MTH4100 Calculus I

Taster Lecture

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What is Calculus I

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  - one real variable
  - many variables (Calculus II)

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- Geometric view: graph of a function
  - slope $\leftrightarrow$ derivative
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  - area $\leftrightarrow$ integral
- many techniques
- many applications

Functions and Their Graphs

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- Important: rule is unique, only one value $f(x)$ for every x

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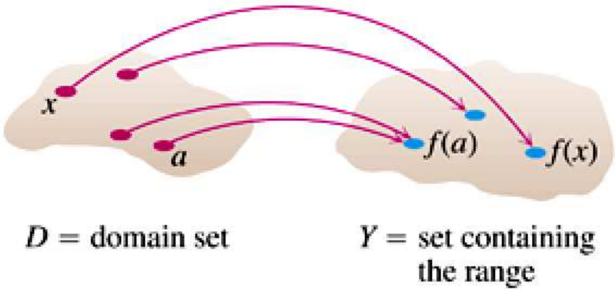
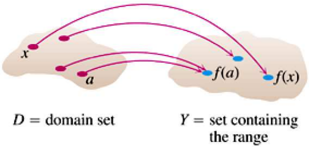


Further definitions and notations



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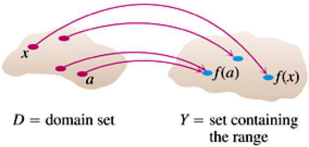

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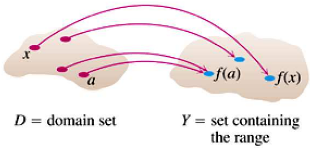

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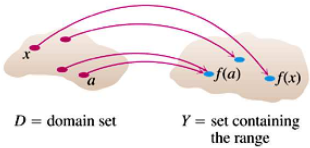

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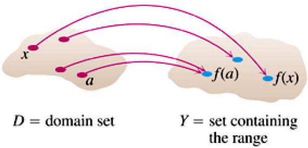

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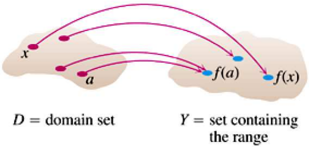

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Note the different arrow symbols used

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Function	Domain (x)	Range (y)
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$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
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Note: a function is specified by the rule f and the domain D

$$f : x \mapsto x^2 \,, \quad D(f) = [0, \infty)$$

and

$$f : x \mapsto x^2 \,, \quad D(f) = (-\infty, \infty)$$

are *different* functions

Graphs of functions

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If f is a function with domain D, its graph consists of the points (x, y) whose coordinates are the the input-output pairs for f:

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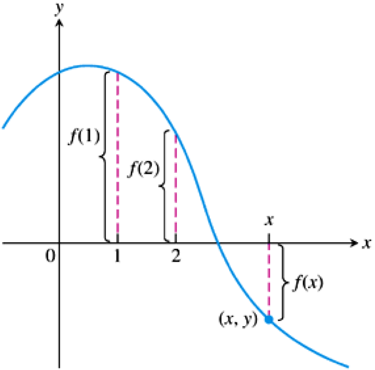
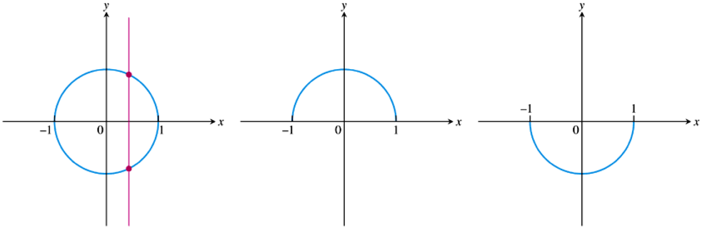


Curves that are graphs of functions

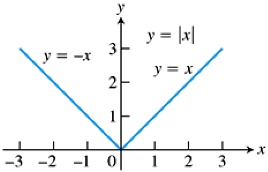
The vertical line test



(a) $x^2 + y^2 = 1$ (b) $y = \sqrt{1 - x^2}$ (c) $y = -\sqrt{1 - x^2}$

Piecewise defined functions

$$f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



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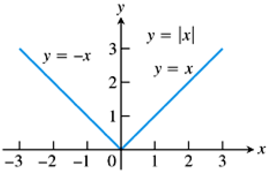
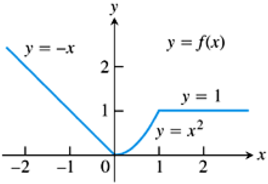


$$f(x) = \begin{cases} -x & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$



Revision so far: Functions and their Graphs

- Definition of a function
- Domain and range of a function
- Graph of a function
- Piecewise defined functions

Compositions of functions

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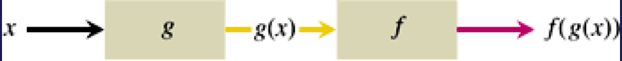

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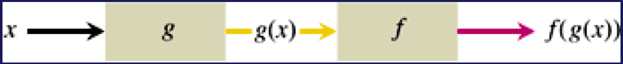

The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f, i.e.

$$D(f \circ g) = \{x | x \in D(g) \text{ and } g(x) \in D(f)\}$$

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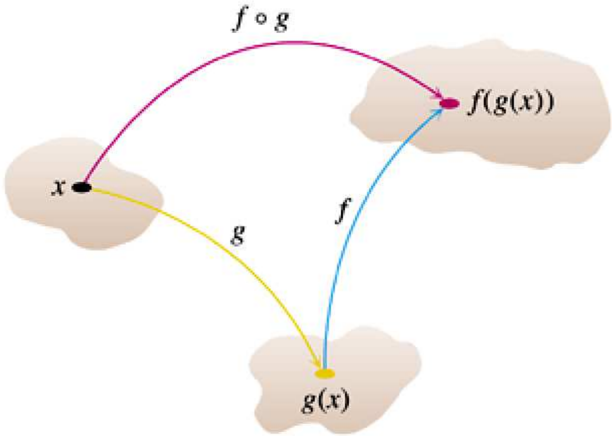

Examples

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Revision

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- Composition of functions

Thank you for coming today!