

Smoothing seismicity techniques applied to seismic source characterization and probabilistic seismic hazard analysis in Brazil

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Intraplate seismicity is a hard problem to solve in tectonics, mainly during the lack of observed seismicity in many areas where seismic hazard assessment are also difficult and the methods applied are often close correlated to the geometries defined by the experts with valuable criteria.

Three smoothing seismicity techniques are reviewed and their application to the Brazilian seismicity and a seismic hazard assessment are discussed.

All these smoothing methods are based on a Kernel Density Estimation (KDE). The main difference between them is the bandwidth selection process.

The method of *Frankel* [1995] uses a *fixed* bandwidth in space. *Woo* [1996] suggests a bandwidth as function of the *magnitude*. *Helmstetter and Werner* [2012] propose a *locally-adaptive* kernel bandwidths in space and time, chosen for each event.

This last one was proposed initially to take care of the background seismicity rate in the context of long-term earthquake forecast and the time-independent assumption about the annual background seismic rate could be used (not indiscriminately REFERENCE PAGANI WEATHERILL) as a grid of points seismic sources.

The results shown that all methods clearly define some known seismic regions and could be used in the Brazilian seismicity. All methods improve the spatial resolution of the GSHAP model, the only previously available for the region.

The Last two methods was implemented and are available as a free software on the Hazard Modeling Toolkit (HMTK) from GEM Foundation.

All the hazard calculations was performed on OpenQuake [*Pagani et al.*,
2014] open source software.

1. Introduction

1.1. Intraplate Seismicity

1.2. Previous Brazilian Studies

Despite some studies of seismicity rates and attempts at defining the main seismic zones [Berrocal *et al.*, 1984], 2000, Assumpção in [Talwani, 2014] (REFERENCE), Brazil has no seismic hazard map at national scale.

The Brazilian seismic hazard building code [ABNT, 2006] was based on the GSHAP [Giardini *et al.*, 1999] results for South America [Shedlock and Tanner, 1999], which is the last published model for the whole region. However, some significant seismic areas (Porto dos Gaúchos [Barros *et al.*, 2009]), are not represented in the GPSHAP-based building codes.

A significant number of earthquakes has been recorded in the last decades [BSB, 2014] allowing a better definition of seismic zones. For this reason an updated hazard map is needed.

1.3. Previous Smoothing Seismicity (Frankel, Woo, Helmstetter...)

Although the standard *Cornell* [1968] and *McGuire* [1976] methodology could be used to assess seismic hazard using expert pre-defined seismic zones, the goal of this paper is to review a few smoothing seismicity methods based on KDE which results in a grid of smoothed seismicity without define any seismic zone polygon.

The seismicity KDE or the smoothed seismicity grid allow to define a grid of seismic point sources. The seismic rate (a -value) associated to each source is defined by the smoothed seismicity methods, but the other parameters, like b -values, minimum and maximum magnitudes, rupture and depth distributions, must be assigned in other ways.

To perform the KDE, both its shape and bandwidth needs to be defined. Most kernels are Gaussian or power-laws or one of their variants. The main distinction between the methods described here is the bandwidth selection process.

Frankel [1995] smoothing method applies a Gaussian *Nadaraya* [1964]-*Watson* [1964] smoothing technique with a *fixed* smoothing-distance bandwidth to estimate the seismicity rate. *Woo* [1996] proposes a *magnitude-dependent* kernel bandwidth based on the nearest neighbor mean distance in magnitude bins. *Helmstetter and Werner* [2012] proposes a *locally-adaptive* bandwidth, based on kernel estimations for the background seismicity rate in long-term forecasts on both space and time dimensions.

Using one-year forecast and assuming time-independence of the seismicity rate, the same assumption of PSHA, it is possible to use these seismicity values on seismic sources characterization. (REVER E COMENTAR)

1.4. Hazard computation

To perform the hazard computation, the OpenQuake suite was chosen. The modelling tool was the HMTK also available by GEM scientific board. The Frankel method was already implemented on the toolkit and the other two was implemented in the context of this paper and they are free available on a public repository.

The earthquake catalog data comes from the Brazilian seismological research authorities.

The Brazilian seismicity data is presented including some overview plots. Next the method's theory is presented follow by the specific decisions and optimizations performed on each method modelling. And last, the results, conclusions and further considerations are discussed.

1.5. Specific Purpose

2. Brazilian seismicity data

Data on intraplate seismicity in Brazil is currently the Brazilian Seismic Bulletin maintained mainly by the Seismological Center of the University of São Paulo [BSB, 2014]. This catalog is a joint compilation from different sources.

