

RANDOM VARIABLE (R.V):

A random variable is a numerical description of the outcome of a statistical experiment.

→ The probability distribution for a random variable describes how the probabilities are distributed over the values of the random variable.

→ In other or simple way,

Random variable is a variable that can take any outcome of a random experiment.

Ex:

Example: $X: SS \rightarrow \{0, 1, 2\}$

R.E \rightarrow TOSSING TWO COINS

S.S $\rightarrow [\{H, H\}, \{H, T\}, \{T, H\}, \{T, T\}]$

R.V \rightarrow 'X' - COUNTING NUMBER OF HEADS

$$P(X=2) \rightarrow \{H, H\} = \frac{1}{4}$$

$$P(X=1) \rightarrow [\{H, T\}, \{T, H\}] = \frac{1}{2}$$

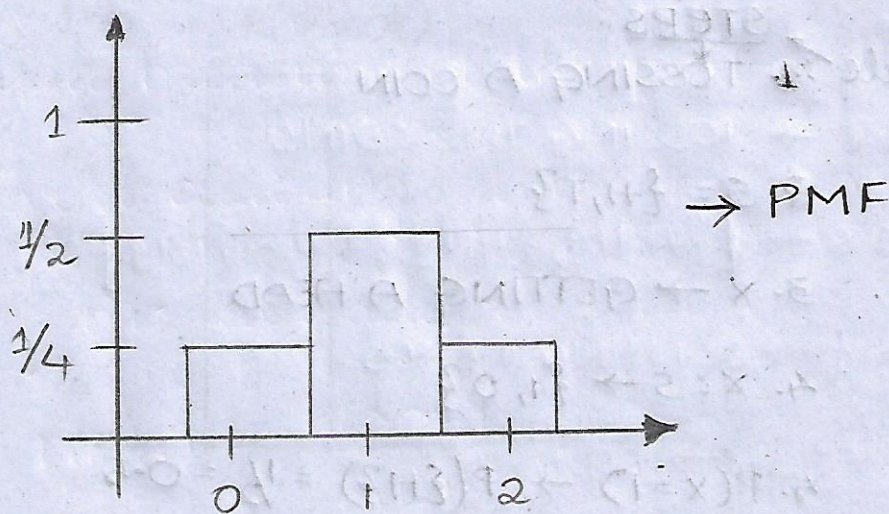
$$P(X=0) \rightarrow \{T, T\} = \frac{1}{4}$$

PROBABILITY MASS FUNCTION (PMF):

A function that gives the probability that a discrete random variable is exactly equal to some value.

↳ Some times it is called as the DISCRETE DENSITY FUNCTION.

↳ A PDF must be integrated over an interval to yield a probability.



BERNOULLI RANDOM VARIABLE:

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A random variable is called as a Bernoulli random variable that has only two outcomes usually called a 'SUCCESS' or a 'FAILURE'.

→ The variance of Bernoulli random variable is

$$\text{Var}[X] = p(1-p)$$

→ A Bernoulli distribution is a discrete Probability distribution of for a Bernoulli trial - SUCCESS or FAILURE.

Example ^{STEPS} ~~1~~ TOSsing A COIN

2. $S = \{H, T\}$

3. $X \rightarrow$ GETTING A HEAD.

