

**MTH5105 Differential and Integral Analysis**  
**MID-TERM TEST**

*Date: 25 Feb 2011 Time: 12:10–12:50*

**Complete the following information:**

<b>Name</b>	
<b>Student Number (9 digit code)</b>	

The test has THREE questions. You should attempt ALL questions. Write your calculations and answers in the space provided. Cross out any work you do not wish to be marked.

Question	Marks
1	
2	
3	
Total Marks	

**Nothing on this page will be marked!**

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**Question 1.**

- (a) State the formula for the Taylor polynomial  $T_{n,a}$  of degree  $n$  of a function  $f$  at  $a$ . [10 marks]

Let  $g(x) = -\log(1 - x)$ .

- (b) Prove by mathematical induction that  $g^{(n)}(x) = \frac{(n-1)!}{(1-x)^n}$  for  $n \in \mathbb{N}$ . [10 marks]  
Hence compute the Taylor polynomial  $T_{4,0}$  of degree 4 of  $g$  at  $a = 0$ . [10 marks]

Let  $h(x) = \log(1 + x + x^2)$ .

- (c) By factorising  $1 - x^3$ , or otherwise, determine the Taylor polynomial  $T_{4,0}$  of degree 4 of  $h$  at  $a = 0$ . [10 marks]

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**Answer 1.**

**Answer 1.** (*Continue*)

**Question 2.**

(a) Give the definition of  $f : \mathcal{D} \rightarrow \mathbb{R}$  being differentiable at a point  $a \in \mathcal{D}$ . [10 marks]

(b) Using this definition, show that  $g(x) = 1/x$  is differentiable at  $a = 1$  and find its derivative. [10 marks]

(c) Suppose that the function  $f$  is continuous at 0. Show that the function  $h$  defined by

$$h(x) = xf(x)$$

is differentiable at zero and find its derivative. [10 marks]

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**Answer 2.**

**Answer 2.** (*Continue*)

**Question 3.**

(a) State the Mean Value Theorem. [15 marks]

(b) Let  $g$  be differentiable on  $[0, 1]$  with

$$g'(x) = 0$$

for all  $x \in [0, 1]$ , and let  $g(0) = 1$ . Using the Mean Value Theorem, or otherwise, prove that

$$g(x) = 1$$

for all  $x \in [0, 1]$ . [15 marks]

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**Answer 3.**

**Answer 3.** (*Continue*)