2.1. Hypocenters

For events up to 1981, the catalog corresponds to *Berrocal et al.* [1984] which is a comprehensive compilation of historic and instrumental data with participation of Universidade de São Paulo (USP), Universidade de Brasilia (UnB), Universidade Federal do Rio Grande do Norte (UFRN), Observatório Nacional (ON) and Instituto de Pesquisas Tecnológicas do Estado de São Paulo (IPT). From 1982 to 1995 the information comes from the annual bulletins published by Revista Brasileira de Geofísica. Since 1995, the Bulletin is maintained mainly by USP under the same cooperation network in addition to Universidade Estadual Paulista (UNESP). Earlier versions of this catalog were used to compose the well-known *CERESIS* [1985, 1995] catalogs.

Today the catalog is distributed in two ways. One, which could be called *raw*, contains all compiled information, such as: events outside the country which were felt in Brazil, errors in previous versions and also non-seismic events (known quarry blasts, sonic boom, etc). The other way could be called *clean*, where all non-seismic and low quality events were removed. This last one was used in this paper and the total number of entries is [CALCULAR O NUMERO DE EVENTOS NO CATALOGO].

This catalog only cover crustal events. Andean subduction earthquakes deeper than 50km in the Brazil-Peru border was discarded. The epicenters of the catalog are shown in figure 1.

2.2. Magnitudes

Most magnitudes were computed as mb type or the equivalent mR [Assumpção, 1983] regional magnitude. For historical events magnitudes were estimated from macroseismic data (felt area and maximum intensity) [Berrocal *et al.*, 1984]. Since most GMPE are based on M_W moment scale magnitude, in this present study, we converted all mb magnitudes to M_W values following the guidelines on table 1. REFERENCIA PARA STEPHANE/ASSUMPCAO WORK IN PROGRESS. (Johnston)

2.3. Catalog checking

Figure 2 shows the distribution of depths, day of the week and origin hour. Most events have no depth determination and the well determined depths (from pP phases or local networks) are mostly less than 10km.

Figures 2b and 2c show an almost uniform distribution of day of the week, and a slightly trend to record earthquakes during the night. This could suggest some influence of daily noise level.

The annual number of earthquakes since 1900 (figure 3) shows a steady increase since 1960 and two peaks in 1986 and 1998 derived from two large seismic sequences: João Câmara [Takeya *et al.*, 1989] and Porto dos Gaúchos [Barros *et al.*, 2009]. The increasing number of records after 1960 is due to the operation of global, regional and local stations.

2.4. Declustering procedures

To preserve the Poissonian assumption about the independence of the events, several declustering algorithms were tried and the results as shown in figure 4a. The window-method [Gardner and Knopoff, 1974] was tested with three different windows: Gardner and Knopoff [1974], Uhrhammer [1986] and Grüenthal [van Stiphout *et al.*, 2012]. In

addition the algorithm (AFTERAN) proposed by *Musson* [1999] was also tested using Grüenthal distance window.

In the case of Brazilian catalog the methods did not present large differences at the final number of earthquakes (fig. 4a) and the Gardner-Knopof/Uhrhammer combination was chosen to maximize the number of events.

The map of figure 4b locates all clusters in the catalog. It is an *a-priori* evidence of regions in Brazil where long earthquake series tend to occur more frequently.

2.5. Frequency-magnitude distribution

Figure 6 presents the catalog Magnitude Frequency Distribution (MFD). A general $b = 1$ fit is also plotted, and the completeness magnitude on the incremental distribution is about 3.0.

2.6. Catalog completeness

The completeness of Brazilian seismicity data has evident time and space dependence. The number and quality of stations increased over time. The population density, previously concentrated along the coast increased in other regions.

The histories of most regional stations are not easy available which hinders application of methodologies, such as *Mignan et al.* [2011]. For this reason, spatial completeness was not considered and temporal completeness parameters for the whole country was defined by expert criteria (figure FIGURA COMPLETENESS)

- completeza é problema temporal mas nesse caso tambem espacial nao negligenciavel
- se conhecia metodos para lidar com completeza espacial
- também se conhece o método multiescala (provavel bom solucionador)
- conhece-se o método de Stepp
- mas usa-se uma completeness temporal simples

3. Smoothing methods

Three smoothing methods are now briefly explained.

