MAS205 Complex Variables 2004-2005

Sample Test

Question 1: [15 marks]

(a) Find all solutions $z \in \mathbb{C}$ of the equation

$$z^4 = 16 \; .$$

(b) Find all solutions $z \in \mathbb{C}$ of the equation

$$e^{3z} = i \; .$$

Express all solutions in standard and polar form, and draw diagrams showing their location in the complex plane.

Question 2: [15 marks]
Consider the transformation

$$z \mapsto w = z^2 - 1 \; .$$

(a) Find the equation of the image of the line $\Re(z) = 1$ and sketch the image.

(b) What is the image of the upper half plane $\{z \in \mathbb{C} : \Im(z) \geq 0\}$?

Question 3: [15 marks]
Find the Möbius transformation $f(z) = (az+b)/(cz+d)$ which maps $0 \mapsto -1$, $1 \mapsto 0$, and $-1 \mapsto \infty$.

Question 4: [15 marks]
Evaluate

$$\text{(a)} \quad \lim_{z \to i} \frac{z^2 + 4iz - 3}{z^2 + 1} \qquad \text{(b)} \quad \lim_{z \to \infty} \frac{(1-z)(2-3z)}{iz^2 + 4 + 2i}$$

Question 5: [10 marks]
Show that $\lim_{z \to 0} (\overline{z} - z)/z$ does not exist.

Question 6: [15 marks]
At what values of $z = x + iy$ is the function $f(x+iy) = y^3 + y^2$ differentiable?

Question 7: [15 marks]
Let $f(z) = 1/(1-5z)$. Determine the Taylor series $\sum_{n=0}^{\infty} a_n z^n$ for f around the point $z_0 = 0$. What is the radius of convergence of this Taylor series?

This test will be the basis of a review exercise class on November 5.

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