Author Response to Reviewers for the paper 'Mitigating object prior-bias in sparse projection tomographic reconstructions'

We thank all the reviewers for their detailed feedback on our work. Based on the comments, we have incorporated significant changes in our revised paper, and have discussed these in this document. We also request the reviewers to read these comments along with the changes in the paper, especially related to the comments in Section 6 (Results), these being the most significant ones.

1 Comments from AE

While the reviewers have acknowledged improvements compared to the earlier manuscript, they have raised some important concerns about the manuscript in its current form. While addressing each of the reviewer specific comments, please pay specific attention to the following:

- *Characterization of deep learning methods and its relation to the presented work*

  We have now included a discussion of deep-learning (DL) methods in 'Related Work', Section 2, of our paper, and have cited a few notable references pertaining to training neural networks to directly predict full-view sinogram and reconstruct the test object from its limited-view measurements. We also discuss how our work is complementary to learning-based teachniques and can fit alongside a recent, state of the art, deep learning framework, as seen in Fig. 2 of the paper. Hence, the ability of DL to uncover new features is not being disputed. Instead, our focus is on a novel discrete method.

- *Presenting all relevant details so the work in the paper can be re-produced without ambiguity*

  In addition to the previously shared code for 2D reconstructions, we have now added code for 3D reconstructions in our github repository. This ensures reproducibility of our results. The repository is currently not a public repository since the paper is still in the review process.

As an additional note, based on the suggestion of R3, we have made a slight change to the title of the paper using the word “Mitigating” instead of “Eliminating”.

2 R1: AQ - Publish With Minor, Required Changes

Comments: The authors have significantly improved their paper when compared to the earlier rejected submission. The results are now much clearer. I have a few relatively minor comments below.

[R1-1,2]: *In the abstract, I feel you are underselling your contributions. You may want to add a line or two that quantifies the fidelity of your reconstructions for a given percentage of the total number of views (as required by Nyquist sampling). Your method is very relevant for scenarios such as Flash X-ray CT where we only are able to measure 5-20 views. Hence, some quantifying statement that appeals to the very few view CT community may be useful. In the abstract, the sentence "In this work, we mitigate this problem by first estimating the location of new regions and then imposing object-prior in only old regions which are similar to the prior." introduces terms such as "new regions" and "old regions" that don't carry any meaning. Please rewrite.*

Response:

1. *. . . You may want to add a line or two that quantifies the fidelity of your reconstructions . . .* While we appreciate the suggestion on the quantification of our results, we felt that (as styles go) it would be better to keep a lower profile because of the dependency on various factors such as the number of views, type of imaging geometry and structural complexity of the object data itself.
2. *. . . introduces terms such as "new regions" and "old regions" that don't carry any meaning. . . .* Duly noted the issue of “old-regions”, we have defined and re-written the abstract.

[R1-3]: *In page 3 and lines 50-53 right column, you mention that your code will be released. However, I recommend including a link to github or other website with the code before publication.*

Response: We had earlier shared code for 2D reconstructions in our github repository. To this, we have now added code for 3D reconstructions for all datasets.

[R1-4]: *In page 7 and lines 39-42 left column, what are the weights for? I don't see any equation where these can be substituted for quantifying results. Also, RMSE and SSIM have their own definitions.*

Response: This point has come up in R2 as well (in a larger context), and we defer our detailed response to that section **[R2-1]**. In general, we agree that our earlier usage of the term 'weights' with respect to SSIM might have been confusing since we also refer to 'weights-map' throughout our paper.

[R1-5, 6]: *In page 7 and lines 52-56 left column, I don't understand your tuning of λ_1. How is SSIM optimized when ground-truth is unknown for the test volume? It appears that the weight λ_2 must be chosen carefully and sufficiently high to ensure that we achieve the very fidelity reconstruction shown in Fig. 11 (h) with just 6 views. A lower incorrect choice for λ_2 will certainly bring back the artifacts. Even the parameter k probably plays an important role in achieving Fig. 11(h). Thus, your explanation for choosing λ_2 (in page 7 and lines 8-10 right column) and k (in page 7 and lines 23-27 right column) is inadequate. If you base your choice of λ_2 and k on "tolerance for noise" and/or "artifacts", how do you even quantify the relevant noise and artifacts? If λ_2 and k are chosen incorrectly, I am sure your method won't work. Hence, this detail needs to be discussed.*

Response:

1. Use of ground-truth:
   Since the aim of our experiments is to demonstrate the benefit of using weighted prior reconstruction, we adjust these parameters by assuming the ground-truth to be known. However, we do not assume the Region Of Interest (RoI) of the test to be known.

