MTH5105 Differential and Integral Analysis 2009-2010

Midterm Test

Problem 1: (a) State the formula for the Taylor polynomial $T_{n,a}$ of degree n of a function f at a, and state the Lagrange form of the remainder term R_n. [*10 marks*]

Let $f(x) = 1/\sqrt{1+x}$.

(b) Determine the Taylor polynomials $T_{2,0}$ and $T_{3,0}$ of degree 2 and 3, respectively, for f at $a = 0$. [*15 marks*]

(c) Using the Lagrange form of the remainder term, or otherwise, show that

$$T_{3,0}(x) < f(x) < T_{2,0}(x) \quad \text{for all } x > 0 \ .$$

[*10 marks*]

Problem 2: (a) Give the definition of $f : \mathbb{R} \to \mathbb{R}$ being differentiable at a point $a \in \mathbb{R}$. [*10 marks*]

(b) Let $f : \mathbb{R} \to \mathbb{R}$ satisfy

(i) $f(x) = f(x-y)f(y)$ for all $x, y \in \mathbb{R}$, and

(ii) $f(x) - 1 = xg(x)$ with $\lim\limits_{x\to 0} g(x) = 1$.

Show that f is differentiable and that $f'(a) = f(a)$ for all $a \in \mathbb{R}$. [*20 marks*]

Problem 3: (a) State Rolle's Theorem. [*15 marks*]

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be twice differentiable with

$$f(0) = f(1) = f(2) = 0 \ .$$

Show that there exists a $c \in (0, 2)$ such that $f''(c) = 0$. [*20 marks*]

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