A Pricing-Based Rate Allocation Game in TVWS Backhaul and Access Link for Rural Broadband

Suman Ghosh, Sandip Karar, and Abhirup Das Barman

***Abstract*—The traditional cellular technology is not profitable to serve the sparsely populated and clustered remote rural areas because of huge network set-up cost and lesser return from low density of rural population. To provide broadband service in those remote rural areas, in this paper, we consider a TV white space (TVWS)-based hierarchical network architecture where sparsely located clustered rural pieces of user equipment (UEs) are connected to far-off central base station (CBS) via intermediate TVWS nodes, known as the pieces of customer premises equipment (CPEs) through TVWS link. The CBS has direct broadband connectivity to the Internet through optical fiber backhaul. In such a network architecture, we adopt a two-stage market model among UEs, CPEs, CBS, and propose a pricing-based Stackelberg game to distribute the total available data rate at CBS among CPEs and then to the UE. Extensive simulations have been carried out to study the data-rate distribution, and the behavior of utilities of CPE, CBS, and UE with variation of different parameters for a particular network scenario. The study of the outcome suggests that using this simple price exchange mechanism, it is possible, based on each UE's willingness-to-pay and the total available data rate, to distribute data rate optimally among the UEs where each of the network entities behaves selfishly.**

***Index Terms*—Backhaul, particle swarm optimization (PSO), pricing policy, rural broadband architecture, Stackelberg game, TV white space (TVWS).**

I. Introduction

EXPONENTIAL progress in information and communication technology and modern-age smartphones play an important role to form a well-connected nation by connecting each corner of the nation including remotest villages through high-speed broadband services. It has several potential benefits, such as new economic opportunities, growth and support in agriculture, small-business development, health-care delivery, education, etc. To connect the rural areas to the Internet, different national incentive plans [1] have been initiated by the governments of different nations. Digital India Program [2] is one such initiative by the Government of India to connect the large villages that have administrative bodies usually known as *Gram Panchayat* (GP) in Indian lingo. Under this program 112 360 GPs have already been covered with 264 033 km of optical fiber [3] by February 2018, laid under the national optical fiber networks. But due to huge cellular network set-up cost and low return from small density of rural population, many villages that are remote from the GP are still disconnected from the Internet services. Thus, setting up a network in those remote rural areas is a challenging task from an economic point of view. Moreover, the whole system should be power efficient as there is always a problem of intermittent electricity connections with low-voltage power supply. Conventional Wi-Fi access points have limitations in providing broadband services to the rural users due to their small coverage and long-distance wired backhaul requirement. Wireless backhaul using microwave frequency can also be a viable alternative in this scenario, but it has some drawbacks [4], such as spectrum scarcity, high power requirement to cover large distance, interference, etc. Also, implementing backhaul using multihop long-distance Wi-Fi links or WiMAX [5] is not a power-efficient solution as well as its implementation requires good amount of initial investment.

In the meantime, different spectrum occupancy studies [6]–[9] show that a significant portion of TV bandwidth remains unused in Asian, African, and European countries due to the switchover from analog to digital technology in TV broadcasting. These unused TV channels are known as *TV white space* (TVWS). Federal Communications Commission (FCC) and UK regulatory body Ofcom have decided to open up the UHF TVWS band for cognitive application [10], [11]. This TVWS band has sufficient bandwidth and good propagation characteristics to attain good coverage. Thus, a device operating over TVWS, termed as *customer premises equipment* (CPE), can be used as a router in remote villages that receive the data packets from *central base station* (CBS) at GP through TVWS backhaul and forward the packets to the *pieces of user equipment* (UEs) under its coverage zone also through TVWS link.

In this paper, we consider a hierarchical network architecture using such pieces of CPE (CPEs) operating over TVWS acting as an intermediate node between rural UEs and CBS, which is generally far away from UEs. The CPEs are primarily controlled by third-party operator, which manages accounting of traffic usage of end users. UEs such as smartphones, tabs, etc., are also assumed to be equipped with TVWS transceivers for communicating with the CPE. The CBS is directly connected to the Internet through optical fiber link. As a single optical fiber serves as the backhaul of a large number of other GPs too, the available bandwidth/data rate for each GP is bounded by a maximum limit depending on the number of village clusters under the GP and the population density. Therefore, a fair distribution of the available data rate among the CPEs and then to the UEs is a key challenge for such a two-stage network architecture and is the main focus of this investigation. To allocate the total available data rate, a game-theoretic approach is adopted because

Manuscript received July 21, 2017; revised March 1, 2018; accepted April 14, 2018. *(Corresponding author: Sandip Karar.)*

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Digital Object Identifier 10.1109/JSYST.2018.2842140

it can perfectly model the selfishness and rational behavior of the players (viz., CBS, CPEs, and UEs). A pricing policy is introduced to model the competition among the players. CBS distributes the data rate among CPEs in exchange of a price when total available data rate is upper bounded by a maximum limit. Accordingly in a similar manner, each CPE distributes the data rate among its UEs in exchange of a price. A leader–follower relationship therefore exists among the players that fit to the model of Stackelberg competition.

A. Related Works

Several applications of TVWS are available in the literature [12]–[19], which include rural broadband services, wireless backhaul medium, device-to-device communications, due to sufficient bandwidth and good coverage. To provide rural broadband service using TVWS, different architectures [20]–[22] have been proposed in the literature. The network architecture used in this paper follows the one given in [22]. The feasibility and cost consideration of such a two-phase TVWS network fitted with 5G cellular architecture is well studied in [23]. To allocate the available data rate among the rural areas, a number of works [24]–[26] have considered pricing-based data-rate allocation scheme using game-theoretic approach including formulation of both one-stage and two-stage Stackelberg game. Loumiotis *et al.* [27] have formulated a Stackelberg game similar to our work for a hierarchical resource allocation, however, the game in [27] does not consider any kind of pricing mechanism to optimally allocate the transmit power and resource blocks among the players while our approach to distribute the available bandwidth (or data rate) among the players is motivated by a pricing mechanism, which forms a two-stage buyer–seller game model.

B. Contributions

The main contributions of this paper can be summarized as follows.

