Studying Mathematics

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School of Mathematical Sciences
Queen Mary, University of London

University of London Open Day Presentation 2009

Topic Outline

Outline

Why You Should Study Mathematics

Good Reasons for Studying Mathematics

- You are really good at maths
- You like problem solving
- You could get into business school (or law, or ...)
- You want to keep your career options open

Bad Reasons for Studying Mathematics

- Your language skills are really weak
- You like memorising formulas
- Your marks are to weak to get you into ...
- You haven't yet figured out what you're good at

The Best Reason for Studying Mathematics

- You love doing maths

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- just "doing things with numbers and letters and other symbols"
- just a collection of facts and rote recipes
- just computational and arithmetic skills

Mathematics is

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- the language of science
- a creative discipline
- a source of pleasure and wonder
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Transferable Skills

- Analytical abilities
- Ability to work independently
- Ability to manage your own time
- Highly developed numerical skills
- Effective communication skills
- Apply mathematical modelling to real-world problems
- Practical computational skills

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Career Opportunities

- Academic Research
- Aerospace
- Biotechnology
- Business and Finance
- Chemicals
- Construction
- Defence
- Electronics
- Energy
- Environment
- Health Care
- Management
- Marketing
- Materials
- Pharmaceuticals
- Retail
- Teaching
- Transport

Career Opportunities

- Academic Research
- Aerospace
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  - Accountant: 19-25K
  - Actuary: 23-28K
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  - Market Researcher: 19-24K
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- Teaching
  - Teacher 20-25K
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Career Opportunities

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  - PhD Scholarship: 15K
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Three-Year BSc Degree Courses (Example: QMUL)

Title	Code	Req.
Mathematics	G100	320
Pure Mathematics	G110	320
Mathematics and Statistics	GG31	320
Mathematics, Statistics, and Financial Economics	GL11	320
Mathematics with Finance and Accounting	G1N4	320
Mathematics with Business Management	G1N1	320
Mathematics with Business Management and Finance	GN13	320
Mathematics and Computing	GG14	320
Mathematics and Physics	FG31	320

A=120, B=100

Other Degree Courses (Example: QMUL)

Degree	Years	Title	Code	Req.
MSci	4	Mathematics	G102	340
MSci	4	Mathematics with Statistics	G1G3	340
BSc	*3*	*Computer Science with Mathematics*	*GG41*	*320*
BSc	*3*	*Economics, Mathematics, and Statistics*	*LG11*	*340*
	1	Science & Eng. Foundation Programme	FGH0	180

- All University of London degrees are honours (including pass degrees)
- Course unit system instead of joint or combined honours

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Course Unit System

Advantages:

- Flexibility
- Opportunities to take modules in other departments
- Freedom to shape your programme of study
- Specialisation in penultimate and final year

Typically,

- take 8 modules in first year (no choice)
- choose 8 of 16 modules in second year
- choose 8 of 24 modules in third year

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Seven Million Dollars Prize Money

7 Prize Problems, selected by Clay Mathematics Institute in 2000

- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

These are hard problems (it might be easier to rob a bank...)

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Solved Problems in Mathematics

Some “recently” proved problems:

- Fermat’s last theorem (1637, proved 1994): If an integer n is greater than 2, then the equation

$$a^n + b^n = c^n$$

  has no solutions in non-zero integers a, b, and c.

  For $n = 2$, this is of course possible, for example

$$3^2 + 4^2 = 5^2 \; .$$

- The four colour theorem (1852, proved 1976): Given any plane separated into regions, such as a political map of the states of a country, the regions may be coloured using no more than four colours.

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Some unsolved problems:

- Goldbach's conjecture (1742): every even integer greater than 2 can be written as the sum of two primes.

  For example, $18 = 5 + 13 = 7 + 11$.

- The twin prime conjecture (300 BC): there are infinitely many primes p such that $p + 2$ is also prime.

  For example, 17 and 19 are twin primes.

- How many different Sudoku squares of size $n \times n$ are there? There are

  $$6,670,903,752,021,936,960$$

  valid 9×9 Sudoku squares. The problem is to find a formula for general n.

There are many more well-known open problems, see e.g. http://en.wikipedia.org/wiki/Unsolved_problems_in_mathematics

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> *"The history of mathematics is a history of horrendously difficult problems being solved by young people too ignorant to know that they were impossible."*

Freeman Dyson, "Birds and Frogs", AMS Einstein Lecture 2008

The 3n+1 Problem

Consider the following operation on an arbitrary positive integer:

- If the number is even, divide it by two.
- If the number is odd, triple it and add one.

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Form a sequence by performing this operation repeatedly, beginning with any positive integer.

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The Conjecture is:

This process will eventually reach the number 1, regardless of which positive integer is chosen initially.

