

MTH5105 Differential and Integral Analysis

2009-2010

Exercises 1

There are two sections. Questions in Section 1 will be marked and will form your coursework mark. Questions in Section 2 are voluntary but highly recommended.

1 Exercise for Feedback/Assessment

- 1) Using the definition of the derivative of a function, investigate for which values of x each of the following two functions is differentiable, and find the derivatives, if they exist.

(a) $f : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto x^2|x|,$ [10 marks]

(b) $g : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto x|x-1|,$ [10 marks]

2 Extra Exercises

- 2) Prove that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 \sin(1/x^2) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is differentiable at zero and find $f'(0)$.

Find $f'(x)$ for $x \neq 0$ assuming that $\sin' = \cos$.

Give a rough sketch of the curve $f'(x)$ for small x and mark $f'(0)$ clearly on your sketch.

- 3) Let $f : [-1, 1] \rightarrow \mathbb{R}$ be continuous on $[-1, 1]$, differentiable at zero and $f(0) = 0$. Show that the function

$$g(x) = \begin{cases} f(x)/x & x \neq 0 \\ f'(0) & x = 0 \end{cases}$$

is continuous at zero.

Is g continuous for $x \neq 0$?

Deduce that there is some number M such that

$$f(x)/x \leq M \quad \text{for all } x \in [-1, 1] \setminus \{0\}.$$

The deadline is 5.00pm (strict) on Monday 25th January. Please hand in your coursework to the red coursework box on the ground floor.