MAS205 Complex Variables 2005-2006

Exercises 6

Exercise 21: Find the radius of convergence of the following power series

$$(a) \quad \sum_{n=1}^{\infty} \frac{z^n}{(2+2i)^n}\ , \quad (b) \quad \sum_{n=0}^{\infty} (n+1)^7 z^n\ , \quad (c) \quad \sum_{n=0}^{\infty} z^n \exp(n)\ ,$$

$$(d) \quad \sum_{n=0}^{\infty} \frac{z^n}{(n!)^2}\ , \quad (e) \quad \sum_{n=0}^{\infty} (-1)^n n! z^n\ .$$

Exercise 22: Give an example, if possible, of power series with the following properties:

(a) centred at $z_0 = i$, with radius of convergence $R = 0$

(b) centred at $z_0 = -2 + 2i$, with radius of convergence $R = 2$

(c) centred at $z_0 = 1$ and convergent for all z with $\Re(z) < 2$ but divergent for all z with $\Re(z) > 4$

(d) centred at $z_0 = (1+i)/\sqrt{2}$, with radius of convergence $R = \infty$

(e) centred at $z_0 = 0$ and convergent for all z with $\Im(z) = 2$ but divergent for all other $z \in \mathbb{C}$

(Proofs are not necessary, but if you can't find an example you should explain why.)

Exercise 23: Let $f(z) = (2+z)/(1-z)$. Determine the Taylor series $\sum_{n=0}^{\infty} a_n z^n$ for

$$(a) \quad f(z) = \frac{1}{1+z} \quad \text{around } z_0 = 0,$$

$$(b) \quad f(z) = \frac{1}{1+z} \quad \text{around } z_0 = 1,$$

$$(c) \quad f(z) = \frac{1}{(1+z)(1-z)} \quad \text{around } z_0 = 0.$$

In each of the cases, give the radius of convergence of the Taylor series.

Exercise 24: Let $D = \{z : |z+3i| < 2\}$. Suppose that $f : D \to \mathbb{C}$ is defined by

$$f(z) = \sum_{n=0}^{\infty} \frac{(z+3i)^n}{(2i)^n}\ .$$

Calculate the Taylor series for f at the point $z_0 = 0$ and determine its radius of convergence.

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 10:30am Wednesday 23rd November

Thomas Prellberg, November 2005