MAS205 Complex Variables 2005-2006

Key Objectives

- To understand and be able to use basic algebraic and geometric properties of complex numbers - modulus, argument, complex conjugate, addition, multiplication and division of complex numbers, and basic properties of complex functions - computation of images of lines, curves, and regions of the plane under complex maps, in particular Möbius transformations. To be able to evaluate limits of sequences of complex numbers.
- To know, and be able to use, the definition of complex differentiation, and to know, be able to derive and be able to use, the Cauchy-Riemann equations.
- To know the statements of the theorems concerning the existence and convergence of Taylor series and Laurent series, and be able to compute Taylor and Laurent expansions for combinations of rational, exponential and trigonometric functions, and be able to state the regions of the complex plane for which these expansions are valid. To know what are the possible types of isolated singularities, to be able to distinguish these types in examples, and to be able to compute residues.
- To know how to parametrise a contour, and to define and compute an integral of a complex function along a contour. To know the statement of Cauchy's Theorem and be able to use it to prove that appropriate contour integrals are zero.
- To know Cauchy's Integral Formula and be able to use it both to compute contour integrals and to evaluate functions. To know the Residue Theorem and be able to use it to evaluate integrals around closed contours.

Learning Outcomes

- To understand and be able to use basic algebraic and geometric properties of complex numbers
- To understand complex functions as maps of the complex plane
- To be able to evaluate simple limits of sequences of complex numbers
- To be able to use the definition of complex differentiation, and to understand the connection to the Cauchy-Riemann equations
- To be able to test a Taylor or Laurent series for convergence and to be able to compute Taylor and Laurent expansions for combinations of rational, exponential and trigonometric functions
- To know what are the possible types of isolated singularities, and to be able to distinguish these types in examples; to be able to compute residues
- To know how to parametrise a contour, and to define and compute an integral of a complex function along a contour.
- To know Cauchy's Integral Formula and be able to use it both to compute contour integrals and to evaluate functions.
- To know the Residue Theorem and be able to use it to evaluate integrals around closed contours.

Warnings

1. The above is intended as a MINIMAL list to be mastered in order to be reasonably sure of PASSING the examination. For a more detailed list of major results of the course, see the "Summary of main results" handout: statements and proofs of all these main results are examinable (except where explicitly stated otherwise), as are examples related to them.
2. Just because knowledge of a particular definition, formula or statement of a theorem is in the list of 'Key Objectives' above does not guarantee that it will be on the examination paper. However, a good proportion will be, so they are worth knowing well.

Examination

The examination lasts for 2 hours. The examination paper will contain a questions in Section A and b questions in Section B. The rubric will state:
This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section. SECTION A: Each question carries x marks. You should attempt ALL questions. SECTION B: Each question carries y marks. You may attempt all questions. Except for the award of a bare pass, only marks for the best n questions will be counted. Calculators are NOT permitted in this examination.
Overall credit on this course will be computed using the algorithm:

10% for best 6 courseworks, plus 10% for test, plus 80% for final exam.

If you missed the test with a valid excuse then the algorithm is modified to:

10% for best 6 courseworks, plus 90% for final exam.

Thomas Prellberg, September 2005