MAS115 Calculus I 2006-2007

Learning Outcomes

On completion of this course students will be expected to

1. know and use elements of set theory notation in the context of real line;
2. be able to solve algebraic equations and inequalities involving the square root and modulus function, e.g.
$$|x-3|+|2x-4|<10, \quad 2\sqrt{x-4}\le x+1, \quad \frac{|2x-4|}{x}<x+2;$$
3. understand the difference between equations and identities, and be able to prove simple identities and inequlities, e.g. $a^2+b^2\ge 2|ab|$;
4. know addition and double-angle formulas for trigonometric functions and use them to express values of trigonometric functions in the surds form, e.g. $\cos(\pi/12)=(1+\sqrt{3})/(2\sqrt{2})$ and $\cos^2(\pi/8)=(2+\sqrt{2})/4$;
5. be able to recognize odd, even, periodic, increasing, decreasing functions, e.g. is $f(x)=\sin(x^3)/\cos(x+\pi/2)$ odd, even or neither?;
6. understand the operation of composition of functions and the concept of functional inverse;
7. to able to recognize linear, quadratic, power, polynomial, algebraic, rational, trigonometric, exponential, hyperbolic and logarithmic functions and sketch their graphs; given the graph of $f(x)$ sketch the graph of $|f(ax+b)|$ or $af(|x|)+b$.
8. be able to manipulate piece-wise defined functions;
9. be able to calculate limits by substitution and by eliminating zero denominators, e.g.
$$\lim_{x\to -5}\frac{x-\sqrt{x+6}}{3-x}, \quad \lim_{x\to 3}\frac{x-\sqrt{x+6}}{3-x}, \quad \lim_{x\to 1}\frac{x^3-1}{x-1};$$
10. be able to calculate limits at infinity of rational functions and rational algebraic, e.g.
$$\lim_{x\to+\infty}\frac{100x^5+1}{x^{10}+4}, \quad \lim_{x\to+\infty}\frac{\sqrt{2x+1}-\sqrt{x}}{\sqrt{x}};$$
11. be able to calculate limits in indeterminate forms by a repeated use of l'Hopital rule, including limits involving $\frac{\sin x}{x}$ and $(1+\frac{1}{x})^x$, e.g.
$$\lim_{x\to 0}\frac{1-\cos^2(2x)}{\sin(x^2)}, \quad \lim_{x\to+\infty}x^2e^{-x}, \quad \lim_{x\to 0}(1+\sin^2 x)^{1/\sin(x^2)};$$
12. understand the concepts of rate of change and instantaneous rate of change;
13. know derivatives of power, trigonometric, exponential, hyperbolic, logarithmic and inverse trigonometric functions; know the basic rules of differentiation and use them to find derivatives of products and quotients;
14. know the chain rule and use it to find derivatives of composite functions, e.g.
$$\cos(1+e^{-x^2}), \quad \frac{\ln(1+\sin^2 x)}{\sin(x^2)};$$
15. be able to use derivatives to find intervals on which the given function is increasing or decreasing, find maxima and minima of functions;
16. be able to find tangents and normals to graphs of functions given in explicit, implicit and parametric forms;
17. be able to estimate change with differentials;

18. be able to sketch graphs of rational functions including finding asymptotes;
19. understand the concept of indefinite integral as anti-derivative;
20. know standard indefinite integrals and basic rules of indefinite integration;
21. be able to evaluate integrals by substitution with and without suitable hints, e.g.
$$\int x\sqrt{2x^2+1}dx, \quad \int \frac{dx}{\sqrt{4+x^2}}, \quad \int \sqrt{4-x^2}dx, \quad \int \frac{xdx}{\sqrt{x^2-3x+2}};$$
22. be able to evaluate integrals of rational functions by partial fractions;
23. be able to evaluate integrals by a repeated use of integration by parts, e.g.
$$\int x^2 \ln x dx, \quad \int e^{2x} \sin x dx;$$
24. be able to solve separable differential equations and first-order linear differential equations;
25. understand the concept of definite integral and know the basic properties of definite integrals;
26. know the Fundamental Theorem of Calculus and be able to use it for evaluating definite integrals and derivatives of integrals with variable limits of integration;
27. understand the concept of area of regions with curvilinear boundaries, be able to find area between curves;
28. be able to find volume of solids by slicing and volume of solids of revolutions;
29. understand the concept of length of a planar curve and be able to find the length of parametric curves;
30. be able to convert cartesian coordinates in polar coordinates and vice versa, know the polar equation for lines, circles, circular sectors, annuli, ellipses, parabolas and hyperbolas.
31. be able to sketch simple polar curves, e.g. $r = 1 - \cos\theta$ or $r = \sin(\theta/2)$.

Warnings

1. The above is intended as a MINIMAL list to be mastered in order to be reasonably sure of PASSING the examination.
2. Just because knowledge of a particular definition, formula or statement of a theorem is in the list of 'Learning outcomes' above does not guarantee that it will be on the examination paper. However, a good proportion will be, so they are worth knowing well.

Examination

The examination lasts for 2 hours. The rubric will state:
You should attempt all questions. Marks awarded are shown next to the questions.
Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.
Overall credit on this course will be computed using the algorithm:
20% for two tests and coursework, plus 80% for final exam.

Thomas Prellberg, September 2006