LECTURE - 36

$\frac{\sigma}{\sqrt{n}}$ is also called as standard error.

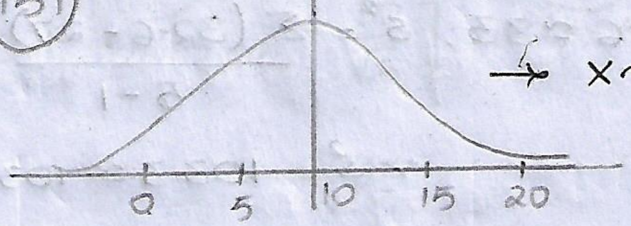
→ $\mu \in \left[\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}\right]$ with __ % confidence.

if z^* is 1.96 (actual value) then it is 95% confidence

commonly used confidence intervals of z^* are :

	CONFIDENCE INTERVAL	z^*
1.	68%	±1
2.	90%	±1.65
3.	95%	±1.96
4.	99%	± 2.58
5.	99.7%	± 3

z^* or z score states how many std away a point is from the mean.

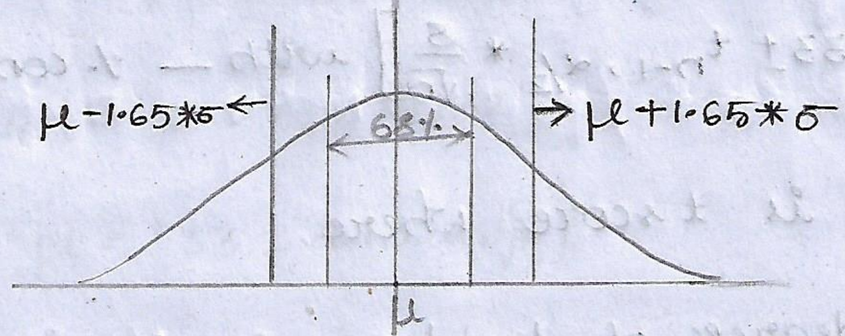


$\rightarrow X \sim N(10, 5)$

$$z^* = \frac{\text{datapoint} - \mu}{\sigma}$$

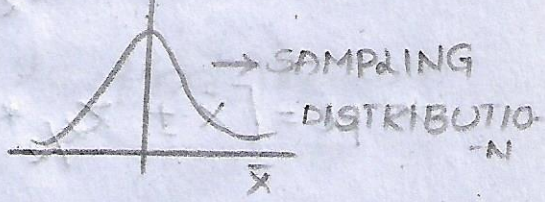
$$\Rightarrow z^* = \frac{15-10}{5} = \frac{5}{5} = 1$$

$$\Rightarrow z^* = \frac{5-10}{5} = \frac{-5}{5} = -1$$



$\rightarrow \mu \pm 1.65\sigma = 90\%$

1. Distribution of all samples means it follows Normal distribution.



2. Mean of sampling distribution $\mu_{\bar{x}} \simeq \mu$

3. Std of sampling distribution $= \frac{\sigma}{\sqrt{n}}$

EXAMPLE: Taking salary details of 5 persons

3.25 → 27K
3.4 → 28K
3 → 25K
4 → 33K
6 → 50K

$n=5, \quad \bar{x} = 32.6 \simeq 33, \quad s^2 = \frac{\Sigma(32.6-27)^2}{5-1}$

$\Rightarrow s^2 = 103.3 \simeq 103$

$\Rightarrow s = 10.15 \simeq 10$

$\mu = \left[33 \pm z^* \frac{\sigma}{\sqrt{n}}\right]$ with — % confidence

So, $\mu = \left[33 \pm t_{n-1, \alpha/2} * \frac{s}{\sqrt{n}}\right]$ with — % confidence

$t_{n-1, \alpha/2}$ is t score where

$n-1 \rightarrow$ degrees of freedom = sample size − 1

$\alpha/2 \rightarrow$ critical value.

z score $<$ z^*, $z_{\alpha/2}$

$\Rightarrow \mu = \left[\bar{x} \pm z_{\alpha/2} * \frac{\sigma}{\sqrt{n}}\right] \rightarrow 0.95$ confidence.

The error made by the above one is:

$0.95 = 1-\alpha = 1-0.95 = 0.05 \rightarrow$ critical value.

5% error $\Rightarrow \alpha = 0.05$

CONFIDENCE LEVEL	z^*	CRITICAL VALUE
68%	± 1	$1-\alpha = 0.68$ $\alpha = 0.32$
99%	± 2.58	$1-\alpha = 0.99$ $\alpha = 0.01$

$\mu = [33 \pm t_{4,0.05} * \frac{10}{\sqrt{5}}]$ with 90% confidence.

$\Rightarrow 1-\alpha = 0.90$

$\Rightarrow \alpha = 0.10$

The 't' values are taken from the "students t distributions table"

$\Rightarrow t_4$ for $0.05 := 2.132$

$\Rightarrow \mu = [33 \pm 2.132 * \frac{10}{\sqrt{5}}]$

$\Rightarrow \mu = [\underset{(-)}{23.46}, \underset{(+)}{42.53}]$ with 90% confidence.

$(-) \rightarrow$ lower bound, $(+) \rightarrow$ upper bound.

If 'σ' is given, it is easy to find μ, then we use $\mu = [\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}]$

$\Rightarrow \mu = [33 \pm z^* \frac{2}{\sqrt{25}}]$ with 90% confidence.

z^* value for 90% confidence is 1.65

$\Rightarrow \mu = [33 \pm (1.65 \times \frac{2}{\sqrt{25}})]$.

$\Rightarrow \mu = [\underset{(-)}{32.34}, \underset{(+)}{33.66}]$

$\therefore$ There is a lot of difference when σ is given and σ is not given.

NOTE:

↳ As degree of freedom increases (upto 30) it tends to be/fall a Normal Distribution