Mathematics at Queen Mary

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School of Mathematical Sciences Queen Mary, University of London

College Open Day Presentation 2009



Topic Outline

- Why Mathematics?
 - Why You Should Study Mathematics
 - What is Mathematics
 - Transferable Skills
 - Career Opportunities
- Mathematics at Queen Mary
 - Three-Year BSc Degree Courses
 - Other Degree Courses
 - Example: G100 Mathematics
 - The Academic Year
- Mathematical Problems
 - Some Million Dollar Problems
 - Examples of Solved and Open Problems
 - The 3n+1 Problem



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- 2 Mathematics at Queen Mary
- Mathematical Problems

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- You are really good at maths
- You like problem solving
- You could get into business school (or law, or ...)
- You want to keep your career options open

Bad Reasons for Studying Mathematics

- Your language skills are really weak
- You like memorising formulas
- Your marks are to weak to get you into ...
- You haven't yet figured out what you're good at

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- just "doing things with numbers and letters and other symbols"
- just a collection of facts and rote recipes
- just computational and arithmetic skills

- a way of thinking
- the language of science
- a creative discipline
- a source of pleasure and wonder
- a means of problem solving



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- Ability to work independently
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- Highly developed numerical skills
- Effective communication skills
- Apply mathematical modelling to real-world problems
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Career Opportunities

- Academic Research
- Aerospace
- Biotechnology
- Business and Finance
- Chemicals
- Construction
- Defence
- Electronics
- Energy

- Environment
- Health Care
- Management
- Marketing
- Materials
- Pharmaceuticals
- Retail
- Teaching
- Transport

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- Business and Finance
 - Accountant: 19-25K
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 - Three-Year BSc Degree Courses
 - Other Degree Courses
 - Example: G100 Mathematics
 - The Academic Year
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Three-Year BSc Degree Courses

Title	Code	Req.
Mathematics	G100	320
Pure Mathematics	G110	320
Mathematics and Statistics	GG31	320
Mathematics, Statistics, and Financial Economics	GL11	320
Mathematics with Finance and Accounting	G1N4	320
Mathematics with Business Management	G1N1	320
Mathematics with Business Management and Finance	GN13	320
Mathematics and Computing	GG14	320
Mathematics and Physics	FG31	320

A=120, B=100



Other Degree Courses

Degree	Years	Title	Code	Req.
MSci	4	Mathematics	G102	340
MSci	4	Mathematics with Statistics	G1G3	340
BSc	3	Computer Science with Mathematics	GG41	320
BSc	3	Economics, Mathematics, and Statistics	LG11	340
	1	Science & Eng. Foundation Programme	FGH0	180

- All Queen Mary degrees are honours (including pass degrees)
- Course unit system instead of joint or combined honours



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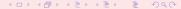
Course Unit System

Advantages:

- Flexibility
- Opportunities to take modules in other departments
- Freedom to shape your programme of study
- Specialisation in penultimate and final year

Typically,

- take 8 modules in first year (no choice)
- choose 8 of 16 modules in second year
- choose 8 of 24 modules in third year



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Example: G100 Mathematics

Study Programme Structure

- Core modules
- + Optional core modules
- + Elective modules

Streams withing G100 include

- Algebra and Discrete Mathematics
- Analysis and Geometry
- Probability and Statistics
- Applied Mathematics



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G100 Mathematics – First Year

Semester 1

- Calculus I
- Probability I
- Geometry I
- Introduction to Mathematical Computing

Semester 2

- Calculus II
- Introduction to Statistics
- Differential Equations
- Introduction to Algebra

Four modules per semester - no choice



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G100 Mathematics - Second Year

Semester 3

Linear Algebra I

Take two of

- Convergence and Continuity
- Calculus III
- Dynamics of Physical Systems
- Probability II
- Mathematical Writing

Semester 4

Take three of

- Algebraic Structures I
- Complex Variables
- Geom. II: Knots and Surfaces
- Statistical Modelling I
- Intro. to Numer. Comp.
- Algorithmic Graph Theory
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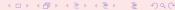
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- Combinatorics
- Algebraic Structures II
- Chaos and Fractals
- Linear Algebra II
- Intro. to Math. Finance
- Relativity
- Metric Spaces
- Linear Operators and Differential Equations
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Semester 6

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- Coding Theory
- Complex Analysis
- Fields and Galois Theory
- Number Theory
- Cryptography
- Mathematical Problem Solving
- Design of Experiments
- Fluid Dynamics
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The Academic Year

late September	Teaching Semester A
mid December	(12 weeks)
early January	Teaching Semester B
late March	(12 weeks)
late April	Examination Period
early June	(6 weeks)

