Reasoning about Retirement Investment Options

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Abstract

The goal of this document is to clarify the difference between different kinds of retirement investments. It's about understanding how taxes interact with geometric (exponential) growth, mathematically. I'm not attempting to verify or explain the tax laws. This is more to say: *if* a certain kind of tax-privileged retirement account option existed, when would it be worth your while to use it? That being said, I focused on working out cases I think do exist in the USA today. I've tried to unpack the equations to make this accessible to those less comfortable with math. This discussion is probably only relevant to people in a middle bracket of income and wealth, who have enough income that they can afford to save for retirement, and maybe some existing savings, but not too much more than that. I found this kind of fun as a math exercise and somewhat pragmatically useful, but it's not fundamentally interesting.

I am not a financial advisor or tax expert. This is math, not financial advice.

0.1 The points I will illustrate

In Sections 1-4, I put equations to the widely appreciated facts that investments grow exponentially, that taxes can take a huge bite out of that, and that one can substantially mitigate the effect of taxes by putting long-term savings in a tax-advantaged retirement account. I also put math behind the conventional advice (but less-appreciated fact) that there's no difference between tax-deferred and tax-free retirement accounts, other than the question of when in your life your tax rate is likely to be lower. Some employers offer the option of putting after-tax dollars into tax-deferred retirement accounts. In Section 5, I argue that this is a terrible deal (and explain why I think the option nevertheless exists). I sum up the actionable implications in Section 6. In Section 7, I work through in detail a special case that isn't much discussed: what if you are employed and making enough to pay your bills, but no more; but you have some savings in regular (taxable) accounts. Should you contribute to retirement accounts from your paycheck, if you'd have to dip into your savings to replace that income? In my experience financial advisors often don't know or get this wrong. In Section 8, I plug in some hypothetical numbers to concretize what it all means. If your interest is pragmatic, you can read that first to see if these factors make a big enough difference for you care.

0.2 Preliminaries

For the sake of comparisons, I will assume you would make the identical investments regardless of which type of account the investments are held in. The variable r refers to the growth rate *actually realized* between the initial investment time and time you withdraw the funds, given the market fluctuations and inflation that actually occur over that time. This is not something you can know in advance. But whatever this turns out to be in the end, it would be the same across all the alternatives we are comparing. To define some terms:

- Let the initial amount invested be M
- Let the annualized growth be r (where 1 is no growth)
- Let the years between investment and withdrawal be t
- Let the tax rates be T_{now} and T_{future}
- Let the fraction you keep after tax be $K \equiv 1 - T$,
  i.e. $K_{now} \equiv 1 - T_{now}$, or $K_{future} \equiv 1 - T_{future}$
- Let the amount available to spend at the time of withdrawal be M_{future}

T_{now} and T_{future} refer to the applicable tax rates applied now (at the time of investment) and in the future (at the time of withdrawal). Those could be either income tax rates or capital gains tax rates depending on the scenario. Future tax rates and your own future tax bracket are matters of speculation, but whatever they turn out to be, they would be the same across the compared conditions.

1 Baseline: no taxes

To provide a common point of reference to compare all other options, consider the case in which there is no tax: $T_{now} = T_{future} = 0$. In this case the expected value at time t is simply:

$$M_{future} = Mr^t \tag{1}$$

In the following sections we will refer to this as $M_{future,baseline}$.

2 Naive Long-term Investing

The straightforward approach to savings would be to pay income tax on your income (because you have to), and then invest some of it in an ordinary investment account. Then you will pay long-term capital gains tax at the time of withdrawal, on all but the initial investment M. For simplicity I'll assume all gains are "realized" (taxed) at the time of the final withdrawal, but this assumption is not critical. In this case, T_{now} is your current income tax rate, and T_{later} is your future capital gains tax rate.

$$M_{future,naive} = MK_{now}r^t K_{future} + MK_{now}T_{future} \tag{2}$$

To walk through that slowly: MK_{now} is the amount you initially invest after you pay income tax on M, and r^t is the multiplicative factor by which it grows in time t. If you had to pay capital gains tax on all of that upon withdrawal, you would keep K_{future} fraction of that. But you don't have to pay tax on the initial investment, so the second term refunds that amount of the tax.

