MTH5105 Differential and Integral Analysis 2008-2009

Exercises 1

Exercise 1: Investigate differentiability of

(a) $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto x|x+1|$, [3 marks]

(a) $g : \mathbb{R} \to \mathbb{R}$, $x \mapsto (x+1)|x+1|$, [3 marks]

and find the derivatives, if they exist. [4 marks]

Exercise 2: Prove that the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 \sin(1/x^2) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is differentiable at zero and find $f'(0)$. [4 marks]

Find $f'(x)$ for $x \neq 0$ assuming that $\sin' = \cos$. [3 marks]

Give a rough sketch of the curve $f'(x)$ for small x and mark $f'(0)$ clearly on your sketch. [3 marks]

Exercise 3: Let $f : [-1,1] \to \mathbb{R}$ be continuous on $[-1,1]$, differentiable at zero and $f(0) = 0$. Show that the function

$$g(x) = \begin{cases} f(x)/x & x \neq 0 \\ f'(0) & x = 0 \end{cases}$$

is continuous at zero. [4 marks]

Is g continuous for $x \neq 0$? [3 marks]

Deduce that there is some number M such that

$$f(x)/x \leq M \quad \text{for all} \quad x \in [-1,1] \setminus \{0\} .$$

[3 marks]

The deadline is 12.15 on Monday, 19th January. Please hand in your coursework at the end of Monday's lecture or to my office MAS113 immediately afterwards.

Thomas Prellberg, January 2009