

MAS330 Mathematical Problem Solving 2007-2008

Level 3: Semester 6

<http://www.maths.qmul.ac.uk/~tp/MAS330>

Lecturers: Prof S. Majid and Dr T. Prellberg (Mathematics)

Class times: Tuesday 2-3 (Eng 317), Friday 1-2 (Eng 317)

Office hours: Tue 12:30-1:30 (SM), Wed 12-1 (SM), Thu 11:30-12:30 (TP), Fri 2:30-3:30 (TP)

1 Welcome

Welcome to the problem solving course.

The first objective of the course is to give you the possibility of working on a challenging problem for a week, rather than spending an hour or so on the problems set as course work in other courses.

If you work steadily at the course you should expect to obtain marks roughly in line with the marks you obtain in other courses. Some students become very keen on the course and get much higher marks on the problem solving than in any other course, and rather more do not have the self discipline to keep working at the problems, and drop out; so the course in practice is primarily a test of your commitment.

The problems do not in general assume any mathematical knowledge that is not covered in basic A level or first year undergraduate mathematics. A secondary objective of the course is to enable you to fill gaps in your elementary mathematical techniques; you may find yourself consulting text books.

For some people this is the most exciting course on the books; so enjoy it, and remember the sage advice of Viscount Mumbles in *Essays on the Imagination*:

‘Often would the deaf man know the answers had he but the faculty of hearing the questions. Likewise would the unimaginative man guess wisely at the answers had he but the wit of posing to himself the appropriate questions.’

2 General Guidelines

You will not be allowed any choice in the problems that you are asked to solve, except that we will not offer you one of the few problems requiring specialist knowledge unless you have covered the relevant material in a course that you have passed.

You may get help from any source that you acknowledge. There will be an oral examination, and we will give no credit for a solution that we feel you do not fully understand if no help has been acknowledged. You do not need to acknowledge help received during the classes as we will take note of the help we have given you. Failure to acknowledge other help is an examination offence. If you get help from a member of staff it is your responsibility to see that they send me (tp) an e-mail outlining the help they have given you. You should not expect to receive help from members of staff not teaching the course; but they may help you if they wish.

Anything you quote from another source; a text book, the web or lecture notes; must be made explicit with the source mentioned.

If we suspect that you have had undeclared help in solving a problem we reserve the right to determine in the oral examination whether or not the solution is your own work, and to give you no credit when we are convinced that you do not understand the solution well enough to have done the work yourself. **IN TAKING THE COURSE YOU ACCEPT THIS CONDITION.**

We will give you a mark for each question depending on the difficulty of the problem, the quality of the solution, the help received, and the understanding displayed in the oral. Do not ask us how many marks you will get for a solution. This cannot in any way be determined before the oral, and the preliminary mark then given is confidential, and may be changed, as the external examiner has a major say in the final assessment of the course.

Again you will have to accept the examiners' judgement, and **IN TAKING THE COURSE YOU AGREE TO ACCEPT THIS JUDGEMENT.**

The general standard of presentation should be much higher than is normal for course work. We do not require your solutions to be word processed, though it would help. Try to make your use of English up to book quality. We will understand if English is not your first language; but you should still be able to observe the rules of basic English.

When you hand in a solution to a problem we may suggest that you continue to work on the problem. We will normally do this if we find a significant error that we think you could correct. If we accept a solution without suggesting that you continue to work on it it may simply mean that we do not believe that it is in your interest to continue to struggle with this particular problem.

WE WILL NOT TELL YOU HOW WELL YOU ARE DOING. You will be able to tell for yourself whether you are making a reasonable job of the course.

You are expected to hand in at least one solution per week. **IF YOU GET THREE WEEKS BEHIND YOU MAY BE DEREGISTERED.**

When you complete a question you will be given another. You may return a question after a week or more and have it replaced by another. You may be marked down slightly if you do this too often.

The oral examination will be held around the beginning of the examination period.

