MAS205 Complex Variables 2004-2005

Exercises 5

Exercise 18: Find the radius of convergence of the following power series

$$(a)\quad \sum_{n=1}^{\infty} \frac{z^n}{n^{10}}\,, \quad (b)\quad \sum_{n=0}^{\infty} \frac{z^n}{(3i)^n}\,, \quad (c)\quad \sum_{n=0}^{\infty} z^n \exp(-n)\,, \quad (d)\quad \sum_{n=0}^{\infty} n! z^n\,.$$

Exercise 19: Give an example, if possible, of power series with the following properties:

(a) centred at $z_0 = 4i$, with radius of convergence $R = 4$

(b) centred at $z_0 = 1 + 2i$, with radius of convergence $R = 0$

(c) centred at $z_0 = -2$ and convergent for all z with $\Im(z) < 4$ but divergent for all z with $\Im(z) > 4$

(d) centred at $z_0 = -4i/3$, with radius of convergence $R = \infty$

(e) centred at $z_0 = 0$ and convergent for all z with $\Re(z) = 8$ but divergent for all other $z \in \mathbb{C}$

(Proofs are not necessary, but if you can't find an example you should explain why.)

Exercise 20: Compute the product of the Taylor series of $(1 - z)^{-1}$ and $(1 + z)^{-1}$ at $z_0 = 0$ and show that the result is equal to the Taylor series of $(1 - z^2)^{-1}$ at $z_0 = 0$.

Exercise 21: Let $D = \{z : |z + 3i| < 4\}$. Suppose that $f : D \to \mathbb{C}$ is defined by

$$f(z) = \sum_{n=0}^{\infty} \frac{(z + 3i)^n}{(4i)^n}\,.$$

Calculate the Taylor series for f at the point $z_0 = 0$ and determine its radius of convergence.

Exercise 22: Let

$$f(z) = \frac{1}{(z + 1)(z - 3)}\,.$$

(a) Write down the Laurent series for f on $\{z : |z| > 3\}$.

(b) Write down the Laurent series for f on a punctured disk centred at $z = -1$. For what values of z dues this series converge?

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 11am Tuesday 16th November

Thomas Prellberg, November 2004