4) $T(n) = \begin{cases} 2T(n-1) & \text{if } n>0 \\ 1 & \text{otherwise} \end{cases}$

Here where $n=0$

$T(n) = 1$

recurrence relation Analysis

for $n>0$:

$T(n) = 2T(n-1)$

$T(n) = 2T(n-1)$

$T(n) = 2T(n-2)$

$T(n-2) = 2T(n-3)$

$T(1) = 2T(0)$

for this pattern

$T(n) = 2 \cdot 2 \cdot 2 \cdots 2^{T}(0) = 2^n T(0)$

since $T(0) = 1$, we have $T(n) = 2^n$

The recurrence relation is $T(n) = 2T(n-1)$ for $n>0$ and $T(0)=$
is $T(n) = 2^n$

5) Big O notation. show that $f(n) = n^2 + 3n + 5$ is $O(n^2)$

$f(n) = O(g(n))$ means $c>0$ and $n_0 \geq 0$

$f(n) \leq g(n)$ for all $n \geq n_0$

Given $f(n) = n^2 + 3n + 5$

$c>0$, $n_0 \geq 0$ such that $f(n) \leq cn^2$

$f(n) = n^2 + 3n + 5$

Lets choose $c = 2$, $f(n) \leq 2n^2$

$f(n) = n^2 + 3n + 3 \leq n^2 + 3n^2 + 5n^2 = 9n^2$

so $c = 9$, $n \geq 1$, $f(n) \leq 9n^2$ for all n^2

$f(n) = n^2 + 3n + 5$ is $O(n^2)$