1) A two-stage buyer–seller-based market framework is adopted for our network scenario, which can easily model the complex interactions among CBS, CPEs, and UEs.
2) We formulate a simple pricing-based Stackelberg game to optimally distribute the total available data rate at CBS among the CPEs and then to the UEs taking into account each UE's willingness-to-pay and the selfish behavior of all network entities.

To the best of our knowledge, no work on pricing-based game-theoretic application on this type of rural broadband architecture has been carried out where data-rate allocation in backhaul and access link is jointly taken care of.

Rest of the paper is organized as follows. Section II describes the system model and pricing-based data-rate distribution scheme with two-stage market model. The Stackelberg game is formulated in Section III and Nash Equilibrium solution is sought in Section IV. Simulation results are given in Section V. Finally, conclusion is drawn in Section VI.

II. System Model and Rate Allocation Scheme

Here, we consider a village scenario, as shown in Fig. 1, which is assumed to be out of coverage or under poor coverage of traditional cellular service. The broadband internet service is available up to CBS at the GP via its optical fiber backhaul located far

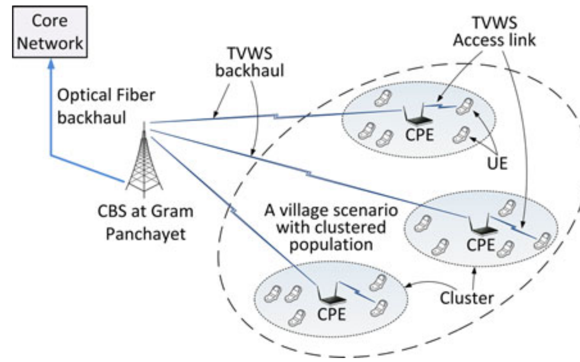

Fig. 1. Adopted system model for providing rural broadband service.

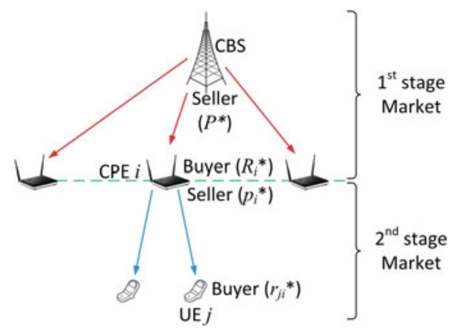

Fig. 2. Proposed two-stage buyer–seller based market model.

away from the village. The village comprises segmented clusters of population gathered around some facilities, such as markets, health center, primary schools, etc., with long-stretched fields, lakes, etc., in between them. Broadband access to the users in different clusters is provided through CPEs installed somewhere at the center of each cluster. CPEs are basically software-defined TVWS radios equipped with two radio transceivers and UHF TV antennas; one is used for communicating to the CBS as backhaul and the other is used for providing access to the UEs. Both communications are assumed to be carried out over different TVWS frequencies. Frequency assignment of the different links between CBS-CPEs as well as CPE-UEs is assumed to be performed by different frequency-assignment algorithms available in the literature such as [28]–[30]. The communication between CPE-UEs as well as CBS-CPEs follows TVWS protocol 802.11af with OFDM at PHY layer and TDMA at MAC layer. The whole work is focused only on the downlink rate distribution. However, uplink transmission can be carried out by coordinated multipoint technology, where multiple CPEs transmit the same signal to the CBS. Here, we consider M number of population clusters in the rural scenario with one CPE installed in each cluster. The number of UEs within ith cluster is N_i where $i = 1, 2, \ldots, M$ and among them n_i numbers of UEs are assumed to be connected with the ith CPE where $n_i \in \{1, 2, \ldots, N_i\}$. The data-rate demand of the jth UE connected to the ith CPE is r_{ji}. Let R_{MAX} be the maximum data rate available at CBS through its fiber backhaul, which is to be allocated among the UEs as per their demand and accordingly among CPEs for its backhaul to support the data-rate demand of their associated UEs.

To solve this problem of rate allocation, a two-stage buyer–seller-based market model scheme is proposed as depicted in Fig. 2. The first-stage market exists between the CBS as seller and the CPEs of different clusters as buyer. CBS allocates backhaul data rate to each CPE in exchange of price P per unit amount of data rate decided by the CBS. It is assumed that the seller (CBS) first sets the price and depending on that price the buyers (CPEs) decide the data rate. The second-stage market exists between each CPE of a different cluster as seller and the UEs within that cluster as a buyer. Let the CPE in the ith cluster allocates r_{ji} data rate to the jth UE in exchange of price p_i per unit amount of data rate decided by the CPE. Thus, a Stackelberg game can be formulated in each stage where each player, i.e., UE, CPE, and CBS try to maximize its utility functions which are defined in Section III.

III. Game-Theoretic Formulation for Two-Stage Buyer–Seller-Based Market Model

In this section, the utility functions for CBS, CPE, and UE are defined and a two-stage Stackelberg game is formulated using the buyer–seller-based market model in each stage. The utility function of a player in a game indicates the net benefit that it can gain from the game.

A. Utility of UE

First we define a term called *throughput* for each UE, which expresses the user satisfaction. Following the model in [31], the throughput for the jth UE in the ith cluster, denoted by S_{ji}, is defined as a logarithmic function of its rate r_{ji} which is written as

$$S_{ji} = \ln(r_{ji}) + \alpha \tag{1}$$

where α is a constant. The logarithmic function is used in the expression of UE throughput to suggest that the throughput slows down with higher data rate. Now, the utility function U_{ji} of the jth UE in the ith cluster is derived from its throughput and the price it has to pay to the ith CPE, which can be written as

$$U_{ji} = k_i S_{ji} - p_i r_{ji}, \text{ where } i = 1, 2, \dots, M \quad j = 1, 2, \dots, N_i. \tag{2}$$

