Open Problems in Mathematics

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Topic Outline

1. Some Thoughts about Mathematics
   - Why You Should Study Mathematics
   - What is Mathematics
2. Mathematical Problems
   - Some Million Dollar Problems
   - Examples of Solved and Open Problems
3. The 3n+1 Problem
   - Statement of the Problem
   - Some Examples
   - Why the Conjecture should be True
   - Extending the Problem
   - The Ulam Spiral

Outline

Top 10 Reasons to Study Maths

10 So you'll know that a negative number isn't a number with an attitude problem.
9 When the teacher talks about an acute angle, you'll know she's not referring to how attractive the angle is.
8 So you'll realize that a factor tree is not the oak next to the school parking lot.
7 Because studying π is simply delicious!
6 To learn that irrational numbers really do make sense.
5 So you'll realize that place value doesn't refer to how close your desk is to the pencil sharpener.
4 So you'll understand that the distributive property has nothing to do with real estate.
3 Because solving word problems could lead to solving the world's problems.
2 To learn that Dewey didn't invent the decimal.
1 If you can't count to a million, how will you know if you've become a millionaire?

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- You are really good at maths
- You like problem solving
- You could get into business school
- You want to keep your options open

Bad Reasons for Studying Mathematics

- Your language skills are really weak
- You like memorising formulas
- You can't get into business school
- You haven't yet figured out what you're good at

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What is Mathematics

Mathematics is not

- just "doing things with numbers and letters and other symbols"
- just a collection of facts and rote recipes
- just computational and arithmetic skills

Mathematics is

- a way of thinking
- the language of science
- a creative discipline
- a source of pleasure and wonder
- a means of problem solving

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Seven Million Dollars Prize Money

7 Prize Problems, selected by Clay Mathematics Institute in 2000

- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

These are hard problems (it might be easier to rob a bank...)

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Solved Problems in Mathematics

Some recently proved problems:

- Fermat's last theorem (1637, proved 1994): If an integer n is greater than 2, then the equation

$$a^n + b^n = c^n$$

has no solutions in non-zero integers a, b, and c.

For $n = 2$, this is of course possible, for example

$$3^2 + 4^2 = 5^2 \; .$$

- The four colour theorem (1852, proved 1976): Given any plane separated into regions, such as a political map of the states of a country, the regions may be coloured using no more than four colours.

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Open Problems in Mathematics

Some unsolved problems:

- Goldbach's conjecture (1742): every even integer greater than 2 can be written as the sum of two primes.

  $$\text{For example, } 18 = 5 + 13 = 7 + 11.$$

  Goldbach's ternary conjecture, proved 2013: every odd integer greater than 5 *can be written as the sum of three primes.*

- The twin prime conjecture (300 BC): there are infinitely many primes p such that $p + 2$ is also prime.

  For example, 17 and 19 are twin primes.

- How many different Sudoku squares of size $n \times n$ are there? There are

  $$6,670,903,752,021,936,960$$

  valid 9×9 Sudoku squares. The problem is to find a formula for general n.

more at http://en.wikipedia.org/wiki/Unsolved_problems_in_mathematics

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  $$\text{For example, } 18 = 5 + 13 = 7 + 11.$$

  Goldbach's ternary conjecture, proved 2013: every odd integer greater than 5 *can be written as the sum of three primes.*

- The twin prime conjecture (300 BC): there are infinitely many primes p such that $p + 2$ is also prime.

  For example, 17 and 19 are twin primes.

- How many different Sudoku squares of size $n \times n$ are there? There are

  $$6,670,903,752,021,936,960$$

  valid 9×9 Sudoku squares. The problem is to find a formula for general n.

more at http://en.wikipedia.org/wiki/Unsolved_problems_in_mathematics

"The history of mathematics is a history of horrendously difficult problems being solved by young people too ignorant to know that they were impossible."

Freeman Dyson, "Birds and Frogs", AMS Einstein Lecture 2008

Outline

Statement of the Problem

Consider the following operation on an arbitrary positive integer:

- If the number is even, divide it by two.
- If the number is odd, triple it and add one.

$$f(n) = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ 3n+1 & \text{if } n \text{ is odd.} \end{cases}$$

Form a sequence by performing this operation repeatedly, beginning with any positive integer.

- Example: $n = 6$ produces the sequence

$$6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \ldots$$

The Collatz conjecture is:

This process will eventually reach the number 1, regardless of which positive integer is chosen initially.

