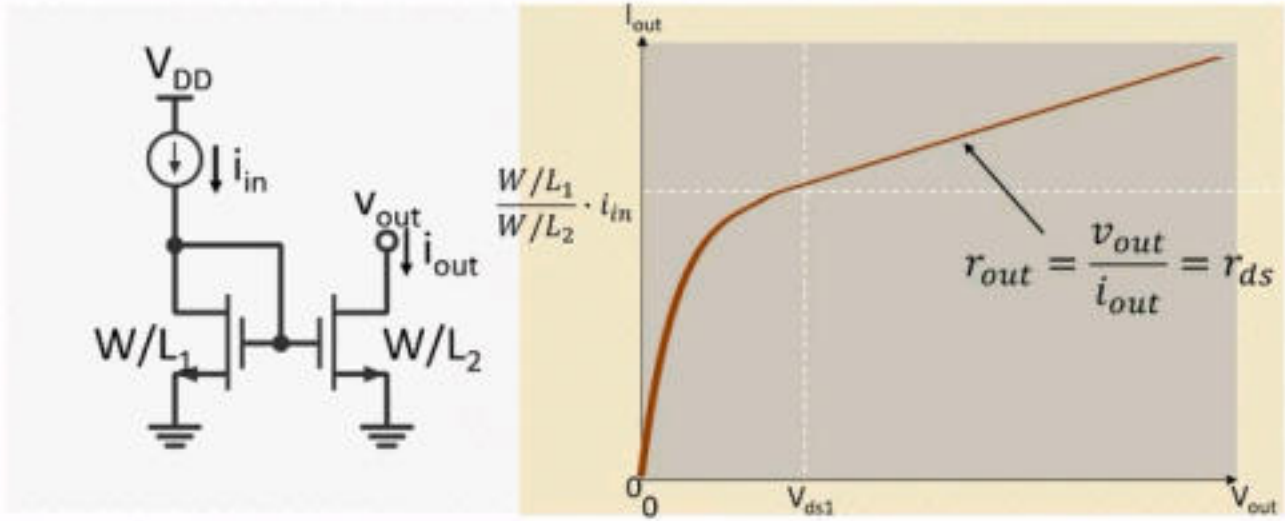


Recall

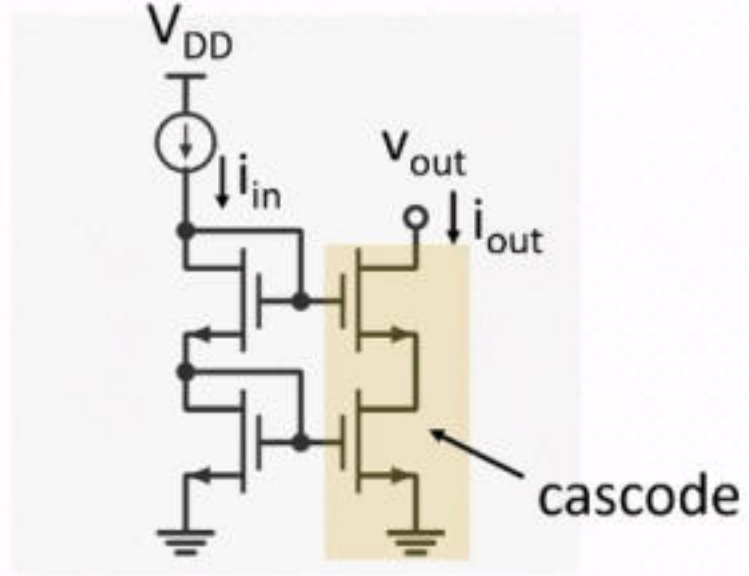
Topology	Gain	Rout	Rin
CM source	high	high	high
CM gate	high	high	low
CM drain	low	low	high

Basic Current Mirror



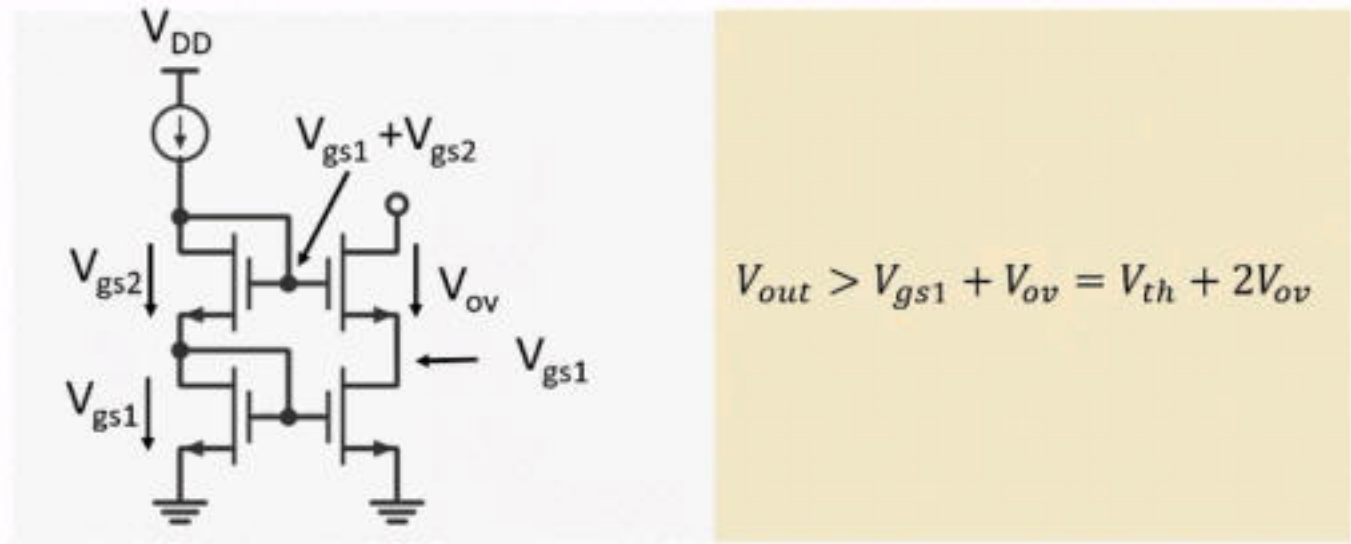
For an ideal current mirror the output current is independent of V_{out} .
 ➔ large r_{out} desired

Cascode Current Mirror



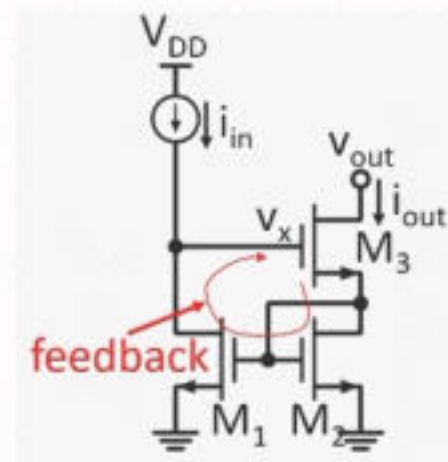
Increased output resistance due to cascoding.

Cascode Current Mirror



Reduced swing compared to basic current mirror.

Wilson Current Mirror



坏角解释 ✓

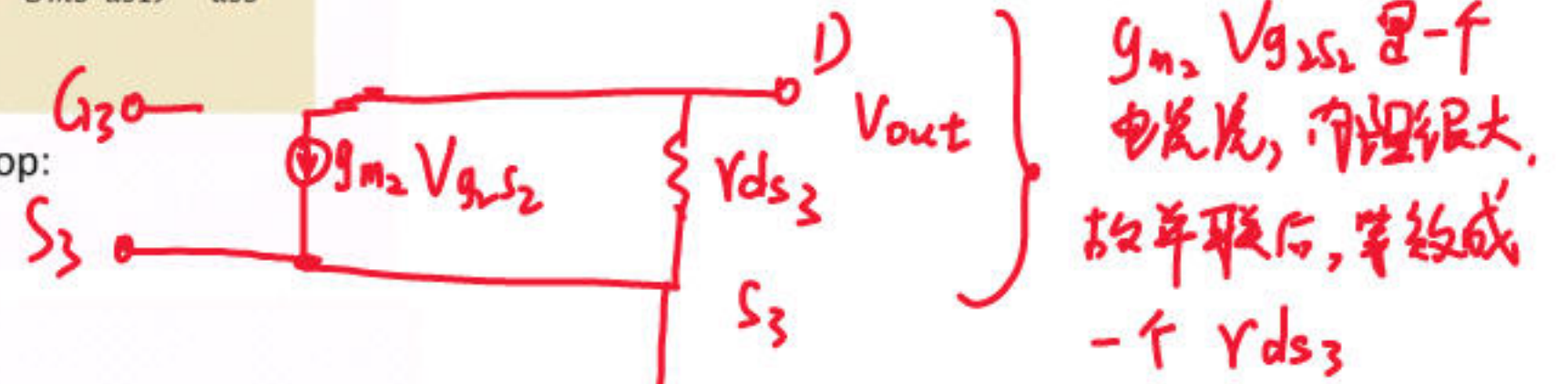
$$i_{out} = \frac{v_{out}}{r_{ds3} + 1/g_{m2}} - i_{out} r_{ds1} g_{m3}$$

feedback 反馈量

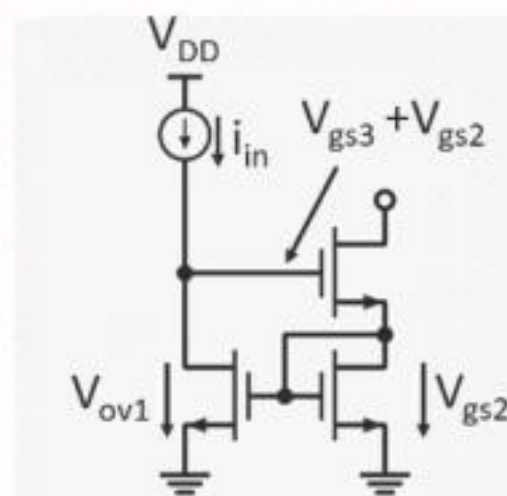
$$\Rightarrow r_{out} \approx \frac{v_{out}}{i_{out}} = (1 + g_{m3} r_{ds1}) \cdot r_{ds3}$$

