



<http://algs4.cs.princeton.edu>

## 4.2 DIRECTED GRAPHS

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- ▶ *introduction*
- ▶ *digraph API*
- ▶ *digraph search*
- ▶ *topological sort*
- ▶ *strong components*



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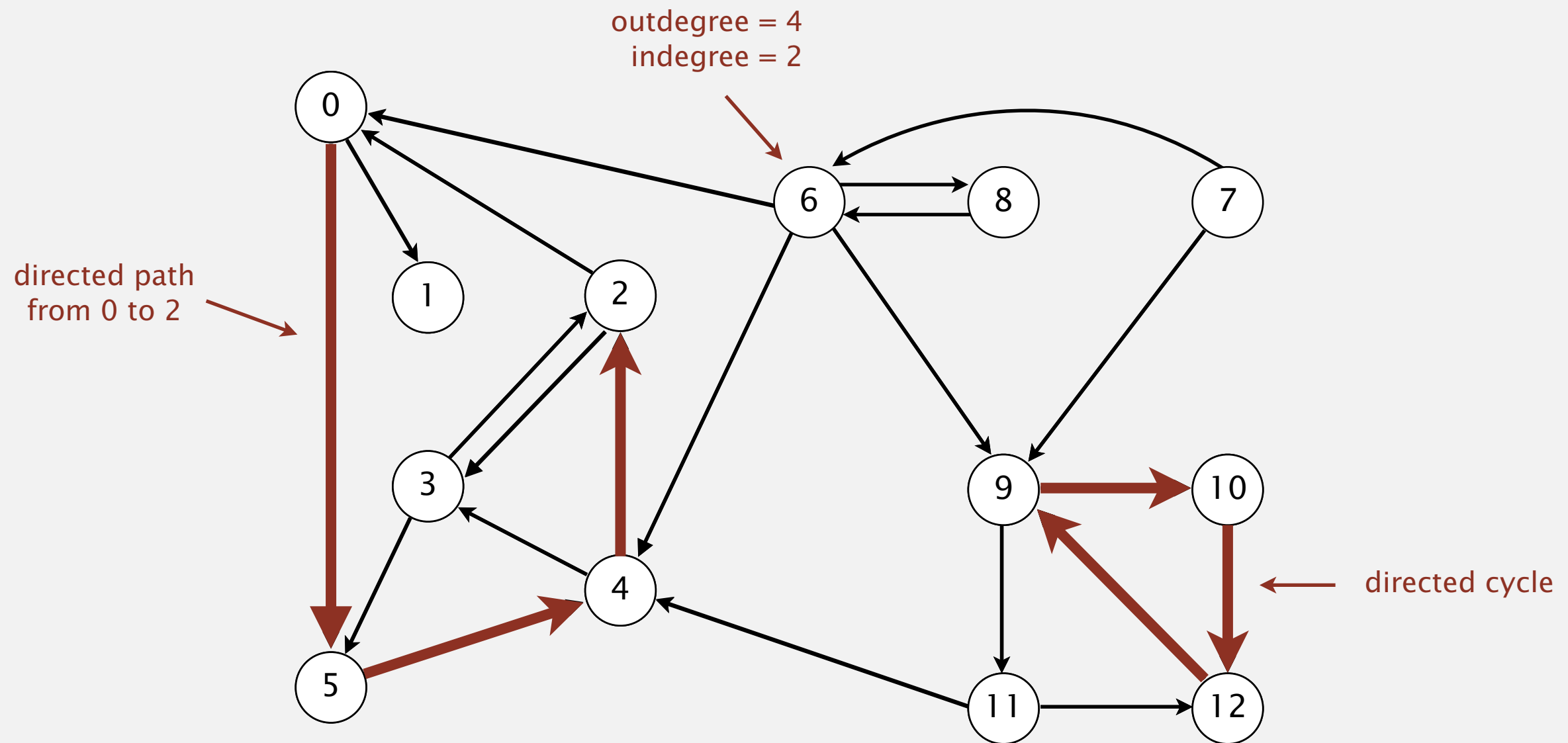
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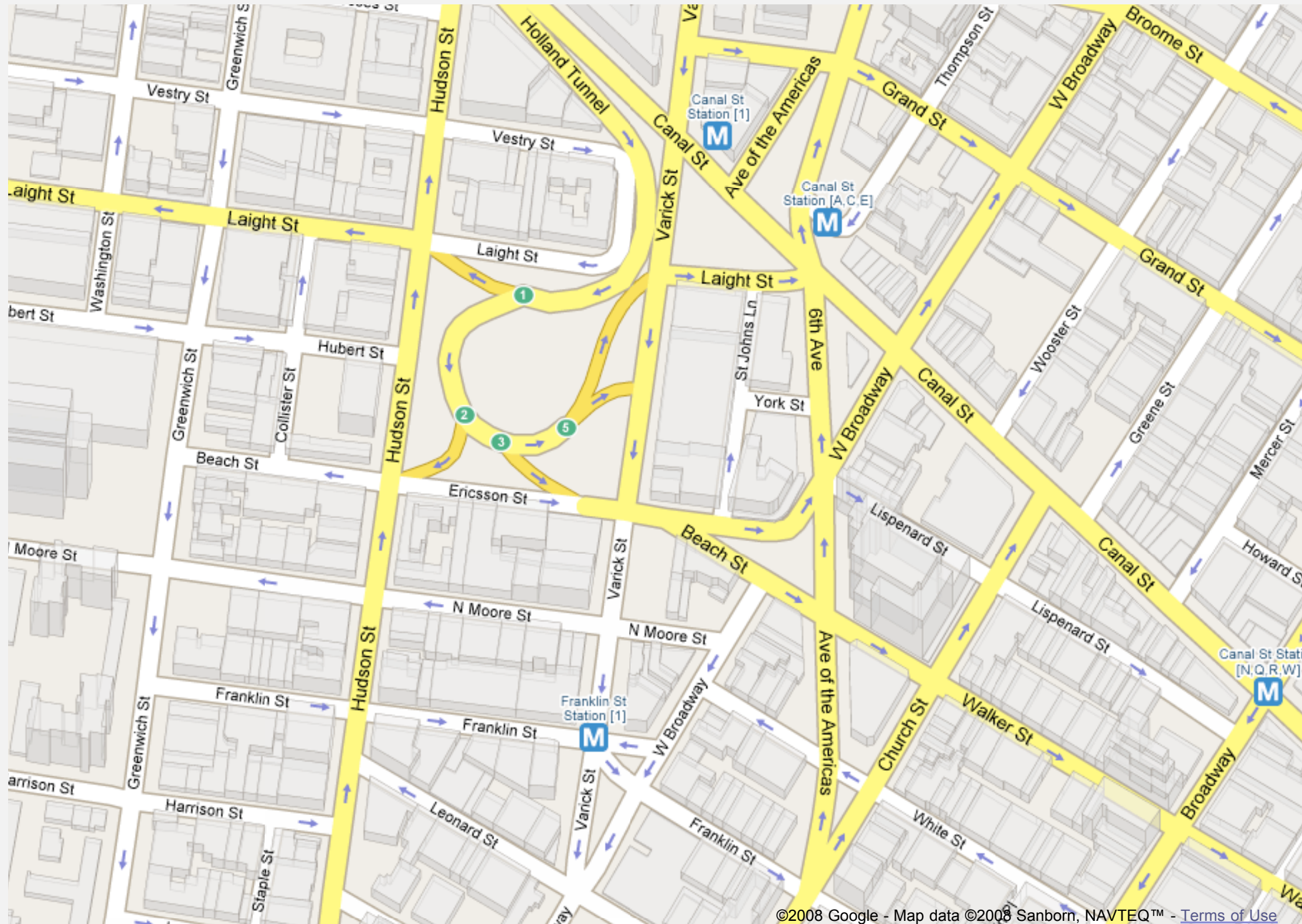
# Directed graphs