3.1. Frankel: fixed bandwidth

Frankel [1995] smoothing seismicity proposal consisted originally in using a "correlation distance" d_F as the *fixed* kernel bandwidth and applying the Nadaraya-Watson [*Nadaraya*, 1964; *Watson*, 1964] estimator to smooth a 2D seismicity histogram using a Gaussian kernel:

$$\tilde{n}_j = \frac{\sum_i n_i e^{-\left(\frac{d_{ij}}{d_F}\right)^2}}{\sum_i e^{-\left(\frac{d_{ij}}{d_F}\right)^2}}, \quad (1)$$

where \tilde{n}_j is the smoothed seismicity (number of earthquakes with magnitude m above the minimum magnitude M_d in the catalog) on cell j . n_i is the earthquake counting in each other cell i and d_{ij} is distance between grid cells i and j .

3.2. Woo: magnitude-dependent bandwidth

Woo [1996] suggested to evaluate the contribution of each earthquake i , located at \mathbf{r}_i , to the seismicity rate R (number of earthquakes per year and unity area) on the cell centered in \mathbf{r} , depending on the magnitude m :

$$R(\mathbf{r}, m) = \sum_{i=1}^N \frac{K(\mathbf{r} - \mathbf{r}_i, m)}{T(\mathbf{r}_i)}, \quad (2)$$

where N is the number of earthquakes i in the magnitude bin $m \pm dm$, and $T(\mathbf{r}_i)$ is the completeness time of magnitude m observed on \mathbf{r}_i .

Any kernel could be applied on that definition. I used used the *Kagan and Knopoff* [1980] kernel for infinite spatial domain as described by *Woo* [1996]:

$$K_{KK}(\mathbf{r}, m | a_W) = \frac{a_W - 1}{\pi h(m)^2} \left[1 + \frac{\mathbf{r}^2}{h(m)^2} \right]^{-a_W}, \quad (3)$$

where a_W is fractal dimension factor, generally about 1.5 and 2 [*Vere-Jones*, 1992].

VERIFICAR SE É O CASO DE APRESENTAR OUTRAS FORMAS DE KERNEL.

DAR PREFERENCIAS AOS REALMENTE UTILIZADOS.

To compute the magnitude-dependent bandwidth function $h(m)$, *Woo* [1996] suggested the follow relation

$$h(m|a_0, a_1) = a_0 e^{a_1 m}, \quad (4)$$

where a_0 and a_1 are computed by the regression of the mean nearest distance h from each magnitude bin $m \pm dm$, as illustrated on figure 7.

Just to compare, in figure 7 it is shown the functions computed by *Beauval* [2003] for Norway and Spain. The angular coefficient (a_1) is in the same range of the others but the general mean distance between events into the same magnitude bin is higher than other countries. It is in some way expected. Brazil is quite large than mentioned countries.

3.3. Helmstetter and Werner: locally-adaptive space and time bandwidths

Even work on a forecast perspective, the background seismicity rate for a long-term time-independent forecast could be used to characterize a diffused seismicity under the common assumption that this background seismic rate is invariant over time.

Helmstetter and Werner [2012] proposes a seismicity model space and time dependent using kernels for space and time independently:

$$R(\mathbf{r}, t) = \sum_{i=1}^N \frac{1}{h_i d_i^2} K_t \left(\frac{t - t_i}{h_i} \right) K_r \left(\frac{\|\mathbf{r} - \mathbf{r}_i\|}{d_i} \right), \quad (5)$$

where $R(\mathbf{r}, t)$ is the seismic rate located \mathbf{r} distant from the earthquake occurred on the instant t , K_t is the time domain kernel function, where t_i is the time location of earthquake i and h_i is the temporal bandwidth for earthquake i , K_r is the space domain kernel function, where \mathbf{r}_i is the spatial location of earthquake i and d_i is the spatial bandwidth for earthquake i .

As they are interested in forecast, just past time $t_i < t$ need to be considered. Also the observation completeness is taken in account by a set of weights w in this follow way:

$$R(\mathbf{r}, t) = R_{min} + \sum_{t_i < t} \frac{2 w(\mathbf{r}_i, t_i)}{h_i d_i^2} K_t \left(\frac{t - t_i}{h_i} \right) K_r \left(\frac{\|\mathbf{r} - \mathbf{r}_i\|}{d_i} \right), \quad (6)$$

where R_{min} is the minimum seismic rate, positive, allowing earthquakes to occur where their never occurred yet.

