Research

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1 Research Accomplishments

I am an applied mathematician who has significantly contributed to a number of different research areas as diverse as dynamical systems and combinatorics of lattice models.

Dynamics of non-uniformly expanding maps

My doctoral research concerned itself with the investigation of intermittency. For this, the thermodynamic formalism was extended to deal with non-uniformly expanding dynamics. This work introduced key ideas which after 12 years are still relevant. I recently returned to this area with a paper in which I managed to answer some questions which had stayed open for a decade. Subsequently I have embarked on an ongoing collaboration with Peter Kleban from the Unversity of Maine on connections to certain number-theoretic spin models.

Interestingly, applications of this work have surfaced in average case analysis of algorithms. Here, algorithms are seen as dynamical systems, which can be analysed by the same tools used in my doctoral dissertation. I was participating in a German-French Project on utilizing these connections.

Interacting Self-Avoiding Walks and Vesicles

This is arguably my most productive research area, and the subject of my *habilitation* thesis. I have considered interacting self-avoiding walks and lattice models of vesicles using a variety of techniques ranging from exact solutions of toy models to large-scale simulations of analytically intractable cases. I have published the most accurate estimate of the length scale exponent of self-avoiding walks in three dimensions to date. Results include development of scaling theory, calculation of critical exponents and scaling functions, discussion of a novel "pseudo-first-order" phase transition, application of conformal field theory to oriented polymers, and development of algorithms for self-avoiding walks.

Stochastic Growth Algorithms

Starting from my work in self-avoiding walk simulations, I have adapted and developed stochastic growth algorithms. These algorithms have potential in simulations of polymers, proteins, percolation, and other areas. I have recently developed a "Flat-Histogram" version of a stochastic growth algorithm, which enables one to determine the whole density of states in one single simulation. This algorithm has been successfully applied to the study of a variety of systems.

Asymptotic Analysis and Iterative Functional Equations

The methods which I acquired to calculate scaling functions have enabled me to accurately compute the asymptotic behavior of counting problems describable by a certain class of iterative functional equations, leading to the Takeuchi-Prellberg constant. Here, I make central use of analytic iteration theory near parabolic fixed points.

Monototicity Properties of Partition Functions

Over the course of time, the techniques I developed in solving combinatorial lattice models have proven to be successful in purely mathematical applications. For instance, I recently solved with Dennis Stanton a long-standing conjecture in additive number theory using q-series techniques I learned when studying combinatorial lattice models.

Asymptotic Enumeration of Incidence Matrix Classes

Similarly, I recently collaborated with Peter Cameron and Dudley Stark on asymptotic enumeration problems in connection with incidence matrix classes, using a variety of methods from group theory, probability theory, and enumerative combinatorics.

Some Excursions into Physics

While at Syracuse, I published a Phys. Rev. Letter on a model of viscoelastic depinning of driven systems. This paper was motivated by experimental evidence of hysteresis in flux line depinning.

In a collaboration with physical chemists at Clausthal, I contributed to a paper on the application of quartz crystal resonators for surface analysis. Essentially, the measured effect of the surface contact on the oscillation of the resonators had to be converted to material parameters via inversion of an operator.

2 Research Plan

My main research for the next few years is a direct consequence of the research grant that I have held recently. The development and application of a new algorithm of a stochastic growth algorithm with uniform sampling is the goal of an ongoing research activity with the ARC Centre of Excellence in the Mathematics and Statistics of Complex Systems in Australia, where I am an associate investigator. This research is computing intensive and necessitates access to a small Beowulf cluster.

This algorithm is suited for the study of various other problems. For example, simulations of various polymer systems in restricted geometries and at low temperature are envisaged (with Aleks Owczarek and Jaroslaw Krawczyk, University of Melbourne). In applications of this algorithm I see definite scope for further grant proposals. Currently planned simulations include

- Simulations of extended Domb-Joyce models for testing of field-theoretical models for polymer collapse
- The phase diagram of adsorbing and collapsing stiff polymers
- The critical behavior of polymers in restricted geometries

and on the more mathematical side

- Simulations of pattern-avoiding permutations
- Understanding the cycle-decomposition of permutations in latin squares

As the algorithm is essentially combinatorial in nature, and can be understood as weighted incomplete enumeration, it is applicable within a purely combinatorial framework. Here, I see definite scope for developing local collaborations.

On the analytical side, I am considering extending exact solutions for various polymer models. Results of mine which were derived for a toy model have recently shown to be applicable in a much more general setting (with Buks van Rensberg, York University, and Andrew Rechnitzer, Melbourne). Hereby the so-called Kernel method is employed to solve certain many-variable functional equations.

Another interest of mine (in collaboration with Peter Kleban, Maine) is in number-theoretic spin-chains and their relation to dynamical systems. Coupling these spin chains to an external field leads to a highly interesting scenario, which provides a playground for testing renormalization group hypotheses. My methods can provide rigorous results against which one can test RG assumptions.

Finally, given that several of my recent publications were largely accidental in origin (i.e. started by informal discussions over meals), I always like to keep an open mind for new collaborations.