MTH5105 Differential and Integral Analysis 2009-2010

Exercises 8

There are two sections. Questions in Section 1 will be marked and will form your coursework mark. Questions in Section 2 are voluntary but highly recommended.

1 Exercise for Feedback/Assessment

1) Let the sequence of functions $g_n : \mathbb{R} \to \mathbb{R}$ $(n \in \mathbb{N})$ be given by

$$g_n(x) = \frac{x}{1+nx^2} \ .$$

(a) Compute $g(x) = \lim_{n\to\infty} g_n(x)$. [4 marks]

(b) Show that g_n converges to g uniformly. [6 marks]

(c) Compute $h(x) = \lim_{n\to\infty} g_n'(x)$. [5 marks]

(d) Does $g'(x) = h(x)$ hold? [2 marks]

(e) Why does Theorem 9.5 not apply here? [3 marks]

2 Extra Exercises

2) For $x \in \mathbb{R}$, compute

$$f(x) = \sum_{n=1}^{\infty} \frac{x}{(1+x^2)^n} \ .$$

Show that the convergence is not uniform.

3) (a) Show that the following sequences of functions converge uniformly on the given intervals.

(i) $u_n(x) = (1-x)x^n$, $[0,1]$;

(ii) $v_n(x) = \dfrac{x^2}{1+nx^2}$, $\mathbb{R}$.

(b) Which of the following sequences of functions converge uniformly to $s(x) = 1$ on the interval $[0,1]$?

(i) $f_n(x) = (1+x/n)^2$,

(ii) $g_n(x) = 1 + x^n(1-x)^n$,

(iii) $h_n(x) = 1 - x^n(1-x^n)$.

The deadline is 5.00pm (strict) on Monday 29th March. Please hand in your coursework to the red coursework box on the ground floor.

Thomas Prellberg, March 2010