

MAS205 Complex Variables 2005-2006

Exercises 2

Exercise 5: Using Euler's formula $e^{i\theta} = \cos \theta + i \sin \theta$ for $\theta \in \mathbb{R}$, show that

(a) $e^{i\theta} = e^{i(\theta+2n\pi)}$ for $\theta \in \mathbb{R}$ and $n \in \mathbb{Z}$

(b) $e^{i\theta}e^{i\phi} = e^{i(\theta+\phi)}$ for $\theta, \phi \in \mathbb{R}$

(c) $1/e^{i\theta} = e^{-i\theta}$ for $\theta \in \mathbb{R}$

Using (b) and mathematical induction, show that

(d) $(e^{i\theta})^n = e^{in\theta}$ for $\theta \in \mathbb{R}$ and $n \in \mathbb{Z}$

Exercise 6: Find all complex solutions of the following equations:

(a) $e^z = i$ (b) $e^{2z} = 1$ (c) $\sinh z = 0$ (d) $\cos z = 0$

Exercise 7: Consider the transformation

$$z \mapsto w = (z - 1)^2.$$

(a) Find the equation of the image of the line $\Re(z) = 0$ and sketch the image.

(b) What is the image of the upper half plane?

Exercise 8: (a) Find the region in the w -plane which is the image of the upper half of the z -plane under the transformation

$$w = 1 + 1/z$$

(b) Find the regions in the z -plane which map to the left half of the w -plane under the transformation

$$w = z^4$$

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 10:30am Wednesday 19th October

Thomas Prellberg, September 2005