MAS205 Complex Variables 2005-2006

Exercises 1

Exercise 1: Let $z_1 = \frac{3}{2} - 2i$ and $z_2 = 4 + 3i$. Compute (in standard $x + iy$ form):

$$(a) \quad z_1 z_2 \qquad (b) \quad \frac{1}{z_1} \qquad (c) \quad \frac{z_2}{z_1} \qquad (d) \quad \frac{1}{z_1} + \frac{1}{z_2}$$

Compute the moduli:

$$(a) \quad |z_1| \qquad (b) \quad \left|\frac{z_1}{z_2}\right| \qquad (c) \quad |z_1 z_2|$$

Exercise 2: Express the following complex numbers in polar exponential form:

$$(a) \quad -1 \qquad (b) \quad 2i \qquad (c) \quad 1+i \qquad (d) \quad 1-\sqrt{3}i \qquad (e) \quad 1/(1-i)$$

Exercise 3: Solve for the roots of the equation

$$z^3 + 8i = 0$$

Express all the roots in standard and polar form, and draw a diagram showing their location in the complex plane.

Exercise 4: Describe graphically the sets of points in the complex plane defined by the following equations and inequalities:

(a) $|z - 2i| < 2$

(b) $\Im(z^2) = 0$

(c) $1 \leq \Im(z + 1) < 2$

(d) $z^2 = -2$

Notation: $\Re(z)$ and $\Im(z)$ denote the real and imaginary parts of z, respectively.

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 10:30am Wednesday 12th October

Thomas Prellberg, September 2005