Teaching

- 4 modules per semester
- 3 hours lectures + 1 hour exercise class (per module and week)
- 4 × 4 = 16 timetabled hours per week

Assessment

- Modules count 1:3:6 to final degree
- 20% in-term assessment + 80% final exam
- 3 attempts (resit exam or retake module)

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- Personal Academic Advisers (academic matters)
- Study Programme Directors
- Pastoral Tutor (pastoral matters)
- Senior Tutor, Director of Undergraduate Studies
- PASS (Peer Assisted Study Support)

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- Health Centre
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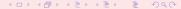


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- Navier-Stokes Equations
 - P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
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Some "recently" proved problems

• Fermat's last theorem (1637, proved 1994): If an integer *n* is greater than 2, then the equation

$$a^n + b^n = c^n$$

has no solutions in non-zero integers a, b, and c.

For n = 2, this is of course possible, for example

$$3^2 + 4^2 = 5^2 .$$

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Some unsolved problems:

• Goldbach's conjecture (1742): every even integer greater than 2 can be written as the sum of two primes.

For example,
$$18 = 5 + 13 = 7 + 11$$
.

• The twin prime conjecture (300 BC): there are infinitely many primes p such that p + 2 is also prime.

For example, 17 and 19 are twin primes.

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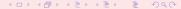
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Some Million Dollar Problems
Examples of Solved and Open Problems
The 3n+1 Problem

"The history of mathematics is a history of horrendously difficult problems being solved by young people too ignorant to know that they were impossible."

Freeman Dyson, "Birds and Frogs", AMS Einstein Lecture 2008

Consider the following operation on an arbitrary positive integer:

- If the number is even, divide it by two.
- If the number is odd, triple it and add one.

$$f(n) = \begin{cases} n/2 & \text{if n is even} \\ 3n+1 & \text{if n is odd.} \end{cases}$$

Form a sequence by performing this operation repeatedly, beginning with any positive integer.

• Example: n = 6 produces the sequence

$$6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \dots$$

The Conjecture is:

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Some Examples

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• n = 11 produces the sequence

11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

• *n* = 27 produces the sequence 27,82,41,124,62,31,94,47,142,71,214,107,322,161,484,242,121, 364,182,91,274,137,412,206,103,310,155,466,233,700,350,175, 526,263,790,395,1186,593,1780,890,445,1336,668,334,167,502, 251,754,377,1132,566,283,850,425,1276,638,319,958,479,1438, 719,2158,1079,3238,1619,4858,2429,7288,3644,1822,911,2734, 1367,4102,2051,6154,3077,9232,4616,2308,1154,577,1732,866, 433,1300,650,325,976,488,244,122,61,184,92,46,23,70,35,106, 53.160,80,40,20,10.5,16.8,4,2,1.

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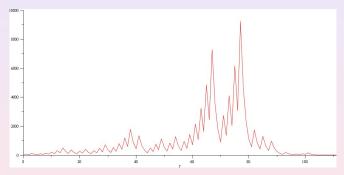
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Graphing the Sequences

A graph of the sequence obtained from n=27

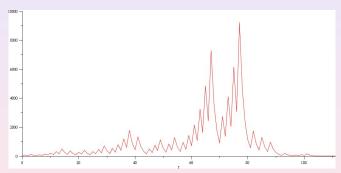


This sequence takes 111 steps, climbing to over 9000 before descending to 1.



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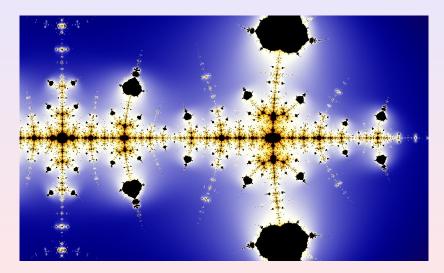
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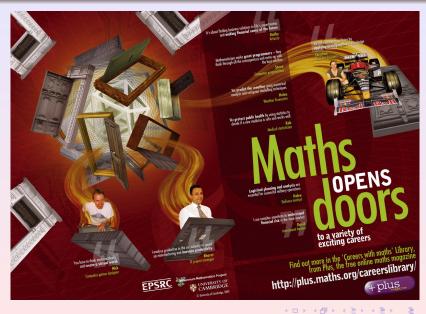


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Iterating on Complex Numbers





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The End