**Digraph.** Set of vertices connected pairwise by **directed** edges.



# Road network

Vertex = intersection; edge = one-way street.

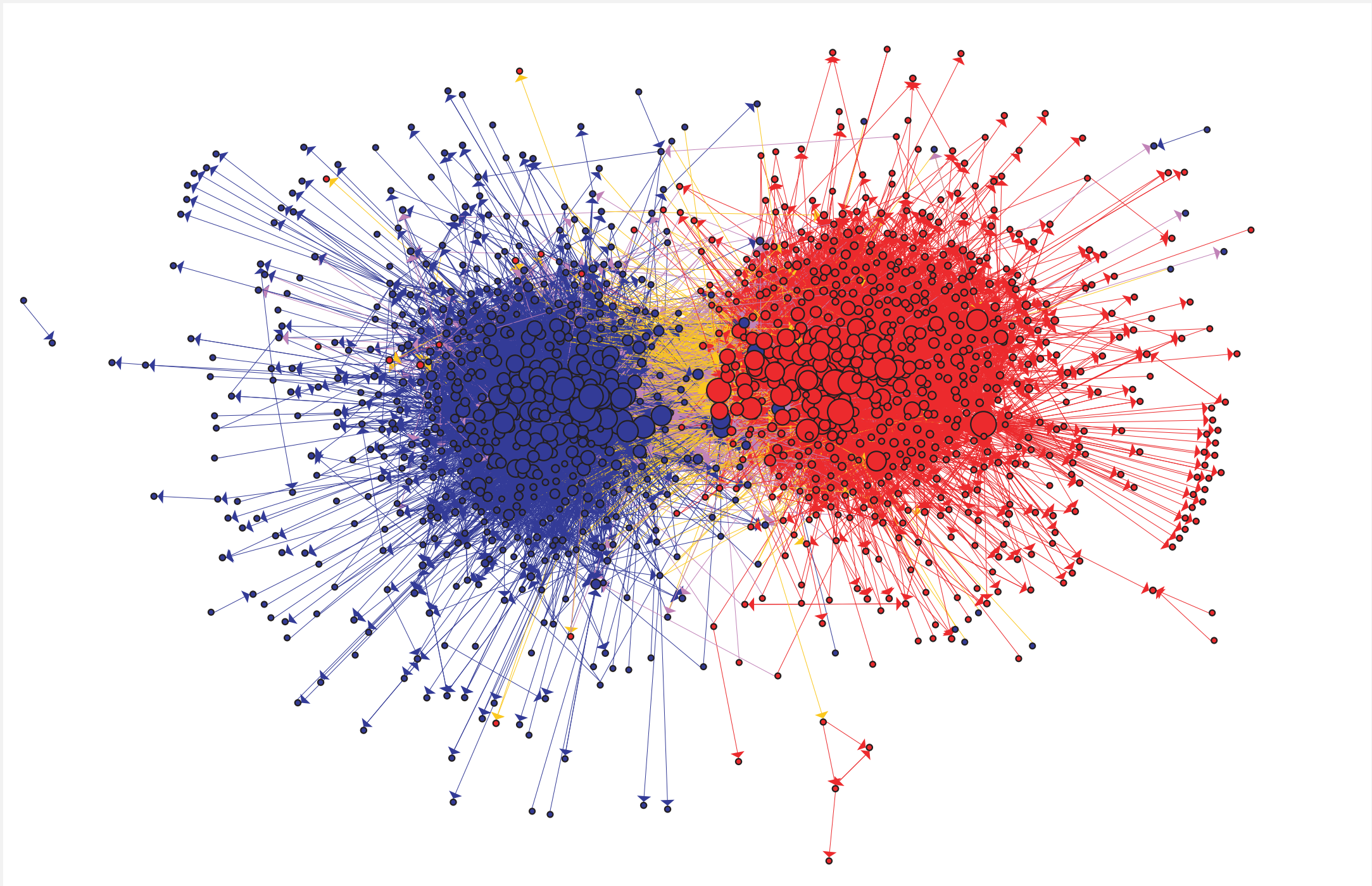




# Political blogosphere graph

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