Step-by-Step Construction of the Cost Matrix

To understand how the `scr` matrix in the Victor–Purpura algorithm is filled, we start with a simple example.

Example 1

Let

$$T_1 = \{1, 3\}, \quad T_2 = \{2\}, \quad q = 1$$

We initialize a (3×2) matrix (since $|T_1| = 2$, $|T_2| = 1$):

$$\begin{bmatrix} 0 & 1 \\ 1 & ? \\ 2 & ? \end{bmatrix}$$

Now we compute each cell:

- $scr(1,1) = \min\{scr(0,1) + 1, scr(1,0) + 1, scr(0,0) + q \cdot |1 - 2|\} = \min\{2, 2, 1\} = 1$
- $scr(2,1) = \min\{scr(1,1) + 1, scr(2,0) + 1, scr(1,0) + q \cdot |3 - 2|\} = \min\{2, 3, 2\} = 2$

Final matrix:

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Example 2

We compute the Victor–Purpura distance between:

$$T_1 = \{1, 15, 113.5, 116, 119\}, \quad T_2 = \{1.5, 2, 15.1, 114, 118, 1100\}, \quad q = 0.5$$

So $nspi = 5$, $nspj = 6$. The cost matrix `scr` will be 6×7. We'll fill it step by step.

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Step 0: Initialization (Insertions and Deletions Only)

We initialize the borders with insertion and deletion costs:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & & & & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \end{bmatrix}$$

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Step 1: Compute `scr[1,1]`

Options: Delete: $scr[0,1] + 1 = 1 + 1 = 2$

Insert: $scr[1,0] + 1 = 1 + 1 = 2$

Shift: $scr[0,0] + 0.5 \cdot |1 - 1.5| = 0 + 0.25 = 0.25$

$\Rightarrow scr[1,1] = 0.25$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & \textbf{0.25} & & & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \end{bmatrix}$$

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Step 2: Compute `scr[1,2]`

Options: Delete: $scr[0,2] + 1 = 2 + 1 = 3$

Insert: $scr[1,1] + 1 = 0.25 + 1 = 1.25$

Shift: $scr[0,1] + 0.5 \cdot |1 - 2| = 1 + 0.5 = 1.5$

$\Rightarrow scr[1,2] = 1.25$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0.25 & \textbf{1.25} & & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \end{bmatrix}$$

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Step 3: Compute `scr[1,3]`

Options: Delete: $scr[0,3] + 1 = 3 + 1 = 4$

Insert: $scr[1,2] + 1 = 1.25 + 1 = 2.25$

Shift: $scr[0,2] + 0.5 \cdot |1 - 15.1| = 2 + 7.05 = 9.05$

$\Rightarrow scr[1,3] = 2.25$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0.25 & 1.25 & \textbf{2.25} & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \end{bmatrix}$$

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Step 4: Compute `scr[2,1]`

$$
\begin{aligned}
\text{Options:}\quad & \text{Delete: } scr[1,1] + 1 = 0.25 + 1 = 1.25 \\
& \text{Insert: } scr[2,0] + 1 = 2 + 1 = 3 \\
& \text{Shift: } scr[1,0] + 0.5 \cdot |15 - 1.5| = 1 + 6.75 = 7.75 \\
\Rightarrow & scr[2,1] = 1.25
\end{aligned}
$$

$$
\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
1 & 0.25 & 1.25 & 2.25 & & & \\
2 & 1.00 & \textbf{1.25} & & & & \\
3 & & & & & & \\
4 & & & & & & \\
5 & & & & & &
\end{bmatrix}
$$

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Final Cost Matrix

After filling all entries using the recurrence rule, the final cost matrix `scr` is:

$$
\begin{bmatrix}
0.00 & 1.00 & 2.00 & 3.00 & 4.00 & 5.00 & 6.00 \\
1.00 & 0.25 & 1.25 & 2.25 & 3.25 & 4.25 & 5.25 \\
2.00 & 1.25 & 2.25 & 1.30 & 2.30 & 3.30 & 4.30 \\
3.00 & 2.25 & 3.25 & 2.30 & 1.55 & 2.55 & 3.55 \\
4.00 & 3.25 & 4.25 & 3.30 & 2.55 & 2.55 & 3.55 \\
5.00 & 4.25 & 5.25 & 4.30 & 3.55 & 3.05 & \textbf{4.05}
\end{bmatrix}
$$

The Victor–Purpura distance is given by the final cell in the matrix:

$$
\boxed{D_{\mathsf{VP}}(T_1, T_2) = 4.05}
$$

This reflects the minimal cost of transforming T_1 into T_2 using spike insertions, deletions, and time shifts with cost parameter $q = 0.5$.