Assignment - 11

Big omega notation Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

$g(n) \geq cn^3$

$g(n) = n^3 + 2n^2 + 4n$

for finding constants c and n_0

$n^3 + 2n^2 + 4n \geq cn^3$

Divide both sides with n^3

$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

Here $\frac{2}{n}$ and $\frac{4}{n^2}$ approches 0

$1 + 2/n + 4/n^2$

example $c = 1/2$

$1 + 2/n + 4/n^2 = \frac{1}{2} \Rightarrow 1 + 2/n + 4/n^2 \geq 1 = 1 + \frac{2}{n} + 4/n^2 \geq \frac{1}{2}$

Thus $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

By theta notation: determine another $h(n) = 4n^2 + 3n$ is $\theta(n^2)$ or not.

$c = n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper Bound $(O(n^2))$:

$h(n) = 4n^2 + 3n \Rightarrow h(n) \leq 2n^2$

$4n^2 + 3n \leq c_2 n^2$

$4n^2 + 3n \leq 5n^2$

lets $c_2 = 5$

divide both sides by n^2

$4 + \frac{3}{n} \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$

lower bound:

$h(n) = 4n^2 + 3n$

$h(n) \geq cn^2$

$4n^2 + 3n \geq c_1 n^2$

lets $c_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

divide both sides by n^2

$4 + \frac{3}{n} \geq 4$, $h(n) = 4n^2 + 3n$ $(c_1 = 4, n_0 = 1)$ is $\Omega(n^2)$