The equation can be simplified to:

$$M_{future,naive} = MK_{now}(r^t K_{future} + T_{future}) \tag{3}$$

This differs from the baseline by a factor of

$$\frac{M_{future,naive}}{M_{future,baseline}} = \frac{MK_{now}(r^t K_{future} + T_{future})}{Mr^t} \tag{4}$$

which simplifies to

$$\frac{M_{\text{future,naive}}}{M_{\text{future,baseline}}} = K_{\text{now}} \left(K_{\text{future}} + \frac{T_{\text{future}}}{r^t} \right) \tag{5}$$

At the outset, when $t = 0$, this ratio reduces to K_{now}. In the limit as $t \to \infty$, it reduces to $K_{now}K_{future}$.

This is substantially worse than baseline. Therefore, a government might try to incentivize saving for retirement by waiving either T_{now} or T_{future} on retirement savings accounts, in exchange for limiting access to those funds until retirement age.

3 Tax-deferred Retirement Accounts

Suppose you can contribute to a retirement account with pre-tax funds, meaning that your contributions are deducted from your total income before calculating your income tax bill. The catch is, you have to pay *income tax* on the *entire* amount you withdraw, at the time you withdraw t at the tax rate of T_{future}. In such an account, $T_{now} = 0$. I'll call this a "tax-deferred retirement account". In this case, at time t you will have:

$$M_{future,deferred} = Mr^t K_{future} \tag{6}$$

This differs from the no-tax baseline by a multiplicative factor of

$$\frac{M_{future,deferred}}{M_{future,baseline}} = \frac{Mr^t K_{future}}{Mr^t} = K_{future} \tag{7}$$

That's worse than if there were no taxes, but significantly better than naive investing (paying taxes twice). Note that this multiplicative factor does not depend on M, r, or t.

4 Tax-free Retirement Accounts

Suppose instead you can contribute to a retirement account with after-tax dollars (pay income tax up front), but then all earnings within the account thereafter are tax-free. In this case T_{now} is your current income tax rate, and $T_{future} = 0$. This might seem like a much better deal, because the funds can grow exponentially inside the account, and all of that growth within the account will be tax-free. Mathematically, this looks like:

$$M_{future,taxfree} = Mr^t K_{now} \tag{8}$$

which differs from the baseline by a multiplicative factor of

$$\frac{M_{future,taxfree}}{M_{future,baseline}} = K_{now} \tag{9}$$

Comparing the two kinds of retirement accounts to each other:

$$\frac{M_{future,deferred}}{M_{future,taxfree}} = \frac{(1 - T_{future})}{(1 - T_{now})} = \frac{K_{future}}{K_{now}} \tag{10}$$

Thus, if your tax rate is the same now as in the future, it makes no difference which kind of retirement account you use. The key intuition is this: if you pay taxes at the end, you'll have to pay tax on a much larger amount of income; but if you pay taxes up front, you lose all the exponential growth you would have had on the part you gave up. These exactly cancel out.

The only reason it makes any difference which of the two you choose is if your tax rate changes. It is advantageous to pay the taxes at whichever end you your income tax rate is lower. If you think your income tax rate will be the same at both times, then it makes absolutely no difference which you do.

5 After-tax contributions to tax-deferred accounts

Some employers allow you to make *after-tax* contributions to a traditional *tax-deferred* retirement account. This is similar to naive investing, in that you pay tax up front *and* when you cash out (Equation 3). But in this case you pay *income tax* on the gains when you withdraw, and that rate is generally higher than long-term capital gains tax. So it's usually worse than investing naively.

