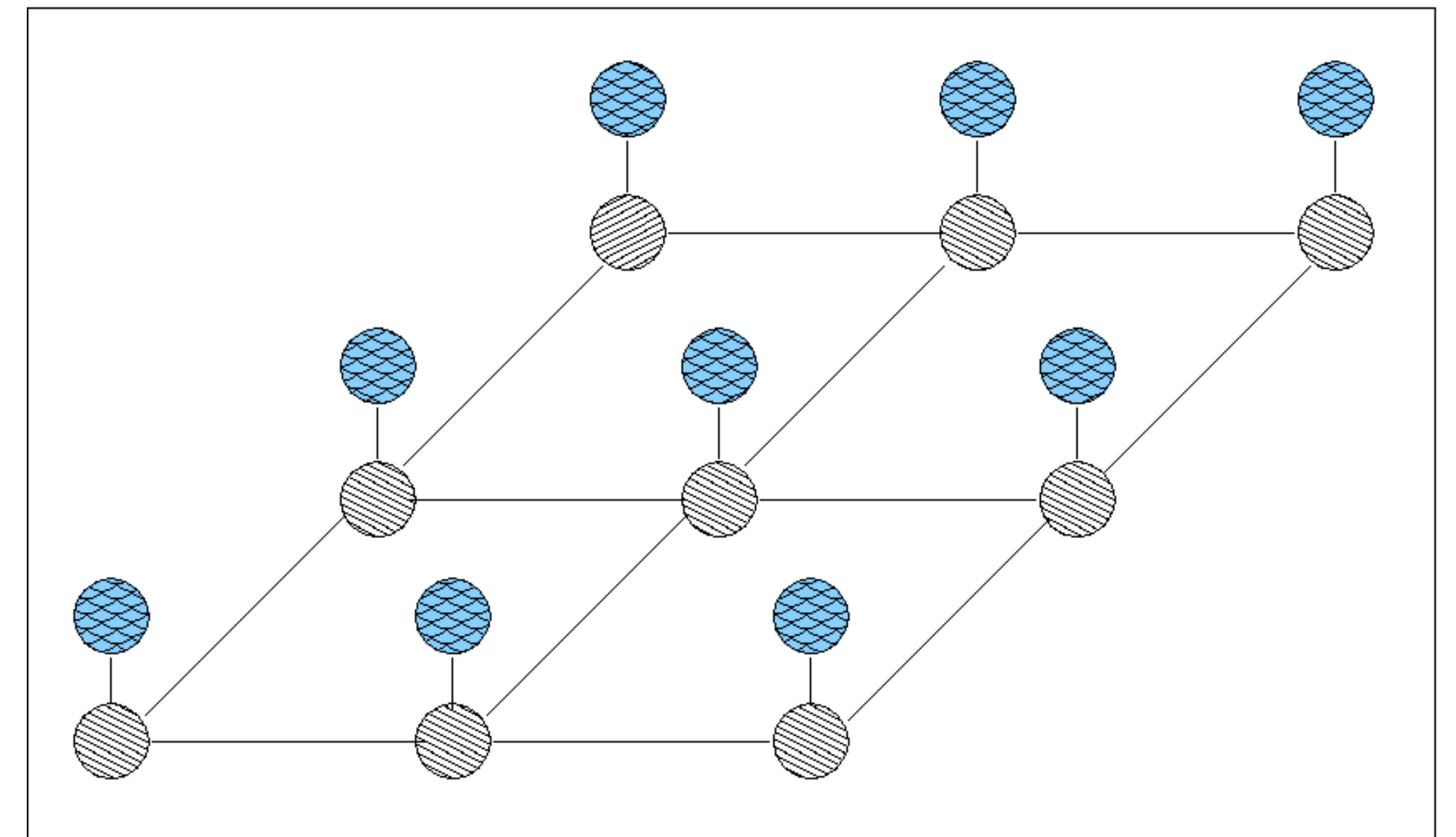
Boltzmann Machine for Denoising Images

- U : Set of nodes
 - Values $\{\text{off, on}\} \mapsto \{-1, 1\}$
 - H : hidden nodes
 - H_i : true value of pixel i
 - Local connections to adjacent nodes
 - X : observed nodes
 - X_i : value of pixel i affected by noise
 - Only connected to H_i
- θ : Weights in the connections



