MAS205 Complex Variables 2004-2005

Exercises 2

Exercise 5: Find all complex solutions of the following equations:

$$(a) \quad e^z = 1 \qquad (b) \quad e^{2z} = -1 \qquad (c) \quad \cosh z = 0 \qquad (d) \quad \sin z = 0$$

Exercise 6: Consider the transformation

$$z \mapsto w = iz^2 + 1 \ .$$

    (a) Find the equation of the image of the line $\Im(z) = 1$ and sketch the image.

    (b) Sketch the image of the curve $z\bar{z} = 1$.

Exercise 7: For each of the following transformations, find the regions in the z-plane which map to the left half of the w-plane:

    (a) $w = 1 + 1/z$

    (c) $w = z^3$

Exercise 8: Find the Möbius transformation $f(z) = (az + b)/(cz + d)$ which maps $1 \mapsto 1$, $i \mapsto 0$, and $-1 \mapsto i$.

    (a) What is the image of $z = 0$

    (b) Which point is mapped by f to $-i$?

    (c) What is the image of the left half plane under f?

Exercise 9: Prove that if $g(z) = (az + b)/(cz + d)$ and $h(z) = (a'z + b')/(c'z + d')$, then $h \circ g(z) = (a''z + b'')/(c''z + d'')$ where

$$\begin{pmatrix} a'' & b'' \\ c'' & d'' \end{pmatrix} = \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 11am Tuesday 19th October

Thomas Prellberg, October 2004