MAS115 Calculus I 2006-2007

Problem sheet for exercise class 3

- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

Problem 1: Compute the following limits:

$$\text{(a)} \quad \lim_{x\to 4} \frac{4-x}{5-\sqrt{x^2+9}}\,, \qquad \text{(b)} \quad \lim_{u\to 1} \frac{u^4-1}{u^3-1}\,.$$

Problem 2: **Two wrong statements about limits.** Show by example that the following statements are wrong.

(*a) The number L is the limit of $f(x)$ as x approaches x_0 if $f(x)$ gets closer to L as x approaches x_0.

(b) The number L is the limit of $f(x)$ as x approaches x_0 if, given any $\epsilon > 0$, there exists a value of x for which $|f(x) - L| < \epsilon$.

Explain why the functions in your examples do not have the given value of L as a limit as $x \to x_0$.

Problem 3: Use the graph of the greatest integer function $y = \lfloor x \rfloor$ to determine the limits

$$\text{(*a)} \quad \lim_{\theta\to 3^-} \frac{\lfloor\theta\rfloor}{\theta}\,, \qquad \text{(b)} \quad \lim_{t\to 4^+} (t - \lfloor t \rfloor)\,.$$

Problem 4: Compute the following limits:

$$\text{(*a)} \quad \lim_{x\to\infty} \frac{x+\sin x + 2\sqrt{x}}{x+\sin x}\,, \qquad \text{(b)} \quad \lim_{x\to\infty} \frac{x^{2/3}+x^{-1}}{x^{2/3}+cos^2 x}\,.$$

Extra: **Roots of a quadratic equation that is almost linear.** The equation $ax^2 + 2x - 1 = 0$, where a is a constant, has two roots if $a > -1$ and $a \neq 0$, one positive and one negative:

$$r_+(a) = \frac{-1+\sqrt{1+a}}{a}\,, \quad r_-(a) = \frac{-1-\sqrt{1+a}}{a}\,.$$

(a) What happens to $r_+(a)$ as $a \to 0$? As $a \to -1^+$?

(b) What happens to $r_-(a)$ as $a \to 0$? As $a \to -1^+$?

(c) Support your conclusions by graphing $r_+(a)$ and $r_-(a)$ as functions of a. Describe what you see.

Thomas Prellberg, October 2006