3 Grading of the Problems

Problems are graded as no star, one star and two star. We will not give you a one star problem until you have solved at least five no star problems reasonably successfully, and we will not give you a two star problem until you have solved at least three one star problems reasonably successfully. It is not sensible to attempt the harder questions unless you are doing well at the course. Generally speaking we will expect candidates hoping for an A to attempt two two-star questions. The easy way to obtain a B is to do five no star questions and five one star questions. The easy way to get a C is to do ten no star questions well.

These guide lines are very rough. You can upgrade many of the questions by producing intelligent generalisations. If we decline to give you a one or two star question because we do not believe that you have been successful enough at the easier questions **YOU MUST ACCEPT THIS JUDGEMENT.** We believe that we can judge these things better than you can. If you are capable of solving harder problems you must prove this by solving easier problems well. You can upgrade the questions you have been given by generalising them. Whatever your level of mathematical skill you will score more marks per minute by solving easier problems. If you are a first class candidate you should be able to do the easier questions at a rate of two per week, thus giving yourself more time for the harder questions. It is a good strategy to solve ten questions reasonably quickly, and then try to upgrade your score by solving harder questions or producing better solutions to easier questions. Your final score will depend on your best ten solutions. We will give you some indication of whether your solutions are correct.

4 How to Approach Problem Solving

We cannot train computers to solve general mathematical problems because we do not know how we do it. However there are some basic hints.

0. You need to wrestle with the problem. The worst thing you can do, when given a problem, is to look at it for three minutes and then come rushing up and saying ‘I don’t understand the question; what exactly am I supposed to do?’.

I will put the other (less important) hints in the context of the following problem. Prove that every even integer greater than 2 is the sum of two primes. This is the famous Goldbach conjecture, and remains unproved. Of course we won’t ask you to solve famous unsolved problems, but supposing that we don’t know how impossibly hard this problem is, how might you be expected to attack it?.

1. Consider simple examples. So express each even number 4,6,8 up to say 50 as the sum of two primes. Observe that 2 is not the sum of two primes. Look at the expressions you obtain. Make the trivial observation that, except for the case of 4 we must express $2n$ as the sum of two odd primes.

2. Try induction. So to prove that $2n$ is a sum of two primes you can assume that $2k$ is the sum of two primes for all $k < n$. This does not seem to help.

3. Try changing the problem. In how many ways can a given even integer be expressed as the sum of two primes? Does the number of ways in which $2n$ can be expressed as the sum of two primes tend to infinity with n ? Here some slight competence at programming would help. We do not assume that you are a programmer, but it would be very easy to get experimental evidence for this problem using Maple. Here is an example of your brushing up (or acquiring) basic first year skills.

4. Think about the problem. You are trying to prove that $2n$ is the sum of two primes. Where are these primes coming from? The inductive assumption tells you that every even number less than $2n$ is a sum of two primes. Are the primes arising in this way enough to solve the problem? Clearly not, as that would imply that only finitely many primes would be need to express all even numbers as the sum of two primes. So where do primes come from? Any proof that there are infinitely many primes must produce a source of prime numbers; so look at proofs of this fact.

5. Still stuck, so try changing the problem again, and try to to prove that every even number is the sum of at most four primes. (Why is three primes silly?) Or try to prove that every odd number greater than 5 is the sum of three primes. Still stuck.

6. Assume that the result is false and try to get a contradiction. You should have tried this earlier, but alas it does not help.

7. Write down all these ideas, and the result of all these computations in a clear and neat way, hand them in at the next exercise class and say ‘I have tried all these ideas, and I’m stuck’.

You will get credit for having had a good bash at the problem, and (if the problem is not the Goldbach conjecture) we can then give you a useful hint. If you do not go through some version of the above process, and simply say ‘could you get me started on question 37?’ (in other words ‘I think the question would come out better if you did it’) we may simply say ‘Try induction’ and observe that you had not thought of this yourself. If you indicate clearly that you have thought of using induction (or the binomial theorem, or differentiation) you will at least get credit for that, and we will give you a hint that moves you on from where you have got to. WHEN YOU ASK FOR HELP PLEASE HAND IN AN ACCOUNT OF WHAT IDEAS YOU HAVE TRIED. PLEASE KEEP A COPY OF EVERYTHING YOU HAVE HANDED IN.