2. Choosing paramters in real-life scenario:
   We note that in practice, however, since ground-truth will never be available, these parameters must be tuned using only the set of templates available. One of the templates can be assumed to be a pseudo-test, and a grid search can be performed to get the best possible hyperparameter values. These values can then be used for reconstructing the 'real' test volume. Since the available templates are of high quality, this is a reasonable technique.

3. Our choice of λ_1 and λ_2 for demonstration:
   As mentioned in Sec.6-B of the paper, λ_1 was tuned to maximize the SSIM of the complete reconstructed test volume using the TV-only regularizer. This of course favours the TV-only regularizer method in contrast to not favouring our method. With the already chosen optimal λ_1, we apply our method for a range of λ_2 values and select the one that gives the best SSIM with reference to the ground truth.

4. Choosing k:
   As far as the stability of parameter k is concerned, in Fig. 8 of our paper, we show how the quality of reconstructions vary for different values of k. We see that the reconstructions are similar for a significant range of k values (viz. (5-40) for Okra). For

the purposes of demonstrating the benefit of our method, we choose k omnisciently assuming known ground truth of test (and unknown RoI). In real-world, k must also be tuned by using one of the templates as pseudo-test and performing a grid search to select the best value.

3 R2: RQ - Review Again After Major Changes

Comments: The paper has greatly improved with the new focus! I would like to thank the authors for the extra work. I do have a few remaining comments:

[R2-1]: *I highly suggest to use standard metrics for comparison, instead of the custom metrics currently used. For SSIM, reporting with the standard weightings is important (if the authors want, of course they can show both the standard weightings and the custom ones!). For RSME, I don't understand why the authors rescale images before computing the metric – this can greatly alter the metrics, making them difficult to interpret and compare across papers. For the RMSE, it is best to not use any scaling at all (or, a fixed scaling for all reconstructions only based on the ground truth).*

Response:

1. Rescaling

   For computation of both SSIM and RMSE, we find that rescaling is necessary because the different optimization routines generate solutions whose ranges vary from that of the ground truth. For example, for reconstruction of the potato data from 18 views, Table 1 shows the minimum and maximum intensity values of the groundtruth and reconstructed volumes (in the case of TV regularization). The pernicious effect of this is obviously invisible in the qualitative results where the display program autoadjusts showing TV superior to FDK. In a quantitative setting it becomes necessary to rescale, otherwise TV (using a prior) will appear to be inferior to FDK (without a prior), when it's obviously not the case. The same issue persists in the reconstruction using the external optimization program mentioned.

	Ground truth	**FDK**	**TV**
Minimum	0.00	-0.19	0.00
Maximum	1.00	0.24	0.24

Table 1: Example. Minimum and maximum intensity values of groundtruth and reconstructed volumes of potato data reconstructed from 18 views.

2. [Meta comment: This section is also in response to R1 (*. . . SSIM have their own definitions. . .*]

[Standard vs custom:] We have used the commonly used SSIM metric, for which, we have now provided reference to the original paper where the metric has been defined. As is well know, SSIM comprises of three factors: Luminance, Contrast and Structure. These factors (now termed 'coefficients', earlier 'weights') for these three terms are adjustable to suit the application at hand. Since SSIM inherently offers a flexibility to set coefficients based on what we want to evaluate, we believe that having a set of coefficients that focus on measuring structural preservation is better suited for our goal.

However, in the revised version, *we have now included quantitative results with both custom (structure-focussed) SSIM and default SSIM. The benefit of the proposed method is seen with* ***either*** *setting of the metric.*

Since the discussion on SSIM has persisted from the previous version, we take the liberty of providing a more detailed response.