Output resistance increased via feedback loop:

$$V_{out} \uparrow \rightarrow I_{out} \uparrow \rightarrow v_x \downarrow$$



Wilson Current Mirror



$$V_{out} > V_{gs2} + V_{ov3} = V_{th} + 2V_{ov}$$

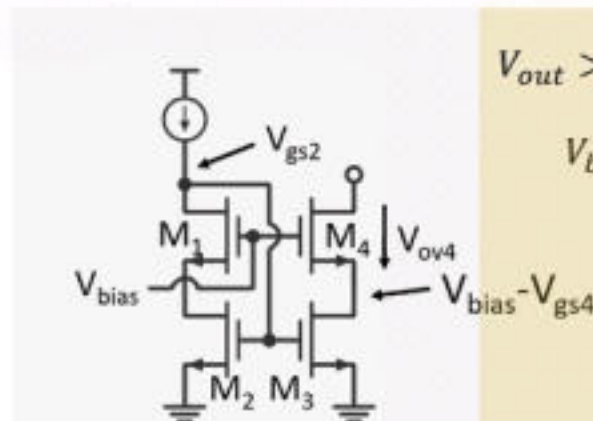


$G_2 = D_2$
即 $g_{m2} V_{gs2}$ 是一个等效电阻 $R' = \frac{V_{gs2}}{g_{m2} V_{gs2}} = \frac{1}{g_{m2}}$
故 $i_{out} = \frac{V_{out}}{r_{ds3} + \frac{1}{g_{m2}}}$

总结 未反馈时

How to increase the output swing?

High Swing Cascode Current Mirror



$$V_{out} > V_{bias} - V_{gs4} + V_{ov4} = V_{bias} - V_{th}$$

$$V_{bias} > V_{gs1} + V_{ov2}$$

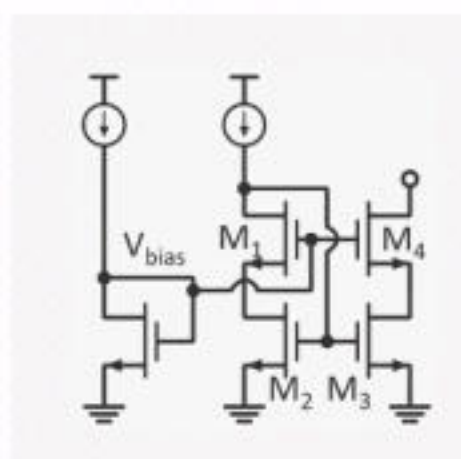
$$\Rightarrow V_{out} > V_{ov1} + V_{ov2} = 2V_{ov}$$

Output swing increased by V_{th} .

How to generate V_{bias} ?

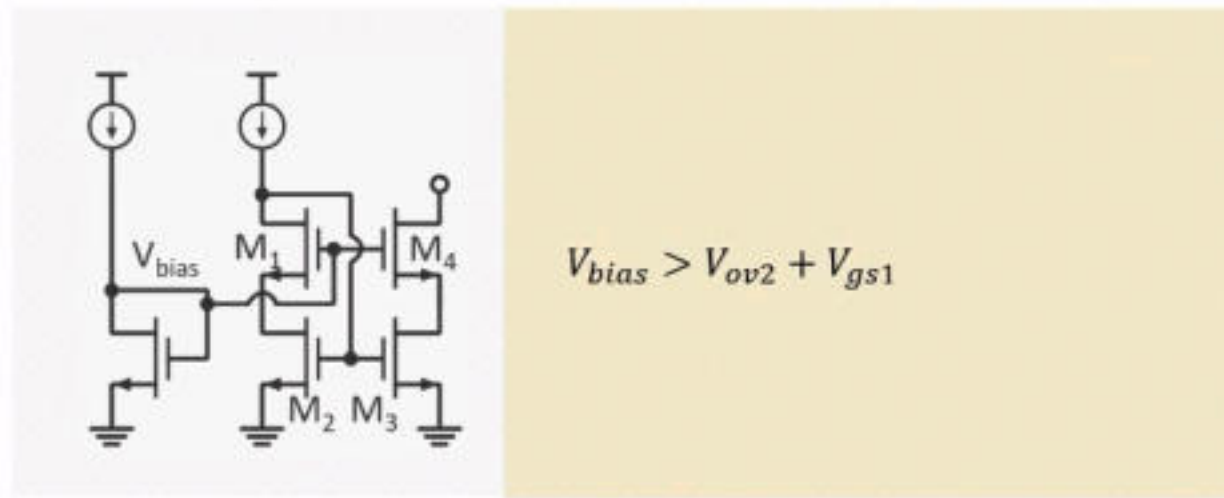
1/2

High Swing Cascode Current Mirror



Use long ($L > W$) diode connected transistor to generate V_{bias} .

High Swing Cascode Current Mirror



Use long ($L > W$) diode connected transistor to generate V_{bias} .

Recall

Current Mirror	Rout	Swing
Basic	-	++
Cascode	+	-
Wilson	+	-
High Swing	+	+

反馈 (Feedback)

Effects of Poles & Zeros on Frequency Response(1)

- Consider a general system transfer function:

$$H(s) = \frac{P(s)}{Q(s)} = b_0 \frac{(s-z_1)(s-z_2)\dots(s-z_N)}{(s-\lambda_1)(s-\lambda_2)\dots(s-\lambda_N)}$$

zeros at z_1, z_2, \dots, z_N
poles at $\lambda_1, \lambda_2, \dots, \lambda_N$
- The value of the transfer function at some complex frequency $s=p$ is:

$$H(s)|_{s=p} = b_0 \frac{(p-z_1)(p-z_2)\dots(p-z_N)}{(p-\lambda_1)(p-\lambda_2)\dots(p-\lambda_N)}$$

$$= b_0 \frac{(r_1 e^{j\phi_1})(r_2 e^{j\phi_2})\dots(r_N e^{j\phi_N})}{(d_1 e^{j\theta_1})(d_2 e^{j\theta_2})\dots(d_N e^{j\theta_N})}$$

Effects of Poles & Zeros on Frequency Response(2)

- Therefore the magnitude and phase at $s=p$ are given by:

$$H(s)|_{s=p} = b_0 \frac{r_1 r_2 \dots r_N}{d_1 d_2 \dots d_N}$$

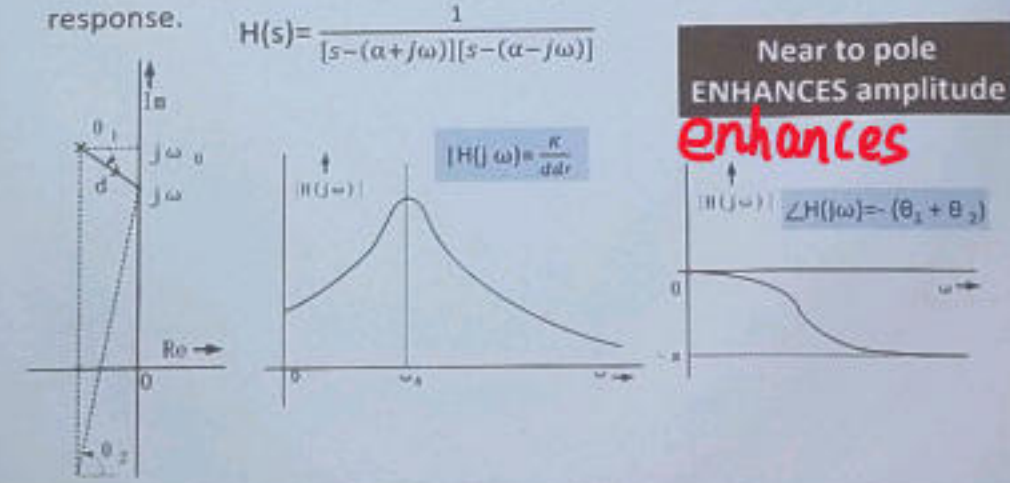
$$= b_0 \frac{\text{product of the distances of zeros to } p}{\text{product of the distances of poles to } p}$$

$$\angle H(s)|_{s=p} = (\phi_1 + \phi_2 + \dots + \phi_N) - (\theta_1 + \theta_2 + \dots + \theta_N)$$

