MAS205 Complex Variables 2005-2006

Exercises 7

Exercise 25: Let

$$f(z) = \frac{1}{(z-1)(z+2)} \, .$$

(a) Compute the partial fraction decomposition of f.

(b) Write down the Laurent series for f on $\{z : |z| > 2\}$.

(c) Write down the Laurent series for f on a punctured disk centred at $z_0 = 1$. For what values of z does this series converge?

Exercise 26: Find the Laurent series of the function

$$f(z) = \frac{1}{(z-1)(z+2)^2}$$

on a punctured disk centred at the point $z_0 = -2$.
Where is this Laurent series valid (i.e. absolutely convergent)?
What is the principal part of this Laurent series?
What type of singularity does f have at $z_0 = -2$?
What is the residue of f at $z_0 = -2$?

Exercise 27: Locate the singularities for each of the following functions, and determine the nature of each singularity:

$$(a) \quad \frac{1}{z^4+4} \qquad (b) \quad \frac{1}{(z+1)^3} - e^{-1/z} \qquad (c) \quad z(e^{-1/z}+1) \qquad (d) \quad \frac{\sin(z^3)}{z^3}$$

Exercise 28: (a) List the singularities of the function $f(z) = e^z/(z^2+\pi^2)$ and determine the nature of each singularity. Compute the residue of f at each singularity.

(b) List the singularities of the function $f(z) = z^5 e^{1/z}$ and determine the nature of each singularity. Compute the residue of f at each singularity.

Note: determining the type of singularity means finding out whether it is a pole (if so, which order?), an essential singularity, or a removable singularity.

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 10:30am Wednesday 30th November

Thomas Prellberg, November 2005