

A Comparison of the New Specification (First 'A' Level Certification June 2005) with Specification (Curriculum) 2000 for Edexcel 'A' Level Pure and Applied Mathematics

Specification (Curriculum) 2000:

'A' level pure and applied mathematics comprised 6 modules or units, three of which (P1, P2 & P3) contained 'core' pure mathematics material and were compulsory. The remaining three units were selected from a set of Applications units (Mechanics M1, M2 & M3, Statistics S1, S2 & S3 and Decision D1 & D2) with the condition that at least one of these units must be M2, M3, S2, S3 or D2. Selecting M1, S1 & D1 was thus forbidden but some of the permitted combinations of Applications units were:

1. M1, M2 & M3
2. S1, S2 & S3
3. S1, S2 & M1
4. S1, S2 & D1
5. M1, M2 & S1

Since all units had equal weighting, i.e. the elementary P1, M1, S1 and D1 units carried the same weight as the much harder P3, M3 and S3 units, weaker students and teachers alike opted for combinations that effectively replaced the difficult M3 and S3 units with M1, S1 or D1 units, e.g. selections 3, 4 & 5 instead of 1 & 2 in the above list.

Thus students could exploit this loop-hole, so to speak, and obtain high grades in 'A' level mathematics by avoiding the more difficult units.

New Specification (Replacing Specification 2000):

The new version of 'A' level pure and applied mathematics also comprises 6 units, each carrying equal weighting, but there are 4 major changes which can have only a deleterious effect on the quality of examination as a whole.

The first is that the core content, that had originally been covered in units P1, P2 and P3, has now not only been reduced (Appendix 1) but also spread over FOUR units, C1, C2, C3 & C4, i.e. there is now even less core material per unit and thus even less to cram into short-term memory. Below are two examples which illustrate how simple some of this 'A' level core material has become:

- Factorization of a cubic expression has been spread over *two units* i.e.
 - C1: Factorization of polynomials of degree n , $n \leq 3$, where x is a factor, e.g. $x^3 + 4x^2 + 3x$ and the Factor Theorem is *not* required.
 - C2: Use of the Factor Theorem and Remainder Theorem. Candidates may be required to factorize cubic expressions such as $x^3 + 3x^2 - 3$ and $6x^3 + 11x^2 - x - 6$.

- A.P.s and G.P.s, which had been examined together previously in unit P1, have now been separated with A.P.s in unit C1 and G.P.s in unit C2.

The second change is a consequence of the first and is the REMOVAL of M3 and S3 (Appendix 2) from the choice of Applications units. So instead of closing the loop-hole present in Specification 2000 that allowed these hard units to be dodged, these units have been elevated to the *further* mathematics syllabus, and the harder further mathematics units M6 and S6 have disappeared from the 'A' level altogether to accommodate this change (Appendix 3).

The third change has probably arisen from the blinkered obsession with 'choice and flexibility' over content and depth and is the removal of the condition that at least one Applications unit must be S2, M2 or D2. Since all units *still* have equal weighting, there is little incentive (and now no requirement) for teachers or students to select one of these harder units for study, e.g. the selection of M1 & S1 over M1 & M2 is thus inevitable.

Thus students can now obtain a grade A in 'A' level pure and applied mathematics knowing less core material than for Specification 2000 and with only a rudimentary understanding of applied topics. A new 'A' level comprising C1, C2, C3, C4, M1 & S1 is of a *lower* standard than that of a Specification 2000 'A' level comprising P1, P2, P3, M1, M2 & M3.

Finally, the requirement that candidates may resit any individual unit once only has been scrapped, i.e. there is now no restriction on the number of times a unit may be attempted, and the best result will count towards the final award. Employers and university admissions tutors will therefore be unable to distinguish between the weak student, who obtained grades A in the easy C1 and C2 units after four attempts, and the very bright student who achieved them on his first attempt.

One does question the motives of those who have modified the specification because their changes have lead to a further reduction in the content and standard of 'A' level pure and applied mathematics and have made it even easier for weak and unsuitable candidates to embark on science, engineering and mathematics courses at university.

As a tutor of 'A' level pure and applied mathematics and further mathematics, Dr. Ramsay has taught the old, high-quality and highly valued 'A' level syllabuses and is now reduced to teaching the new, watered-down, emasculated 'A' level units - the product of successive 'modifications' following the introduction of the G.C.S.E. examinations in 1988. The recent drive to increase choice and accessibility has culminated in a shallow and lack-lustre 'A' level that is both boring to teach and boring to study. One finds it increasingly difficult to feel genuine satisfaction when students achieve a grade A in such an embarrassingly easy examination.

