Extended Domb-Joyce Model

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A careful definition of the extended Domb-Joyce model.

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INTRODUCTION

We formulate the Domb-Joyce model as a discrete version of a field-theoretic model.

THE MODEL

We consider random walks $\omega = (\vec{\omega}_0, \vec{\omega}_1, \ldots, \vec{\omega}_N)$ on a lattice. The number of visits to each site $\vec{x}$ induces a density ϕ_ω on the lattice sites $\vec{x}$ via

$$\phi_\omega(\vec{x}) = \sum_{i=0}^{N} \delta_{\vec{\omega}_i, \vec{x}} \; .$$

Interpreting the density ϕ as a field, we now write the energy of the field ϕ as

$$E(\phi) = a \sum_{\vec{x}} \phi(\vec{x}) + b \sum_{\vec{x}} \phi^2(\vec{x}) + c \sum_{\vec{x}} \phi^3(\vec{x}) + \ldots \; .$$

If the field is given by a particular random walk configuration ω, i.e. $\phi = \phi_\omega$, the first term in this expression is simply related to the length N of the random walk, as

$$\sum_{\vec{x}} \phi(\vec{x}) = N + 1 \; ,$$

so that a is related to a chemical potential. In the conventional Domb-Joyce model, the energy functional is truncated after the quadratic term,

$$E_{DJ}(\phi) = a \sum_{\vec{x}} \phi(\vec{x}) + b \sum_{\vec{x}} \phi^2(\vec{x}) \; .$$

For $b = 0$ we have pure random walk, while for $b < 0$ the model is weakly self-avoiding. If one chooses $b > 0$, the model leads to an extreme collapse, dominated by configurations occupying few lattice sites with very high density. Thus, while this model is capable of modelling the swollen polymer regime, further terms in the energy functional need to be taken into consideration to model "realistic" polymer collapse. We therefore introduce an extended Domb-Joyce model, given by

$$E_{EDJ}(\phi) = a \sum_{\vec{x}} \phi(\vec{x}) + b \sum_{\vec{x}} \phi^2(\vec{x}) + c \sum_{\vec{x}} \phi^3(\vec{x}) \; .$$

Here, we choose $c < 0$ to stabilize the model and explore the behavior as we change b. For $b < 0$ this again leads to a swollen polymer phase. However, the polymer should in some sense be "collapsed" for sufficiently large b, and therefore we expect to find a transition at $b = b_c(c) > 0$.

If $m_r(\omega)$ denotes the number of r-times visited sites for a random walk ω, we can write

$$\sum_{\vec{x}} \phi_\omega^k(\vec{x}) = \sum_r r^k m_r(\omega) \; .$$

This expression we use to define the model studied here as follows. For a random walk ω we define an energy $E(\omega)$ as

$$E(\omega) = b \sum_r r^2 m_r(\omega) + c \sum_r r^3 m_r(\omega) \; .$$

The canonical partition function is then given by

$$Z_N(\beta) = \sum_{|\omega| = N+1} e^{-\beta E(\omega)} \; ,$$

where the sum extend over all random walk configurations with N steps, respectively $N + 1$ sites.

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