Applied Machine Learning

Convolution for Images

Convolution for Images

- Convolution operator
- Pattern detection
- Invalid convolutions
- Strides
- Properties of convolution

Image Classification

- Small local patterns in objects
  - edges
  - corners
  - dominant colors
  - textures
- Composition of patterns
  - small figures or shapes
- More composition
  - larger objects

Convolutional Layer - Pattern Detection

- Domain Knowledge from Image Processiing
- Feature maps
  - Convolutional layers
- Stacks of convolutional layers
  - Learn patterns from patterns learned at previous layers
  - More complex patterns learned at each successive layer

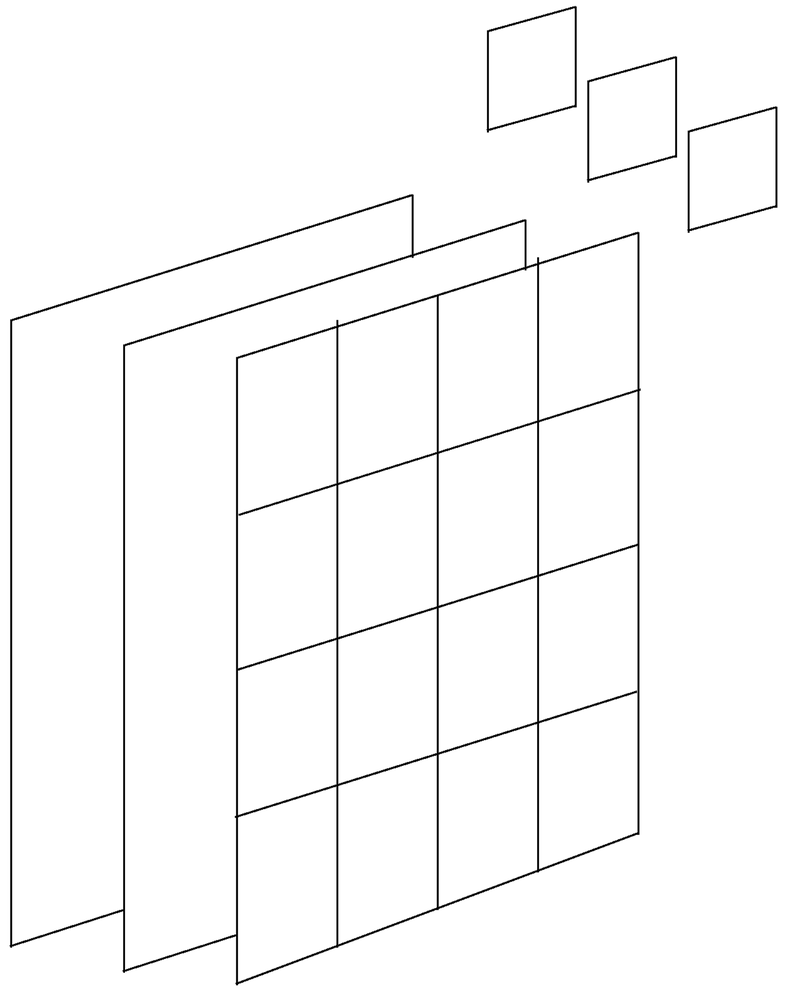
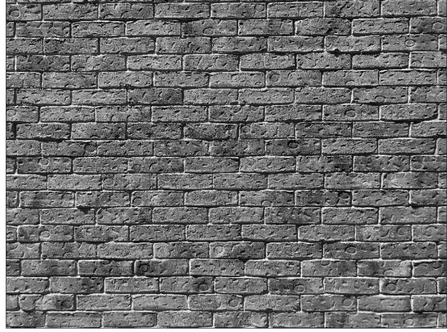


