Ring Taxonomy

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Fields $\subset$ Euclidean domains $\subset$ PIDs $\subset$ UFDs $\subset$ Integral domains

Examples of property X do not apply to listed property $Y \subset X$. Eg, examples of UFDs are not also PIDs

Ideals

- maximal $\implies$ prime

Integral domains

- prime $\implies$ irreducible
- Ex:
- Ex of irreducible but not prime ideal: TODO

UFD

- prime $\iff$ irreducible
- Ex: $\mathbb{Z}[x], \mathbb{Q}[x, y]$

PID

- irreducible $\implies$ prime (also from inclusion in UFD)
- Every nonzero prime ideal is maximal.
- R commutative, $R[x]$ PID $\iff$ R is field. (since (x) is prime $\implies$ maximal)
- Ex:

Euclidean domain

- Ex: $F[x]$

Fields

- Ex: $\mathbb{R}, \mathbb{Q}, \mathbb{R}[x]/(x^2+1) \cong \mathbb{C}$