# MAS205 Complex Variables 2004-2005

Exercises 6

Exercise 23: Find the Laurent series of the function

$$f(z) = \frac{1}{(z+3)(z-2)^2}$$

on a punctured disk centred at the point $z_0 = 2$.
Where is this Laurent series valid (i.e. absolutely convergent)?
What is the principal part of this Laurent series?
What type of singularity does $f$ have at $z_0 = 2$?
What is the residue of $f$ at $z_0 = 2$?

Exercise 24: Locate the singularities for each of the following functions, and determine the nature of each singularity:

$$(a) \quad \frac{1}{z^4-16} \qquad (b) \quad \frac{1}{(z-1)^4} + e^{-1/(z+3)} \qquad (c) \quad z^2(e^{1/z^2}-1) \qquad (d) \quad \frac{\sin(z^2)}{z^2}$$

Exercise 25:

(a) List the singularities of the function $f(z) = e^{-iz}/(z^2 - \pi^2)$ and determine the nature of each singularity. Compute the residue of $f$ at each singularity.

(b) List the singularities of the function $f(z) = e^{1/z}/(1+z)$ and determine the nature of each singularity. Compute the residue of $f$ at each singularity.

Exercise 26: Let $f$ and $g$ be holomorphic on a disk $D$ centred at $z_0$, and let $h$ be holomorphic on the punctured disk $D' = D \setminus \{z_0\}$. Suppose $f$ and $g$ both have zeros of order $m \geq 1$ at $z_0$ and $h$ has a pole of order $n \geq 1$ at $z_0$.

(a) Does $fh$ have a zero or pole at $z_0$? If so, what is its order?

(b) Does $f + g$ have a zero or pole at $z_0$? If so, what is its order?

Exercise 27: Let $E$ denote the set of all entire functions.

(a) Is $E$ a group under addition (i.e. $(f + g)(z) = f(z) + g(z)$)?

(b) Is $E$ a group under multiplication (i.e. $(fg)(z) = f(z)g(z)$)?

(c) Is $E$ a group under composition (i.e. $(f \circ g)(z) = f(g(z))$)?

Prove your answers.

Note: determining the type of singularity means finding out whether it is a pole (if so, which order?), an essential singularity, or a removable singularity.
Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 11am Tuesday 23rd November

Thomas Prellberg, November 2004