

MAS205 Complex Variables 2005-2006

Exercises 3

Exercise 9: Find the Möbius transformation $f(z) = (az + b)/(cz + d)$ which maps $1 \mapsto 0$, $i \mapsto 1$, and $0 \mapsto i$.

- (a) What is the image of $z = -1$?
- (b) Which point is mapped by f to $2i$?
- (c) What is the image of the unit disk under f ?

Exercise 10: Evaluate the following limits:

$$(a) \quad \lim_{z \rightarrow \infty} \frac{(iz - 1)(z + 3)^2}{(z + i)^2(z - 1)} \quad (b) \quad \lim_{z \rightarrow 2-2i} \frac{z^5 + 1}{z^2 + 8i} \quad (c) \quad \lim_{z \rightarrow i} \frac{z^2}{z^3 - 1}$$

Exercise 11: (a) Give an example of a function $f : \mathbb{C} \rightarrow \mathbb{C}$ such that

$$\lim_{z \rightarrow i} f(z) = \infty \quad \text{and} \quad \lim_{z \rightarrow 1} f(z) = 0.$$

(b) Suppose

$$f(z) = \frac{p(z)}{z^2 - 4}, \quad \text{where } p(z) = az + b \text{ for some } a, b \in \mathbb{C}.$$

If $\lim_{z \rightarrow -2} f(z) = 1$, what is $p(z)$?

(c) Suppose

$$f(z) = \frac{p(z)}{z^2 + 1}, \quad \text{where } p(z) \text{ is a quadratic polynomial.}$$

If $\lim_{z \rightarrow -i} f(z) = 1$ and $\lim_{z \rightarrow \infty} f(z) = 1$, what is $p(z)$?

Exercise 12: For each of the following functions, decide at which values of z the function is continuous and at which values it is not continuous. Give reasons, but detailed proofs are not expected.

- (a) $f(z) = z + i\Re(z)$
- (b) $f(z) = (z/\bar{z})^3$ for all non-zero z , and $f(0) = 1$.

Please hand in your solutions (to the yellow Complex Variables box on the ground floor) by 10:30am Wednesday 26th October

Thomas Prellberg, October 2005