MTH5105 Differential and Integral Analysis 2008-2009

Exercises 5

Exercise 1: Let $f(x) = \exp(\sqrt{x})$, $g(x) = \sin(\pi x)$, and $P = \{0, 1, 4, 9\}$.

    (a) Find the upper and lower sums $U(f, P)$ and $L(f, P)$ of f for the partition P. Use these sums to give bounds for $\int_0^9 f(x)\, dx$.   [5 marks]

    (b) Find the upper and lower sums $U(g, P)$ and $L(g, P)$ of g for the partition P. Use these sums to give bounds for $\int_0^9 g(x)\, dx$.   [5 marks]

Exercise 2: Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} 0 & x \neq 0 \\ 1 & x = 0 \,. \end{cases}$$

    (a) Given a partition P of $[-1, 1]$, what is $L(f, P)$? What is $\underline{\int_{-1}^1} f(x)\, dx$?   [3 marks]

    (b) For fixed $\epsilon > 0$, find a partition P of $[-1, 1]$ such that $U(f, P) < \epsilon$. What is $\overline{\int_{-1}^1} f(x)\, dx$?   [5 marks]

    (c) Is f integrable on $[-1, 1]$? If so, what is its integral?   [2 marks]

Exercise 3: Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Consider the equidistant partitions P_n of $[0, 1]$ into n subintervals.

    (a) Find $U(f, P_n)$. What can you say about $\overline{\int_0^1} f(x)\, dx$?   [4 marks]

    (b) Find $L(f, P_n)$. What can you say about $\underline{\int_0^1} f(x)\, dx$?   [4 marks]

    (c) Is f integrable on $[0, 1]$? If so, what is its integral?   [2 marks]

    [Hint: $\sum_{j=1}^n j^2 = \frac{1}{6}n(n + 1)(2n + 1)$.]

Please note that I couldn't get the notation I used in class to typeset properly here: $\int_{*a}^b f(x)\, dx$ looks worse than $\underline{\int_a^b} f(x)\, dx$. Therefore, I've used lines above and below the integral sign to denote upper and lower integrals.

The deadline is 12.15 on Monday, 2nd March. Please hand in your coursework at the end of Monday's lecture or to my office MAS113 immediately afterwards.

Thomas Prellberg, February 2009