With μ_x, μ_y and σ_x^2, σ_y^2 being the averages and variances of pixel values along dimensions x and y respectively, σ_{xy} being the covariance along x and y, L being the dynamic range, c_1 and c_2, and c_3 being defined to be $(k_1L)^2$, $(K_2L)^2$ and $c_2/2$ repectively, with $k_1 = 0.01$ and $k_2 = 0.03$, the three components of SSIM metric: luminance l, contrast c and structure s are defined as:

$$l(x,y) = \frac{2\mu_x\mu_y + c_1}{\mu_x^2 + \mu_y^2 + c_1} \tag{1}$$

$$c(x,y) = \frac{2\sigma_x\sigma_y + c_2}{\sigma_x^2 + \sigma_y^2 + c_2} \tag{2}$$

$$s(x,y) = \frac{\sigma_{xy} + c_3}{\sigma_x\sigma_y + c_3} \tag{3}$$

Fianlly, SSIM is defined as

$$SSIM = \frac{1}{n}\sum_x\sum_y [l(x,y)^\alpha.c(x,y)^\beta.s(x,y)^\gamma], \tag{4}$$

where n denotes the total number of pixels. The coefficients α, β and γ are set to 1 as default, but need to be adjusted to suit the application goal.

[R2-2]: *The computation comparison is not very convincing to me. In practice, given a certain number of projections, a user might want to compare possible reconstruction algorithms to decide what is the best one to use from a computation time / reconstruction* *quality tradeoff point of view. Therefore, it would be good to compare times between methods for the same number of projections (it is of course possible to show results for a few different cases, e.g. many views, moderate views, and few views). It is also useful to include more detailed information about the data size (detector size, reconstruction size, number of projections, etcetera). Furthermore, please show timing results for all steps of the algorithm, including the computation of the low quality reconstructions, pilot reconstructions, and building of eigenspaces.*

Response:

1. *. . . therefore it would be good . . .* Indeed, this is precisely what we have done. Sadly, our earlier column header of Table-6 conveyed a different impression. We intended to indicate that FDK is good for sufficient views, and TV is sufficent for moderate views. We ended up misleading the reviewer.

   Hence, we have edited the caption and Table-field titles of Table-6 (Page 10) to accurately describe details of measurement.

2. *. . . all steps . . .* We also provide the details required. We note that the TV reconstruction *is* our pilot reconstruction and hence the computation times are equal.

   Having specified the exact compute times in our machine, we also note that the type of machine and the degree of parallelism available in the current times make these timing measurements, especially for iterative solutions, subjective.

[R2-3]: *In several parts of the paper, the authors claim that existing object-prior based methods have problems, e.g.: "A common problem with these techniques is the strong influence of object-priors in the reconstruction of new regions in the test". At the moment, this hard claim is not supported in the paper by references or (even better) comparison results. It would be good to show, on one (or more) of the datasets used, that this is indeed the case, and that the proposed methods solves this problem.*

Response: We have now included a reference (Pg.2, Line 3 under Sec.1-A) to a PhD thesis. This thesis discusses the problem of object-priors biasing the reconstruction of new test regions. A sample result (Figures 1-3) from the thesis is shown here. It illustrates how a direct application of object-prior can adversely affect the reconstruction of new regions in the test.

[R2-4]: *The discussion of deep learning methods in Section 7B is not very strong in my opinion. There are many existing techniques that have shown quite convincing results for reconstructing CT images with a low number of projections, even without taking into account any static parts across time. These methods rely on learning image-based correlations instead of the "given intensities of a voxel at various time instants in a longitudinal setting and partial measurements of the voxel at the current time, what will be the intensity of the*

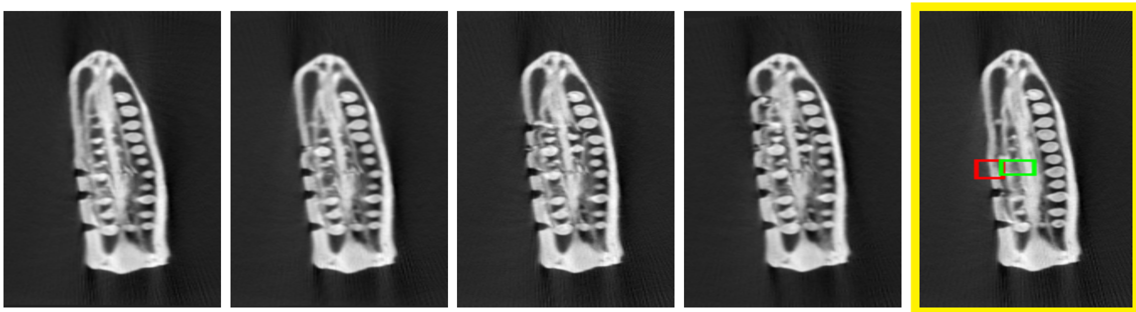
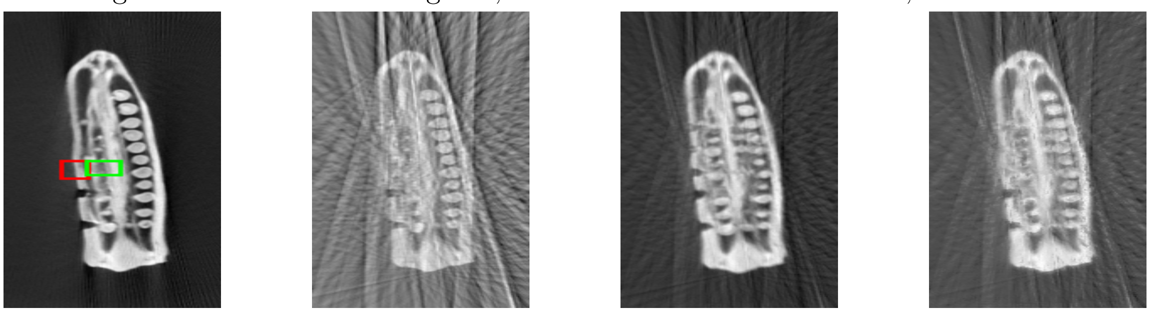