The only reason I can think of to make after-tax contributions to a traditional tax-deferred account is if you can immediately move them into a tax-free account (conversion or "rollover"). The only reason this would be necessary is if there are limits on after-tax contributions to tax-free accounts, in which case this mechanism would allow you to circumvent those limits. At the moment in the USA those two conditions often apply, so this mechanism is often useful.

6 Bottom Line So Far

The main take home lesson is: given there is income tax, if you are able to set aside any money for retirement, it's significantly advantageous to put it either in a tax-deferred or tax-free retirement account, to avoid paying tax twice. It's much less important which of the two you choose. The optimal choice depends on whether you think your taxes will be higher or lower when you retire. If you're not sure, you can hedge your bets by using both mechanisms. There are limits on how much you are allowed to save in this way, but if you are able to set aside more than that, there are ways to circumvent those limits.

7 Existing taxable investments

7.1 Moving taxable investments into retirement accounts

Suppose you already have some savings in an ordinary taxable investment account. Maybe you paid income tax on the funds you originally invested there, but that's water under the bridge – a sunk cost – so it does not factor into decisions about what to do next. If you could move these savings into an after-tax, tax free retirement account, should you?

If the savings are in cash, by all means do this immediately, to the extent the tax code allows. Other things being equal, $T_{future} = 0$ is always better than $T_{future} > 0$. Likewise, if the investments currently have the same value as when you bought them, or less (net loss), there's no down side to selling them and putting the cash in an after-tax, tax-free account, if you can.

It is less obvious what to do if the taxable account contains investments, such as stocks, that have appreciated in value since you bought them. In that case, to put them into a tax-free account you would first have to sell the investments, and you'll pay tax on the gains when you do so. Therefore the amount you deposit in the tax-free retirement account will be less than you had in the original account. Does it ever make sense to do this?

Let's say the value of the taxable account is M. The assets have a fractional cost basis of b (what you initially paid for them), and a fractional embedded gains g=1-b.

Scenario A: You sell the stocks for M in cash. After paying capital gains tax this leaves you with $M(gK_{now} + b)$ you can invest in your tax-free account. When you withdraw in retirement at time t, this will have grown by a factor of r^t, with no further taxes to pay. So you will have:

$$M_{future,A} = Mr^t(gK_{now} + b) \tag{11}$$

Scenario B: You leave M in the taxable account and liquidate it at time t, paying tax on the gains at that time. This option looks like:

$$M_{future,B} = Mr^tK_{future} + MbT_{future} \tag{12}$$

It's similar to the equation for naive investing (Equation 3), except we don't pay any (additional) tax now, and when we sell, the part we *don't* have to pay tax on is only the *basis* portion of M. Equation 12 can be rearranged to:

$$M_{future,B} = Mr^t(K_{future} + \frac{bT_{future}}{r^t}) \tag{13}$$

which makes it easier to compare the two alternatives, because Mr^t cancels out:

$$\frac{M_{future,A}}{M_{future,B}} = \frac{Mr^t(gK_{now} + b)}{Mr^t(K_{future} + \frac{bT_{future}}{r^t})} = \frac{gK_{now} + b}{K_{future} + \frac{bT_{future}}{r^t}} \tag{14}$$

Bottom line: selling-to-reinvest (Scenario A) is advantageous if

$$gK_{now} + b > K_{future} + \frac{bT_{future}}{r^t} \tag{15}$$

This looks complicated but it's simpler if we consider two extremes. Any real investment account has some non-zero basis cost, and we're only considering the case where there are at least some gains (because if there are no gains, we already know the transaction is strictly advantageous). So the situation will lie somewhere between the extremes of $b = 0$ and $g = 0$.