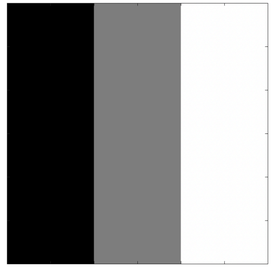
Image Processing

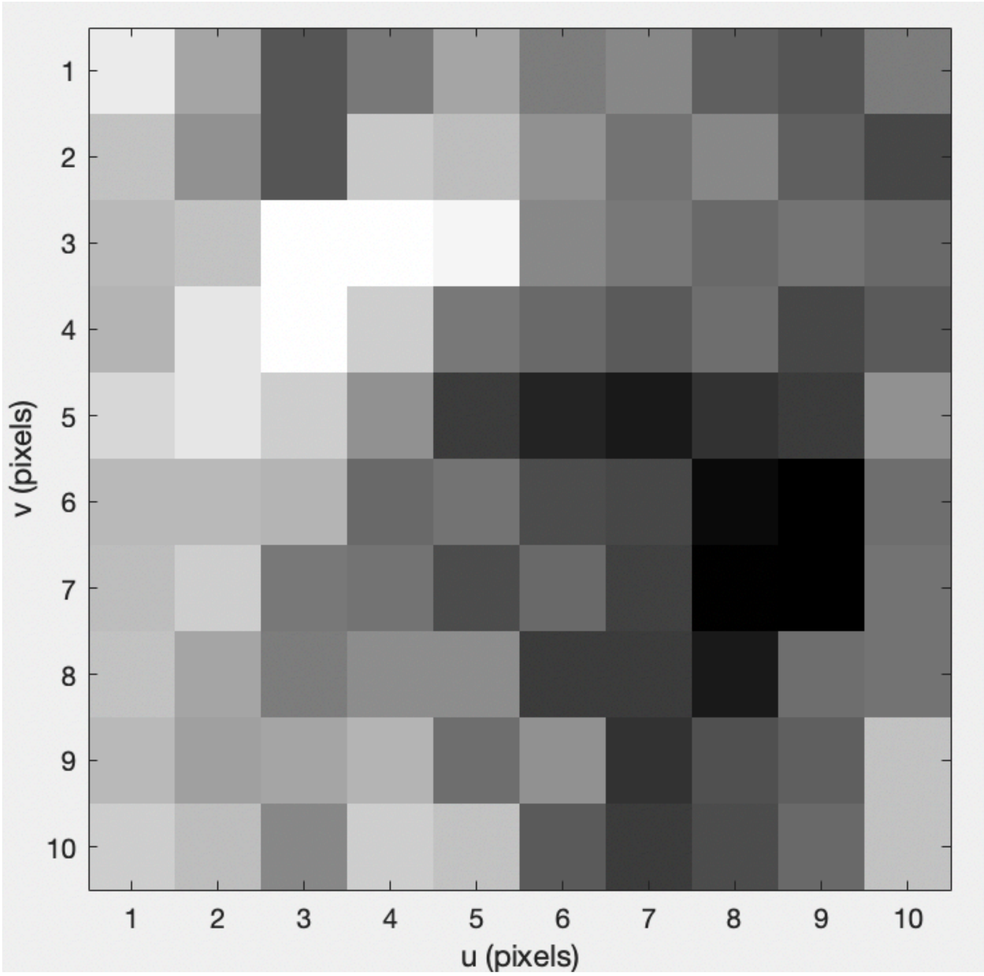


- Image: matrix I
  - Pixels: $I_{u,v}$
- Pattern: Kernel: matrix K
  - Weights: $K_{u,v}$

65	50	33	40	50	41	44	35	33	41
56	46	33	58	55	46	39	44	35	30
54	56	69	69	67	44	40	37	39	37
53	64	69	59	40	37	34	38	30	34
61	64	59	46	27	22	20	25	27	46
54	54	53	37	39	31	30	17	14	38
55	59	40	39	31	37	28	15	14	39
56	50	41	45	45	27	27	20	38	39
54	49	50	53	38	46	25	32	35	56
59	55	44	59	56	34	27	31	37	56