= sum of zero angles to p - sum of pole angles to p

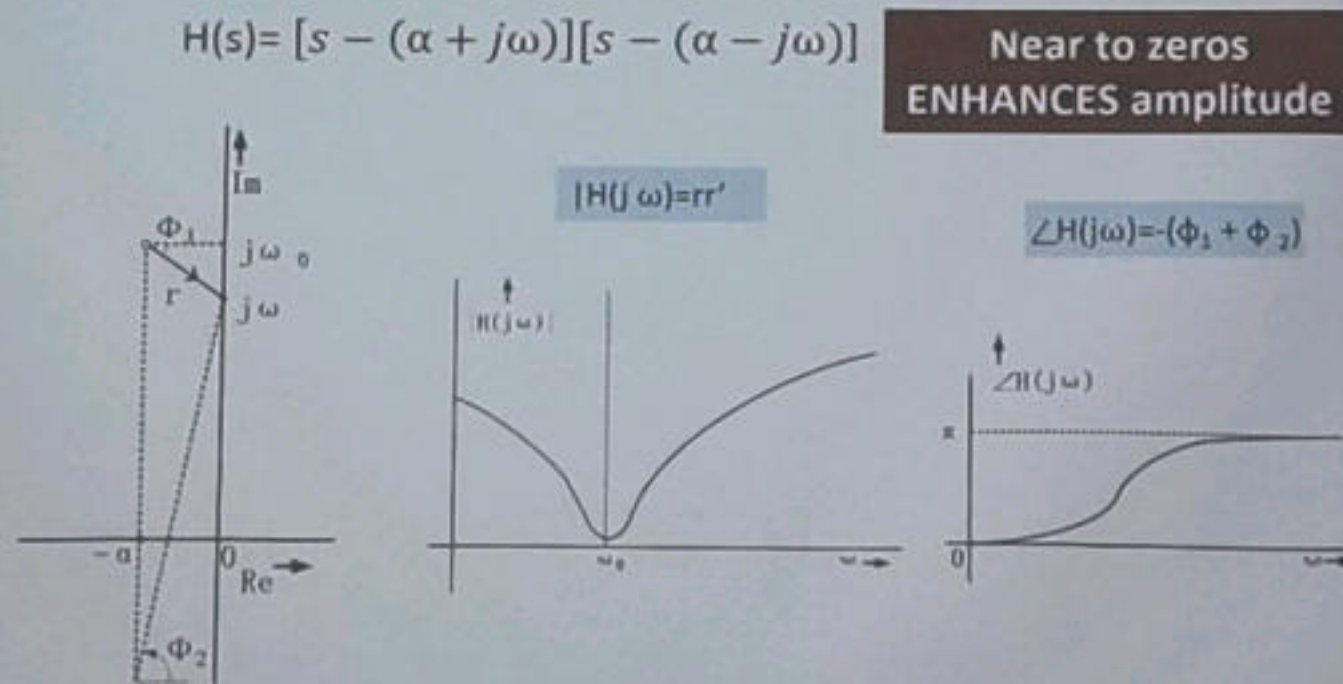
Effects of Poles & Zeros on Frequency Response(3)

- Frequency Response of a system is obtained by evaluating $H(s)$ along the $j\omega$ -axis (i.e. taking all value of $s=j\omega$).
- Consider the effect of two complex poles on the frequency response.



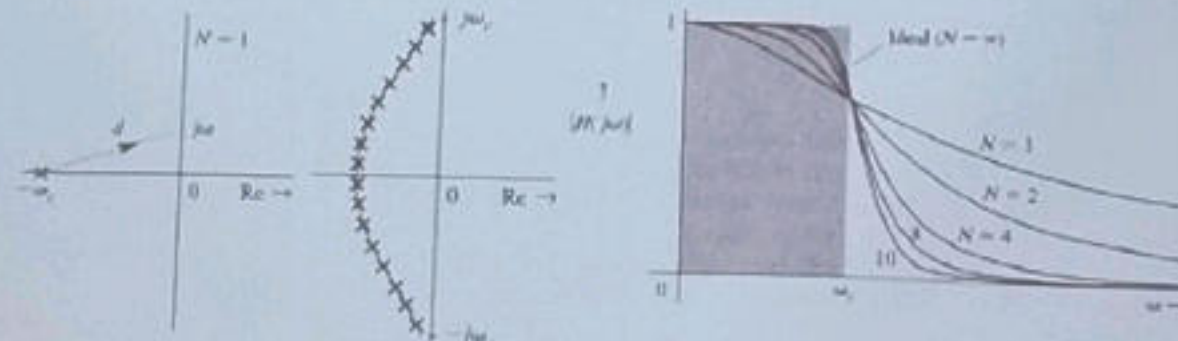
Effects of Poles & Zeros on Frequency Response(4)

- Consider the effect of two complex zeros on the frequency response.



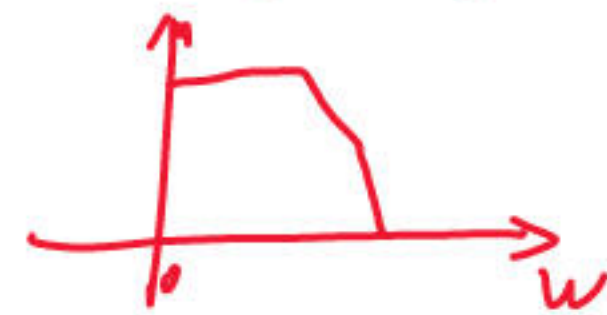
Poles & Low-pass filter

- Use the enhancement and suppression properties of poles & zeros to design filters
- Low-pass filter (LPF) has maximum gain at $\omega = 0$, and the gain decreases with ω
- Simplest LPF has a single pole on real axis, say at $(\omega = -\omega_c)$. Then $H(s) = \frac{1}{s + \omega_c}$ and $|H(j\omega)| = \frac{\omega_c}{d}$
- To have a "brickwall" type of LPF (i.e. very sharp cut-off), we need a WALL OF POLE as shown, the more poles we get, the sharper the cut-off



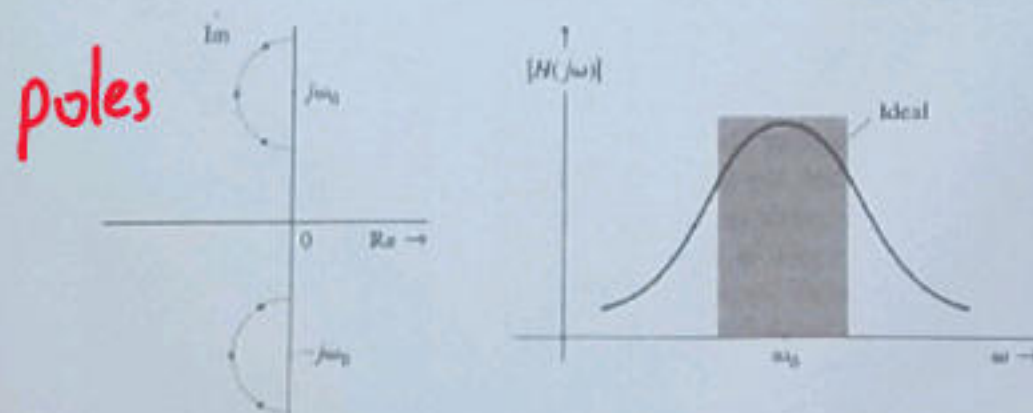
低通滤波器
在分母做极点

$$H(s) = \frac{1}{(s-a)(s-b)}$$



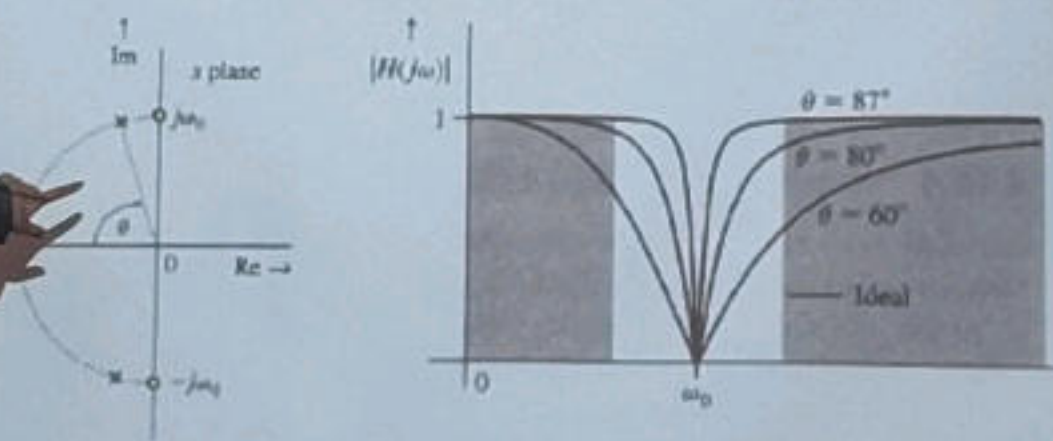
Poles & Band-pass filter 带通

- Band-pass filter has gain enhanced over the entire passband, but suppressed elsewhere
- For a passband centred around ω_0 , we need lots of poles opposite the imaginary axis in front of the passband center at ω_0