Figure 1: One slice each from the templates and one from the test volume (extreme right). In the regions marked in red and green, while all slices have deformities, the test has none.



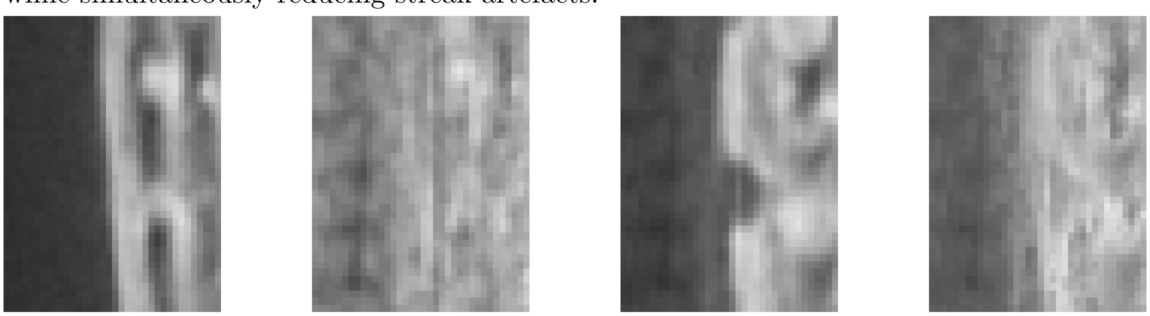
(a) Test

(b) FDK, no prior

(c) Only object-prior

(d) Spatially varying object-prior

Figure 2: Reconstruction from 10% views: (b) strong streak artefacts, (c) no new information detected (prior dominates – the deformity from the prior shows up as a false positive) and (d) new information detected (no deformities corresponding to red and green regions) while simultaneously reducing streak artefacts.



(a) Test

(b) FDK, no prior

(c) Only object-prior

(d) Spatially varying object-prior

Figure 3: Zoomed RoI of Fig. 2 (b) strong streak artefacts, (c) no new information detected (prior dominates – the deformity from the prior shows up as a false positive) and (d) new information detected (no deformities).

same voxel at the current time instant?" given by the authors. These image-based methods often don't need 'hundreds of labeled data', sometimes being able to accurately learn from just one time step. In the paper, these methods should be discussed, and proper references should be included.

Response: We acknowledge that there has been significant work pertaining to training neural networks to directly predict full-view sinogram and reconstruction of the test object from its limited-view measurements. We have now included a discussion of deep-learning methods in 'Related Work', Section 2, of our paper, and have now cited a few notable references. We also discuss how our work is complementary to learning-based teachniques and can fit alongside a deep learning framework, as seen in Fig. 2 of the revised paper. That said, the focus of our work is on a new discrete algorithm that builds structural maps that can and does point to new features.

We also believe that information from longitudinal data, if available, is helpful and must be used to better inform reconstruction of the test. This is especially the case in applications where longitudinal data is naturally available as part of the process as in radio frequency ablations. It is in this context that our paper presents an analytical method to fuse the existing prior data with test measurements in order to construct a 'weights map'. This map gives clear insights about regions of change within the object over time.

