

# MAS205 Complex Variables 2004-2005

## Exercises 3

Exercise 10: Evaluate the following limits:

$$(a) \quad \lim_{z \rightarrow \infty} \frac{((2+i)z+1)(z+3)^3}{(2z-i)^2(3z-4)^2} \quad (b) \quad \lim_{z \rightarrow 1+i} \frac{z^6}{z^2-2i} \quad (c) \quad \lim_{z \rightarrow \infty} \frac{z^2}{z^3-1-i}$$

Exercise 11: (a) Give an example of a function  $f : \mathbb{C} \rightarrow \mathbb{C}$  such that

$$\lim_{z \rightarrow i} f(z) = 2 \quad \text{and} \quad \lim_{z \rightarrow 1} f(z) = \infty.$$

(b) Suppose

$$f(z) = \frac{p(z)}{z^2-1}, \quad \text{where } p(z) = az+b \text{ for some } a, b \in \mathbb{C}.$$

If  $\lim_{z \rightarrow 1} f(z) = 1$ , what is  $p(z)$ ?

(c) Suppose

$$f(z) = \frac{p(z)}{z^2+1}, \quad \text{where } p(z) \text{ is a quadratic polynomial.}$$

If  $\lim_{z \rightarrow i} f(z) = i$  and  $\lim_{z \rightarrow \infty} f(z) = 2$ , what is  $p(z)$ ?

(d) Find a polynomial  $p(z)$  such that

$$\lim_{z \rightarrow 0} \frac{p(z)}{z(z-i)} = 3i, \quad \lim_{z \rightarrow -i} \frac{p(z)}{z(z-i)} = 0, \quad \lim_{z \rightarrow 3+i} \frac{p(z)}{z(z-i)} = 0.$$

Exercise 12: For each of the following functions, decide at which values of  $z$  the function is continuous and at which values it is not continuous. Give reasons, but detailed proofs are not expected.

(a)  $f(z) = z^2 + i\bar{z} - 1$

(b)  $f(z) = i(\bar{z}/z)^4$  for all non-zero  $z$ , and  $f(0) = i$ .

Exercise 13: Starting from the definition of the derivative of a complex function as a limit,

(a) find the derivative of  $f(z) = z^3 - 2z$  at  $z = i$ ;

(b) find the derivative of  $f(z) = z^2 - 1$  for all  $z \in \mathbb{C}$ ;

(c) prove that  $f(z) = z\bar{z} - 2z$  does not have a derivative at  $z_0$  unless  $z_0 = 0$ .  
What is the value of  $f'(0)$ ?

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 11am Tuesday 26th October

Thomas Prellberg, October 2004