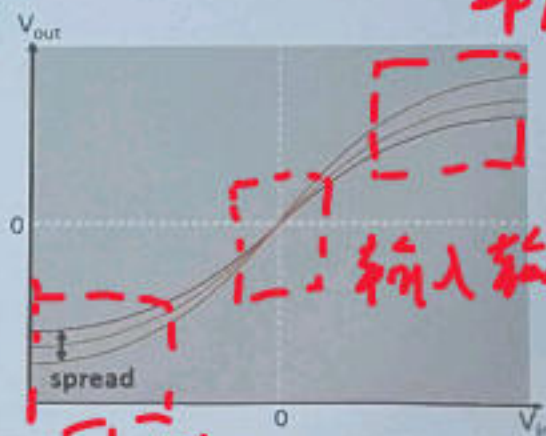
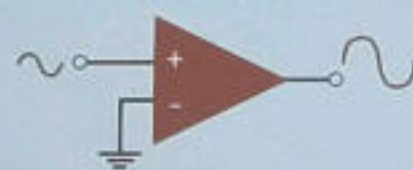


Notch filter 非线性

- Notch filter could in theory be realised with two zeros placed at $\pm j\omega_0$. However, such a filter would not have unity gain at zero frequency, and the notch will not be sharp
- To obtain a good notch filter, put two poles close to the two zeros on the semicircle as shown. Since the both pole/zero pair are equal-distance to the origin, the gain at zero frequency is exactly one. Same for $\omega = \infty$.



External Feedback 非线性



Open loop amplifiers are typically nonlinear and have process spread.

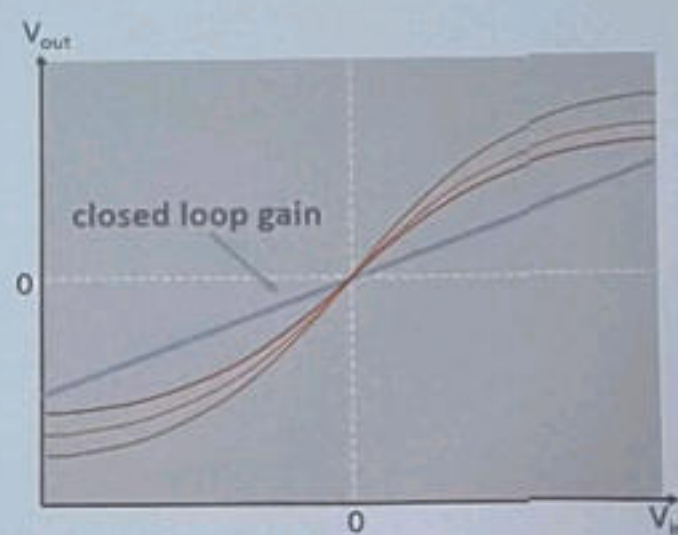
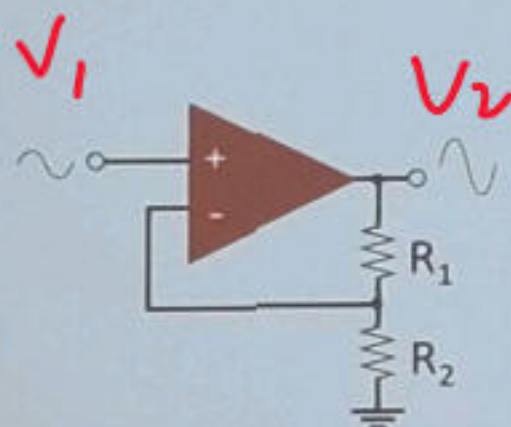
非线性

分散

↓ 引入负反馈

Feedback.

External Feedback



Apply feedback for linearization:

$$V_{out} = A \cdot \left(V_{in} - \frac{R_2}{R_1 + R_2} V_{out} \right) \Rightarrow V_{out} = \frac{A \cdot V_{in}}{1 + A \cdot \frac{R_2}{R_1 + R_2}} \approx \left(1 + \frac{R_1}{R_2} \right) \cdot V_{in}$$

gain is only defined by external resistors \Rightarrow very linear & deterministic

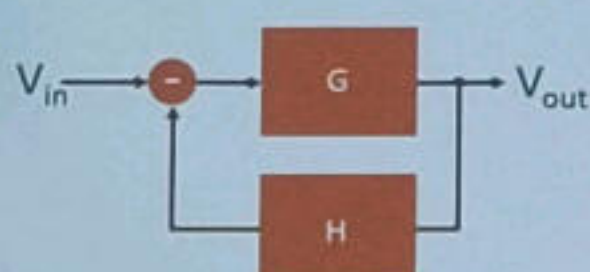
确定

$$V_1 \Rightarrow V_2 \cdot \frac{R_2}{R_1 + R_2}$$

$$\frac{V_2}{V_1} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

Black's Formula

Let's generalize:



$$A_{cl} = \frac{V_{out}}{V_{in}} = \frac{G}{1 + G \cdot H}$$

$$G \gg \frac{1}{H} \Rightarrow A_{cl} \approx \frac{1}{H}$$

$$G \ll \frac{1}{H} \Rightarrow A_{cl} \approx G$$

In our example: $G = A$, $H = \frac{R_2}{R_2 + R_1} = \frac{V_{in} \text{ 的反馈}}{V_{in} \text{ 的输入}} = \frac{V_{in}}{V_{out}}$

$$\Rightarrow A_{cl} \approx \frac{1}{H} = 1 + \frac{R_1}{R_2} \text{ for } G \gg \frac{1}{H}$$

\Rightarrow Build amplifiers with high open loop gain to satisfy $G \gg 1/H$.

