MAS205 Complex Variables 2004-2005

Exercises 4

Exercise 14: For each of the following functions $f(x+iy) = u(x,y) + iv(x,y)$, find the set of all points (x,y) at which u and v satisfy the Cauchy-Riemann differential equations $(\partial u/\partial x = \partial v/\partial y$ and $(\partial u/\partial y = -\partial v/\partial x)$.

(a) $f(x+iy) = y^2 + 2ixy$

(b) $f(x+iy) = 2xy - ix + 2iy^3/3$.

Exercise 15: Let $f(z) = e^{z^2/2}$. Write $f(z)$ as $u(x,y)+iv(x,y)$ and show that u and v satisfy the Cauchy-Riemann differential equations. Write

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

and use this to express $f'(z)$ as a function of z.

Exercise 16: Use the Cauchy-Riemann differential equations to find at which values of z the following functions are differentiable. Find the derivative of the functions at these points.

(a) $f(x+iy) = 3x^2y - y^3 + i(3xy^2 - x^3)$

(b) $f(x+iy) = 3x^2y - y^3 + i(x^3 - 3xy^2)$

(c) $f(x+iy) = 2xy^2 + i(x + 2y^3/3)$

(d) $f(z) = (z+\overline{z})(z-\overline{z})^2$.

Exercise 17: Let f and g denote functions $\mathbb{C} \to \mathbb{C}$. For each question below, give either a proof or a counterexample to justify your answer.

(a) If f and g are both differentiable at z_0, does it follow that $g - f$ is continuous at z_0?

(b) If f and g are both non-differentiable at z_0, does it follow that fg is non-differentiable at z_0?

(c) If f and g are both differentiable at z_0, does it follow that $f \circ g$ is differentiable at z_0?

(d) Suppose f is non-differentiable at $3+i$, but differentiable everywhere else, and g is non-differentiable at $2+i$, but differentiable everywhere else. Is $f+g$ differentiable at $4+2i$?

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 11am Tuesday 2nd November

Thomas Prellberg, October 2004