MTH5105 Differential and Integral Analysis 2008-2009

Exercises 7

Exercise 1: Let $f : [a, b] \to \mathbb{R}$ be continuous. Show that if

$$\int_a^b f(x)\,dx = 0$$

then there exists a $c \in (a, b)$ such that $f(c) = 0$.

[Hint: use an antiderivative of f.]

[10 marks]

Exercise 2: Evaluate

$$\lim_{n\to\infty} \int_0^{\pi/2} \frac{\sin(nx)}{nx}\,dx\ .$$

[Hint: Choose an $\epsilon > 0$ and consider the intervals $[0, \epsilon]$ and $[\epsilon, \pi/2]$ separately.]

[8 marks]

Exercise 3: Compute $\lim_{n\to\infty} f_n(x)$ and $\lim_{n\to\infty} f_n'(x)$ for the following functions:

(a) $f_n : \mathbb{R} \to \mathbb{R}$,

$$x \mapsto \frac{\sin(nx)}{\sqrt{n}}\ .$$

(b) $f_n : \mathbb{R} \to \mathbb{R}$,

$$x \mapsto \frac{1}{n}(\sqrt{1 + n^2x^2} - 1)\ ,$$

(c) $f_n : \mathbb{R} \to \mathbb{R}$,

$$x \mapsto \frac{1}{1 + nx^2}\ .$$

If the limit doesn't exist, please indicate clearly for which values of x this is the case and give a brief indication why (no complete proof necessary).

[12 marks]

The deadline is 12.15 on Monday, 16th March. Please hand in your coursework at the end of Monday's lecture or to my office MAS113 immediately afterwards.

Thomas Prellberg, March 2009