

MAS205 Complex Variables 2005-2006

Exercises 4

Exercise 13: For each of the following functions, decide at which values of z the function is continuous and at which values it is not continuous. Give reasons, but detailed proofs are not expected.

- (a) $f(z) = |z|$
- (b) $f(z) = z^3/\bar{z}$ for all non-zero z , and $f(0) = 0$.

Exercise 14: Let f and g denote functions $\mathbb{C} \rightarrow \mathbb{C}$. For each question below, give either a proof or a counterexample to justify your answer.

- (a) If f and g are both continuous at z_0 , does it follow that $g - f$ is continuous at z_0 ?
- (b) If f and g are both discontinuous at z_0 , does it follow that fg is discontinuous at z_0 ?
- (c) If f and g are both continuous at z_0 , does it follow that $f \circ g$ is continuous at z_0 ?
- (d) Suppose f is discontinuous at $3 + i$, but continuous everywhere else, and g is discontinuous at $2 + i$, but continuous everywhere else. Is $f + g$ continuous at $4 + 2i$?

Exercise 15: Starting from the definition of the derivative of a complex function as a limit,

- (a) find the derivative of $f(z) = iz(1 - 2z)$ at $z = i$;
- (b) find the derivative of $f(z) = z^3 + z$ for all $z \in \mathbb{C}$;
- (c) prove that $f(z) = |z|^2 + z^2$ does not have a derivative at z_0 unless $z_0 = 0$. What is the value of $f'(0)$?

Exercise 16: For each of the following functions, decide at which values of z the function is differentiable and at which values it is not differentiable. Give reasons, but detailed proofs are not expected.

- (a) $f(z) = |z|$
- (b) $f(z) = z^3/\bar{z}$ for all non-zero z , and $f(0) = 0$.

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 10:30am Wednesday 2nd November

Thomas Prellberg, October 2005