MTH5105 Differential and Integral Analysis 2009-2010

Exercises 2

There are two sections. Questions in Section 1 will be marked and will form your coursework mark. Questions in Section 2 are voluntary but highly recommended.

1 Exercise for Feedback/Assessment

1) Suppose that $f : [0, 1] \to \mathbb{R}$ is continuously differentiable.

   (a) Show that there is some number M such that $|f'(x)| \leq M$ for all x. [8 marks]

   (b) Using the Mean Value Theorem, or otherwise, prove that

   $$|f(x) - f(y)| \leq M|x - y|$$

   for all $x, y \in [0, 1]$. [12 marks]

2 Extra Exercises

2) Let $f, g : \mathbb{R} \to \mathbb{R}$ be differentiable with

   $$f' = g \qquad \text{and} \qquad g' = -f \, .$$

   Show that between every two zeros of f there is a zero of g and between every two zeros of g there is a zero of f.

3) Let $f : \mathbb{R} \to \mathbb{R}$ be twice differentiable ($f'' = (f')'$) with

   $$f(0) = f'(0) = 0 \qquad \text{and} \qquad f(1) = 1 \, .$$

   Show that there exists a $c \in (0, 1)$ such that $f''(c) > 1$.

The deadline is 5.00pm (strict) on Monday 1st February. Please hand in your coursework to the red coursework box on the ground floor.

Thomas Prellberg, January 2010