The term k_i is the proportionality constant that represents the equivalent price per unit throughput and p_i is the price per unit amount of data rate paid to the ith CPE. We assume that the jth UE under the ith CPE can withstand up to a minimum value of utility $U_{\text{th}j,i}$, termed as *utility threshold*. The objective of the UE is to find the optimal value of r_{ji} by maximizing its utility U_{ji} with a lower limit of $U_{\text{th}j,i}$. The optimization problem for the corresponding UE can be written as

$$\max_{r_{ji}} U_{ji} \tag{3a}$$

$$\text{s.t. } U_{ji} \geq U_{\text{th}j,i} \tag{3b}$$

$$\forall i = 1, 2, \dots, M \ \forall j = 1, 2, \dots, N_i.$$

B. Utility of CPE

The utility of each CPE is defined as the revenue it earns from the payment of its associated UEs connected to it minus the price it has to pay to the CBS for claiming the required data rate to satisfy its UEs. Hence, the utility function for the ith CPE is given by

$$U_{\text{CPE}_i} = \sum_{j=1}^{n_i} p_i r_{ji} - P \sum_{j=1}^{n_i} r_{ji}, \ i = 1, 2, \dots, M \tag{4}$$

where n_i is the number of connected UEs to the ith CPE among a total of N_i UEs and P is the price per unit amount of data rate that has to be paid by each CPE to the CBS. The objective of the ith CPE is to find the optimal value of p_i that maximizes its utility U_{CPE_i} can be found by solving the optimization problem written as

$$\max_{p_i} U_{\text{CPE}_i} \tag{5a}$$

$$\text{s.t. } n_i \leq N_i \quad \forall i = 1, 2, \dots, M. \tag{5b}$$

The backhaul data rate required for the ith CPE R_i is given by the sum of individual UE data rate connected to that CPE which is given by

$$R_i = \sum_{j=1}^{n_i} r_{ji}. \tag{6}$$

C. Utility of CBS

The utility of CBS is the revenue it earns from the payment of all the CPEs. Hence, the utility function of CBS is given by

$$U_{\text{CBS}} = \sum_{i=1}^{M} P R_i \tag{7}$$

where M is the number of CPEs connected to the CBS and R_i is the backhaul data-rate requirement for the ith CPE. Thus, the objective of the CBS is to find the optimal value of the price P that maximizes its utility U_{CBS} with the constraint of maximum available R_{MAX} data rate from its fiber backhaul. The optimization problem of the CBS can then be written as

$$\max_{P} U_{\text{CBS}} \tag{8a}$$

$$\text{s.t. } \sum_{i=1}^{M} R_i = R_{\text{MAX}}. \tag{8b}$$

Note that instead of putting the rate constraint on each UE and CPE, we put a constraint on the total available data rate in CBS mentioned in (8b), which automatically bounds the rate demand for the CPEs as well as UEs. The UEs cannot therefore demand arbitrary high data rate due to the rate constraint of the backhaul link of each CPE as there is a rate constraint on the total available data rate.

IV. Analysis of Stackelberg Game

Now, from the utility functions and constraints defined above, we can solve the Stackelberg Game using a *backward induction* technique and find out the *Nash Equilibrium* solutions [32] in which no player has the advantage of maximizing its utility by unilaterally deviating from its own strategy.

A. Calculation of Optimal Data Rate for UE

The optimal data rate of the jth UE connected to the ith CPE for which its utility is maximized is calculated by equating its derivative with respect to r_{ij} to zero

$$\frac{\partial U_{ji}}{\partial r_{ji}} = \frac{k_i}{r_{ji}} - p_i = 0. \tag{9}$$

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Thus, the optimal data rate or the *best response strategy* of the jth UE in the ith CPE is given as

$$r_{ji}^* = \frac{k_i}{p_i}. \tag{10}$$

Thus, the optimal utility of the jth UE in the ith CPE is given by

$$U_{ji}^* = k_i \left(\ln \frac{k_i}{p_i} + \alpha - 1 \right). \tag{11}$$

From (11), it can be observed that as p_i increases U_{ji}^* is decreased. Now, each jth UE under the ith CPE can withstand minimum utility $U_{\mathrm{th}j,i}$. Under a *best response strategy*, the price value corresponding to $U_{\mathrm{th}j,i}$ can be found by replacing $U_{ji}^* = U_{\mathrm{th}j,i}$ in (11) and denoted as $p_{\mathrm{th}j,i}$. Hence,

$$p_{\mathrm{th}j,i} = k_i \exp \left\{ - \left(\frac{U_{\mathrm{th}j,i}}{k_i} - (\alpha - 1) \right) \right\}. \tag{12}$$

In other words, the jth UE is willing to pay at the most $p_{\mathrm{th}j,i}$ unit price per unit amount of data rate to the ith CPE while demanding data rate r_{ji}^*. Beyond that price value $p_{\mathrm{th}j,i}$, the utility of jth UE will fall below $U_{\mathrm{th}j,i}$ and, hence, the UE will not be interested to be a part of the game and get disconnected or leave the game. The notation $p_{\mathrm{th}j,i}$ is termed as *price threshold* of the jth UE under the ith CPE.

B. Calculation of Optimal Price for CPE

Based on the *best response strategy* of UEs, we can rewrite the utility function of the ith CPE as

$$U_{\mathrm{CPE}_i} = \sum_{j=1}^{n_i} p_i r_{ji}^* - P \sum_{j=1}^{n_i} r_{ji}^* = n_i \left(k_i - P \frac{k_i}{p_i} \right). \tag{13}$$

Here, r_{ji} in (4) is replaced by the UE optimal data rate r_{ji}^*. It is observed that for a fixed value of P, the value of U_{CPE_i} initially increases with p_i because the revenue earned from the UEs will increase. Now, while increasing the value of p_i, as it crosses each price threshold $p_{\mathrm{th}j,i}$, the corresponding jth UE will be disconnected from the ith CPE. Hence, n_i decreases by one for each crossing of p_i through the price thresholds. Without loss of generality, it is assumed that the UEs under the ith cluster are arranged in ascending order according to their price thresholds such that $p_{\mathrm{th}l,i} \leq p_{\mathrm{th}l+1,i}\ \forall l = 1, 2, \ldots, (N_i - 1)$. Now n_i can be written as

$$n_i = \begin{cases} N_i, & p_i \leq p_{\mathrm{th}1,i} \\ N_l, & p_{\mathrm{th}(l-1),i} < p_i \leq p_{\mathrm{th}l,i}, \\ & \forall l = 2, \ldots, N_i \\ 0, & p_i > p_{\mathrm{th}N_i,i} \end{cases} \tag{14}$$