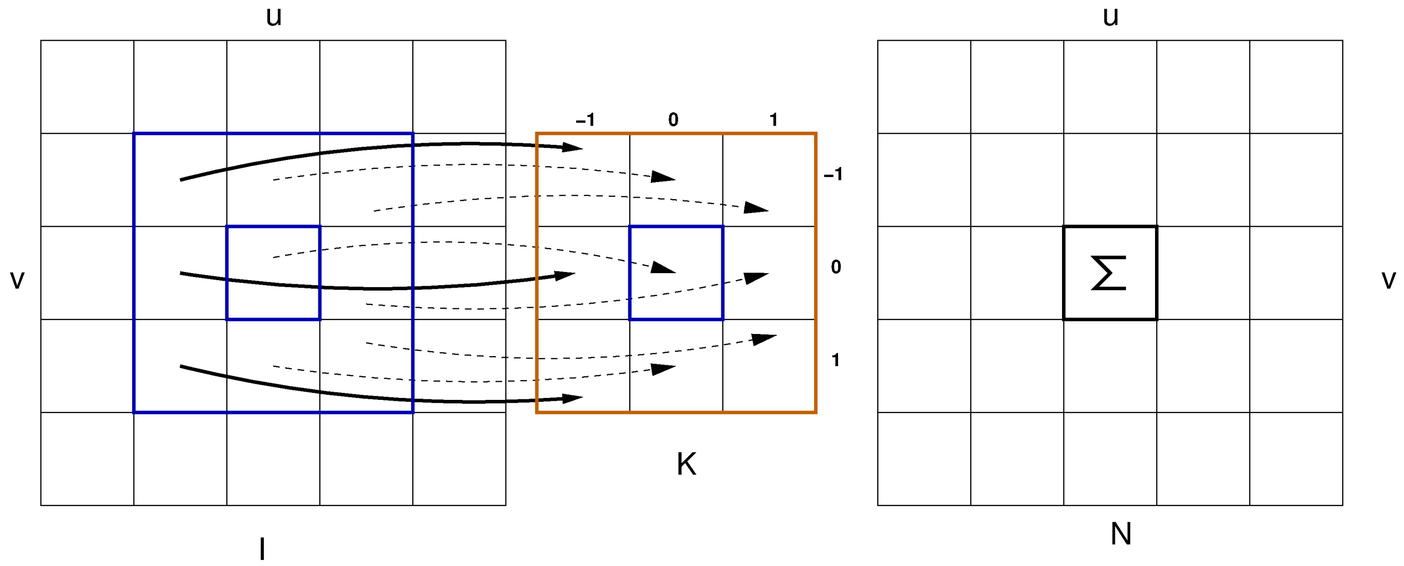
0.5000	0	−0.5000
0.5000	0	−0.5000
0.5000	0	−0.5000