[R2-5]: *What algorithm is used to minimize Eq 3? How many iterations were used for this minimization? How was convergence ensured?*

Response: Eq. 3 consists of alternately solving for Eq.4 and Eq.5. While Eq.5 has a closed form solution, Eq.4 was solved using the solver mentioned in Reference [18]. (as mentioned in page 6, just above Eq.4). For solving Eq.4, maximum number of iterations was set to 200 and tolerance was set to 1e-4.

The number of alternate minimization cycles (between Eq.4 and 5) was set to 3. We observed that further cycles did not improve reconstruction significantly for the extra time consumed.

[R2-6]: *What were the various parameters of the reconstructions show in Fig 11? How were they chosen?*

Response: For 2D reconstructions using ART, SIRT and SART, we set the maximum iterations to 50 and observed that further iterations did not improve the quality of reconstructions.

[R2-7]: *How was the λ_2 parameter empirically chosen?*

Response: Please refer to our response to a similar query in **[R1-5,6]**.

[R2-8]: *In some parts of the paper, the use of language could be improved. Also, I think it is better to not use words like 'paltry'.*

Response: Although we had meant to use 'paltry' in a neutral way to only suggest the use of very few measuremetns, we have noted this and made the change.

4 R3: RQ - Review Again After Major Changes

Comments: The paper presents an image reconstruction method for recovering samples from limited projections provided that the high-quality CT-scans of the same sample exist. This approach could be useful to longitudinal studies in which the patient's previous CT scans are available and the purpose is to reduce the dose. So this is good!

The structure of the paper is good and the authors did a good job in presenting both the theory and the validation with experimental data, demonstrating the potential. of the nice things about the paper is that they use PCA to get the orthonormal basis to represent the image and this both serves as a regularizer as well as a nice way to quantify new features in data.

[R3-1]: *One challenge of the method that I can see is that these datasets needs to be aligned but there is not much discussion on this procedure or how this can be performed in a realistic setting. Are the variation from a past to present scans looking similar? In addition to the variations in scans taken at different times, in the low-quality scan, the features are less pronounced so the registration might pose a challenge even if the sample looks similar. So, it would be good to see the artifacts due to an intentionally less-aligned images and a discussion about the limitations would be nice.*

Response: Yes, registration between the unknown test and templates is important for our method. Registration is also a common problem in many other fields such as MRI/CT/PET data. There is a lot of research on linear and non-linear methods to optimise this. The focus in our paper is not on registration but on building accurate maps that delineate regions of structural changes in already pre-registered data (registered using state-of-the-art methods). Hence, in this work, we do not intend to predict what difficulties registration might present but rather that show that it is possible to blend prior and new views.

We note that in practice, a pilot reconstruction of the test must be registered with similarly imaged, pilot reconstructed templates. This registration is challenging due to the presence of sub-sampling artefacts. We consider the problem of registration as a separate one and intend the practitioners to use any state-of-the-art registration algorithm before applying the proposed method in our paper.

[R3-2]: *Another challenge, as the authors acknowledge, is the high number of hyper-parameters. I believe the paper can benefit a lot from an in depth analysis to get some insights about how this parameters should be chosen. Because otherwise the applicability of the method is limited given the amount of time required to solve the reconstruction problem.*

Response: Please refer to our response to a similar query in **[R1-5,6]**.

[R3-3]: *The title says "eliminating object prior-bias", but I think this is not accurate, because you give a weight to those priors, but not fully eliminate them. A more appropriate wording might be "Prior-informed sparse-projection..." or "Weighted object-priors from past CT-scans for ..." or something like this.*

Response: Thank you for suggesting other possible titles. We agree. We have changed the title to "Mitigating object prior-bias from sparse-projection tomographic reconstructions"

[R3-4]: *As a final remark, the authors can also use this technique for imaging dynamical samples. A discussion would be nice.*

Response: Based on our understanding, dynamic imaging is a technique for acquiring only those measurements that have high entropy. This involves computing the next the measurement location based on the past measurements acquired for the *same* object volume. This is different from our scenario in which we consider measurements from *multiple volumes* of an object, each taken at a different time. The measurement-locations of the test are also not pre-computed while imaging, in our case.

Hence, our method may not be directly applicable for reconstruction from dynamic imaging wherein the acquired measurements of a single object are already non-redundant by design. However, a suitable dynamic imaging reconstruction algorithm (like the Compressed Sensing) can definitely be used as a pilot reconstruction method within our framework.