where $N_l = N_i - l + 1$. Therefore, the expression for U_{CPE_i} can be rewritten as

$$U_{\mathrm{CPE}_i} = \begin{cases} N_i c(p_i, k_i, P), & p_i \leq p_{\mathrm{th}1,i} \\ N_l c(p_i, k_i, P), & p_{\mathrm{th}(l-1),i} < p_i \leq p_{\mathrm{th}l,i} \\ & \forall l = 2, \ldots, N_i \\ 0, & p_i > p_{\mathrm{th}N_i,i} \end{cases} \tag{15}$$

where $c(p_i, k_i, P) = \left(k_i - P \frac{k_i}{p_i} \right)$. From (14), it is noticed that there is a sudden change of U_{CPE_i} when p_i increases over each price threshold value and ultimately the value of U_{CPE_i} becomes zero when no UE is connected. From (15), it is seen that n_i changes only at price threshold values and remains constant in between two price threshold values. As a consequence, the value of U_{CPE_i} increases in between two price threshold values and at price threshold values, it may increase or decrease depending on the value of k_i and P. Thus, optimal value of p_i for which U_{CPE_i} is maximum must lie on any of the price threshold $p_{\mathrm{th}j,i}$ values, since in between two $p_{\mathrm{th}j,i}$ values, the U_{CPE_i} curve always increases monotonically. Therefore, the infinite search space for finding the optimal solution p_i^* is now reduced to a finite set of $p_{\mathrm{th}j,i}$ values. Optimal p_i can thus be found by choosing one of such $p_{\mathrm{th}j,i}$ values for which U_{CPE_i} is maximum and denoted by p_i^*.

Algorithm 1: Constrained PSO Algorithm.

Input: P, U_{CBS}, R_{MAX}.
Output: P^*.
1: Calculate P_{temp}^* using conventional PSO given in [33]
2: Calculate the value of R_i for P_{temp}^* from (6)
3: **if** $\sum_{i=1}^{M} R_i > R_{MAX}$ **then**
  3.1 Set $U_{\mathrm{CBS}}(P_{\mathrm{temp}}^*) = 0$
  3.2 Go to Step 1.
else
  3.3 $P^* = P_{\mathrm{temp}}^*$.
end

C. Calculation of Optimal Price for CBS

For the optimal price p_i^*, the required backhaul data rate R_i of the ith CPE can be calculated using (6), which represents data-rate demand of the corresponding CPE in the first-stage market. Based on the demand of all CPEs, CBS tries to maximize its utility U_{CBS} with a constraint on maximum data-rate supply from its fiber backhaul. From (7), the utility function of CBS can be rewritten as

$$U_{\mathrm{CBS}} = \sum_{i=1}^{M} P R_i = \sum_{i=1}^{M} P \sum_{j=1}^{n_i} r_{ji}^* = \sum_{i=1}^{M} P n_i \frac{k_i}{p_i^*}. \tag{16}$$

For a very small value of CBS price P, the backhaul bandwidth (data rate) demand by the CPEs may exceed maximum limit, and therefore, there exists a minimum limit of the CBS price P. As P increases, the CBS utility U_{CBS} in (16) also increases for small P. For a higher value of P, each CPE price p_i^* increases and correspondingly the number of UEs connected to that CPE, i.e., n_i, decreases. As a consequence, the utility of CBS U_{CBS} decreases. Finally, for a very large value of P, when all the UEs get disconnected, the CBS utility becomes zero. Thus, there exists an optimal price P^* for CBS for which U_{CBS} will be maximum. As *utility thresholds* for each UE are random, the UEs will get disconnected randomly with respect to different CBS price resulting in an abrupt decrease in the CBS utility consequently shows a jagged pattern (see Fig. 7). Thus, the CBS utility as a function of P is not exactly convex and has finitely many local maxima and minima. Therefore, we cannot use the direct solution approach to find the optimal value. Instead a metaheuristic algorithm particle swarm optimization (PSO) [33] is used to find the optimal value of P. To include the constraint in the conventional PSO algorithm [33], Algorithm 1

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TABLE I
SIMULATION PARAMETERS

Parameters	**Values**
α	1
Values of k used in simulation	$\{0.5, 1, 2, 3, 4\}$
Number of CPEs (M)	4
Number of UEs in 1^{st}, 2^{nd}, 3^{rd}, 4^{th} cluster (N_1, N_2, N_3, N_4)	$(5, 3, 4, 6)$
Range for all CPE price (p_1, p_2, p_3, p_4)	$0.1 - 10$ unit/Mbps
Range for CBS price (P)	$0.01 - 100$ unit/Mbps

is used. In Algorithm 1, at first the conventional PSO algorithm is executed and one optimal value of P is found and denoted by P^*_{temp}. Then, total backhaul data rate for all CPEs is calculated for CBS price P^*_{temp} and checked whether the total backhaul data rate lies within the maximum data-rate limit. If it lies within the limit, then P^*_{temp} is taken as optimal price value P^* for CBS; otherwise, the conventional PSO is executed again after setting the value of U_{CBS} at the current CBS price P^*_{temp} to zero, in order to rule out the possibility of selecting it again in the next execution of PSO. This process is repeated until an appropriate P^* value is found for which the total date rate required will maintain the maximum data-rate limit. In this way, the optimal price for the CBS is calculated for which the utility of CBS is maximum.

To implement the proposed scheme, the network operator first informs the CBS about its maximum backhaul data-rate constraint R_{MAX}. Accordingly, the CBS decides about its optimal pricing strategy P^*, by solving the optimization problem (8). This calculation of optimal pricing strategy requires the knowledge of the best response strategies of all the CPEs and UEs, and hence, the knowledge of their utility functions. Initially, the UEs send the information about their utility functions to their respective CPEs and the CPEs forward this information to the CBS. All the CPEs also inform about their utility functions directly to the CBS. Now, for any given P, the CBS can directly calculate its utility value U_{CBS} from (16). CBS can then directly apply the PSO algorithm (Algorithm 1) to solve the optimization problem (8) and calculate the optimal CBS price P^*, which maximizes U_{CBS}. The CBS informs this optimal price value P^* to all the CPEs and accordingly the ith CPE will decide the optimal price p_i^* and informs this optimal price to all the UEs under its coverage. Each UE then demands a data rate depending on the CPE price based on its best response strategy given in (10).