The weights $w(\mathbf{r}_i, t_i)$, computed for each earthquake i , are the Gutenberg-Richter a -value projection. This is the expression used to compute it:

$$w(\mathbf{r}, t) = 10^{b(\mathbf{r}, t)[M_c(\mathbf{r}, t) - M_d]}, \quad (7)$$

where w is the weight factor computed on location \mathbf{r} on instant t , $b(\mathbf{r}, t)$ is the space and time dependent b-value, $M_c(\mathbf{r}, t)$ is the completeness magnitude on location \mathbf{r} and t , M_d is the minimum magnitude value on the catalogue.

These weights increase the seismicity contribution from earthquakes occurred where and when the completeness magnitude M_c was greater than minimum catalog magnitude M_d . They also could easily take into account the space and time completeness and b -value fluctuations.

3.3.1. Local bandwidth computation

The method implemented to compute the space and time bandwidths for each earthquake was the Coupled-Nearest-Neighbor (CNN) [Helmstetter and Werner, 2012] expressed as:

$$h_i, d_i = \arg \min_{\substack{h_i \geq h_k \\ d_i \geq d_k}} [s(h_i, d_i | k_{cnn}, a_{cnn}) := h_i + a_{cnn}d_i], \quad (8)$$

where k_{cnn} is the k^{th} nearest neighbour, a_{cnn} is a space-time coupling factor, d_k is the $\max \{d_j\}, j = 1, \dots, k_{cnn}$ and h_k is the $\max \{h_j\}, j = 1, \dots, k_{cnn}$.

The bandwidths are defined locally by this simple optimization process. It could be small on high earthquake density regions and higher where earthquakes are rarely or regions with information lack.

3.3.2. Stationary seismic rate

After compute the model parameters it is completely defined, and the time-independent or stationary seismic rate \bar{R} on each location \mathbf{r}_0 can be computed on the way proposed by *Helmstetter and Werner* [2012] taking the median value for some location \mathbf{r}_0 over all considered time window:

$$\bar{R}(\mathbf{r}_0) = \text{Median}[R(\mathbf{r}_0, t)]. \quad (9)$$

The median should avoid the seismicity rate fluctuations derived by fore and aftershocks and consequently the decluster procedures. This is one of the most important achievement of this methodology.

3.3.3. Maximum likelihood optimization

The model is completely defined by k_{cnn} , a_{cnn} and R_{min} . To optimize these parameters the catalog is divided in two parts: one for learning about the parameters and other to test them. The target catalog will able us to check the model prediction performance derived from its chosen parameters. The best model parameters will maximize the prediction model capacity likely the target catalog.

If the earthquake occurrence could be modeled by a Poisson process with rate N_p , then the probability to observe exactly n events on the considered time frame is

$$p(N_p, n) = \frac{N_p^n e^{-N_p}}{n!}. \quad (10)$$

Then, over all cells, the log-likelihood, to be maximized, between model (from the learning catalog) prediction and earthquakes observed (on testing catalog) is written as

$$L = \sum_{i_x=1}^{N_x} \sum_{i_y=1}^{N_y} \log p [N_p(i_x, i_y), n(i_x, i_y)] \quad (11)$$

where $N_p(i_x, i_y)$ is the predicted seismic rate on cell (i_x, i_y) and $n(i_x, i_y)$ is the number of target earthquakes observed on cell (i_x, i_y) .

The model parameters R_{min} , a_{cnn} and k_{cnn} should be optimized by the maximization of log-likelihood L .

4. Results

4.1. Simulations

4.1.1. Simulation 1

5. Discussion

Appendix A: Appendix Title

Acknowledgments. The author thanks specially to all IAG-USP seismological team. It is also grateful to Stéphane Drouet and Vincent Guigues for all helpful discussions.

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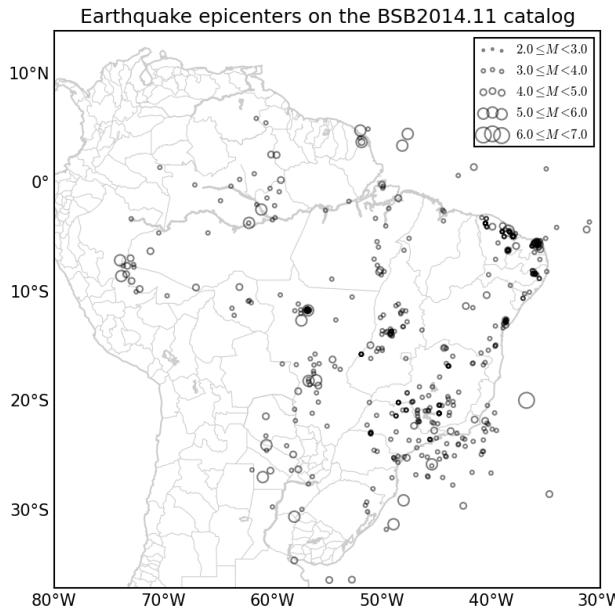


Figure 1. Brazilian seismicity from 1767 to 2014. Source: [BSB, 2014].