Boltzmann Machine for Denoising Images

- U : Set of nodes $\{-1, 1\}$
 - H : hidden nodes X : observed nodes
- θ : Weights in the connections
- Goal: find the values in hidden nodes that maximize
 - Maximum A Posteriori Inference
 - MAP Inference
 - $\log P(H | X, \theta) = -E(H, X | \theta) - \log \sum_H e^{-E(H, X | \theta)}$
 - Variational Inference
 - tractable distribution $Q(H; \theta)$
 - close to $P(H | X)$

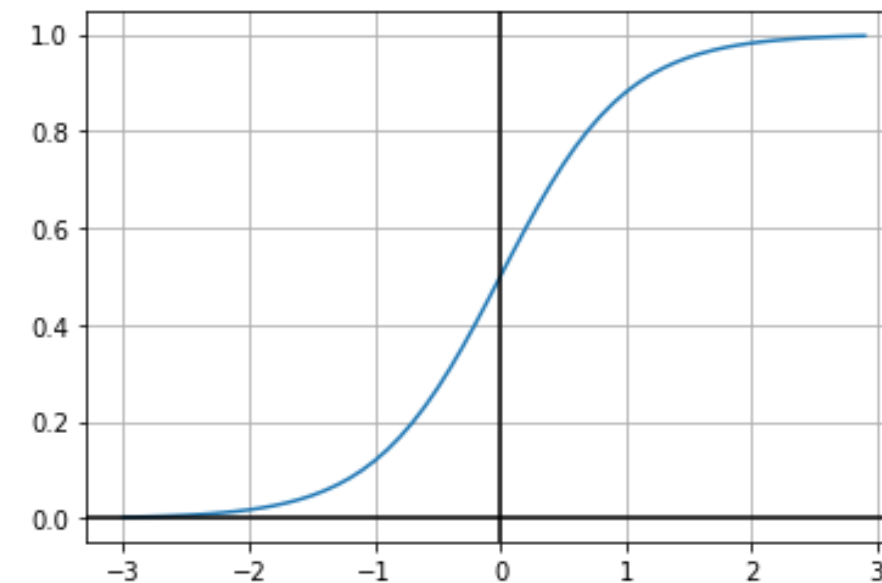


Mean Field Inference

- U : Set of nodes $\{-1, 1\}$
 - H : hidden nodes, X : observed nodes
- θ : Weights in the connections
- Goal: find the values in hidden nodes that maximize