V. RESULTS AND DISCUSSIONS

For simulation purpose, we consider a rural scenario consisting of four clusters with one CPE installed in each cluster. The CPEs in the four clusters are denoted by CPE_1, CPE_2, CPE_3, and CPE_4, respectively. The number of UEs in first, second, third, and fourth cluster are assumed to be 5, 3, 4, and 6, respectively. The k_i values for all clusters are assumed to be the same and each is denoted by k. All the parameters that are used for simulation are listed in Table I. For a fixed k, *utility threshold* values $U_{\text{th}j,i}$ for the jth UE under the ith CPE are chosen from Table II. The values are chosen randomly. For a fixed k, the N_i UEs associated with the ith cluster selects first N_i values from Table II as their price thresholds. Suppose $k = 3$, the *utility threshold* values of four UEs corresponding to the third CPE are as follows: $U_{\text{th}1,3} = 48$, $U_{\text{th}2,3} = 46$, $U_{\text{th}3,3} = 44$, and $U_{\text{th}4,3} = 42$. The corresponding *price threshold* values can be

TABLE II
UTILITY THRESHOLDS FOR DIFFERENT k

k	**Utility Thresholds**
0.5	$\{7.5,\ 7.1,\ 6.7,\ 6.3,\ 5.9,\ 5.5\}$
1	$\{15.5,\ 14.7,\ 13.9,\ 13.1,\ 12.3,\ 11.6\}$
2	$\{31,\ 29.7,\ 28.4,\ 27.1,\ 25.8,\ 24.5\}$
3	$\{48,\ 46,\ 44,\ 42,\ 40,\ 38\}$
4	$\{65,\ 62.6,\ 60.2,\ 57.8,\ 55.4,\ 53\}$

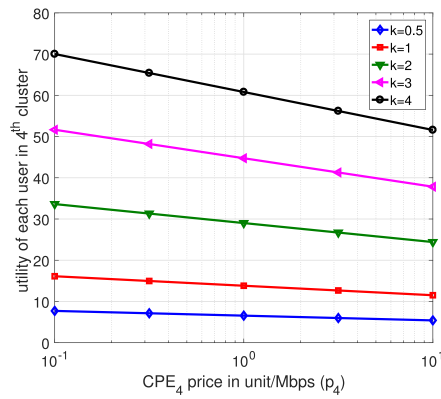

Fig. 3. Variation of UE utilities in the fourth cluster with CPE_4 price for different values of k.

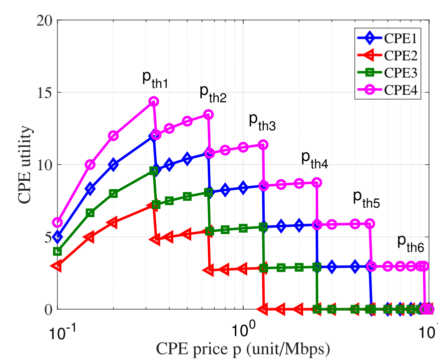

Fig. 4. Variation of CPE utility with CPE price for $k = 3$ and $P = 0.2$ unit/Mb/s.

calculated using (12). Considering third cluster, if the CPE_3 price increases over $p_{\text{th}1,3}$, the first UE will be disconnected from CPE_3. Similarly, when CPE_3 price increases over $p_{\text{th}4,4}$, all UEs in the cluster will be disconnected from CPE_3. Using the parameters given in Tables I and II, the behavior of utility functions of UE, CPE, and CBS can be delineated.

The utility curve of each UE under fourth CPE for the price range of CPE_4, i.e., p_4 as specified in Table I, is shown in Fig. 3 for the different values of k. Since k is the same for every cluster each UE in any other cluster behaves in the same manner. From Fig. 3, it is seen that as the CPE_4 price increases the UE utility decreases because the UEs have to pay more for the same data rate. Again for a fixed price, the UE utility increases with respect to k due to higher equivalent price value per unit throughput of the UE.

Fig. 4 shows the variation of the CPE utility U_{CPE_i} of different clusters with respect to their corresponding CPE price p_i for $k = 3$, while the CBS price P is arbitrarily fixed at 0.2 unit/Mb/s. The *utility threshold* values for all CPEs are chosen from

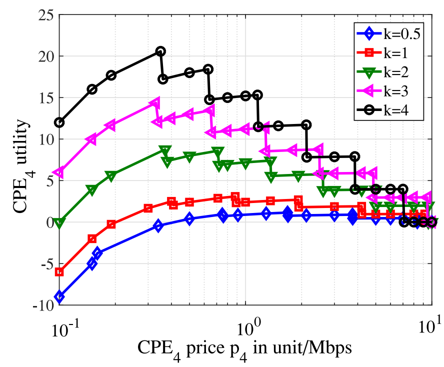

Fig. 5. Variation of CPE$_4$ utility with p_4 for different values of k and fixed $P = 0.2$ unit/Mb/s.

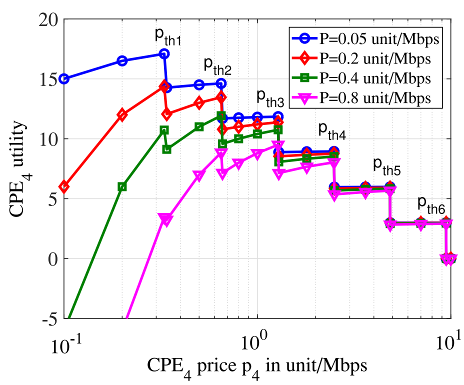

Fig. 6. Variation of CPE$_4$ utility with p_4 for different values of P unit/Mb/s and fixed $k = 3$.

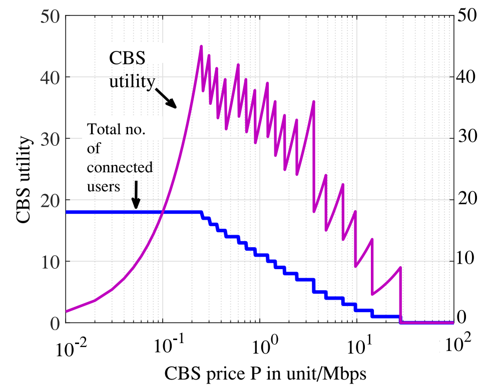

Fig. 7. Variation of CBS utility along with total number of connected UEs with respect to CBS price P for $k = 3$.