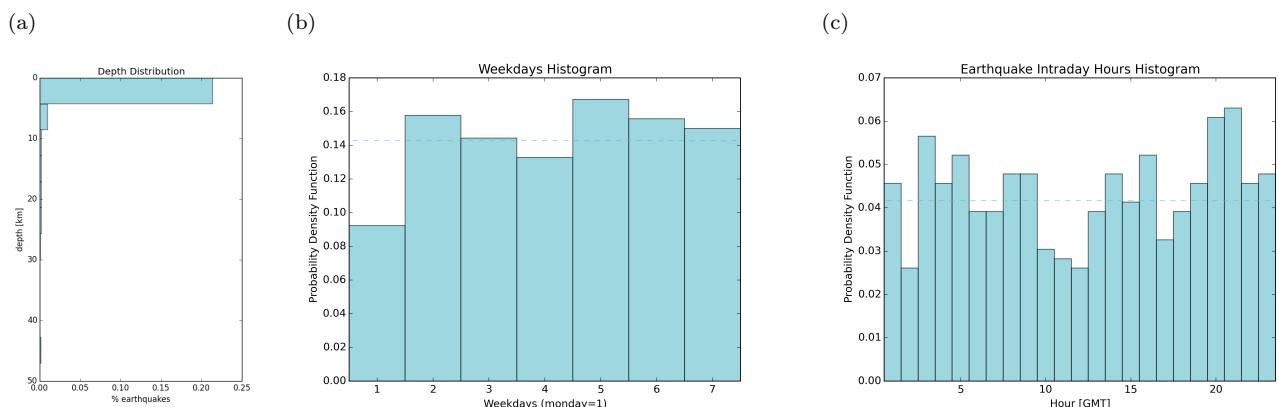


Figure 2. Catalogue overview. The histogram (a) shows the depth distribution. The histograms (b) and (c) represent the distributions of weekday and hour which earthquakes occur respectively. Dashed lines represent the mean value.

Table 1. BSB 2014.11 M_W proxies *ad-hoc* guidelines.

magnitude source	rule	uncertainty
mb or mR	$M_W(m) = 1.121m - 0.76$	0.3
Area felt A_f	$M_W(A_f) = 0.81 + 0.639 \log(A_f) + 0.00084\sqrt{A_f}$	0.4
Maximum intensity I_0	$mb(I_0) = 1.21 + 0.45I_0$ then $M_W(m)$	0.6
mb and A_f	$M_W(m, A_f) = 0.7M_W(m) + 0.3M_W(A_f)$	0.33

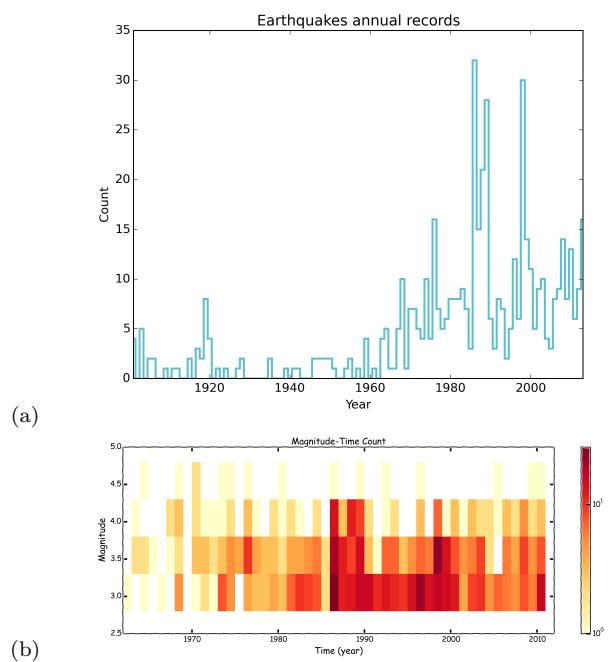


Figure 3. Earthquakes by year (a) and discriminated by magnitude (b).