First, consider the limit in which the investments had no basis cost: $b = 0$, so their value is entirely gains: $g = 1$. In that case Equation 15 reduces to:

$$K_{now} > K_{future} \tag{16}$$

In this extreme, if your current and future capital gains tax rates are identical, it's a wash. You will take the hit for capital gains tax on your embedded gains now, but you would've had to pay it later anyway, so it doesn't matter. If your current tax rate is lower, it's best to pay tax on the gains-to-date at this time, and if you expect your tax rate to go down later, it's better to pay taxes then.

Now consider the opposite extreme in which the original investments had no embedded gains, $g = 0$, $b = 1$. Then Equation 15 reduces to:

$$1 > K_{future} + \frac{T_{future}}{r^t} \tag{17}$$

which (recalling that $K_{future} = 1 - T_{future}$), can be rearranged to:

$$r^t > 1 \tag{18}$$

This means that in the limit of no embedded gains, the transaction will prove to be advantageous as long as there is *any growth at all* between now and the time of withdrawal; the more growth, the more advantageous it becomes.

Thus, regardless of the amount of embedded gains in the taxable account, if you expect your future capital gains tax rate to be the same as (or higher than) your current capital gains tax rate, and you expect your investments to grow at all in the long run ($r > 1$), selling assets in an existing taxable investment account in order to deposit the funds in an after-tax, tax-free account *is always a good idea*.

The greater total growth r^t you expect in the future (higher return rate and/or longer time), the more advantageous this is. Therefore it seems like an especially good idea to do this to move the highest risk, highest return part of a balanced investment portfolio into an after-tax, tax-free retirement account. Putting it in a retirement account means you can't cash it out until later in life – but that's exactly what you want for investments that are expected to be volatile on short timescales but outperform in the long run.

The closer you are to the no-embedded-gains extreme, the greater the advantage. Therefore if you only want to move part of your savings, it's optimal to sell and move the assets with the least embedded gains.

The edge case is if you think your future capital gains tax rate is very likely to be *lower* later *and* your current taxable account has high fractional embedded gains. Then the advantage for the basis part of the investment may or may not outweigh the disadvantage of paying a higher tax rate on the gains-to-date. Equation 15 can be used to determine this.

The failure case is if your investments turn out to have a net loss at the time of withdrawal in retirement ($r < 1$). In that case, you would have been better off not paying taxes on gains up front, because there would not have been any taxes to pay at the withdrawal time. But this discussion is about long-term savings for retirement, and if you invest at all reasonably, a net loss is unlikely over the long run.

We already showed that tax-deductible contributions to a tax-deferred retirement account are equivalent to after-tax contributions to a tax-free retirement account, other than the change in tax rate between the two times. Likewise, all the arguments in favor of selling taxable investments to invest them in a tax-free retirement account generalize to the case of selling taxable assets to make a *deductible* contribution to a tax-deferred retirement account.

7.2 Spending your savings in order to save

The initial impetus for this entire analysis was the following question: does it make sense to contribute part of your salary to a workplace retirement account, if that would force you to dip into your (taxable, stock-invested) savings to support your living expenses? If your employer makes matching contributions, this is obviously a win. But otherwise? Two different professional investment advisors told me no. I believe they were wrong.

The analysis I provided in Section 7.1 assumed you would deposit the cash proceeds of a taxable stock sale into an after-tax, tax-free retirement account. But the math would be exactly the same if you just kept the cash you got from selling the stocks, and had the same amount of your paycheck deposited to the after-tax retirement account. Yes, you'd be paying income tax on the new income before it went into the retirement account, but you were going to pay that income tax anyway.

Therefore, if it makes sense to sell stocks from a taxable investment account to deposit the cash in a retirement account – and it almost always does – then it also makes sense to deposit your salary into a retirement account, even if you have to sell taxable investments to replace that income.

This is a consequential extension of the argument above because there tend to be low annual contribution limits for direct cash contributions to retirement accounts (whether deductible, tax-deferred or after-tax, tax-free); but much higher allowances for payroll deduction contributions. Therefore, if you want to shift savings from taxable into tax-advantaged retirement accounts, you might be able to shift considerably more through payroll deductions than you would be able to otherwise.