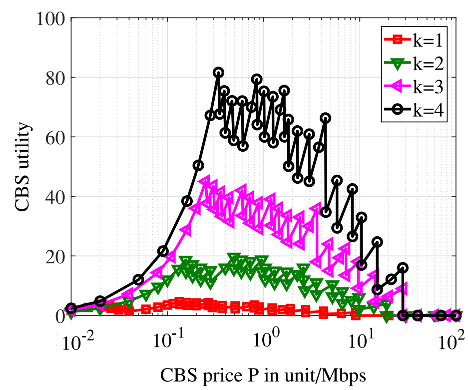

Fig. 8. Variation of CBS utility with CBS price P for different value of k.

Table II for $k = 3$. It can be noticed that for a fixed value of k and CBS price P, as the ith CPE price p_i increases, the ith CPE utility $U_{\mathrm{CPE},i}$ also increases initially due to more revenue earned from its UE increases, but as the ith CPE price p_i crosses $p_{\mathrm{th}1,i}$, the first UE under the ith CPE will get disconnected from the CPE. Thus, the ith CPE utility $U_{\mathrm{CPE},i}$ drops abruptly at $p_{\mathrm{th}1,i}$ due to the decrease in revenue earned. Thereafter, it will again increase up to the next price threshold $p_{\mathrm{th}2,i}$ due to higher revenue earned from the remaining UEs and then again drops abruptly at next price threshold $p_{\mathrm{th}2,i}$. This happens to each CPE. In this manner, as CPE$_2$ price p_2 crosses $p_{\mathrm{th}3,2}$, utility of CPE$_2$ goes down to zero because all of the three UEs under second cluster get disconnected from CPE$_2$. For the same reason, the utilities of CPE$_1$, CPE$_3$, and CPE$_4$ will be reduced to zero whenever their corresponding CPE price cross $p_{\mathrm{th}5,1}$, $p_{\mathrm{th}4,3}$, and $p_{\mathrm{th}6,4}$, respectively. Moreover, for a fixed value of CPE price and fixed k value, the CPE with higher number of UEs earns higher revenue and consequently gets higher utility value. All the CPEs therefore try to choose such price so that it maximizes the number of UEs that can be connected to it, but the situation may change with smaller values of k.

Instead of taking fixed k and P, in Fig. 5, the variation of a fourth CPE utility U_{CPE_4} with respect to corresponding CPE price p_4 for different k and fixed P is shown. It depicts that for a fixed p_4 as k increases, the utility of CPE$_4$ also increases because with increasing k value the UE demand also increases [see (10)], and the revenue earned from the UE also increases. For different values of k, *price threshold* values are different. It can be noticed that for smaller (higher) values of k, the maximum of CPE$_4$ utility occurs at higher (lower) CPE$_4$ price because for smaller (higher) value of k, UE demand is low (high) and thus higher CPE utility occurs for higher (lower) CPE price. Fig. 6 shows the variation of CPE utility U_{CPE_4} of fourth cluster with CPE price p_4 for different P and fixed k. The *price threshold* values will remain the same as k is fixed. From Fig. 6, it is shown that as P increases the CPE$_4$ utility decreases because of which CPE has to pay more to the CBS for the same amount of data rate. As P increases, CPE$_4$ also has to increase its own price, in order to maximize its utility which results in lesser number of UEs connected to that CPE. So for higher P value, the maximum value of CPE$_4$ utility occurs at higher value of CPE$_4$ price.

Fig. 7 shows the variation of CBS utility with CBS price for $k = 3$. Initially, as the CBS price increases, CBS utility also increases due to higher revenue earned from CPEs, but after a certain price CBS utility starts to decrease because of higher CBS price, the CPEs have to set higher price to maximize their utility (see Fig. 6), and consequently, the UEs associated with the CPEs get disconnected one by one, which in turn results lesser demand of data rate and consequently a reduction in the CBS utility. It is noticed that the CBS utility varies with the CBS price in a jagged pattern because different UEs of different CPEs will get disconnected at different CBS prices, which is also shown in Fig. 7, and disconnection of each UE results in an abrupt decrease in the CBS utility. Fig. 8 shows the variation of the CBS utility with the CBS price P for different k where it is observed that CBS gains higher utility for higher value of k. In Figs. 7 and 8, each local maximum in the CBS utility

TABLE III
OPTIMAL CBS PRICE, OPTIMAL PRICE OF DIFFERENT CPE, AND OPTIMAL DATA RATE OF UES UNDER DIFFERENT CPES FOR DIFFERENT k

k	P^* unit/Mbps	p_i^* unit/Mbps, $i=1,2,3,4$	r_i^* Mbps, $i=1,2,3,4$
0.5	0.04	$\{0.34, 0.34, 0.34, 0.34\}$	$\{1.47, 1.47, 1.47, 1.47\}$
1	0.13	$\{0.4, 0.4, 0.4, 0.4\}$	$\{2.5, 2.5, 2.5, 2.5\}$
2	0.49	$\{0.71, 0.71, 0.71, 0.71\}$	$\{2.82, 2.82, 2.82, 2.82\}$
3	0.25	$\{0.34, 0.34, 0.34, 0.34\}$	$\{8.82, 8.82 8.82, 8.82\}$
4	0.34	$\{0.3, 0.3, 0.3, 0.3\}$	$\{13.33, 13.33, 13.33, 13.33\}$

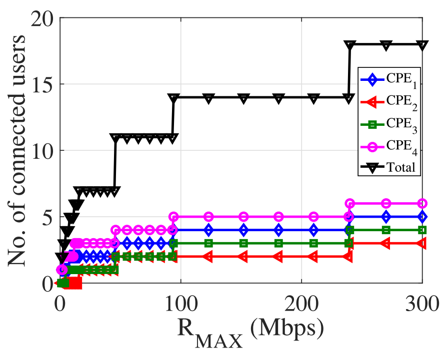

Fig. 9. Number of UEs connected to different CPE R_{MAX} for $k = 4$.