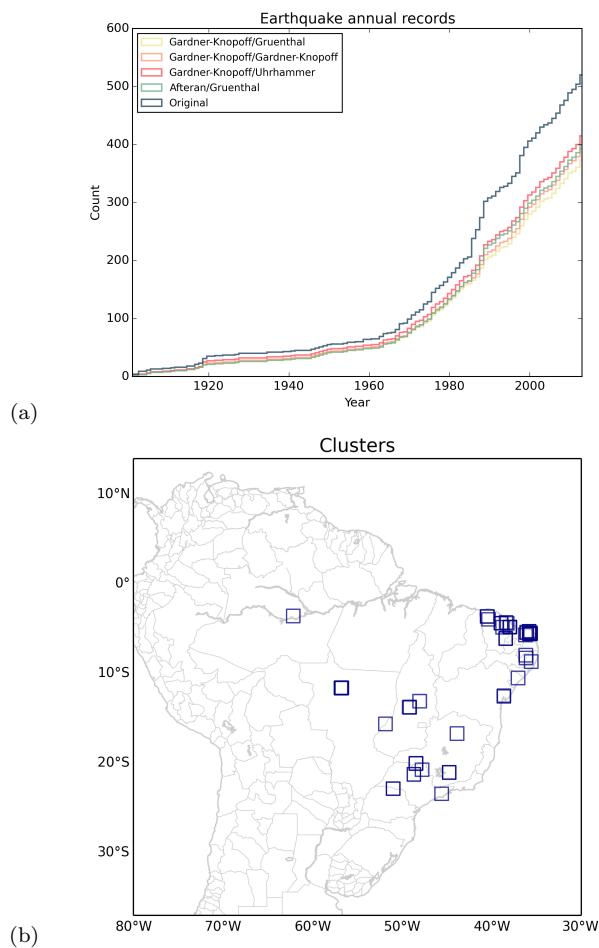


Figure 4. Decluster evaluation (a) and the clusters map (b).

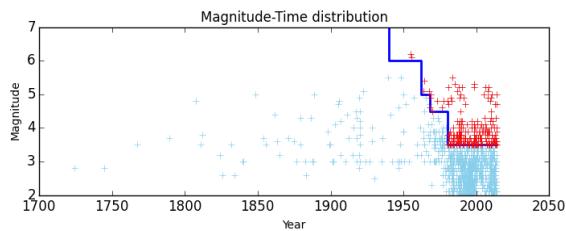


Figure 5. Completeness

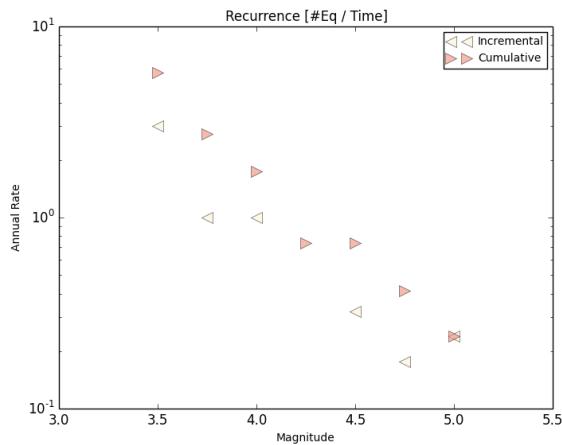


Figure 6. Annual earthquake recurrence.

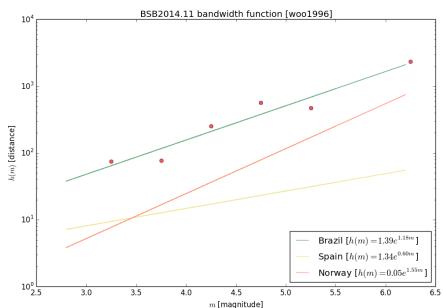


Figure 7. Magnitude dependence bandwidth.

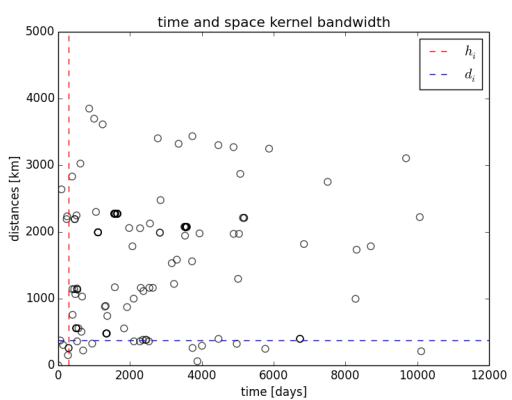


Figure 8. Local bandwidth example.