We have written this article because we feel that employers and university lecturers must realize that the new modified specification for 'A' level mathematics is of a standard below that of Specification 2000, and both are of a standard well below that of the pre-modular 'A' level of 20 years ago. The 'gold standard' in 'A' level mathematics no longer exists and students today are receiving an inferior qualification masquerading as an 'A' level. Students, employers and university lecturers are all being deceived but we are at a loss to know how to bring a halt to this process now that the rot, which set in so many years ago, has taken such a firm hold.

Jonathan Ramsay, BSc PhD (private tutor) and
John Corner, DIC MSc PGCE CMath MIMA (former teacher).

Appendices:

Appendix 1: Topics Removed From the Core Material in the New Specification

1. Identities (equating coefficients).
2. Proof by direct methods.
3. Proof of the quadratic formula.
4. Area under curves requiring $\int x \, dy$.
5. The equation of a circle in the form $x^2 + y^2 + 2gx + 2fy + c = 0$.
6. Odd and even functions.
7. The knowledge and use of identities such as $2\cos A \cos B \equiv \cos(A + B) + \cos(A - B)$ to solve equations and prove identities.
8. Partial fractions with quadratic factors in the denominator.
9. Sketching a curve from its parametric equations.
10. Volumes of revolution requiring $\pi \int x^2 \, dy$.

Appendix 2: Applications Units No Longer Examined in 'A' Level Mathematics

Unit M3 (Elevated to Further Mathematics):

1. Kinematics of a particle moving in a straight line when the acceleration is a function of displacement (x) or time (t).
2. Elastic strings and springs.
3. Hooke's Law.
4. Energy stored in an elastic string or spring.
5. Newton's laws of motion, for a particle moving in one dimension, when the applied force is variable.
6. Simple harmonic motion.
7. Oscillations of a particle attached to the end of an elastic string or spring.
8. Angular speed.
9. Radial acceleration in circular motion.

10. The forms $r\omega^2$ and $\frac{v^2}{r}$ are required.
11. Uniform motion of a particle moving in a horizontal circle.
12. Motion of a particle in a vertical circle.
13. Centre of mass of uniform rigid bodies and simple composite bodies.
14. Simple cases of equilibrium of rigid bodies.

or

Unit S3 (Elevated to Further Mathematics):

1. Distributions of linear combinations of independent Normal random variables.
2. Methods for collecting data. Simple random sampling. Use of random numbers for sampling.
3. Other methods of sampling: stratified, systematic, quota.
4. Concepts of standard error, estimator, bias.
5. The distribution of the sample mean \bar{X} .
6. Concept of confidence interval and its interpretation.
7. Confidence limits for a Normal mean, with variance known.
8. Hypothesis tests for the mean of a Normal distribution with variance known.
9. Use of the Central Limit Theorem to extend hypothesis tests and confidence intervals to samples from non-Normal distributions. Use of large samples results to extend to the case in which the variance is unknown.
10. Hypothesis test for the difference between the means of two Normal distributions with variances known.
11. Use of large sample results to extend to the case in which the population variances are unknown.
12. The null and alternative hypotheses. The use of $\sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2 statistic.
13. Degrees of freedom.
14. Spearman's rank correlation coefficient, its use, interpretation and limitations.
15. Testing the hypothesis that a correlation is zero.

Appendix 3:

Applications Units Removed From 'A' Level Further Mathematics

Unit M6 (Removed):

1. Differentiation of unit vectors in two dimensions. Velocity and acceleration components using Cartesian coordinates, polar coordinates and intrinsic coordinates.
2. Motion of a particle on a smooth curve, given in intrinsic form.
3. Motion under a central force.

4. Motion of projectiles.
5. Motion of centre of mass.
6. Independence of rotational and translation motion.
7. The effect of an impulse on a rigid body which is unconstrained.
Conservation of linear momentum. Conservation of angular momentum.

Unit S6 (Removed):

1. Hypothesis test and confidence interval for β , the gradient of a linear regression model, assuming a Normal distribution.
2. Residuals. The residual sum of squares.
3. Sign test for a population median based on a single sample. Sign test for equality of two distributions based on paired samples.
4. Wilcoxon signed-rank test.
5. Wilcoxon rank sum test.
6. Control charts for mean, range, standard deviation and fraction defective.
7. One-way analysis of variance.
8. Two-way analysis of variance.

Observations on London (Edexcel) 'A' Level Mathematics from 1960 to 2004

In the early 1960s, pure and applied mathematics taken as one 'A' level comprised two 3 hour papers, each containing 10 long questions covering the whole syllabus, from which candidates were expected to answer 8.

Since then, the 'A' level mathematics syllabus has undergone several changes, the most deleterious following the abolition of the C.S.E. and the 'O' level and their replacement with the G.C.S.E. In 1992, modularization created four examination papers (modules) each of which could be taken at separate sittings, 6 months apart if the candidate wished.