TABLE IV
VARIATION OF OPTIMAL RATE ALLOCATION WITH MAXIMUM AVAILABLE BACKHAUL DATA-RATE LIMIT AT CBS FOR $k = 4$

R_{MAX} Mbps	P^* unit/Mbps	$\{p_1^*, p_2^*, p_3^*, p_4^*\}$ unit/Mbps	$\{r_1^*, r_2^*, r_3^*, r_4^*\}$ Mbps	$\{n_1^*, n_2^*, n_3^*, n_4^*\}$
240	0.34	$\{0.3, 0.3, 0.3, 0.3\}$	$\{13.33, 13.33, 13.33, 13.33\}$	$[5, 3, 4, 6]$
200	0.85	$\{0.6, 0.6, 0.6, 0.6\}$	$\{6.67, 6.67, 6.67, 6.67\}$	$[4, 2, 3, 5]$
90	1.65	$\{1.1, 0.6, 1.1, 1.1\}$	$\{3.64, 6.67, 3.64, 3.64\}$	$[3, 2, 2, 4]$
45	4.4	$\{2.1, 1.1, 2.1, 1.1\}$	$\{1.9, 3.64, 1.9, 1.9\}$	$[2, 1, 1, 3]$
14	4.43	$\{2.1, 1.2, 2.1, 2.1\}$	$\{1.9, 0, 1.9, 1.9\}$	$[2, 0, 1, 3]$
10	5.8	$\{2.1, 1.2, 2.1, 3.8\}$	$\{1.9, 0, 1.9, 1.05\}$	$[2, 0, 1, 2]$

indicates the disconnection of an UE from the network. From Figs. 7 and 8, it can be noticed that the CBS utility has many local maximum values. To find the global maximum, the PSO algorithm as mentioned in Algorithm 1 in Section IV is used.

The optimal CBS price P^*, corresponding the CPEs optimal prices p_i^*, $i \in \{1, 2, 3, 4\}$ and optimal data rate for the UEs under different CPEs for different values of k are given in Table III. Note that the optimal data r_{ji}^* of the jth UE under the ith CPE will only depend on optimal price of the ith CPE p_i^* as the values of k are the same for all CPEs and UEs. Hence, from (10), $r_{ji}^* = k/p_i^* \, \forall j \in \{1, 2, \ldots, n_i\}$. Thus, the optimal data rate of each UE under the ith CPE are equal and denoted as r_i^* in Table III. Fig. 9 shows the variation of the number of connected UEs with maximum amount of available backhaul data rate R_{MAX} at CBS is shown for $k = 4$. It can be noticed that initially for sufficient amount of available backhaul data rate, all the UEs of each cluster are connected but as the available backhaul data rate decreases, some of the UEs of each cluster get disconnected from their associated CPEs. Especially, the UEs with higher *utility threshold* will get disconnected first. Table IV shows the variation of optimal CBS price P^*, optimal price of CPEs p^*, number of connected UEs to each CPE n_i, and the optimal data rate of each UE connected to each CPE r_i^* with R_{MAX} for $k = 4$. It can be noticed that as R_{MAX} decreases, the maximum of the CBS utility occurs at higher value of the CBS price. Hence, the optimal CBS price P^* increases. Consequently, the optimal price of each CPEs p^* also increases and the number of connected UEs to each CPE and their data rate gradually decreases.

VI. CONCLUSION

In this paper, a theoretical investigation is carried out on a two-stage network architecture for providing broadband service in sparsely populated remote rural areas, which advocates the use of TVWS frequencies for serving the users in remote areas from CBS via CPE. The allocation of the total available rate at the CBS among the CPEs and then to the UEs is the main focus of our investigation. To solve this issue, a pricing-based two-stage market model is proposed using Stackelberg competition. By solving the Stackelberg game, the UEs' optimal data rates, CPEs' optimal prices, and CBS's optimal price have been found out, considering a maximum limit on available data rate at CBS. Extensive simulation studies have been done to study the behavior of UE utility, CPE utility, and CBS utility with variations of different parameters. Furthermore, variations of number of connected UEs, UEs' optimal data rates, CPEs' optimal prices, and CBS's optimal price with maximum data-rate limit have been also studied. Simulation studies depict the proportionality constant k and the maximum available backhaul data-rate limit as an important parameter to determine the optimal strategies UE, CPE, and CBS, and the number of connected UEs. In summary, this study facilitates a game-theoretic scheme, based on which it is possible to distribute the data rate optimally among the UEs depending on each UE's willingness-to-pay and the total available data rate where each network entity behaves selfishly.

ACKNOWLEDGMENT

The work was undertaken as part of Information Technology Research Academy (ITRA)-Media Lab Asia project entitled "Mobile Broadband Service Support over Cognitive Radio Networks." The first two authors are thankful to ITRA Media Lab Asia, for providing funding to carry out this work.

REFERENCES

[1] Y. Knoll, "WHITE PAPER, enabling rural broadband services with wireless backhaul," Ceragon Networks, Oct. 2009. [Online]. Available: http://www.winncom.com
[2] [Online]. Available: http://www.digitalindia.gov.in/content/vision-and-vision-areas. Accessed on: Jun. 8, 2018.
[3] Jan. 2018. [Online]. Available: http://www.bbnl.nic.in

