

# MAS115 Calculus I 2006-2007

## Problem sheet for exercise class 4

- **Make sure you attend the exercise class that you have been assigned to!**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

Problem 1: Can  $f(x) = x(x^2 - 1)/|x^2 - 1|$  be extended to be continuous at  $x = 1$  or  $x = -1$ ? Give reasons for your answers.

Problem 2: **Limits and continuity.** Which of the following statements are true and which false? If true, say why; if false, give a counterexample (that is, an example confirming the falsehood).

- (\*) a. If  $\lim_{x \rightarrow a} f(x)$  exists but  $\lim_{x \rightarrow a} g(x)$  does not exist, then  $\lim_{x \rightarrow a} (f(x) + g(x))$  does not exist.
- (\*) b. If neither  $\lim_{x \rightarrow a} f(x)$  nor  $\lim_{x \rightarrow a} g(x)$  exists, then  $\lim_{x \rightarrow a} (f(x) + g(x))$  does not exist.
- c. If  $f$  is continuous at  $a$ , then so is  $|f|$ .
- d. If  $|f|$  is continuous at  $a$ , then so is  $f$ .

(\*) Problem 3: **Stretching a rubber band.** Is it true that if you stretch a rubber band by moving one end to the right and the other to the left, some point of the band will end up in its original position? Give reasons for your answer.

(\*) Problem 4: Show that the equation  $x^3 - 15x + 1 = 0$  has three solutions in the interval  $[-4, 4]$ .

Extra: **A function continuous at only one point.** Let

$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

- a. Show that  $f$  is continuous at  $x = 0$ .
- b. Use the fact that every nonempty open interval of real numbers contains both rational and irrational numbers to show that  $f$  is not continuous at any nonzero value of  $x$ .