MAS205 Complex Variables 2005-2006

Exercises 5

Exercise 17: For each of the following functions $f(x+iy) = u(x,y) + iv(x,y)$, find the set of all points (x,y) at which u and v satisfy the Cauchy-Riemann differential equations ($\partial u/\partial x = \partial v/\partial y$ and ($\partial u/\partial y = -\partial v/\partial x$).

(a) $f(x+iy) = y^2 + ixy^2$

(b) $f(x+iy) = 2xy + 2ixy + y^3/3$.

Exercise 18: Let $f(z) = ze^z$. Write $f(z)$ as $u(x,y) + iv(x,y)$ and show that u and v satisfy the Cauchy-Riemann differential equations. Write

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

and use this to express $f'(z)$ as a function of z.

Exercise 19: Use the Cauchy-Riemann differential equations to find at which values of z the following functions are differentiable. Find the derivative of the functions at these points.

(a) $f(x+iy) = 3xy^2 - x^3 + i(y^3 - 3x^2y)$

(b) $f(x+iy) = 3x^2y - x^3 + i(y^3 - 3x^2y)$

(c) $f(z) = z(z - \overline{z})^2$.

Exercise 20: Let f and g denote functions $\mathbb{C} \to \mathbb{C}$. For each question below, give either a proof or a counterexample to justify your answer.

(a) If f and g are both differentiable at z_0, does it follow that gf is continuous at z_0?

(b) If f and g are both non-differentiable at z_0, does it follow that $f+g$ is non-differentiable at z_0?

(c) If f is differentiable for all $z \in \mathbb{C}$ and g is differentiable at z_0, does it follow that $g \circ f$ is differentiable at z_0?

(d) Suppose f is discontinuous at $3+4i$, but continuous everywhere else, and g is discontinuous at $2+i$, but continuous everywhere else. Is $f-g$ differentiable at $1+3i$?

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 10:30am Wednesday 16th November

Thomas Prellberg, November 2005