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Some Examples

Examples:

- $n = 11$ produces the sequence

$$11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.$$

- $n = 27$ produces the sequence
27,82,41,124,62,31,94,47,142,71,214,107,322,161,484,242,121,
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526,263,790,395,1186,593,1780,890,445,1336,668,334,167,502,
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719,2158,1079,3238,1619,4858,2429,7288,3644,1822,911,2734,
1367,4102,2051,6154,3077,9232,4616,2308,1154,577,1732,866,
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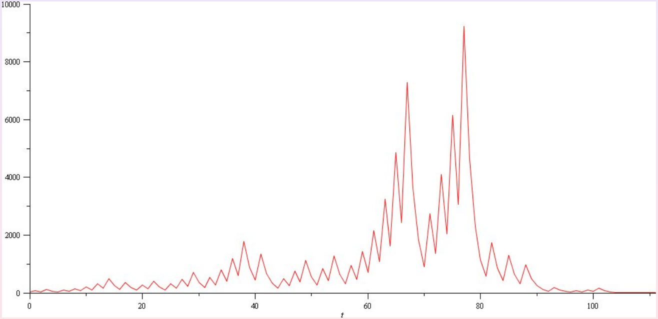
Graphing the Sequences

A graph of the sequence obtained from $n = 27$



This sequence takes 111 steps, climbing to over 9000 before descending to 1.

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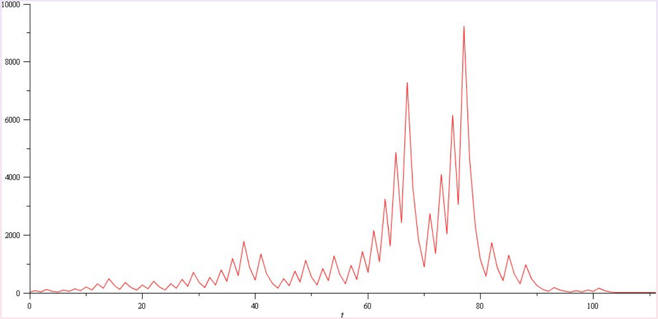

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Supporting Arguments for the Conjecture

- Experimental evidence:

  The conjecture has been checked by computer for all starting values up to $19 \times 2^{58} \approx 5.48 \times 10^{18}$.

- A probabilistic argument:

  One can show that each odd number in a sequence is on average 3/4 of the previous one, so every sequence should decrease in the long run.

  This not a proof because Collatz sequences are not produced by random events.

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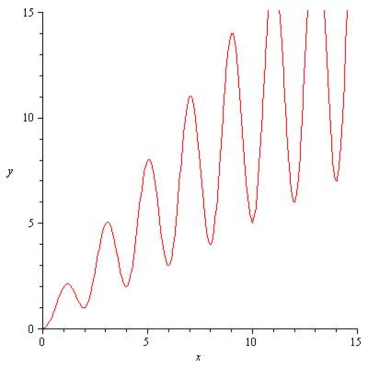
Iterating on Real Numbers



Reduce an orbit by replacing $3n+1$ with $(3n+1)/2$:

$$10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \ldots$$

shortens to

$$10, 5, 8, 4, 2, 1, 2, 1, \ldots$$

The function graphed is given by

$$f(x) = \frac{x}{2}\cos^2\left(\frac{\pi}{2}x\right) + \frac{3x+1}{2}\sin^2\left(\frac{\pi}{2}x\right)$$

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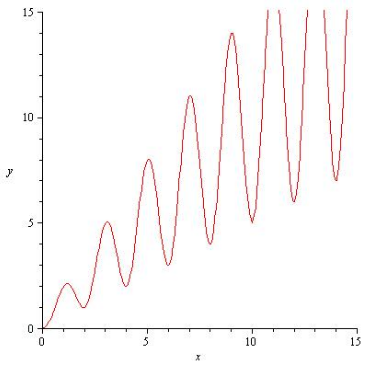

Reduce an orbit by replacing $3n + 1$ with $(3n + 1)/2$:

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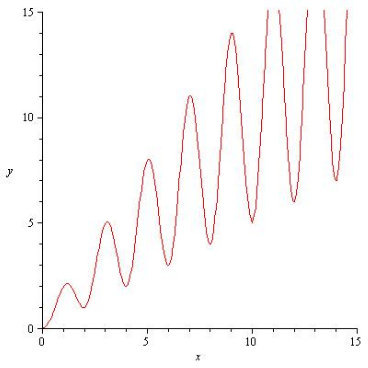

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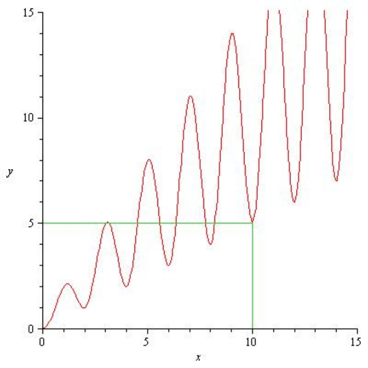

$10 \to 5$

Reduce an orbit by replacing $3n+1$ with $(3n+1)/2$:

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shortens to

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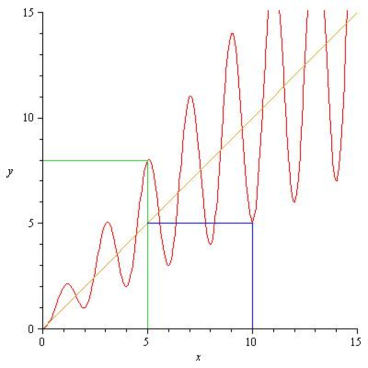