Typically: $1000 < A < 100000$

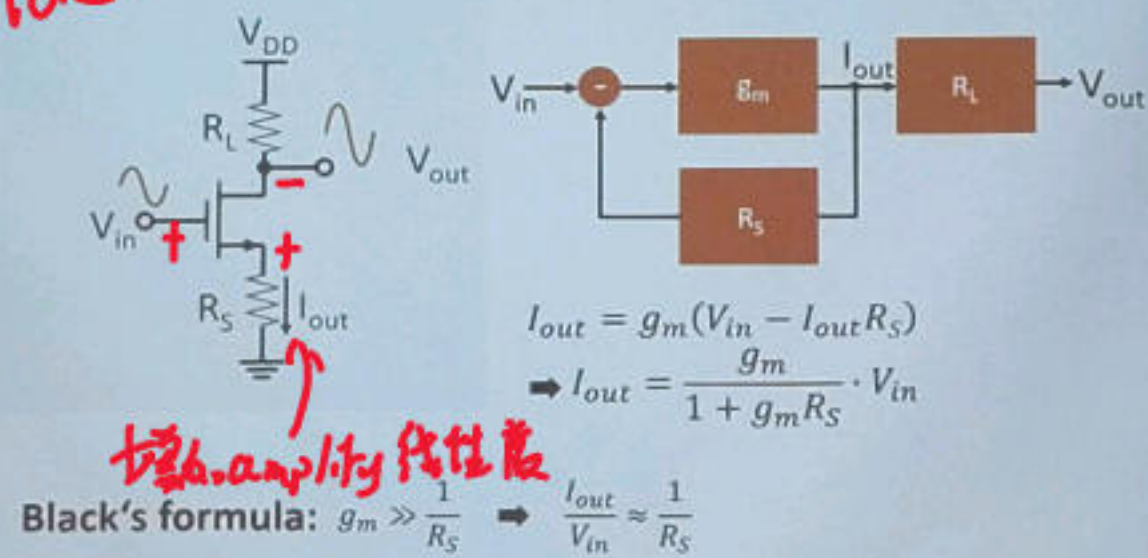
满足

GH 环路增益

1+GH 反馈深度

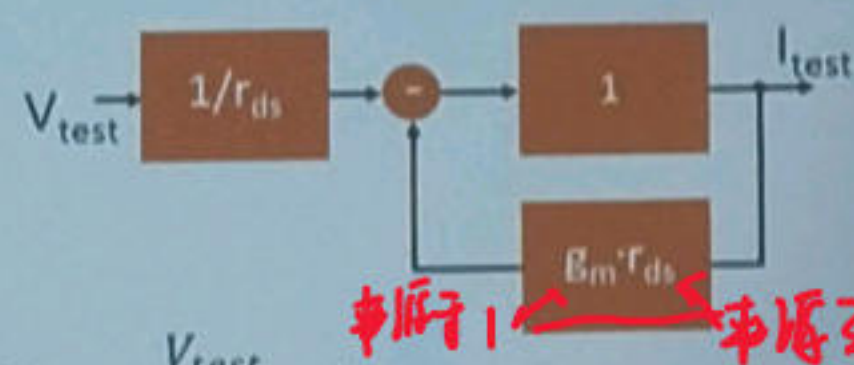
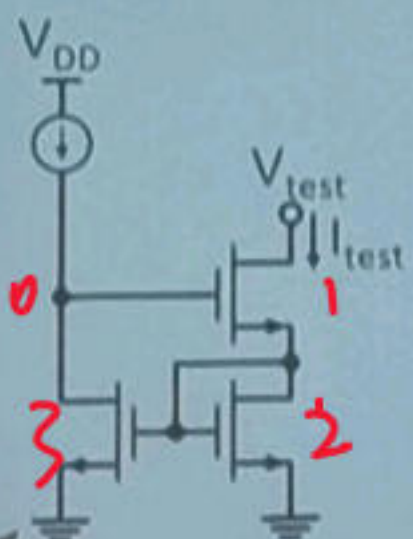
Example: Source Degeneration

反馈输出是 I_{out}



增加amp. 线性度

Example: Wilson Current Mirror



$$I_{test} = \frac{V_{test}}{r_{ds}} - I_{test} \cdot g_m r_{ds}$$

$$I_{test} \approx \frac{V_{test}}{(1 + g_m r_{ds}) \cdot r_{ds}}$$

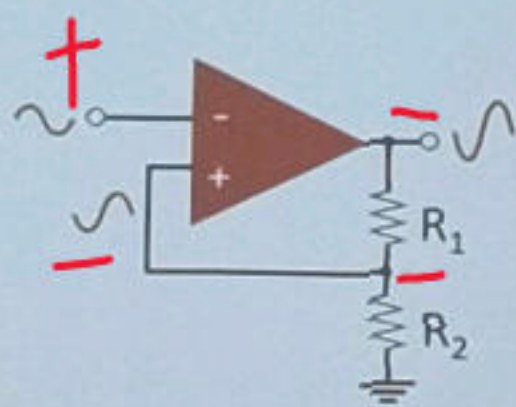
Black's formula: $1 \gg \frac{1}{g_m r_{ds}} \Rightarrow \frac{I_{out}}{V_{test}} \approx \frac{1}{r_{ds}} \cdot \frac{1}{g_m r_{ds}}$

串接于1, 2, 3

$$\Delta I \quad V_{ds3} \approx V_o$$

$$\Delta V_o \cdot g_m = \Delta I$$

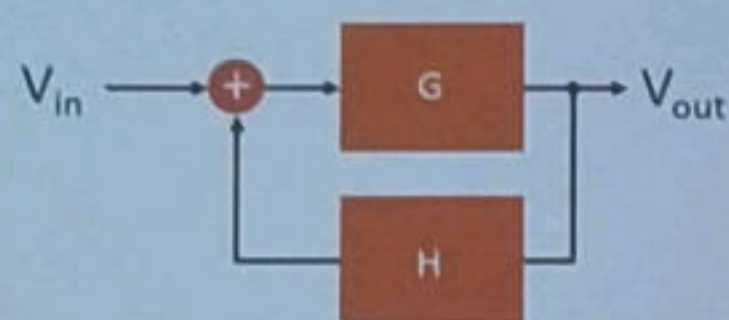
Stability



正反馈

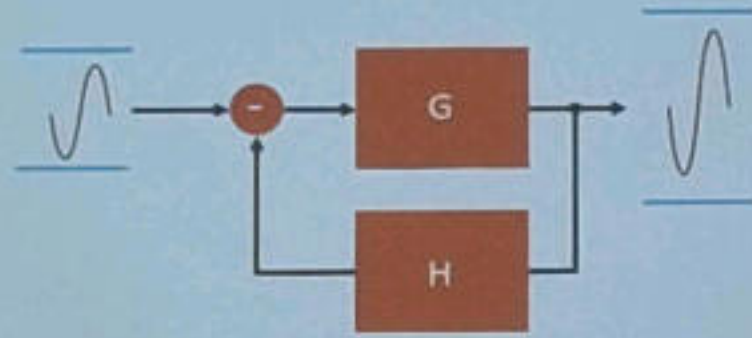
不稳定

Stability



Positive feedback makes the loop unstable.

BIBO

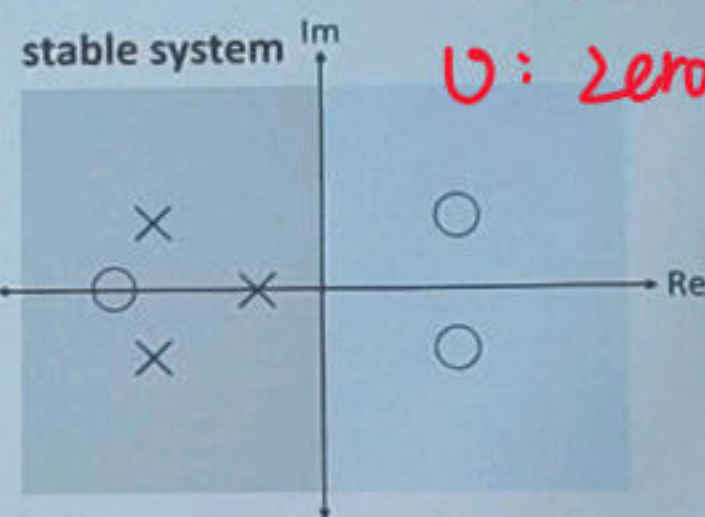


A general stability criteria is that for a bounded input there must be a bounded output (BIBO).

Stability Criteria

原则

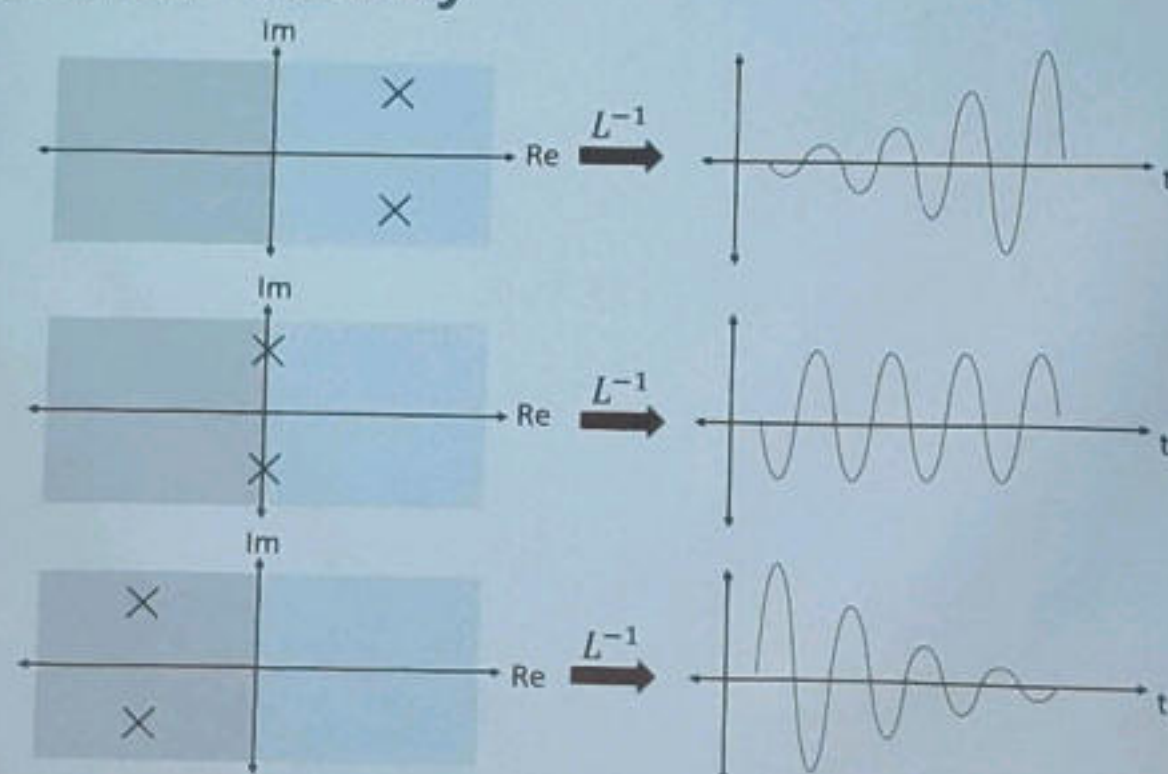
$$A_{cl} = \frac{(s - z_1)(s - z_2) \dots (s - z_n)}{(s - p_1)(s - p_2) \dots (s - p_n)}$$