$$p_i = \sum_{j \in N(i) \cap H} \theta_{i,j}(2\pi_j - 1) + \theta_i^X X_j$$

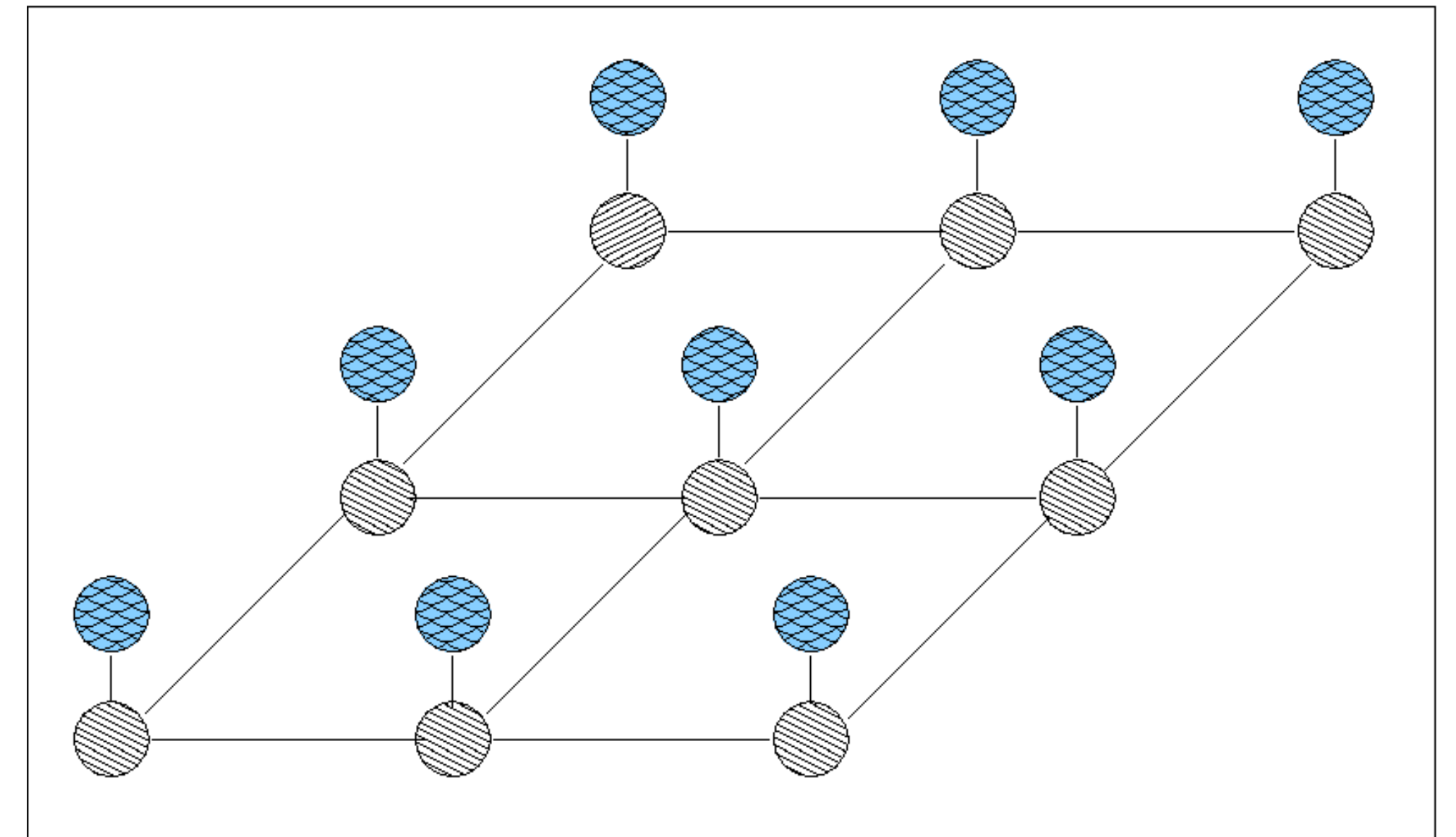
$$\pi_i = \frac{e^{p_i}}{e^{p_i} + e^{-p_i}}$$



- $\theta_{i,j} = k$: positive constant to favor agreement between adjacent nodes

- θ_i^X : based on noise. Probability of flipped pixel value:

$$p_{flip} = \frac{e^{-\theta_i^X}}{e^{-\theta_i^X} + e^{\theta_i^X}}$$



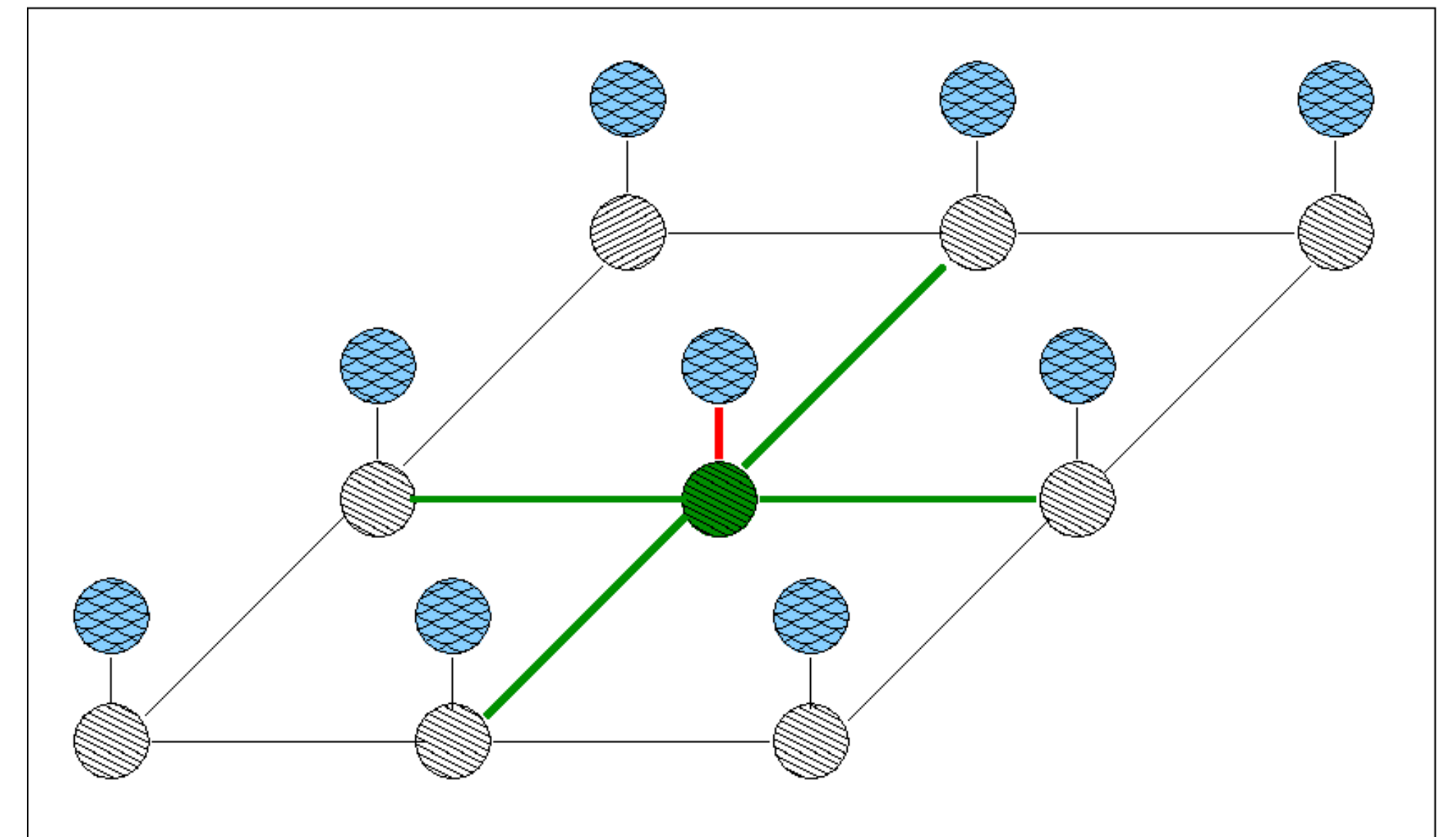
Mean Field Inference for Denoising Images

- U : Set of nodes $\{-1, 1\}$
 - H : hidden nodes, X : observed nodes
- θ : Weights in the connections
- Variational Inference. Mean Field Algorithm
 - Initialize π_i to random uniform distribution $[0, 1]$
 - While (π_i 's change more than some ϵ)

- update each $\pi_i = \frac{e^{p_i}}{e^{p_i} + e^{-p_i}}$

- with $p_i = \sum_{j \in N(i) \cap H} \theta_{i,j}(2\pi_j - 1) + \theta_i^X X_j$

- Estimated $H_i = \begin{cases} 1 & \pi_i \geq 0.5 \\ -1 & \pi_i < 0.5 \end{cases}$



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