# MTH5105 Differential and Integral Analysis 2008-2009

Exercises 2

Exercise 1: Let $f, g : \mathbb{R} \to \mathbb{R}$ be differentiable with

$$f' = g \qquad \text{and} \qquad g' = -f \ .$$

Show that between every two zeros of $f$ there is a zero of $g$ and between every two zeros of $g$ there is a zero of $f$. [8 marks]

Exercise 2: Let $f : \mathbb{R} \to \mathbb{R}$ be twice differentiable ($f'' = (f')'$) with

$$f(0) = f'(0) = 0 \qquad \text{and} \qquad f(1) = 1 \ .$$

Show that there exists a $c \in (0, 1)$ such that $f''(c) > 1$. [10 marks]

Exercise 3:

(a) Let $g : [a, b] \to \mathbb{R}$ be differentiable and $g'(a) < 0 < g'(b)$.
Show that $g$ attains its minimal value for some $c \in (a, b)$. Deduce that $g'(c) = 0$. [8 marks]

(b) Let $f : [a, b] \to \mathbb{R}$ be differentiable and $f'(a) < s < f'(b)$.
Using $g(x) = f(x) - sx$, show that there exists a $c \in (a, b)$ such that $f'(c) = s$. [4 marks]

*This shows that the derivative of differentiable functions satisfies the intermediate value property. Note that the derivative doesn't have to be continuous, so this is different from the intermediate value theorem for continuous functions.*

The deadline is 12.15 on Monday, 26th January. Please hand in your coursework at the end of Monday's lecture or to my office MAS113 immediately afterwards.

Thomas Prellberg, January 2009