x : pole
o : zero

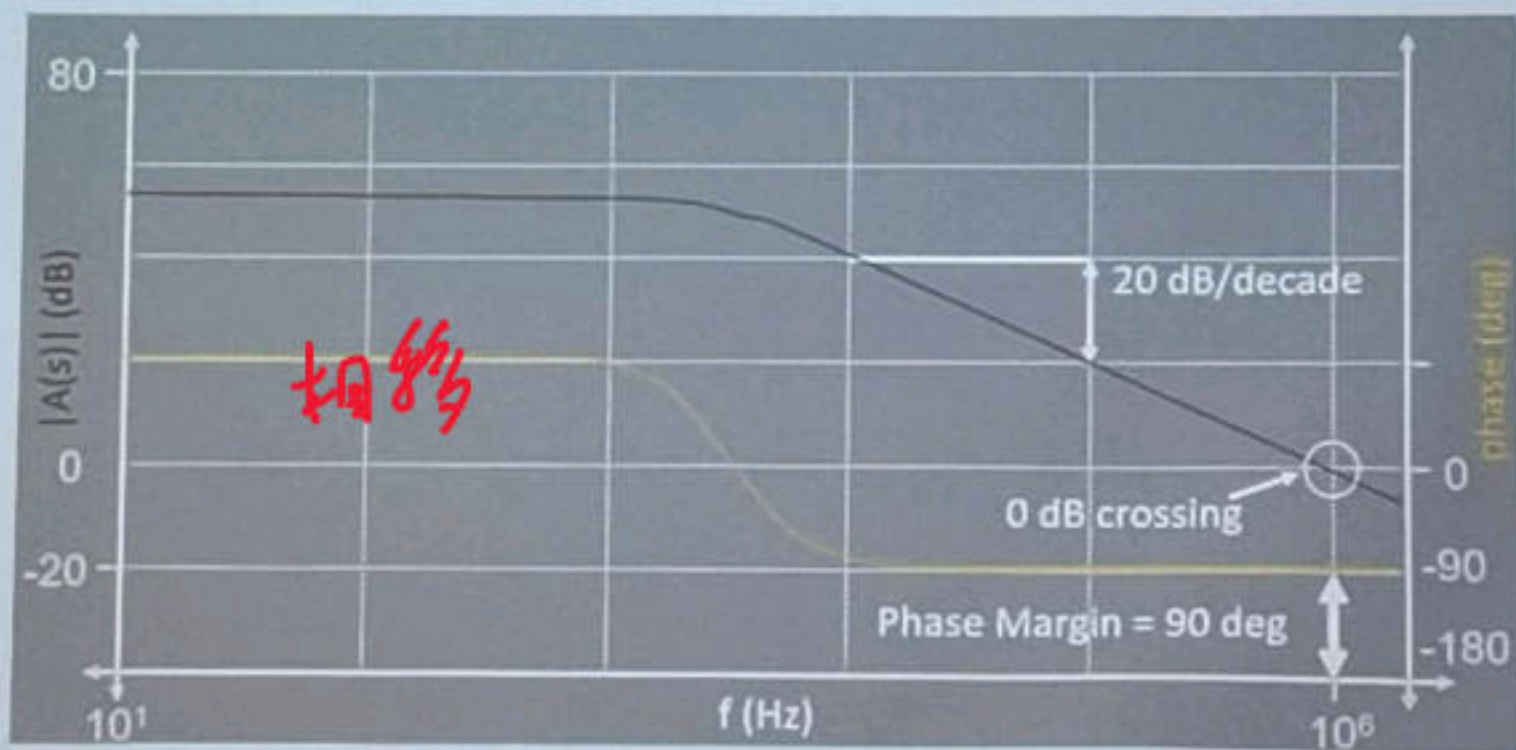
A system is stable if all poles of the closed loop response lie on the left half plane (LHP).
Any pole in the right half plane (RHP) results in instability

Intution on Stability



稳定系统

Bode Plot

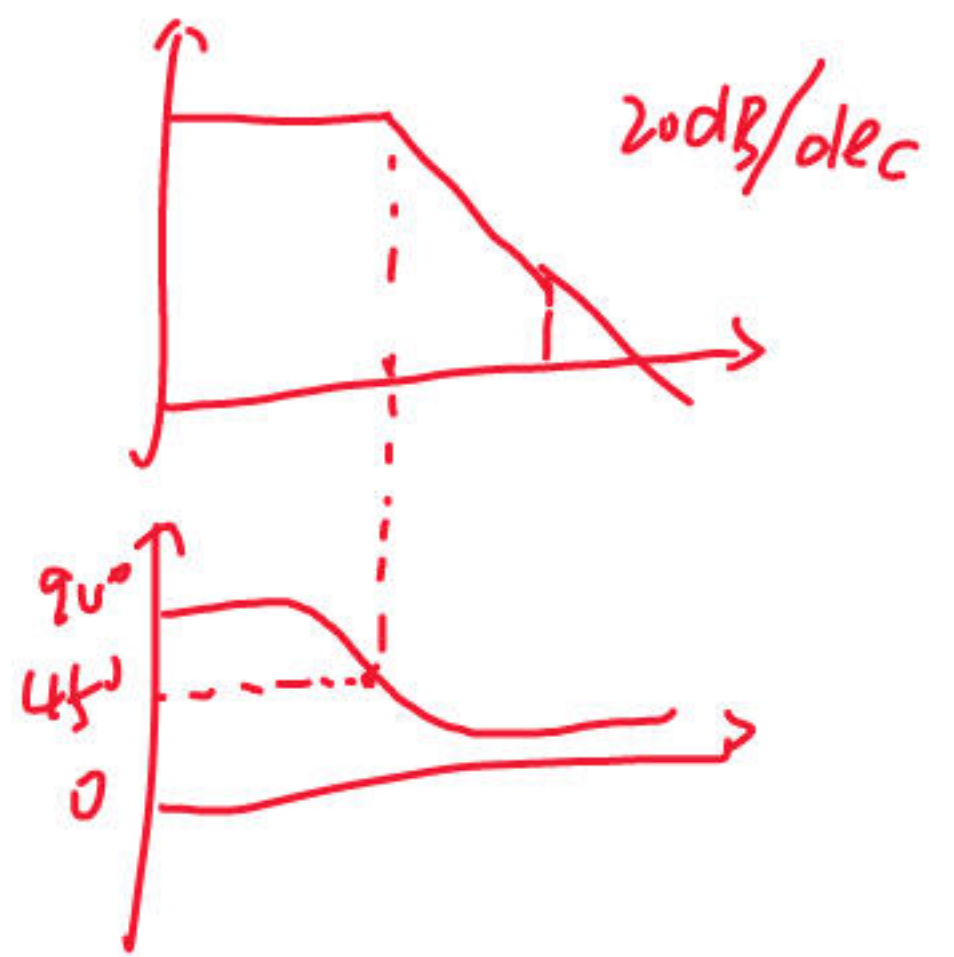


There is a more intuitive way to check on the Bode Plot of the **open loop** response:
 phase margin > 0 @ 0 dB crossing