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Some Examples

Examples:

- $n = 11$ produces the sequence

$$11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.$$

- $n = 27$ produces the sequence
27,82,41,124,62,31,94,47,142,71,214,107,322,161,484,242,121,
364,182,91,274,137,412,206,103,310,155,466,233,700,350,175,
526,263,790,395,1186,593,1780,890,445,1336,668,334,167,502,
251,754,377,1132,566,283,850,425,1276,638,319,958,479,1438,
719,2158,1079,3238,1619,4858,2429,7288,3644,1822,911,2734,
1367,4102,2051,6154,3077,9232,4616,2308,1154,577,1732,866,
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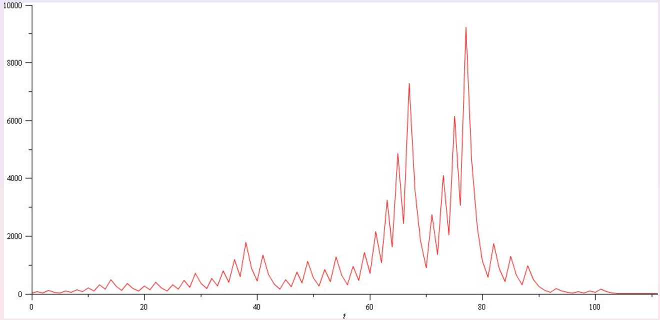
Graphing the Sequences

A graph of the sequence obtained from $n = 27$



This sequence takes 111 steps, climbing to over 9000 before descending to 1.

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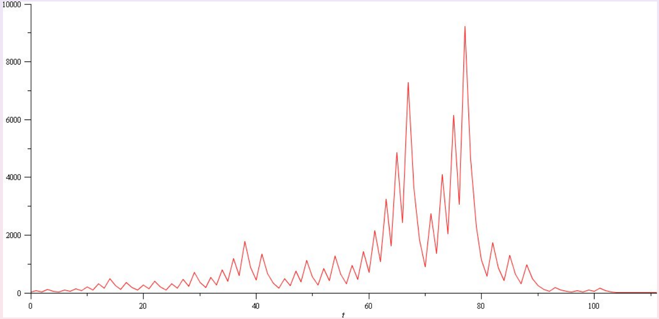

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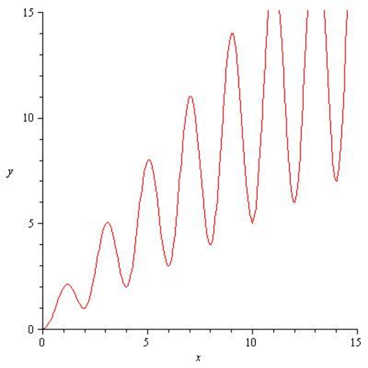
Iterating on Real Numbers



Reduce an orbit by replacing $3n+1$ with $(3n+1)/2$:

$$10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \dots$$

shortens to

$$10, 5, 8, 4, 2, 1, 2, 1, \dots$$

The function graphed is given by

$$f(x) = \frac{x}{2}\cos^2\left(\frac{\pi}{2}x\right) + \frac{3x+1}{2}\sin^2\left(\frac{\pi}{2}x\right)$$

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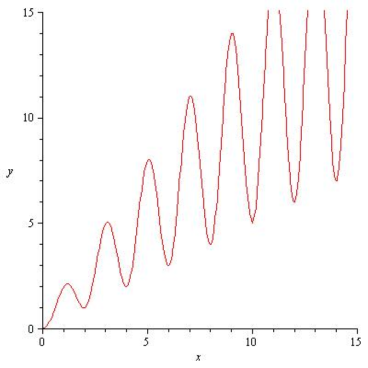

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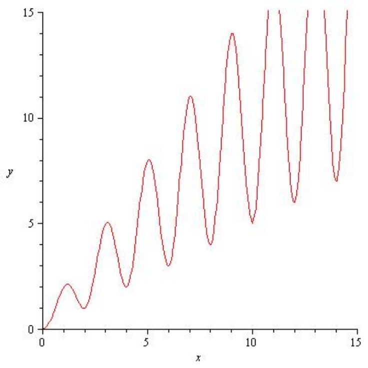

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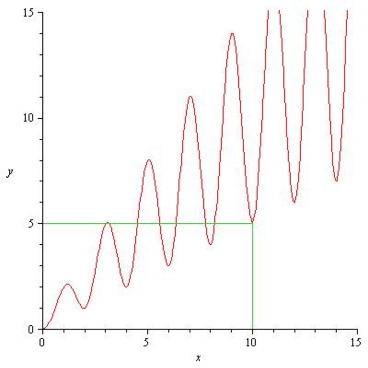

$10 \longrightarrow 5$

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shortens to

$$10, 5, 8, 4, 2, 1, 2, 1, \ldots$$

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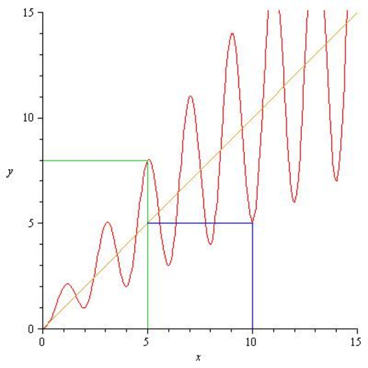

