LECTURE - 28

PROBABILITY: STUDY OF UNCERTAINITY

It is the measure of the likelihood that an event will occur.

↳ We can also say that, the probability of an event is the measure of the chance that the event will occur as a result of an experiment.

↳ The probability that an event will occur is usually expressed as a number between '0' and '1'.

RANDOM EXPERIMENT:

• A process by which we observe something uncertain.

↳ After the experiment, the result of the random experiment is known.

↳ An outcome is a result of a random experiment.

In otherwords, random experiment is defined as "in any process the outcome can't be predicted by uncertainity".

Examples: COIN TOSSING, ROLLING A DIE, WEATHER CONDITION.

SAMPLE SPACE (S.S):

The set of all possible outcomes that we get from a random experiment.

Examples: TOSSING COIN - S = {H, T}

ROLLING A DIE - S = {1, 2, 3, 4, 5, 6}

EVENT: SUB-SECT of sample space.

An event is an outcome or defined as collection of outcomes of a random experiment

Since the collection of all the possible outcomes to a random experiment is called the sample space.

Examples: TOSSING A COIN - {H, T}

$E_1 \rightarrow$ Getting a head - {H}

$E_2 \rightarrow$ Getting a tail - {T}

$E_3 \rightarrow$ Empty set - { }

S.S $\rightarrow$ {H}, {T}, { }

TOSSING TWO COINS.

↓

[{H, H}, {H, T}, {T, H}, {T, T}]

↳ Getting 2 heads = 1/4

↳ Getting excatly one head = 2/4 = 1/2

RANDOM EXPERIMENT (RE): CLIMATIC CONDITION

SAMPLE SPACE (SS): {SUNNY, RAINY, CLOUDY}

Will it rain → 1

P(RAINY) = 1/3 = 0.33

AXIOMS OF PROBABILITY: The heart of this defination are three conditions, called the axioms of probability.

→ 1. The probability of an event P(E) is a real number greater than or equal to zero.

2. $0 \leq P(E) \leq 1$

→ 2. The probability that at least one of all the possible outcomes of a process (such as rolling a die) will occur is 1.

$$P(S) = 1$$

P(S) → Probability of sample space.

S.S → TOSSING A COIN – {H, T}

H → 0.5, T → 0.5

↳ H + T = 0.5 + 0.5 = 1.

→ 3. If two events A and B are mutually exclusive, then the probability of either A or B occuring is the probability of A occuring plus the probability of B occuring.

$P(E_1 \cup E_2 \ldots\ldots E_n) = P(E_1) + P(E_2) + \ldots\ldots + P(E_n)$

$\rightarrow P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i)$

For any sequence of events that are mutually exclusive.

R.E – TOSSING TWO COINS

S.S – [{H,H}, {H,T}, {T,H}, {T,T}]

→ E_1 – One head – [{H,H}, {H,T}, {T,H}]

→ E_2 – One tail – [{H,T}, {T,H}, {T,T}]

$E_1 \cap E_2 = \phi$ → NOT MUTUALLY EXCLUSIVE

→ E_1 – Excatly 2 heads – {H, H} – $P(E_1) = 1/4$

→ E_2 – Excatly 2 Tails – {T, T} – $P(E_2) = 1/4$

→ E_3 – Getting same element on both coins

= [{H, H}, {T, T}]

∴ $E_3 = 2/4 = 1/2 = 0.5$

$E_3 = P(E_1) + P(E_2)$

$\neq$ ~~0.5~~

= {H, H} + {T, T}

$E_3 = P(E_1 \cup E_2)$

QUESTION :

To find a probability of an even number when a die is rolled.

HINT : I CAN SEE A PRIME NUMBER

SOLUTION :

$P(E_1) = \{2, 4, 6\} \rightarrow$ Getting an even no.

$P(E_2) = \{2, 3, 5\} \rightarrow$ Getting a prime no.

$\rightarrow E_1 \cap E_2 = \{2\}$

$\rightarrow P(E_1 \cap E_2) = \frac{1}{6}$.

$$* \; P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{2}{6} = 1/3.$$

CONDITIONAL PROBABILITY :

It is a measure of the probability of an event occuring, given that another event has already occured.

$$\rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)}$$