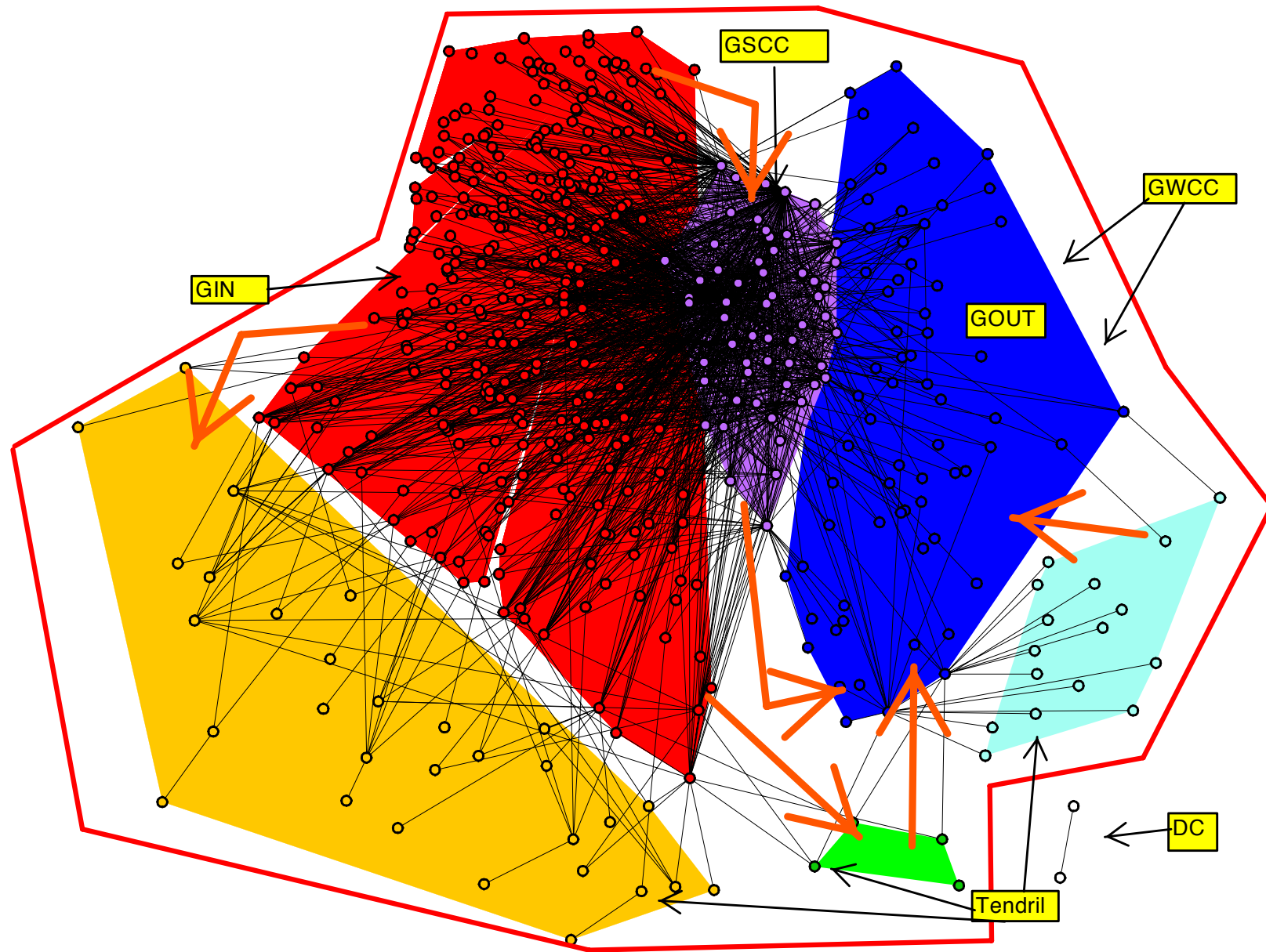
Vertex = political blog; edge = link.



**The Political Blogosphere and the 2004 U.S. Election: Divided They Blog, Adamic and Glance, 2005**

# Overnight interbank loan graph

Vertex = bank; edge = overnight loan.