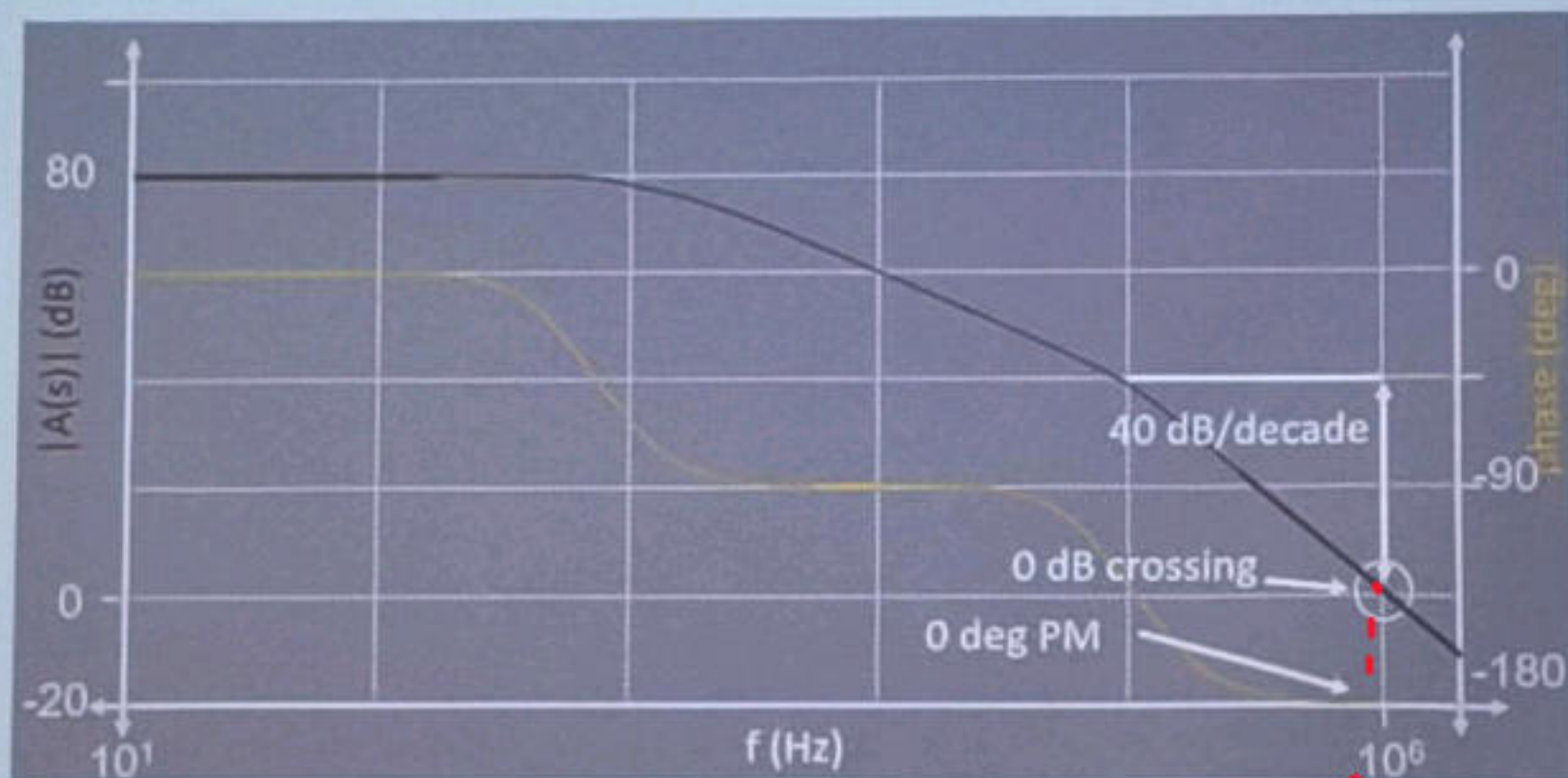
相移

一个 pole 产生 90 度相移

位于坐标轴上的



Bode Plot



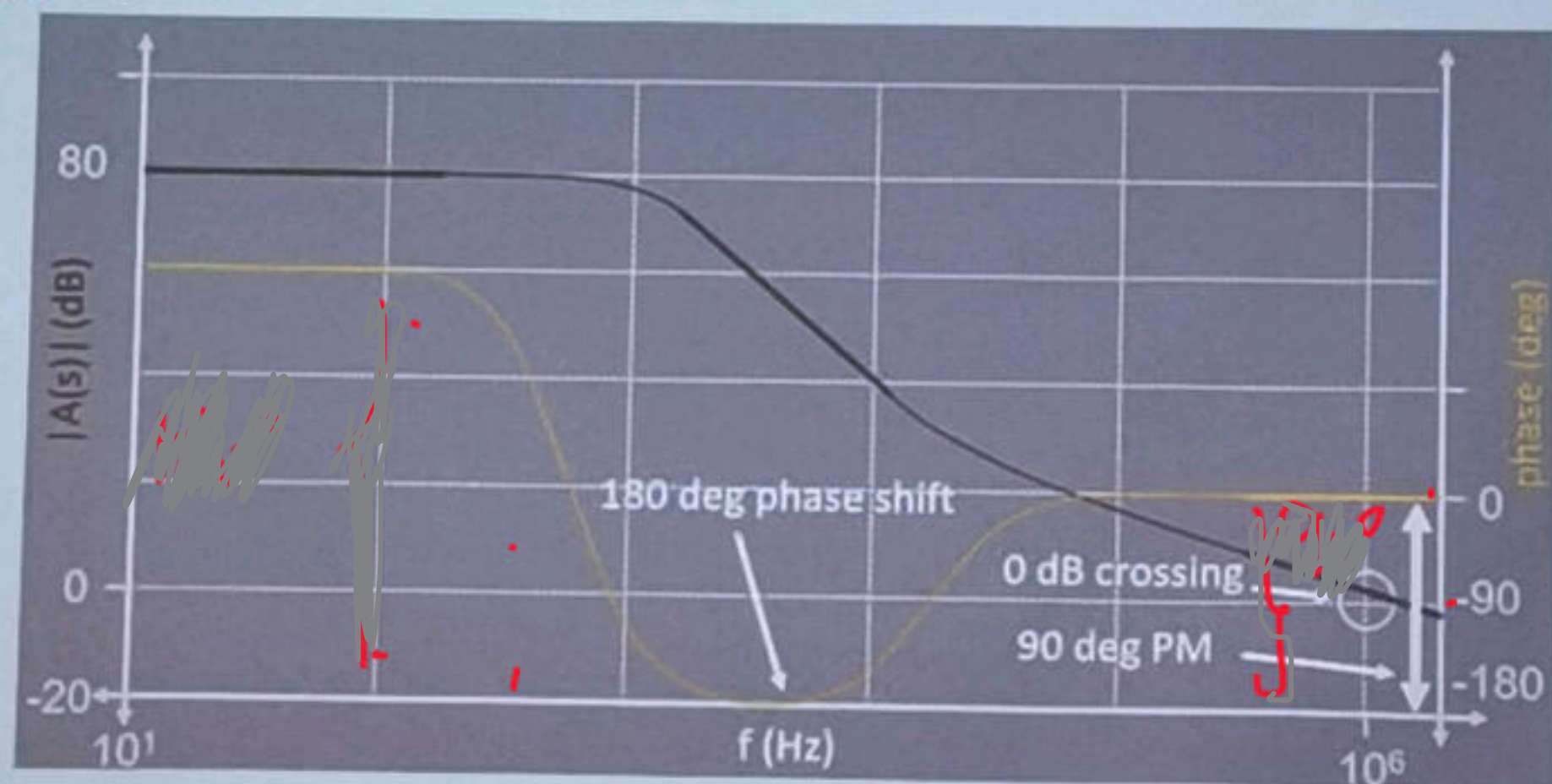
2nd order roll-off creates low phase margin.

增益提高
频率变

gain = 1 时,

相位裕度

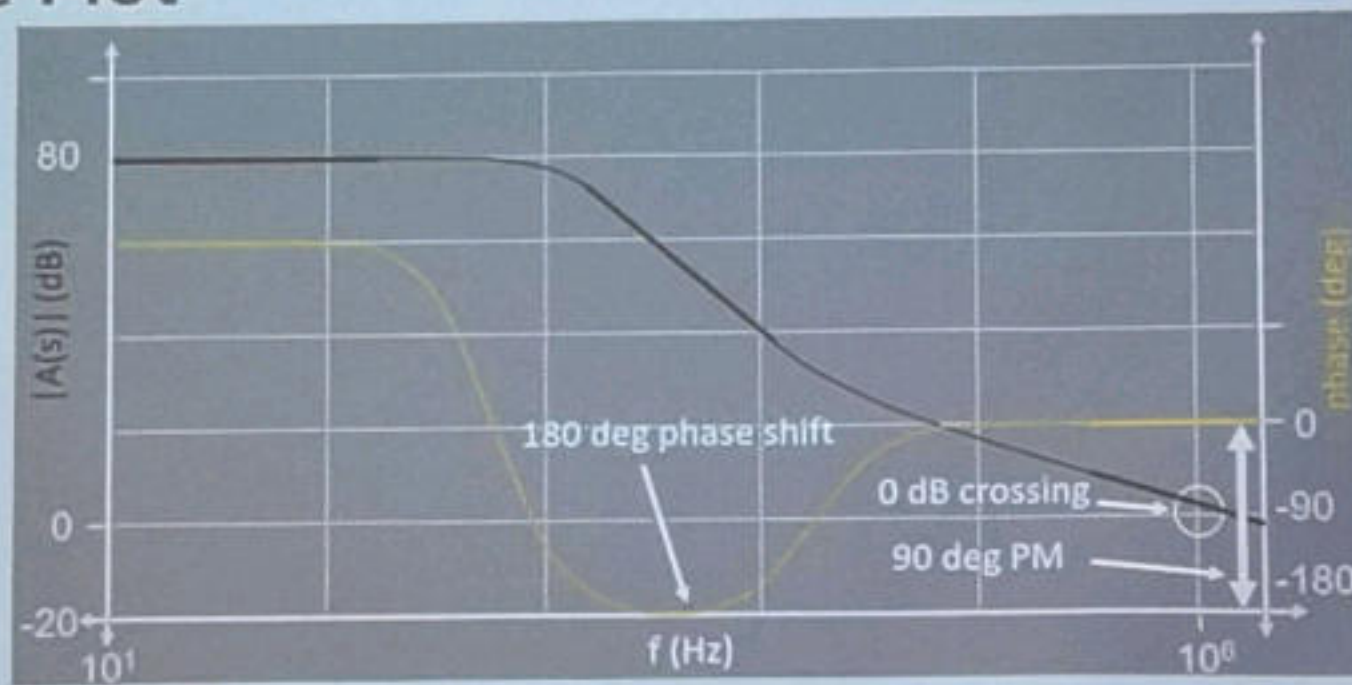
Bode Plot



两组复 pole + 一组复 zero



Bode Plot



Is this stable? Surprisingly yes.