Modularization broke the syllabus into smaller, easier to learn (cram) chunks, with some topics in a module never again to be encountered in a later module. This allowed candidates who possessed little, or only short-term, memory to obtain an 'A' level in mathematics which they could not have obtained under the old system.

Furthermore, by 1995 almost half of the pure mathematics topics had been removed (Appendix 1). In a study of the performance of first year students in an engineering department, K. L. Todd¹, reported that a student with a grade A in mathematics in 2001, on average obtained a score on his test which would have placed that student near the bottom of the cohort 15 years previously.

Since 1991, D. A. Lawson² has performed a diagnostic test on first year students on degree courses with significant mathematical content. He found that a student with a grade B in 'A' level mathematics in 1999 scored similarly to a student with grade N (a fail grade) in 1991.

In these studies by Todd and Lawson, the candidates reviewed had taken their 'A' level mathematics with whatever examination board that their school had subscribed to, not particularly Edexcel.

The Chief Executive of the Qualifications and Curriculum Authority (QCA) agreed that there had been a "backsliding" of standards during the 1990s and that they would now be "reasserting earlier standards". To redress the situation, the QCA introduced Curriculum 2000. This applied to all examination boards and for Edexcel a small number of the topics which had been removed previously has been restored to the syllabus (topics 2, 7, 9, 10, 15 & 18 in Appendix 1) but the examination now has 6 modules (or units), involving even less content per examination paper – less to learn per module or per sitting.

The mathematical skills and topics, which previously formed part of the syllabus for children aged 14 to 16 years, and were examined at C.S.E. and 'O' level, are now set as questions in the early 'A' level units. In particular, finding areas and volumes using calculus, which used to be examined at 'O'

level, are now examined in 'A' level pure mathematics units one (P1) and two (P2) respectively, but it is the 'O' level questions which are harder.

Some applied mathematics C.S.E. papers from the 1970s are almost indistinguishable from the mechanics unit one (M1) 'A' level paper, with some C.S.E. topics even overlapping with unit two (M2). One calculus topic from 'O' level pure mathematics (variable velocity) is now to be found at 'A' level in mechanics unit two (M2).

On comparing the current 'A' level mathematics with the earlier, pre-modular examination, one observes several other worrying trends in the new examination. Its modular nature severely limits the variety and difficulty of questions, which are shorter and more predictable, less algebraically demanding and are often no more than a series of easy drill-type exercises. The examination routinely guides students through the questions with the liberal use of diagrams and splitting the questions into several ordered steps.

'A' level mathematics grades A & B are less reliable indicators of mathematical competence. Universities and employers are finding it very difficult to separate the wheat from the chaff.

Intelligent and mathematically-gifted students are now denied the opportunity of being distinguished from their less able peers. They are also denied the quality of education enjoyed by their forebears.

References:

1. Todd, K. L. 'An Historical Study of the Correlation between G.C.E. Advanced Level Grades and the Subsequent Academic Performance of Well Qualified Students in a University Engineering Department', *Mathematics Today* (IMA), **37**(5):152-156, Oct. 2001.
2. Lawson, D. A. 'What can we expect of 'A' level mathematics students?', *Teaching Mathematics and its Applications*, **16**(4):151-156, 1997.

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Appendix 1:

A Summary of Topics Removed From London (Edexcel) 'A' Level Mathematics by 1995

1. Complex numbers.
2. Vectors in 2 & 3 dimensions.
3. Mathematical induction.
4. Centroids and centres of mass.
5. Permutations and combinations.
6. Arithmetic and geometric means.
7. The equation of a circle.

8. Newton-Raphson iteration.
9. The trapezium rule.
10. The binomial expansion of $(1 + x)^k$ where k is rational.
11. The roots of quadratic equations without solving the equations.
12. Inequalities of the form $\frac{1}{(x-a)} > \frac{x}{(x-b)}$, $|x - b| > k|x - a|$.
13. The approximations $\sin x \approx x$, $\tan x \approx x$, $\cos x \approx 1 - \frac{x^2}{2}$.
14. The product formulae.
15. Expression of $a\cos\theta + b\sin\theta$ in the form $r\cos(\theta + \alpha)$.
16. The general solutions to trigonometric equations.
17. The summation of simple finite series, e.g. $\sum_{r=1}^n r(r+1)$
18. The inverse functions of sine, cosine and tangent, defined over suitable regions and the graphs of these functions.
19. The application of differentiation to small increments and approximations.
20. The integrals of $\frac{1}{1+x^2}$, $\frac{1}{\sqrt{1-x^2}}$.