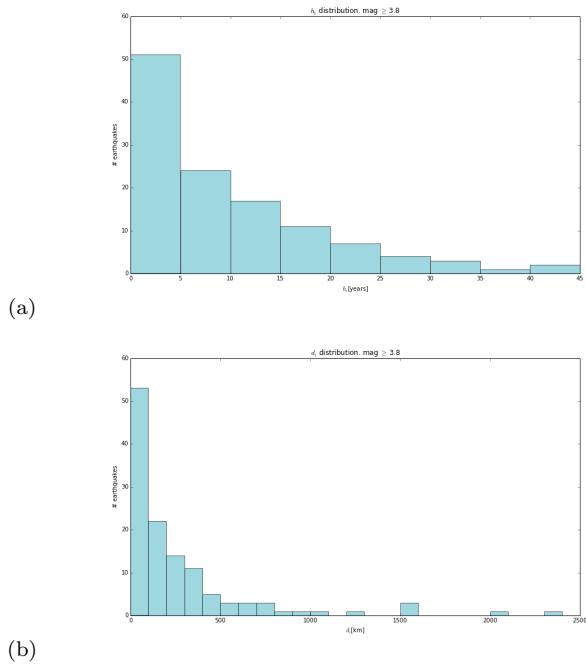


Figure 9. (a) h_i and (b) d_i distributions from BSB.

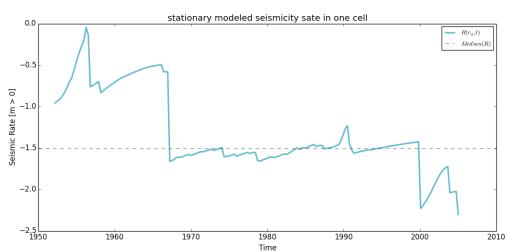


Figure 10. Stationary seismic rate

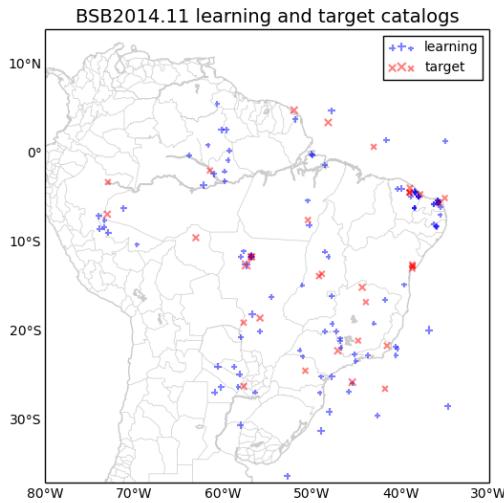


Figure 11. Learning and target catalogs.

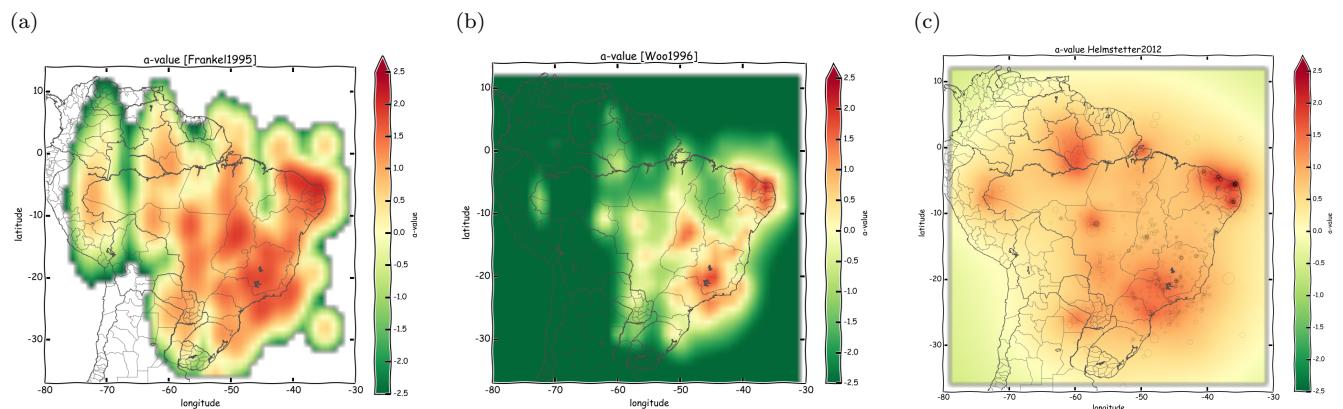


Figure 12. Smoothed rates comparission: (a) shows *Frankel* [1995] method smoothed rate results, (b) and (c) represent rates smoothed by *Woo* [1996] and *Helmstetter and Werner* [2012] methods respectively.