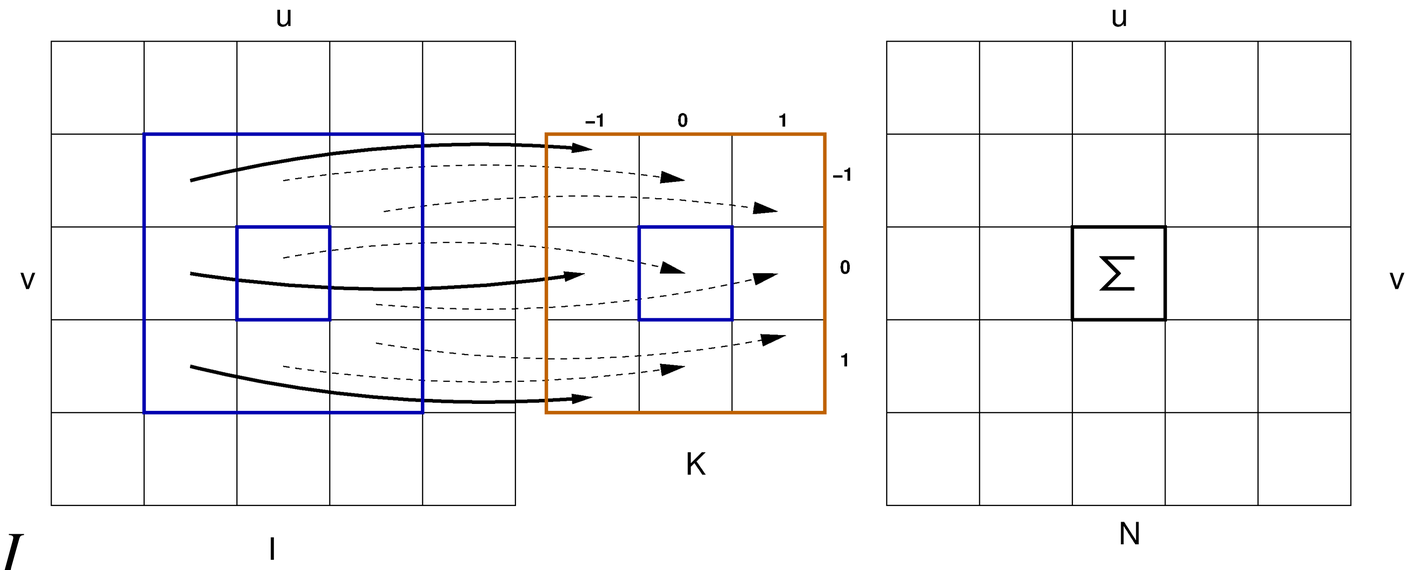
Convolution

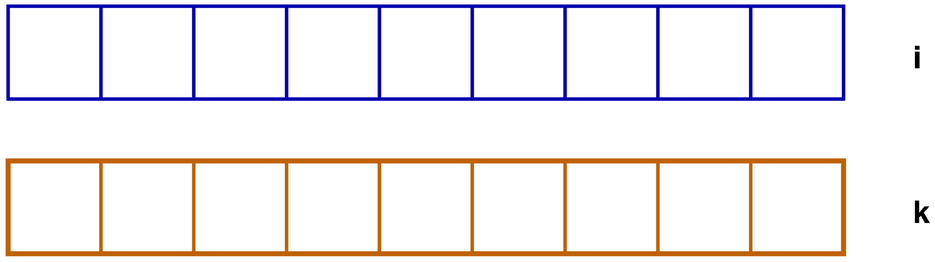
- Convolution
  - Slide kernel K over the image I
  - $N_{u,v} = \sum_{(i,j) \in K} I_{u+i,v+j} K_{i,j}$
  - compute for every pixel $\forall (u, v) \in I$
  - Common representations of convolution:
    - $N = K \otimes I$
    - $N = \text{conv}(K, I)$
- Pixels at the output image encode information around them as defined by the weights in the kernel



Pattern Detection through Convolution

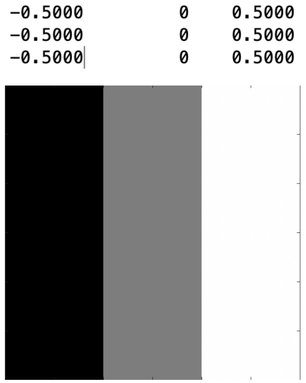
- Convolution
  - Slide kernel K over the image I
  - $$N_{u,v} = \sum_{(i,j)\in K} I_{u+i,v+j} K_{i,j}$$
  - compute for every pixel $\forall (u,v) \in I$
  - $N_{u,v} = \mathbf{i} \cdot \mathbf{k}$
  - $$\begin{cases} \mathbf{i} = \mathbf{k} & N_{u,v} > 0 \\ \mathbf{i} = -\mathbf{k} & N_{u,v} < 0 \end{cases}$$





Pattern Detection through Convolution

- Convolution
  - Slide kernel K over the image I
  - $$N_{u,v} = \sum_{(i,j) \in K} I_{u+i,v+j} K_{i,j}$$
  - compute for every pixel $\forall (u, v) \in I$
- Kernel Templates
  - Edges, Points, Corners, Others
  - Filtering
- Learning Kernels
  - Weights in convolutional Layers