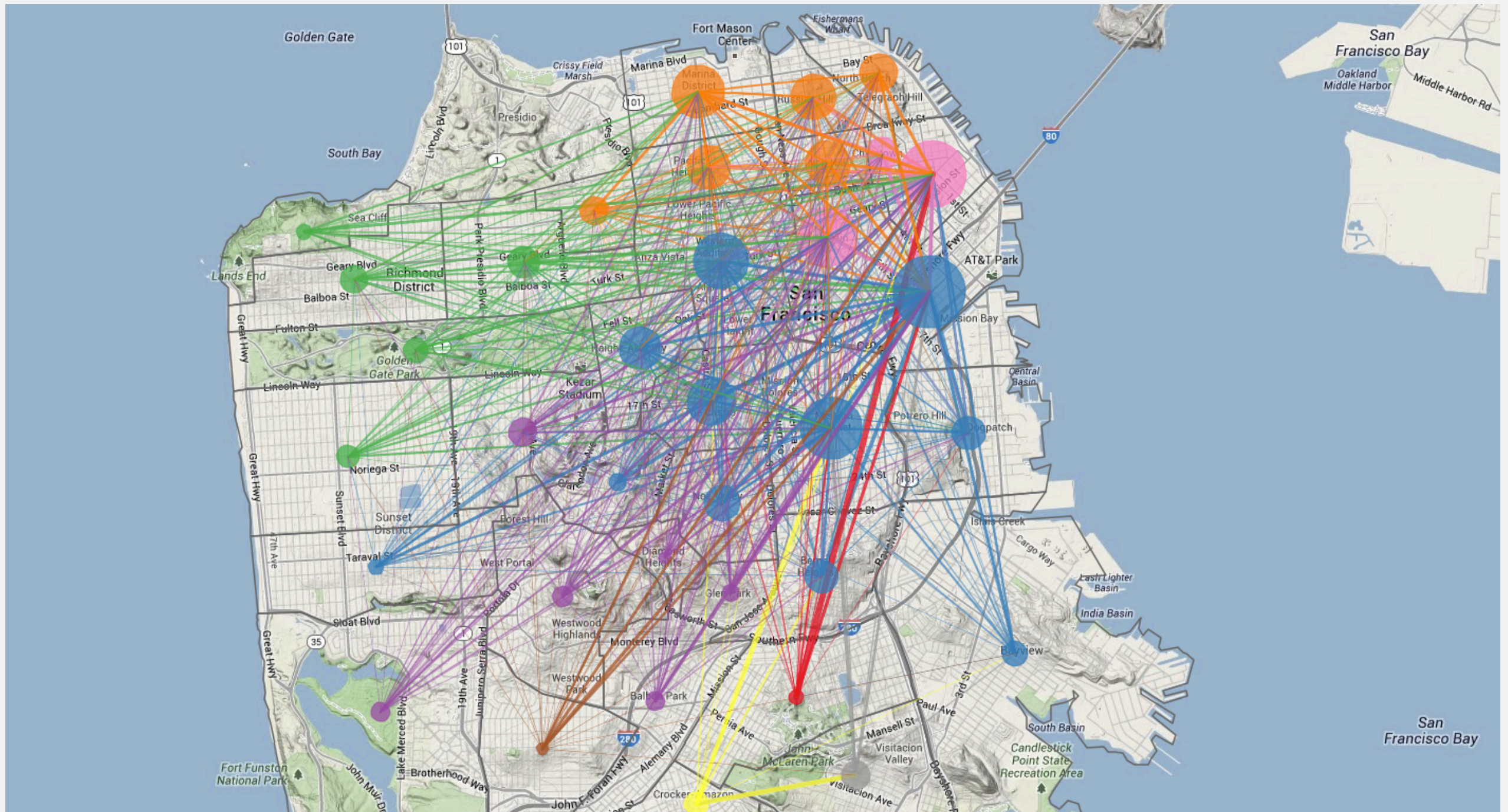


The Topology of the Federal Funds Market, Bech and Atalay, 2008



# Uber taxi graph

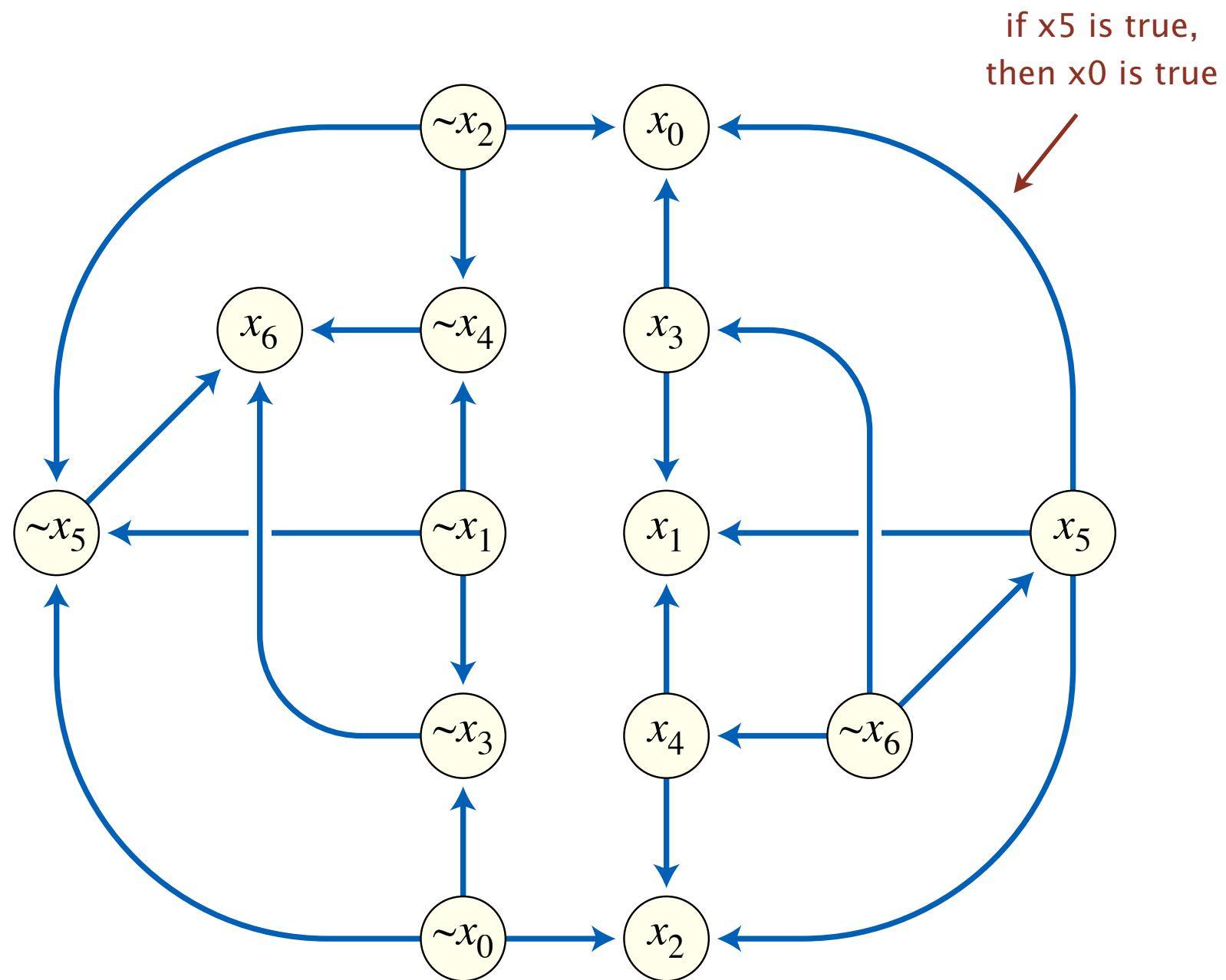
Vertex = taxi pickup; edge = taxi ride.



<http://blog.uber.com/2012/01/09/uberdata-san-franciscocomics/>

# Implication graph

Vertex = variable; edge = logical implication.

