MTH5105 Differential and Integral Analysis 2009-2010

Exercises 5

There are two sections. Questions in Section 1 will be marked and will form your coursework mark. Questions in Section 2 are voluntary but highly recommended.

1 Exercise for Feedback/Assessment

1) Let $f(x) = \exp(-1/\sqrt{x})$, $g(x) = \cos(\pi x/2)$, and $P = \{1, 4, 9, 16\}$.

   (a) Find the upper and lower sums $U(f, P)$ and $L(f, P)$ of f for the partition P. Use these sums to give bounds for $\int_1^{16} f(x)\,dx$. [10 marks]

   (b) Find the upper and lower sums $U(g, P)$ and $L(g, P)$ of g for the partition P. Use these sums to give bounds for $\int_1^{16} g(x)\,dx$. [10 marks]

2 Extra Exercises

2) Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} 0 & x \neq 0 \\ 1 & x = 0\,. \end{cases}$$

   (a) Given a partition P of $[-1, 1]$, what is $L(f, P)$?
   What is $\int_{*-1}^{1} f(x)\,dx$?

   (b) For fixed $\epsilon > 0$, find a partition P of $[-1, 1]$ such that $U(f, P) < \epsilon$.
   What is $\int_{-1}^{*1} f(x)\,dx$?

   (c) Is f integrable on $[-1, 1]$? If so, what is its integral?

3) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Consider the equidistant partitions P_n of $[0, 1]$ into n subintervals.

   (a) Find $U(f, P_n)$. What can you say about $\int_0^{*1} f(x)\,dx$?

   (b) Find $L(f, P_n)$. What can you say about $\int_{*0}^{1} f(x)\,dx$?

   (c) Is f integrable on $[0, 1]$? If so, what is its integral?

   [Hint: $\sum_{j=1}^{n} j^2 = \frac{1}{6}n(n+1)(2n+1)$*.]*

The deadline is 5.00pm (strict) on Monday 8th March. Please hand in your coursework to the red coursework box on the ground floor.

Thomas Prellberg, February 2010