$10 \rightarrow 5 \rightarrow 8$

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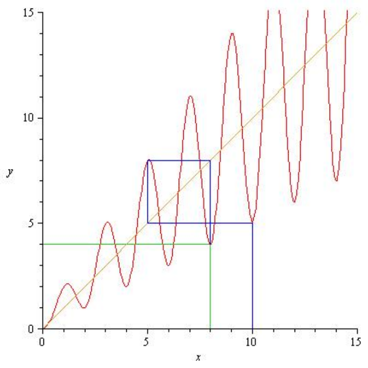

$10 \to 5 \to 8 \to 4$

Reduce an orbit by replacing $3n+1$ with $(3n+1)/2$:

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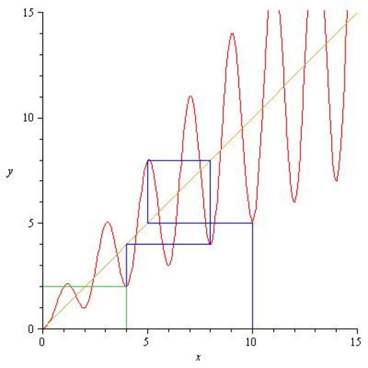

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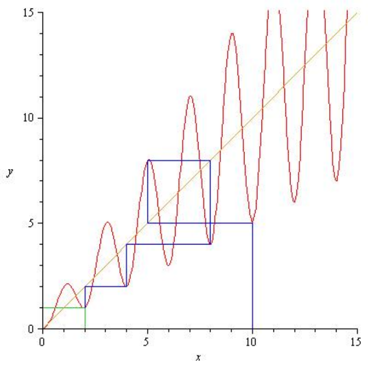

$10 \to 5 \to 8 \to 4 \to 2 \to 1$

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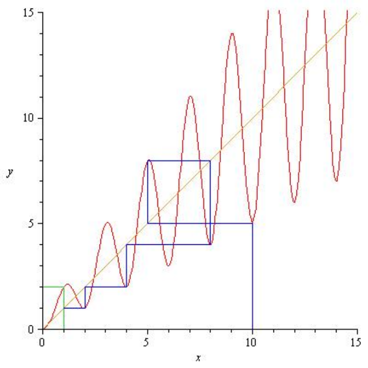

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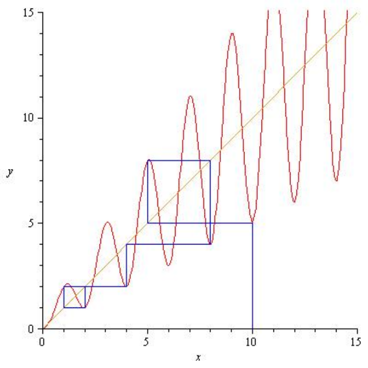

A "cobweb" plot

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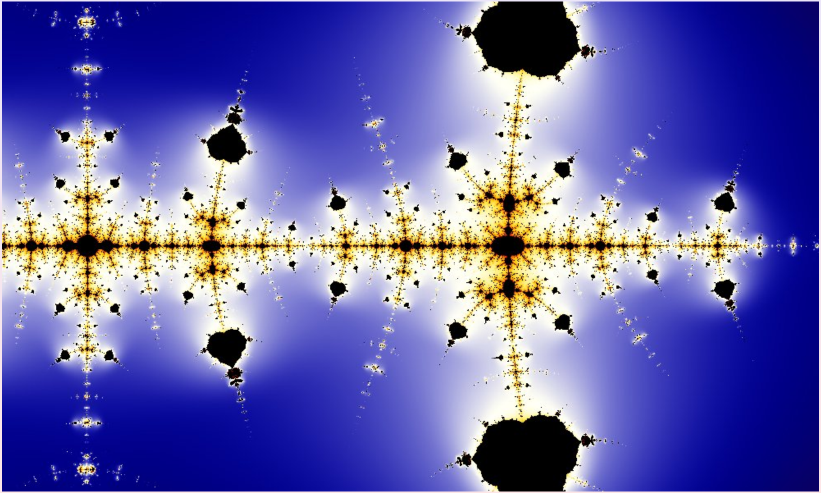
Iterating on Complex Numbers



$3n+1$ on the Ulam Spiral

```
37—36—35—34—33—32—31
|                  |
38  17—16—15—14—13  30
|   |           |   |
39  18   5— 4— 3  12  29
|   |    |     |  |   |
40  19   6   1— 2  11  28
|   |    |        |   |
41  20   7— 8— 9—10  27
|   |                 |
42  21—22—23—24—25—26
|
43—44—45—46—47—48—49...
```

The $3n+1$ iteration on the Ulam spiral

> *"Mathematics is not yet ready for such problems."*

Paul Erdős, 1913 - 1996

The End