MAS205 Complex Variables 2005-2006

Sample Test

A a Show that $\Re(iz) = -\Im(z)$ and $\Im(iz) = \Re(z)$ for any $z \in \mathbb{C}$. [*10 marks*]

b Describe the set of points $z \in \mathbb{C}$ satisfying:
(i) $|z - 2i| = 1$, (ii) $\Im(z - 2i) = 1$, and (iii) $z - 2i = 1$. [*15 marks*]

B a Find all complex solutions of $e^{iz} = -2i$. [*10 marks*]

b Show that under the map $w = iz^2$, lines parallel to the real axis are mapped to parabolas. [*15 marks*]

C a Find the Möbius transformation $f(z) = (az + b)/(cz + d)$ which maps $1 \to 0$, $0 \to 1$, and $i \to -1$. [*15 marks*]

b What is the image of the unit circle (circle about the origin with radius 1) under the transformation f? [*10 marks*]

D a Prove or disprove that $f(z) = \Im(z)/z$ is continuous at $z = 0$. [*10 marks*]

b Prove or disprove that $f(z) = \Re(z)$ is differentiable at $z = 0$. [*15 marks*]

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This test will be the basis of a review exercise class on November 7.

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