$10 \to 5 \to 8$

Reduce an orbit by replacing $3n+1$ with $(3n+1)/2$:

$$10, 5, {\color{red}16}, 8, 4, 2, 1, {\color{red}4}, 2, 1, \ldots$$

shortens to

$$10, 5, 8, 4, 2, 1, 2, 1, \ldots$$

The function graphed is given by

$$\begin{aligned} f(x) = & \frac{x}{2} \cos^2\left(\frac{\pi}{2}x\right) \\ & + \frac{3x+1}{2} \sin^2\left(\frac{\pi}{2}x\right) \end{aligned}$$

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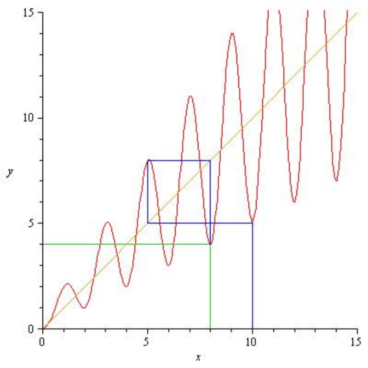

$10 \to 5 \to 8 \to 4$

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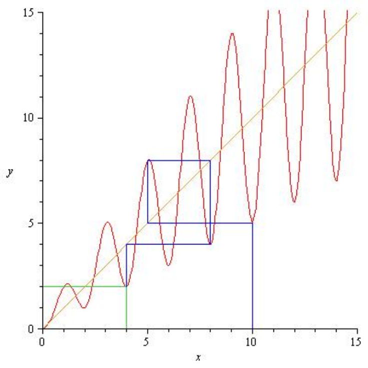

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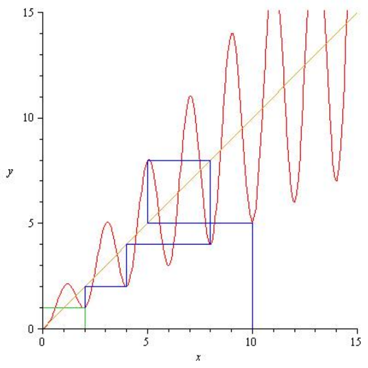

$$10 \to 5 \to 8 \to 4 \to 2 \to 1$$

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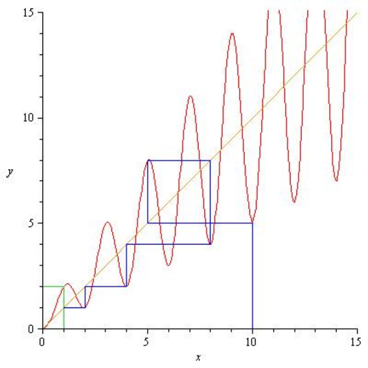

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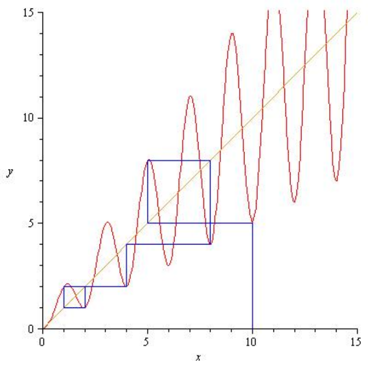

A "cobweb" plot

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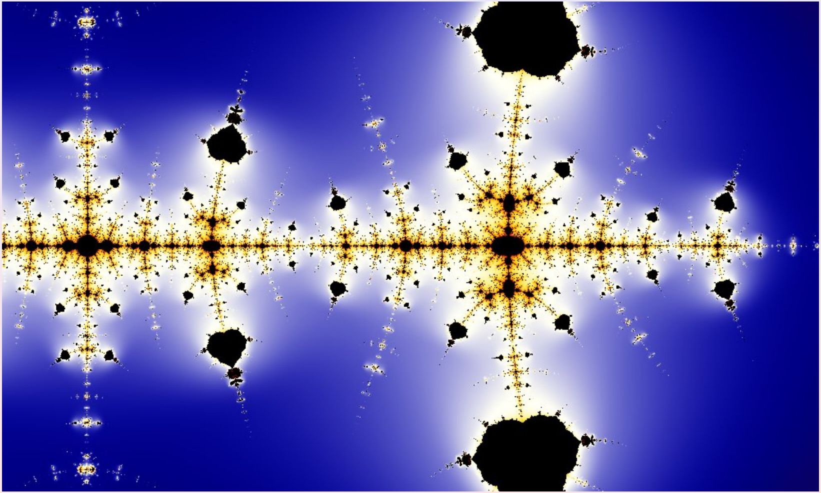
Iterating on Complex Numbers



> *"Mathematics is not yet ready for such problems."*

Paul Erdős, 1913 - 1996

The End