

MAS115 Calculus I 2006-2007

Problem sheet for exercise class 7

- **Make sure you attend the exercise class that you have been assigned to!**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

(*) Problem 1: The average value of an integrable function on the interval $[a, b]$ is defined as

$$\text{av}(f) = \frac{1}{b-a} \int_a^b f(x) dx .$$

(i) If $\text{av}(f)$ really is a typical value of the function $f(x)$ on $[a, b]$, then

$$\int_a^b \text{av}(f) dx = \int_a^b f(x) dx$$

should hold. Does it?

(ii) It would be nice if average values obeyed the following rules on an interval $[a, b]$.

- $\text{av}(f + g) = \text{av}(f) + \text{av}(g)$
- $\text{av}(kf) = k \text{av}(f)$ (any number k)
- $\text{av}(f) \leq \text{av}(g)$ if $f(x) \leq g(x)$ on $[a, b]$.

Do these rules ever hold?

Give reasons for your answers.

Problem 2: Which formula is not equivalent to the other two?

- $\sum_{j=2}^4 \frac{(-1)^{j-1}}{j-1}$
- $\sum_{k=0}^2 \frac{(-1)^k}{k+1}$
- $\sum_{l=-1}^1 \frac{(-1)^l}{l+2}$

Problem 3: L'Hopital's rule does not help with the following limits. Find them some other way:

- $\lim_{x \rightarrow \infty} \frac{\sqrt{x+5}}{\sqrt{x+5}}$
- $\lim_{x \rightarrow \infty} \frac{2x}{x+7\sqrt{x}}$

Extra: Let $f(x)$, $g(x)$ be two continuously differentiable functions satisfying the relationships $f'(x) = g(x)$ and $f''(x) = -f(x)$. Let $h(x) = f^2(x) + g^2(x)$. If $h(0) = 5$, find $h(10)$.