Nyquist is derived from time domain → Start-Up transients are present
→ Higher frequencies present, that drive loop into stability.

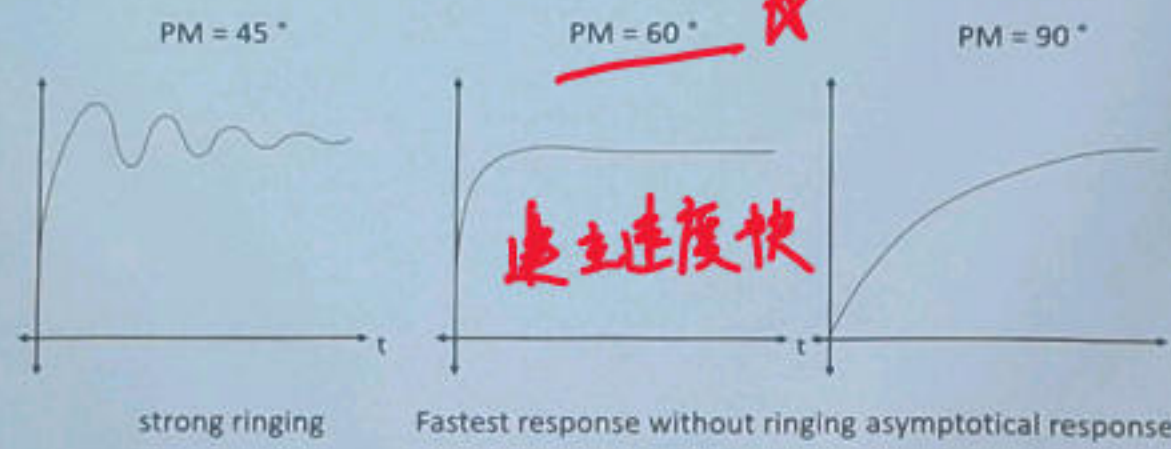
settle time is long

雖然有90°的 margin

但仍是穩定的

Phase Margin

How much phase margin do we need?



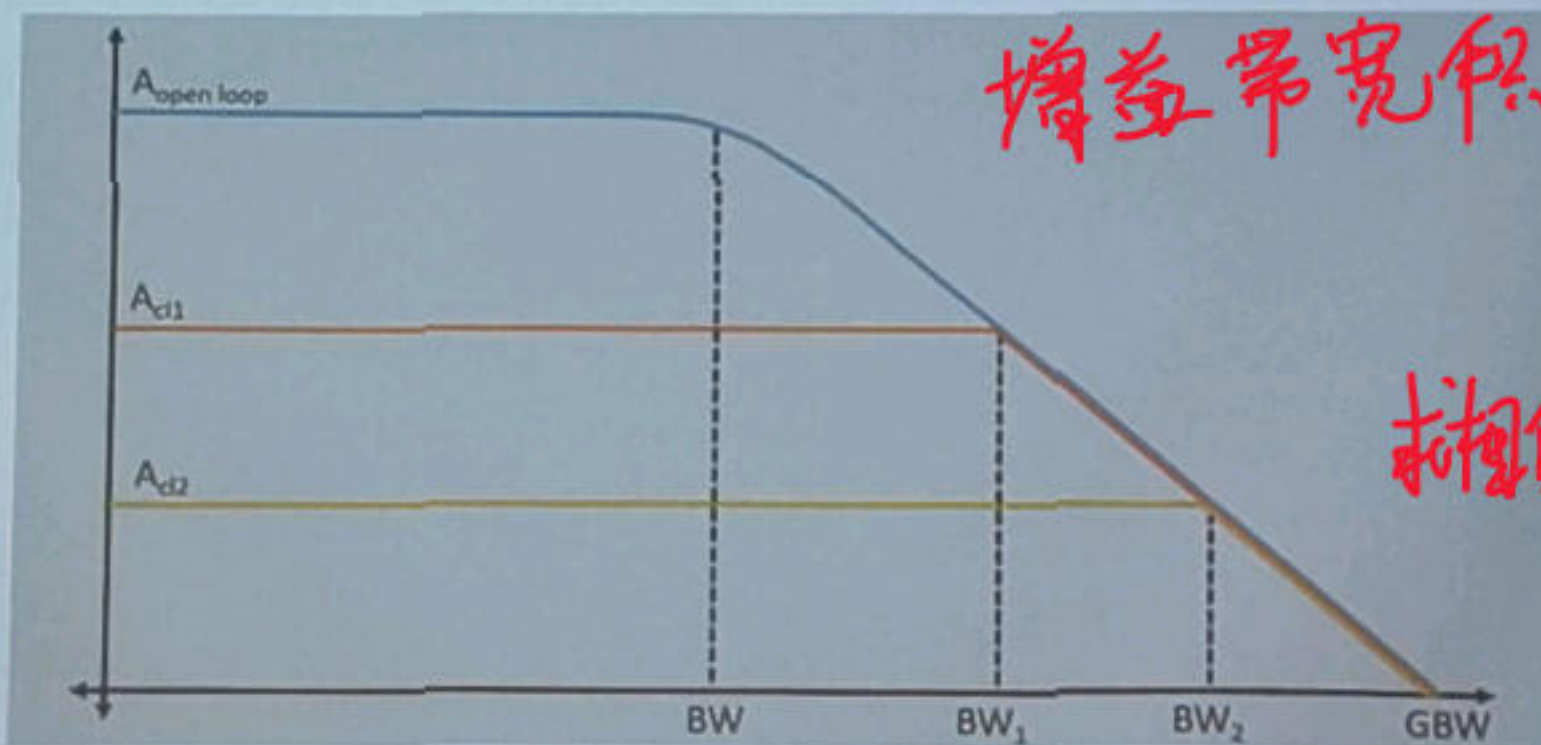
The closer the loop is at instability the more oscillations will be present.

Over stabilization results in slow loop response.

振盪

建立速度快

Gain Bandwidth



Gain Bandwidth (GBW) Product is a constant.

→ Increasing closed loop gain decreases closed loop bandwidth.

Tradeoff: gain ↔ BW

增益帶寬積

和相位裕度 需看 Feedback.

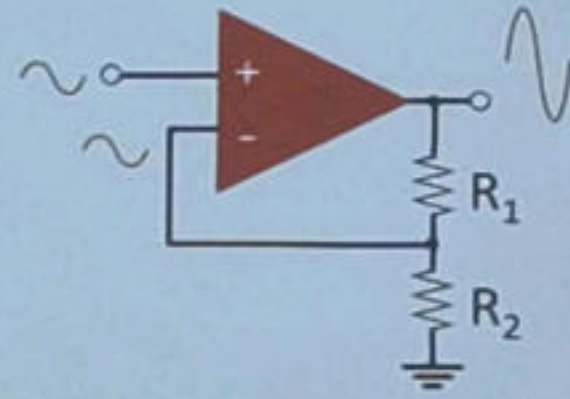
$$\text{Feedback} = \frac{R_2}{R_1 + R_2}$$

$$A = 1 + \frac{R_1}{R_2} > 1, \text{ 取不到 GBW.}$$

則裕度可取到 $A = 1 + \frac{R_1}{R_2}$ 處的相位裕度

Commonly Used Feedback Configurations

Non Inverting Amplifier:



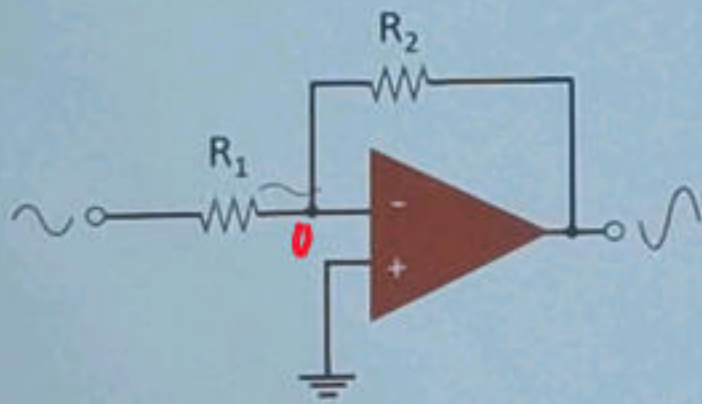
feedback factor β (H):

$$\beta = \frac{R_2}{R_1 + R_2}$$

$$A = \frac{1}{\beta} = 1 + \frac{R_1}{R_2}$$

Commonly Used Feedback Configurations

Inverting Amplifier:



$$\beta = \frac{R_1}{R_2 + R_1}$$

$$A = -\frac{R_2}{R_1}$$

