Examiners Joint Report

Orbital Tutte Polynomials

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The thesis contains work exploring how to define an orbital version of the Tutte polynomial. More precisely, given a graph and a group of automorphisms of the graph, a polynomial is sought that counts orbits of the automorphism group on various objects counted by the Tutte polynomial. Several extensions of the Tutte polynomial are developed. A discussion of how to compute some of these polynomials is also included.

There are many results here that are interesting, new and sometimes difficult to prove. Clearly the candidate has a deep understanding of the subject. The thesis is vast and definitely more than meets the requirements for a PhD in terms of its depth, interest and novelty.

Briefly the first chapter contains discussion on a first version of the Orbital Tutte polynomial which counts orbits of an automorphism group on nowhere zero flows, nowhere zero tensions and vertex colourings. In the second chapter modifications are made so that the polynomial counts orbits on bases, independent sets and spanning sets. This requires consideration of collapsed graphs formed by identifying the vertex and edge cycles induced by an element of the automorphism group and leads to a modified version of the Orbital Tutte polynomial discussed in Chapter 3. In Chapter 4 attempts are made to recover the cycle index from the Orbital Tutte polynomial - this does not seem to be easy to do. The fixed point Tutte polynomial is discussed in Chapter 5. This counts various objects of types counted by the Tutte polynomial that are fixed by every member of an automorphism group and allows the definition of a third Orbital Tutte polynomial that contains more information about the sizes of the orbits. An investigation of deletion and contraction formulae make up Chapter 6. The final chapter investigates exactly which graphs have various desirable properties identified in an earlier chapter.

Unfortunately the thesis in the form presented is marred by the quality of the presentation. There are a number of issues here:

- There is no introduction, so it is hard for the reader to understand the background to the problem, which parts of the thesis influence each other and how the problems were developed. This would be a good place to introduce the main topics of the thesis on which background knowledge is needed, for instance, the Tutte polynomial, matroids and Smith Normal Form.

- There is no mention of any existing literature on the topic. Indeed the thesis is in general inadequately referenced. There are many places where the reader should be referred to background sources for more information.

During the viva, the candidate confirmed our suspicion that very little material on this topic has appeared in the literature beforehand. However this needs to be explained and any existing material referenced.

- The number of typos and small mistakes is not acceptable.
- Chapters are not linked together as they should be. Lengthy arguments are entirely repeated without mentioning that they have previously appeared.
- It would also be nice to have formulas for the polynomials T, OT, OT', OT'', OT_{sets}, OT_{all}, OT_{II}, Z, FPT, OT_{III}, FPT', etc. together somewhere. A glossary of notations used (along with short explanations) could be useful, too.

During the viva the candidate acknowledged that each of these points were valid. He was able to give a very coherent and informative summary of the thesis, explained carefully its relationship with existing the literature and was able to evaluate the contribution that had been made. The candidate dealt with these problems in a confident way and most of the problems outlined above will be rectified by adding to the thesis the contents of the candidate's answers.

Before the viva we had several queries concerning the clarity of particular definitions and proofs. These are outlined in our preliminary reports which we presented to the candidate and are enclosed. The overwhelming majority of these issues were resolved at the viva. The small remainder of problems that we were unable to resolve are relatively unimportant and do not impinge on the rest of the thesis.

Overall the thesis makes a substantial and original contribution to the subject area. The candidate displayed a mastery of the topic and familiarity with related areas. Subject to the completion of a fairly lengthy list of minor corrections, which we have given to the candidate, we strongly recommend the award of a PhD.

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