MAS115 Calculus I 2006-2007

Problem sheet for exercise class 8

- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

(*) Problem 1: Suppose that f has a positive derivative for all values of x and that $f(1) = 0$. Which of the following statements must be true of the function

$$g(x) = \int_0^x f(t)dt \ ?$$

a. g is a differentiable function of x.
b. g is a continuous function of x.
c. The graph of g has a horizontal tangent at $x = 1$.
d. g has a local maximum at $x = 1$.
e. g has a local minimum at $x = 1$.
f. The graph of g has an inflection point at $x = 1$.
g. The graph of dg/dx crosses the x-axis at $x = 1$.

Problem 2: Sometimes it helps to reduce the integral step by step, using a trial substitution to simplify the integral a bit and then another to simplify it some more. Practice this on

$$\int \sqrt{1 + \sin^2(x-1)}\sin(x-1)\cos(x-1)dx \ .$$

a. $u = x - 1$, followed by $v = \sin u$, then by $w = 1 + v^2$
b. $u = \sin(x-1)$, followed by $v = 1 + v^2$
c. $u = 1 + \sin^2(x-1)$

Problem 3: Determine conditions on the constants a, b, c, and d so that the rational function

$$f(x) = \frac{ax+b}{cx+d}$$

has an inverse.

Extra: Prove that

$$\int_0^x \left(\int_0^u f(t)dt \right) du = \int_0^x f(u)(x-u)du \ .$$

(Hint: Express the integral on the right hand side as the difference of two integrals. Then show that both sides of the equation have the same derivative with respect to x.)

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