MTH5105 Differential and Integral Analysis 2008-2009

Exercises 8

Exercise 1: For $x \in \mathbb{R}$, compute

$$f(x) = \sum_{n=1}^{\infty} \frac{x}{(1+x^2)^n} \ .$$

Show that the convergence is not uniform.

[7 marks]

Exercise 2: Let $f_n : \mathbb{R} \to \mathbb{R}$ be a sequence of continuous functions converging uniformly to a function f. Show that if $\lim_{n\to\infty} x_n = x$ then

$$\lim_{n\to\infty} f_n(x_n) = f(x) \ .$$

[8 marks]

Exercise 3: (a) Show that the following sequences of functions converge uniformly on the given intervals.

$$\begin{array}{llr} \text{(i)} & u_n(x) = (1-x)x^n \ , & [0,1] \ ; \\ \text{(ii)} & v_n(x) = \dfrac{x^2}{1+nx^2} \ , & \mathbb{R} \ . \end{array}$$

[6 marks]

(b) Which of the following sequences of functions converge uniformly to $s(x) = 1$ on the interval $[0,1]$?

$$\begin{array}{rl} \text{(i)} & f_n(x) = (1+x/n)^2 \ , \\ \text{(ii)} & g_n(x) = 1 + x^n(1-x)^n \ , \\ \text{(iii)} & h_n(x) = 1 - x^n(1-x^n) \ . \end{array}$$

[9 marks]

The deadline is 12.15 on Monday, 23rd March. Please hand in your coursework at the end of Monday's lecture or to my office MAS113 immediately afterwards.

Thomas Prellberg, March 2009