MAS205 Complex Variables 2004-2005

Exercises 9

Exercise 37: Use the residue theorem to calculate

$$\int_{\mathcal{C}} \frac{1}{z^2(z^2-4)} dz$$

where $\mathcal{C}$ is the positively oriented circle of radius 2 centred at 1.

Exercise 38: Use the residue theorem to calculate

$$\int_{\mathcal{C}} \frac{\sin z}{z(z+1)^2} dz$$

where $\mathcal{C}$ is the positively oriented circle of radius 3 centred at 0.

Exercise 39: Let $f(z) = z^8 - 7z^3 - 4$. Use Rouche's theorem to determine how many zeros of f (counted with multiplicity) have modulus strictly less than one. How many zeros of f (counted with multiplicity) lie in the annulus $\{z : 1 < |z|\}$?

Exercise 40: Let $f(z) = e^{-iz} - 20z + z^6$. Use Rouche's theorem to determine how many zeros of f (counted with multiplicity) lie in the annulus $\{z : 1 < |z| < 2\}$.

And for 50 extra marks (to top up your course work):

Exercise 41: Calculate

$$\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} dx \ .$$

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 11am Tuesday 14th December

Thomas Prellberg, November 2004