MAS205 Complex Variables 2005-2006

Exercises 9

Exercise 33: Let $f(z) = \Re(z)$. Find the values of $\int_{\mathcal{C}_k} f(z)dz$ where

(a) $\mathcal{C}_1$ denotes the straight line from $z_0 = 3$ to $z_1 = 3i$,

(b) $\mathcal{C}_2$ denotes the arc from $z_0 = 3$ to $z_1 = 3i$ along a circle of radius 3 about the origin.

Find a simple closed contour $\mathcal{C}$ for which $\int_{\mathcal{C}} f(z)dz \neq 0$.

Exercise 34: By applying Cauchy's theorem (or otherwise) show that $\int_{\mathcal{C}} f(z)dz = 0$ where $\mathcal{C}$ is the unit circle $\{z \in \mathbb{C} : |z| = 1\}$ and

$$(a) \quad f(z) = \frac{1}{z^2+5} \qquad (b) \quad f(z) = \frac{1}{z^2 - 2iz - 5} \qquad (c) \quad f(z) = \tanh z$$

Exercise 35: Use the Cauchy integral formula to evaluate each of the following integrals, where $\mathcal{C}$ is the positively oriented circle $\{z \in \mathbb{C} : |z-3| = 1/2\}$:

(a)

$$\int_{\mathcal{C}} \frac{(z+2)^3}{(z-3)z^2} dz$$

(b)

$$\int_{\mathcal{C}} \frac{\exp z}{(z-\pi)\cos z} dz$$

Exercise 36: Use the residue theorem to calculate

(a)

$$\int_{\mathcal{C}} \frac{1}{(z^2+4)(z+2+2i)} dz$$

for $\mathcal{C} = \mathcal{C}_1$, the positively oriented circle of radius 3 centred at 1, and
for $\mathcal{C} = \mathcal{C}_2$, the positively oriented square with corners $-3-3i$ and $1+i$;

(b)

$$\int_{\mathcal{C}} \frac{1}{z(z^2-4)(z-2)} dz$$

where $\mathcal{C}$ is the positively oriented circle of radius 2 centred at 1.

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 10:30am Wednesday 14th December

Thomas Prellberg, December 2005