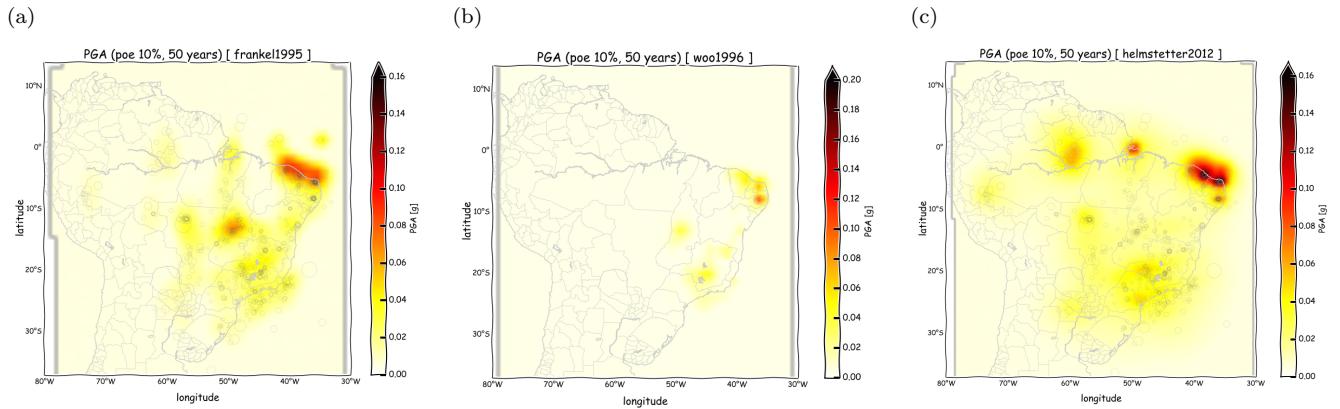


Figure 13. PGA (poe 10%/50y) comparission for (a) *Frankel* [1995], (b) *Woo* [1996] and (c) *Helmstetter and Werner* [2012] methods.

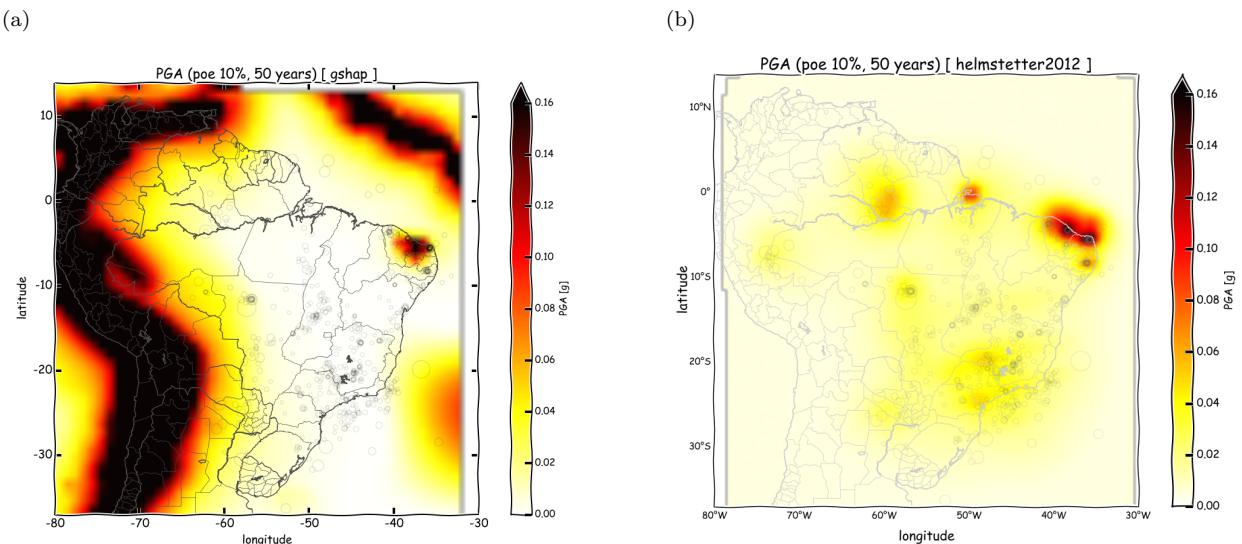


Figure 14. Previous available Brazilian PSHA from (a) GSHAP project and the PSHA proposed by this work (b).