8 A concrete example

Fine, but does all this really make enough difference to care about it? To answer that let's plug in some realistic numbers.

José is a single 25-year-old living in California and earning a salary of $\$150,000$/year. He wants to retire when he is 60, so his savings this year will have $t = 35$ years to grow. He plans on investing in a diversified mutual fund that projects an average annualized return of 10% in the long haul. He plans to retire in Texas, where he's hoping there will still be no income tax and no capital gains tax. Due to some past success dabbling in cryptocurrency, he was also able to put $\$16,000$ into the stock market several years ago, and the value of that account has already grown to $\$20,000$.

Fast forward to 2060. The mutual fund turns out to earn exactly 10% annualized return between 2025 and 2060, and inflation over that time turns out to be 2% over that interval. In that case, his effective, inflation-adjusted annualized growth rate is $r = 1.10/1.02 = 1.0784$, for all the alternatives compared. The time interval is $t = 2060 - 2025 = 35$ years, for all the alternatives compared. For every $M = \$1$ he invested in 2025, how much will he have to spend in 2060?

With an income of $\$150,000$/year his federal income tax rate in 2025 was 24% and state income tax rate in California was 9.3%. His federal long-term capital gains tax rate was 15%, and his state long-term capital gains tax rate was 9.3%. Suppose that in 2060 Texas is still tax-free, and his Federal tax bracket and tax rates are the same as in 2025. Thus:

- $r = 1.0784$
- $t = 35$
- $T_{now,income} = 0.24 + 0.093 = 0.333$
- $T_{now,capital} = 0.15 + 0.093 = 0.243$
- $T_{future,income} = 0.24$
- $T_{future,capital} = 0.15$

therefore,

- $K_{now,income} = 1 - 0.333 = 0.667$
- $K_{now,capital} = 1 - 0.243 = 0.757$
- $K_{future,income} = 1 - 0.24 = 0.76$
- $K_{future,capital} = 1 - 0.15 = 0.85$

8.1 How best to invest

Baseline: by Equation 1, if there had been no taxes, every $1 he saved in 2025 would have grown by 2060 to:

$$M_{future,baseline} = Mr^t = \$1 * 1.0784^{35} = \$14.05 \tag{19}$$

Naive Investing: If he had simply paid income tax on his income, and put money into his taxable investment account in the same mutual fund, then by Equation 3, every $1 he saved in 2025 would have in 2060 been worth:

$$M_{future,naive} = \$1 * 0.667(1.0784^{35} * 0.85 + 0.15) = \$8.07 \tag{20}$$

Pre-tax, Tax-deferred: If instead he had put it on a pre-tax basis into tax-deferred retirement accounts, such as a 401(k) or an Individual Retirement Account (IRA), then according to Equation 6, after paying tax at the time of withdrawal, every $1 he saved in 2025 would in 2060 be worth:

$$M_{future,deferred} = Mr^t K_{future} = \$1 * 1.0784^{35} * 0.76 = \$10.68 \tag{21}$$

After-tax, Tax-free: On the other hand, if he had put it in an after-tax, tax-free retirement account, such as a Roth IRA, then by Equation 8, every $1 he saved in 2025 would in 2060 be worth:

$$M_{future,taxfree} = Mr^t K_{now} = \$1 * 1.0784^{35} * 0.667 = \$9.37 \tag{22}$$

After-tax, Tax-deferred: If he had put the funds into a tax-deferred retirement account on an after-tax basis, then by Equation 2 (with T_{future} being *income* tax), after paying tax at the time of withdrawal, every $1 he saved in 2025 would in 2060 be worth:

$$M_{future,rawdeal} = \$1 * 0.667(1.0784^{35} * 0.76 + 0.24) = \$7.28 \tag{23}$$

