MAS205 Complex Variables 2004-2005

Midterm Test, 8th November 2004, 11.05-11.55am

You should attempt ALL questions. Make sure your name and student number is on EVERY sheet handed in. This is a "closed book" test. Calculators are not allowed.

Question 1: [15 marks]

(a) Find all solutions $z \in \mathbb{C}$ of the equation

$$z^3 = -8i \ .$$

(b) Find all solutions $z \in \mathbb{C}$ of the equation

$$e^{-z} = 1 \ .$$

Express all solutions in standard and polar form, and draw diagrams showing their location in the complex plane.

Question 2: [15 marks]
Consider the transformation

$$z \mapsto w = iz^2 \ .$$

(a) Find the equation of the image of the line $\Im(z) = 1$ and sketch the image.

(b) What is the image of the left half plane $\{z \in \mathbb{C} : \Re(z) < 0\}$?

Question 3: [15 marks]
Find the Möbius transformation $f(z) = (az + b)/(cz + d)$ which maps $0 \mapsto i$, $1 \mapsto 0$, and $-1 \mapsto \infty$.

Question 4: [15 marks]
Evaluate

$$\text{(a)} \quad \lim_{z \to 2i} \frac{z^2 - 5iz - 6}{z^2 + 4} \qquad \text{(b)} \quad \lim_{z \to \infty} \frac{(1 - 2z)(1 + 2z)}{1 + iz^2}$$

Question 5: [10 marks]
Show that $\lim_{z \to 0} (\overline{z} - z)^2/z$ exists.

Question 6: [15 marks]
At what values of $z = x + iy$ is the function $f(x + iy) = x^2 + y^2 - 2xyi$ differentiable?

Question 7: [15 marks]
Let $f(z) = (1-z)/(1+z)$. Determine the Taylor series $\sum_{n=0}^{\infty} a_n z^n$ for f around the point $z_0 = 0$. What is the radius of convergence of this Taylor series?

Thomas Prellberg, October 2004