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Introductio

Function

Composition of functions

Revision

MTH4100 Calculus I

Taster Lecture

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1 July 2013

What is Calculus I

Introduction

- Study of functions of real variables
 - one real variable
 - many variables (Calculus II)

What is Calculus I

Introduction

- Study of functions of real variables
 - one real variable
 - many variables (Calculus II)
- Fundamental: real numbers
- Geometric view: graph of a function
 - slope ↔ derivative
 - area ↔ integral

What is Calculus I

Introduction

- Study of functions of real variables
 - one real variable
 - many variables (Calculus II)
- Fundamental: real numbers
- Geometric view: graph of a function
 - slope ↔ derivative
 - area ↔ integral
- many techniques
- many applications

Functions and Their Graphs

Functions

of function

Revisior

What do we mean when we say

"y is a function of x"?

Functions

of function

Revisic

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$$y = f(x)$$

Functions and Their Graphs

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Functions

of functions

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Functions and Their Graphs

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Functions and Their Graphs

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• Important: rule is unique, only one value f(x) for every x

Definition of a function

Functions

Definition

A function from a set D to a set Y is a rule that assigns a unique (single) element $f(x) \in Y$ to each element $x \in D$.

Definition of a function

Introducti

Functions

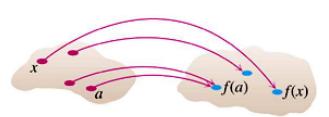
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of function

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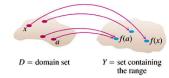


D = domain set

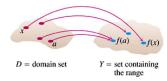
Y = set containing the range

Further definitions and notations

Functions



Functions



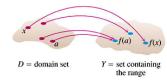
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Functions

of functions

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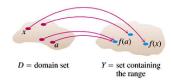
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Functions

of functions

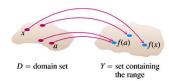
Revisio



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Functions



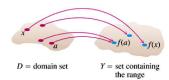
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Revision



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Note the different arrow symbols used

Natural domain

Introduction The *natural domain* is the largest set of real x which the rule f can be applied to.

Composition of function

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Functions

The natural domain is the largest set of real x which the rule fcan be applied to.

Function	Domain (x)	Range (y)
$y = x^2$	$(-\infty, \infty)$	[0, ∞)
y = 1/x	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0) \cup (0,\infty)$
$y = \sqrt{x}$	$[0,\infty)$	$[0,\infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0,\infty)$
$y = \sqrt{1 - x^2}$	[-1, 1]	[0, 1]

Natural domain

Functions

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Note: a function is specified by the rule f and the domain D

$$f: x \mapsto x^2$$
, $D(f) = [0, \infty)$

and

$$f: x \mapsto x^2$$
, $D(f) = (-\infty, \infty)$

are different functions

Graphs of functions

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Functions

Composition of function

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Definition

If f is a function with domain D, its graph consists of the points (x, y) whose coordinates are the input-output pairs for f:

$$\{(x, f(x))|x\in D\}$$

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Functions

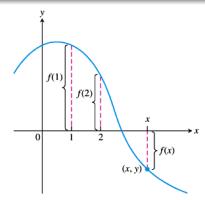
of function

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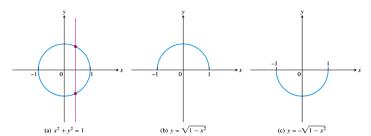
Curves that are graphs of functions

Functions

of function

Revision

The vertical line test

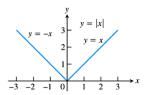


Functions

$$f(x) = |x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$$

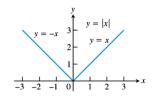
$$y = -x \xrightarrow{3} y = |x|$$

$$y = x$$

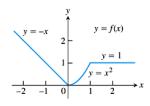


Functions

$$f(x) = |x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$$



$$f(x) = \begin{cases} -x & x < 0 \\ x^2 & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$
 $y = -x$



Revision so far: Functions and their Graphs

Functions

of function

Revisio

- Definition of a function
- Domain and range of a function
- Graph of a function
- Piecewise defined functions

Composition of functions

Definition

If f and g are functions, the composite function $f \circ g$ ("f composed with g'') is defined by

$$(f\circ g)(x)=f(g(x))$$

Introduction

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<u>Composition</u>

of functions

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Definition

If f and g are functions, the composite function $f \circ g$ ("f composed with g") is defined by

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$$x \longrightarrow g \qquad g(x) \longrightarrow f \qquad f(g(x))$$

Compositions of functions

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Composition of functions

Revisio

Definition

If f and g are functions, the composite function $f \circ g$ ("f composed with g") is defined by

$$(f\circ g)(x)=f(g(x))$$

$$x \longrightarrow g \longrightarrow f(g(x))$$

The domain of $f \circ g$ consists of the numbers x in the domain of g for which g(x) lies in the domain of f, i.e.

$$D(f \circ g) = \{x | x \in D(g) \text{ and } g(x) \in D(f)\}$$

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Composition of functions

Revision

$$D(f \circ g) = \{x | x \in D(g) \text{ and } g(x) \in D(f)\}$$

$$f \circ g$$

$$g$$

$$g(x)$$

Composition of functions

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 with $D(f) = [0, \infty)$
 $g(x) = 1 + x$ with $D(g) = (-\infty, \infty)$

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$$\bullet (f \circ g)(x) = f(g(x)) =$$

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Function

Composition of functions

Revisio

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, Domain $[-1, \infty)$

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Revision

Revision

- Functions and their graphs
- Composition of functions

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Introduction

Function

of functions

Revision

Thank you for coming today!