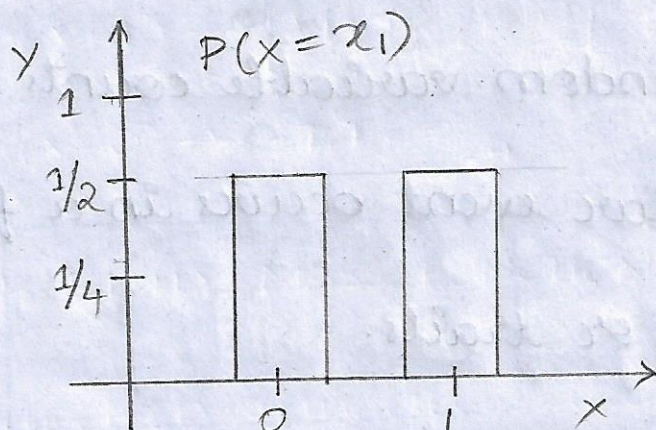
4. $X: S \rightarrow \{1, 0\}$

5. $P(X=1) \rightarrow P(\{H\}) = \frac{1}{2} = 0.5$

6. $P(X=0) \rightarrow P(\{T\}) = \frac{1}{2} = 0.5$

6. Plot the PMF

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Example - 2 : STEPS :-

1. ROLLING A DIE, $S = \{1, 2, 3, 4, 5, 6\}$

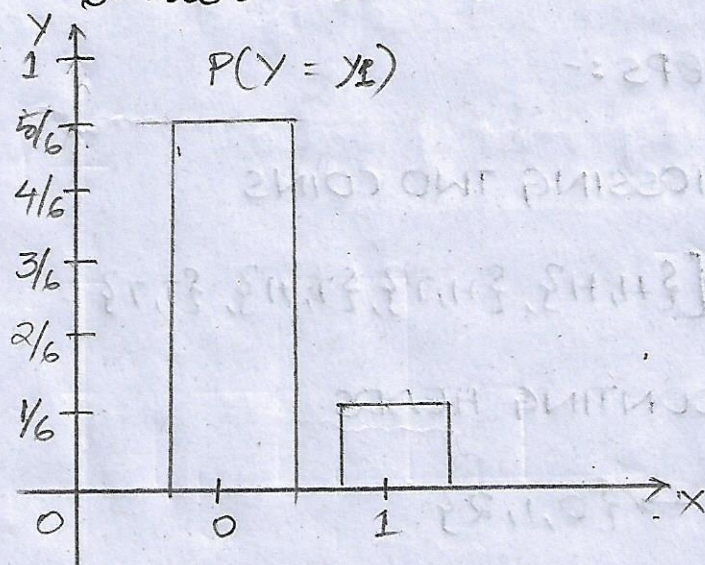
2. $Y \rightarrow$ GETTING '6'

3. $Y: S \rightarrow \{0, 1\}$

4. $P(Y=0) \Rightarrow P(\{1, 2, 3, 4, 5\}) = 5/6$

$P(Y=1) \Rightarrow P(\{6\}) = 1/6$

6. Plot the PMF



PMF FOR BERNOULLI :

$$P(X=1) = p \quad \& \quad P(X=0) = 1-p$$

BINOMIAL RANDOM VARIABLE:

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A binomial random variable counts how often a particular event occurs in a fixed number of trials or trials.

→ The probability of occurrence (or not) is the same on each trial.

→ Trials are independent of one another.

→ In simple,

Binomial random variable is the collection of a Bernoulli Random Variable.

Example: STEPS:-

1. R.E → TOSSING TWO COINS

2. S.S → $\{\{H, H\}, \{H, T\}, \{T, H\}, \{T, T\}\}$

3. X → COUNTING HEADS

4. X : S.S → $\{0, 1, 2\}$

$$5. P(X=0) = {}^2C_0 (0.5)^0 (0.5)^2$$

Probability of success $\left[\because P(X=i) = {}^nC_i \cdot P^i (1-P)^{n-i} \right]$

$$= \frac{2!}{0! 2!} \times 1 \times 0.25$$

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$$= 0.25$$

$$n_{C_i} = \frac{n!}{i!(n-i)!}$$

$$P(X=1) = {}^2C_1 (0.5)^1 (0.5)^{2-1}$$

$$= \frac{2!}{1! 1!} \times (0.5) \times (0.5)$$

$$= 2 \times 0.25$$

$$= 0.5$$

$$P(X=2) = {}^2C_2 (0.5)^2 (0.5)^{2-2}$$

$$= {}^2C_2 (0.5)^2 (0.5)^0$$

$$= \frac{2!}{2! 0!} \times 0.25 \times 1$$

$$= 0.25$$

DISCRETE RANDOM VARIABLE :

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A discrete random variable has a countable number of possible values.

→ The probability of each value of a discrete random variable is between '0' and '1', the sum of all the probabilities is equal to 1.

CONTINUOUS RANDOM VARIABLE :

A continuous random variable is a random variable where the data can take infinitely many values.