MAS205 Complex Variables 2005-2006

Exercises 8

Exercise 29: Let f and g be holomorphic on a disk D centred at z_0, and let h be holomorphic on the punctured disk $D' = D \setminus \{z_0\}$. Suppose f and g both have zeros of order $m \geq 1$ at z_0 and h has a pole of order $n \geq 1$ at z_0.

(a) Does fh have a zero or pole at z_0? If so, what is its order?

(b) Does $f + g$ have a zero or pole at z_0? If so, what is its order?

Exercise 30: Let the curve $\mathcal{C}$ be given by the graph of the function $y = f(x)$ with

$$f(x) = \cosh(x) \qquad (-1 \leq x \leq 1)$$

embedded in $\mathbb{C}$ via $z = x + iy$.

(a) Give a path $\gamma : [a, b] \to \mathbb{C}$ which has the curve $\mathcal{C}$ as its image. Draw a sketch of the curve and indicate the parametrisation.

(b) Compute the length $L(\mathcal{C})$. Evaluate the result numerically and discuss it in view of your sketch (i.e. does your result make sense and why).

Exercise 31: Let $\mathcal{C}$ be the unit circle described counterclockwise. Show that

$$\left| \int_{\mathcal{C}} \frac{e^z}{z^3} dz \right| < 2\pi e \ .$$

Exercise 32: Using the definition of the integral of a complex function f along a contour $\gamma : [a, b] \to \mathbb{C}$ as

$$\int_a^b f(\gamma(t))\gamma'(t)dt \ ,$$

compute the integral of $f(z) = (z - 1)^2$ along

(a) the straight line segment from 0 to i,

(b) the straight line segment from i to $1 + i$,

(c) **[10 bonus marks]** an arc from 0 to $1 + i$ on a circle of radius 1 about 1.

Check your answers by finding an antiderivative F for f and evaluating F at the points $z = 0, i, 1 + i$.

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 10:30am Wednesday 7th December

Thomas Prellberg, November 2005