MAS115 Calculus I 2006-2007

Problem sheet for exercise class 1

- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

(*) Problem 1: Prove that for all positive real numbers x and y (i.e. $x, y \in \mathbb{R}^+$),

$$\frac{2}{\frac{1}{x}+\frac{1}{y}} \leq \sqrt{xy}$$

(a) by direct proof, and
(b) by using the geometric-arithmetic inequality.

(*) Problem 2: Determine the set of all real numbers x (i.e. $x \in \mathbb{R}$) that satisfy

$$|2x-1|+|4x+1|<3$$

(a) by direct computation, and
(b) by plotting the graph.

Problem 3: Determine the set of all real numbers x (i.e. $x \in \mathbb{R}$) that satisfy

$$x^2-3x-4<0$$

(a) by direct computation, and
(b) by plotting the graph of $y = x^2 - 3x - 4$.
Hint: compute the zeros of $x^2 - 3x - 4$.

Problem 4: Determine the set of all real numbers x (i.e. $x \in \mathbb{R}$) that satisfy

$$\sqrt{1-x^2} \leq -x$$

(a) by direct computation, and
(b) by plotting the graphs of $y = -x$ and $y = \sqrt{1-x^2}$.

Extra: Prove that for all real numbers x and y (i.e. $x, y \in \mathbb{R}$)

$$||x|-|y|| = |x+y|+|x-y|-|x|-|y| \ .$$

Thomas Prellberg, September 2006