MTH5105 Differential and Integral Analysis 2008-2009

Midterm Test

Problem 1: Let $f(x) = 1/x$.

(a) Determine the Taylor polynomials $T_{3,1}$ and $T_{4,1}$ of degree 3 and 4 at $a = 1$ for f.

[*15 marks*]

(b) Using the Lagrange form of the remainder, or otherwise, show that

$$T_{3,1}(x) < f(x) < T_{4,1}(x) \quad \text{for all } x > 1 \ .$$

[*15 marks*]

Problem 2: (a) Give the definition of $f : \mathbb{R} \to \mathbb{R}$ being differentiable at a point $a \in \mathbb{R}$.

[*10 marks*]

(b) Using the definition, determine whether or not

$$f(x) = \begin{cases} \dfrac{x}{1 + \exp(1/x)} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is differentiable at $x = 0$. (For this you may wish to consider the left and right derivatives of $f(x)$ at $x = 0$.) Find $f'(0)$, if it exists. [*20 marks*]

Problem 3: (a) State the Mean Value Theorem. [*15 marks*]

(b) Show that for all $x, y \in \mathbb{R}$

$$|\sin(y) - \sin(x)| \leq |y - x| \ .$$

[*25 marks*]

You may assume standard properties of trigonometric functions.

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