MAS205 Complex Variables 2004-2005

Exercises 8

Exercise 33: Use the Cauchy integral formula to evaluate each of the following integrals, where $\mathcal{C}$ is the circle $\{z \in \mathbb{C} : |z-4| = 2\}$ traversed in the positive (anticlockwise) sense:

(a)

$$\int_{\mathcal{C}} \frac{(z+2)^3}{(z-4)z^2} dz$$

(b)

$$\int_{\mathcal{C}} \frac{4}{(z-\pi)\sin(z/2)} dz$$

Exercise 34: Use the residue theorem to calculate

$$\int_{\mathcal{C}} \frac{1}{(z^2-9)(z+2i)} dz$$

for

(a) $\mathcal{C} = \mathcal{C}_1$, the positively oriented circle of radius 5 centred at 1;

(b) $\mathcal{C} = \mathcal{C}_2$, the positively oriented square with corners $-1-3i$ and 2.

Exercise 35: Let $f : \mathbb{C} \to \mathbb{C}$ be holomorphic apart from a simple pole at $3i$ with residue $2\pi i$ and an essential singularity at $2-i$ with residue $3+i$. How many elements are there in the set

$$\left\{ \int_{\mathcal{C}} f(z)dz : \mathcal{C} \text{ is a simple closed curve in } \mathbb{C} \setminus \{3i, 2-i\} \right\} ?$$

Exercise 36: Suppose $f : \mathbb{C} \to \mathbb{C}$ is holomorphic except for singularities at the points $-2i$, 1, and $2i$. Let $\mathcal{C}_1$ be the positively oriented circle of radius 3 centred at 0. Let $\mathcal{C}_2$ be the positively oriented circle of radius 2 centred at i. Let $\mathcal{C}_3$ be the positively oriented circle of radius 3 centred at 3. Suppose $\int_{\mathcal{C}_1} f(z)dz = 1$, $\int_{\mathcal{C}_2} f(z)dz = 2$, and $\int_{\mathcal{C}_3} f(z)dz = 3$.

(a) Calculate the residues of f at each of its three singularities.

(b) Give an explicit example of a function f satisfying the above properties.

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 11am Tuesday 7th December

Thomas Prellberg, November 2004