MAS205 Complex Variables 2004-2005

Exercises 7

Exercise 28: Let the curve $\mathcal{C}$ be given by the graph of the function $y = f(x)$ with

$$f(x) = \frac{x^2}{8} - \log x \qquad (1 \le x \le 2)$$

embedded in $\mathbb{C}$ via $z = x + iy$.

(a) Give a path $\gamma : [a,b] \to \mathbb{C}$ which has the curve $\mathcal{C}$ as its image. Draw a sketch of the curve and indicate the parametrisation.

(b) Compute the length $L(\mathcal{C})$. Evaluate the result numerically and discuss it in view of your sketch (i.e. does your result make sense and why).

Exercise 29: Let $\mathcal{C}$ be the unit circle described counterclockwise. Show that

$$\left| \int_{\mathcal{C}} \frac{\cos z}{z} dz \right| < 2\pi e \,.$$

Exercise 30: Using the definition of the integral of a complex function f along a contour $\gamma : [a,b] \to \mathbb{C}$ as

$$\int_a^b f(\gamma(t))\gamma'(t) dt \,,$$

compute the integral of $f(z) = (z-4)^2$ along the straight line segments

(a) from 0 to 2,

(b) from 0 to $-3i$.

Check your answers by finding an antiderivative F for f and evaluating F at the points $z = 0, 3, -2i$.

Exercise 31: Let $f(z) = \bar{z}$. Find the values of $\int_{\mathcal{C}_k} f(z) dz$ where

(a) $\mathcal{C}_1$ denotes the straight line from $z_0 = 2$ to $z_1 = 2i$,

(b) $\mathcal{C}_2$ denotes the arc from $z_0 = 2$ to $z_1 = 2i$ along a circle of radius 2 about the origin.

Find a simple closed contour $\mathcal{C}$ for which $\int_{\mathcal{C}} f(z) dz \neq 0$.

Exercise 32: By applying Cauchy's theorem (or otherwise) show that $\int_{\mathcal{C}} f(z) dz = 0$ where $\mathcal{C}$ is the unit circle $\{z \in \mathbb{C} : |z| = 1\}$ and

$$(a) \quad f(z) = \frac{1}{z^2+3} \qquad (b) \quad f(z) = \frac{1}{z^2+2iz-5} \qquad (c) \quad f(z) = \frac{1}{\cosh z}$$

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 11am Tuesday 30th November

Thomas Prellberg, November 2004