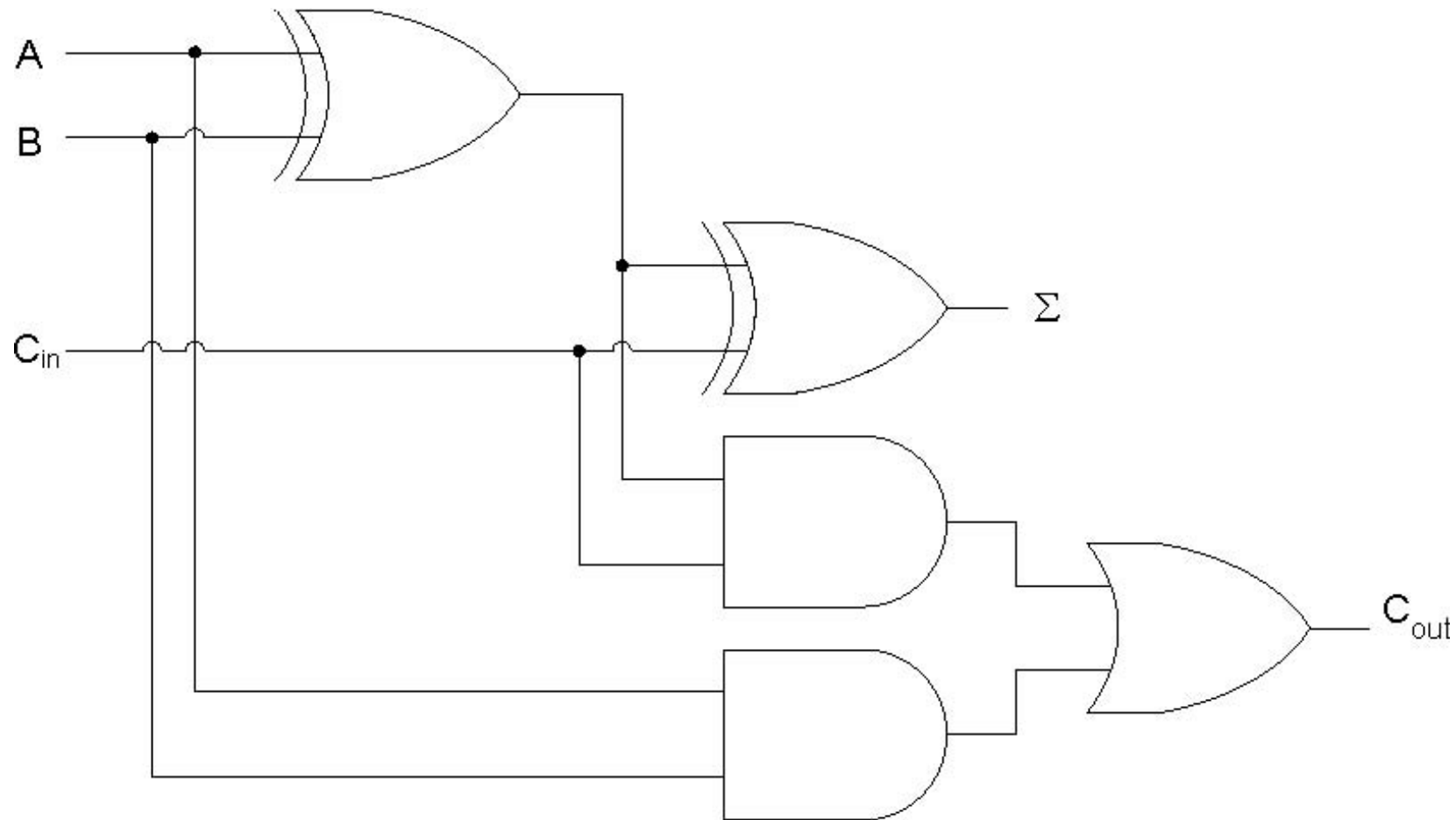




# Combinational circuit

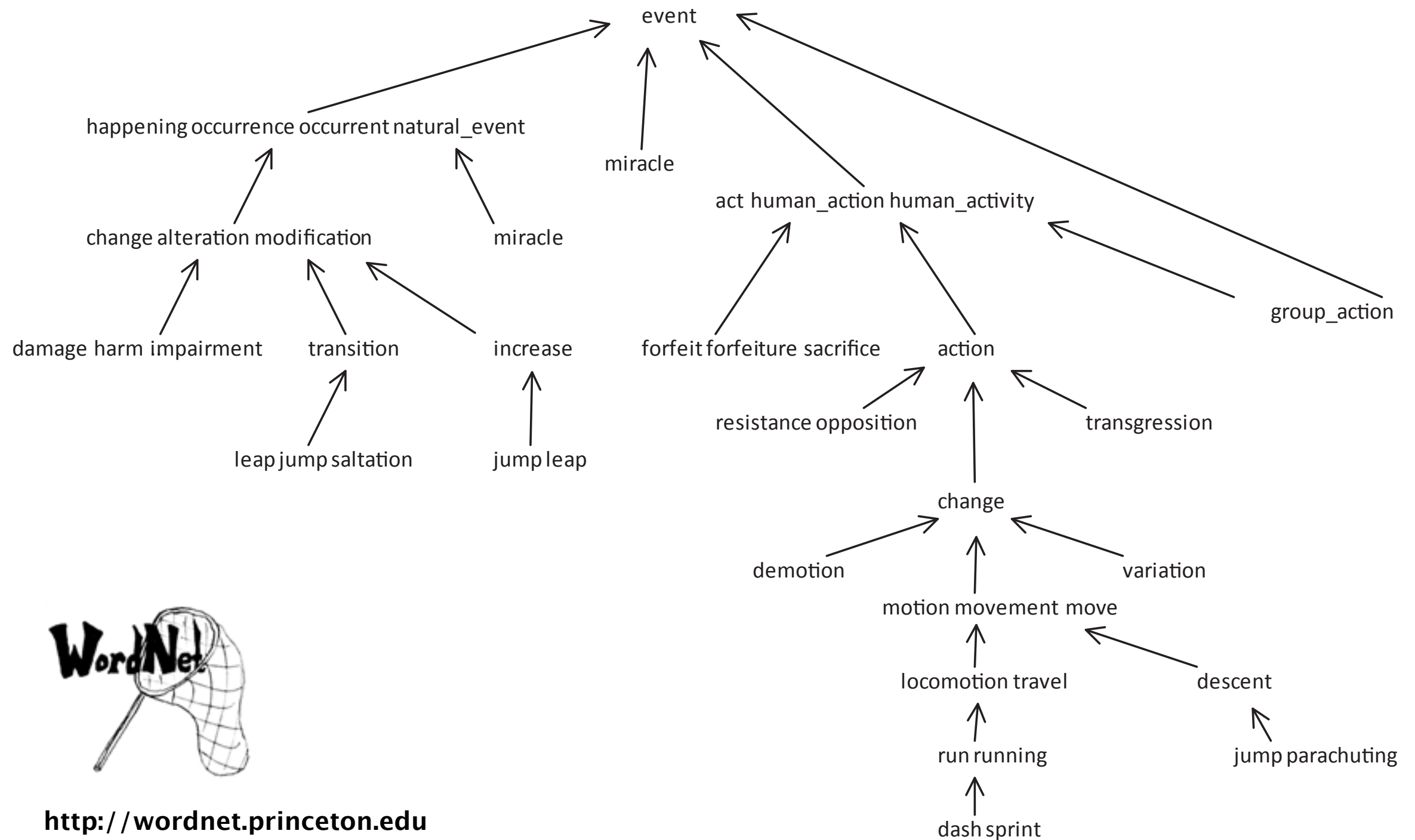
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Vertex = logical gate; edge = wire.



# WordNet graph

Vertex = synset; edge = hypernym relationship.



<http://wordnet.princeton.edu>

# Digraph applications

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digraph	vertex	directed edge
<b>transportation</b>	street intersection	one-way street
<b>web</b>	web page	hyperlink
<b>food web</b>	species	predator-prey relationship
<b>WordNet</b>	synset	hypernym
<b>scheduling</b>	task	precedence constraint
<b>financial</b>	bank	transaction
<b>cell phone</b>	person	placed call
<b>infectious disease</b>	person	infection
<b>game</b>	board position	legal move
<b>citation</b>	journal article	citation
<b>object graph</b>	object	pointer
<b>inheritance hierarchy</b>	class	inherits from
<b>control flow</b>	code block	jump



# Some digraph problems

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problem	description
<b><math>s \rightarrow t</math> path</b>	<i>Is there a path from <math>s</math> to <math>t</math> ?</i>
<b>shortest <math>s \rightarrow t</math> path</b>	<i>What is the shortest path from <math>s</math> to <math>t</math> ?</i>
<b>directed cycle</b>	<i>Is there a directed cycle in the graph ?</i>
<b>topological sort</b>	<i>Can the digraph be drawn so that all edges point upwards?</i>
<b>strong connectivity</b>	<i>Is there a directed path between all pairs of vertices ?</i>
<b>transitive closure</b>	<i>For which vertices <math>v</math> and <math>w</math> is there a directed path from <math>v</math> to <math>w</math> ?</i>
<b>PageRank</b>	<i>What is the importance of a web page ?</i>