[4] X. Huang, Y. J. Guo, A. Zhang, and V. Dyadyuk, "A multi-gigabit microwave backhaul," *IEEE Commun. Mag.*, vol. 50, no. 3, pp. 122–129, Mar. 2012.
[5] J. Simo-Reigadas, E. Municio, E. Morgado, E. M. Castro, and A. Martinez, "Sharing low-cost wireless infrastructures with telecommunications operators for backhauling 3G services in deprived rural areas," in *Proc. IEEE 16th Int. Symp. World Wireless, Mobile Multimedia Netw.*, Jun. 2015, pp. 1–8.
[6] G. Naik, S. Singhal, A. Kumar, and A. Karandikar, "Quantitative assessment of TV white space in India," in *Proc. 20th Nat. Conf. Commun.*, Feb. 2014, pp. 1–6.
[7] R. Kennedy, K. George, O. Vitalice, and W. Okello-Odongo, "TV white spaces in Africa: Trials and role in improving broadband access in Africa," in *Proc. AFRICON*, Sep. 2015, pp. 1–5.
[8] J. van de Beek, J. Riihijarvi, A. Achtzehn, and P. Mahonen, "TV white space in Europe," *IEEE Trans. Mobile Comput.*, vol. 11, no. 2, pp. 178–188, Feb. 2012.
[9] T. Shimomura, T. Oyama, and H. Seki, "Analysis of TV white space availability in Japan," in *Proc. IEEE Veh. Technol. Conf.*, Sep. 2012, pp. 1–5.
[10] FCC, "Third memorandum opinion and order," 2012.
[11] OFCOM, "Statement on cognitive access to interleaved spectrum," Jul. 2009.
[12] K. Patil, K. E. Skouby, and R. Prasad, "Cognitive access to TVWS in India: TV spectrum occupancy and wireless broadband for rural areas," in *Proc. 16th Int. Symp. Wireless Pers. Multimedia Commun.*, Jun. 2013, pp. 1–5.
[13] M. Deshmukh, K. Patif, F. Frederiksen, K. Skouby, and R. Prasad, "Wireless broadband network on TVWS for rural areas: An Indian perspective," in *Proc. 16th Int. Symp. Wireless Pers. Multimedia Commun.*, Jun. 2013, pp. 1–5.
[14] O. Holland, "Some are born with white space, some achieve white space, and some have white space thrust upon them," *IEEE Trans. Cogn. Commun. Netw.*, vol. 2, no. 2, pp. 178–193, Jun. 2016.
[15] C. McGuire, M. R. Brew, F. Darbari, S. Weiss, and R. W. Stewart, "Enabling rural broadband via TV white space," in *Proc. 5th Int. Symp. Commun., Control Signal Process.*, May 2012, pp. 1–4.
[16] S. Kawade and M. Nekovee, "Is wireless broadband provision to rural communities in TV whitespaces viable? A UK case study and analysis," in *Proc. IEEE Int. Symp. Dyn. Spectr. Access Netw.*, Oct. 2012, pp. 461–466.
[17] C. Gerami, N. Mandayam, and L. Greenstein, "Backhauling in TV White Spaces," in *Proc. IEEE Global Telecommun. Conf.*, Dec. 2010, pp. 1–6.
[18] Y. Han, E. Ekici, H. Kremo, and O. Altintas, "Vehicular networking in the TV white space band: Challenges, opportunities, and a media access control layer of access issues," *IEEE Veh. Technol. Mag.*, vol. 12, no. 2, pp. 52–59, Jun. 2017.
[19] G. Ding, J. Wang, Q. Wu, Y. D. Yao, F. Song, and T. A. Tsiftsis, "Cellular-base-station-assisted device-to-device communications in TV white space," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 1, pp. 107–121, Jan. 2016.
[20] A. Kumar, R. Kumar, P. Rathod, and A. Karandikar, "How much TV UHF band spectrum is sufficient for rural broadband coverage?" in *Proc. 13th Int. Symp. Modeling Optim. Mobile, Ad Hoc, Wireless Netw.*, May 2015, pp. 419–426.
[21] M. T. Masonta, T. M. Ramoroka, and A. A. Lysko, "Using TV white spaces and e-learning in South African rural schools," in *Proc. IST-Africa Conf.*, May 2015, pp. 1–12.
[22] S. Roberts, P. Garnett, and R. Chandra, "Connecting Africa using the TV white spaces: From research to real world deployments," in *Proc. 21st IEEE Int. Workshop Local Metropolitan Area Netw.*, Apr. 2015, pp. 1–6.
[23] M. Khalil, J. Qadir, O. Onireti, M. A. Imran, and S. Younis, "Feasibility, architecture and cost considerations of using TVWS for rural internet access in 5G," in *Proc. 20th Conf. Innov. Clouds, Internet Netw.*, Mar. 2017, pp. 23–30.
[24] D. Niyato and E. Hossain, "A game theoretic analysis of service competition and pricing in heterogeneous wireless access networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5150–5155, Dec. 2008.
[25] M.-D. Weng, B.-H. Lee, and J.-M. Chen, "Two novel price-based algorithms for spectrum sharing in cognitive radio networks," *EURASIP J. Wireless Commun. Netw.*, vol. 2013, no. 1, 2013, Art. no. 265.
[26] I. V. Loumiotis, E. F. Adamopoulou, K. P. Demestichas, T. A. Stamatiadi, and M. E. Theologou, "Dynamic backhaul resource allocation: An evolutionary game theoretic approach," *IEEE Trans. Commun.*, vol. 62, no. 2, pp. 691–698, Feb. 2014.
[27] L. Liang, G. Feng, and Y. Jia, "Game-theoretic hierarchical resource allocation for heterogeneous relay networks," *IEEE Trans. Veh. Technol.*, vol. 64, no. 4, pp. 1480–1492, Apr. 2015.
[28] F. Hessar and S. Roy, "Resource allocation techniques for cellular networks in TV white space spectrum," in *Proc. IEEE Int. Symp. Dyn. Spectr. Access Netw.*, Apr. 2014, pp. 72–81.
[29] R. Kumbhkar, M. N. Islam, N. B. Mandayam, and I. Seskar, "Rate optimal design of a wireless backhaul network using TV white space," in *Proc. 7th Int. Conf. Commun. Syst. Netw.*, Jan. 2015, pp. 1–7.
[30] S. Sadr and R. S. Adve, "Partially-distributed resource allocation in small-cell networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 6851–6862, Dec. 2014.
[31] D. Niyato and E. Hossain, "A microeconomic model for hierarchical bandwidth sharing in dynamic spectrum access networks," *IEEE Trans. Comput.*, vol. 59, no. 7, pp. 865–877, Jul. 2010.
[32] J. Nash, "Non-cooperative games," *Ann. Math.*, vol. 54, no. 2, pp. 286–295, 1951. [Online]. Available: http://www.jstor.org/stable/1969529
[33] X. S. Yang, *Engineering Optimization: An Introduction With Metaheuristic Applications*. Hoboken, NJ, USA: Wiley, 2010.

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