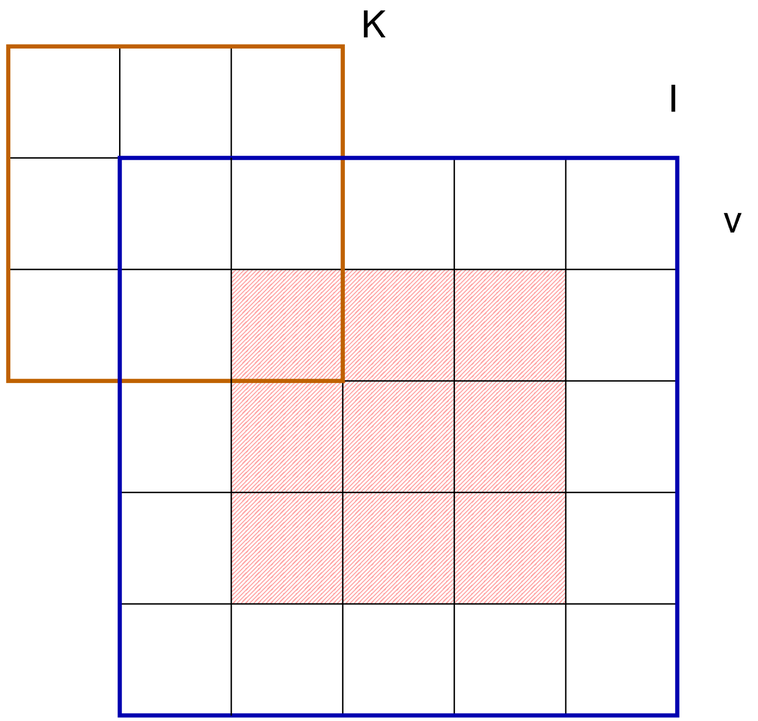


Kernel: Vertical Edge

Bricks on a Wall

Kernel Outside the Image

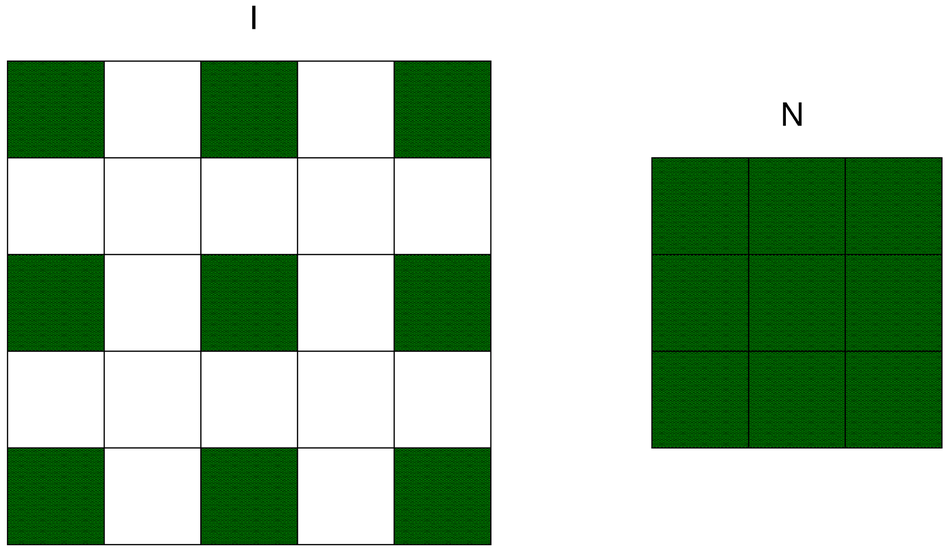
- Pixels with invalid convolution
  - kernel of size $\{2k+1\} \times \{2k+1\}$
  - k rows and columns closest to the borders: invalid convolution
- Handling invalid convolution
  - Valid convolution
    - Input I of size $\{i\} \times \{j\}$
    - Output N of size $\{i-2k\} \times \{j-2k\}$
  - Padding
    - expand I on each side by k pixels
    - assign value of 0 to new rows and columns
    - Input I of size $\{i\} \times \{j\}$
    - Output N of size $\{i\} \times \{j\}$



Invalid

Reducing Redundancy - Strides

- Applying convolution to every pixel leads to some redundancy in the output
  - downsampling reduces the size of the output
  - compute convolution to a subset of the input
- Stride size
  - number of pixels to move the kernel over the input image I
    - Stride of 1: compute convolution for every pixel
    - Stride of 2: move kernel by two pixels, skipping one in each dimension
    - Stride of s: move kernel by s pixels in each dimension
      - Input I of size $\{i\} \times \{j\}$
      - Output N of size $\{\lceil \frac{i}{s} \rceil\} \times \{\lceil \frac{j}{s} \rceil\}$



Properties of Convolution

- Commutative: $A \otimes B = B \otimes A$
- Associative: $A \otimes (B \otimes C) = (A \otimes B) \otimes C$
- Distributive: $A \otimes (B + C) = A \otimes B + A \otimes C$
- Linear: $A \otimes (\alpha B) = \alpha(A \otimes B)$
- Invariant to spatial shift $S(\cdot)$: $A \otimes S(B) = S(A \otimes B)$

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