So to summarize his options:

$1 saved in 2025	Will be worth in 2060
In a world without taxes	$ 14.05
Naive investing	$ 8.07
Pre-tax, Tax-deferred	$ 10.68
After-tax, Tax-free	$9.37
After-tax, Tax-deferred	$7.28

Conclusion: if José could save anything for retirement in 2025, the pre-tax, tax-deferred account would have been his best option, because he was working in a high-tax state, and moving to a state with a lower tax rate in retirement. If he ran up against a limit of how much he could put in such an account, it would still be worth contributing to an after-tax, tax-free account rather than invest in a regular taxable account. Note that if the market had performed differently, or inflation had been higher or lower, or the date of withdrawal were earlier or later, those factors would change the numbers but **not the ratios between them**.

8.2 No income to spare

However, recall that in 2025 he needed his entire income to meet living expenses. Here's where the savings he already had in a taxable account in 2025 become relevant. He could contribute some if his paycheck to retirement accounts, and sell his taxable investments to replace that income. However, he'll have to pay capital gains tax when he sells taxable investments. Is it worth it?

The value of the taxable investment account in 2025 was $\$20,000$, and the basis was $b = 16,000/20,000 = 0.80$, for a gain of $g = 1 - b = 0.20$. If he left it all in the taxable account, then in 2060 after paying capital gains tax he would have (by Equation 13):

$$M_{future} = \$20,000 * 1.0784^{35}(0.85 + \frac{0.8 * 0.15}{1.0784^{35}}) = \$241,284 \tag{24}$$

There'd be nothing left from his take-home salary, because he would have spent it all on living expenses in 2025.

If instead he had sold his entire taxable investment account in 2025, after paying capital gains tax, that would have produced $\$16,000 + 0.757 * \$4,000 = \$19,028$, enabling him to give up that amount of take-home pay to contribute to retirement accounts.

Suppose he was able to find a way to contribute $\$19,028$ of his *after-tax* income to a *tax-free* retirement account, such as a Roth IRA, in 2025. In 2060, by Equation 11, he'd have:

$$M_{future} = \$20,000 * 1.0784^{35} * (0.2 * 0.757 + 0.8) = \$267,381 \tag{25}$$

There'd be nothing left of the original taxable investment account, because he would have spent it all on living expenses in 2025.

He'd rather use a tax-deferred retirement account, though. Every $\$19,028$ of take-home pay in 2025 came from $\$28,527.74$ of pre-tax income ($\$28,527.74 * 0.667 = 19,028$). Therefore if he contributed $\$28,527.74$ on a pre-tax basis to a tax-deferred account, such as a 401(k) plan, his take-home pay would only go down by $\$19,028$, which he could replace by selling the taxable stocks. By 2060, he would have (by Equation 6):

$$M_{future} = \$28,527.74 * 1.0784^{35} * 0.76 = \$304,662 \tag{26}$$

There'd be nothing left of the original taxable investment account, because he would have spent it all on living expenses in 2025.

So to summarize his three options:

Strategy in 2025	Available Funds in 2060
Leave assets in taxable account	\$241,284
Sell taxable assets, put proceeds in a tax-free retirement account	\$267,381
Sell taxable assets, put proceeds in tax-deferred retirement account	\$304,662

With these particular assumptions, it's a pretty clear win to sell the taxable assets in 2025 to enable contributions to his employer's tax-deferred retirement account (such as a 401(k) plan) – even though there were embedded gains, and his capital gains tax was higher in 2025 than it would be at the time of retirement.

8.3 Coda

To explore how other base assumptions would play out, a MATLAB function implementing these equations can be found here.

To reiterate: this document is not meant to be financial advice. There is a lot more to financial planning and tax planning than considered here, and other factors could interact with these decisions. This exercise was solely intended to clarify the differences between these alternatives arising from the factors considered here, *other things being equal*.