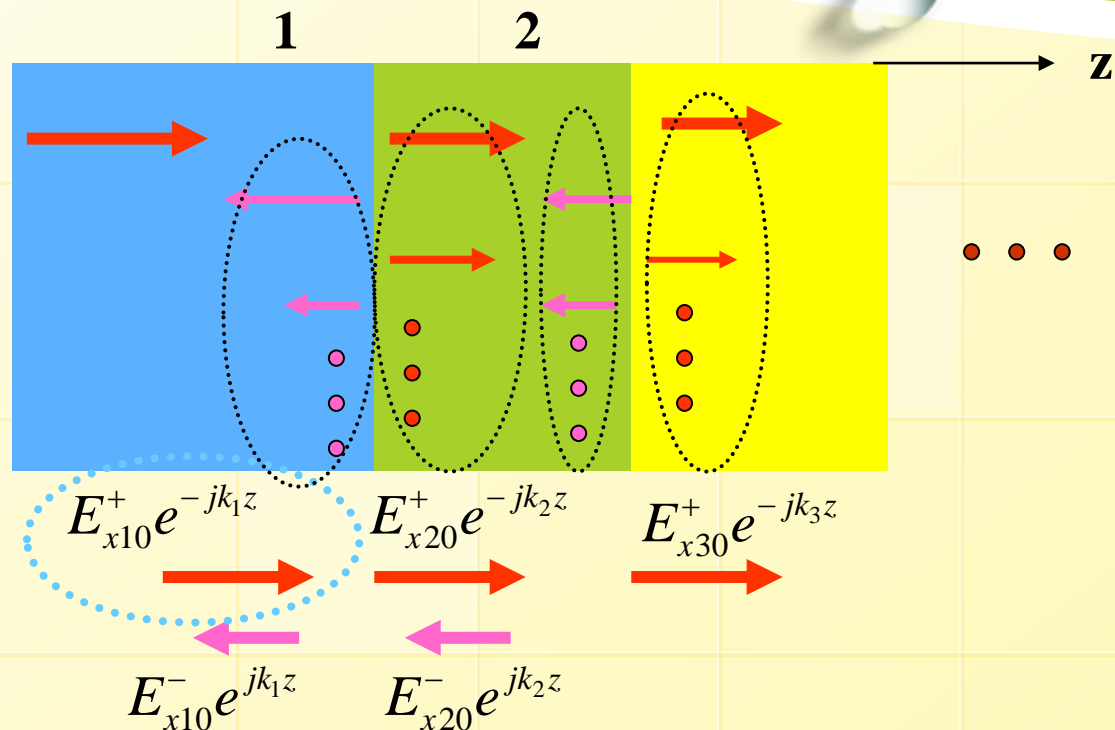


## 4.9 在多层电介质上的垂直入射



### 方法一：直接分析

- (1) 写出已知的入射波
- (2) 写出各层中的正向波和反向波的一般表达式，共有 $2n-1$ 个波， **$2n-2$** 个未知量
- (3) 写出 $n-1$ 个边界上的电场和磁场的共 **$2n-2$** 个边界条件方程
- (4) 解边界条件方程，得到 $2n-2$ 个未知的各区电磁波复振幅

## 4.9 在多层电介质上的垂直入射

### 方法二：总场波阻抗

➤ 单一无界媒质的波阻抗

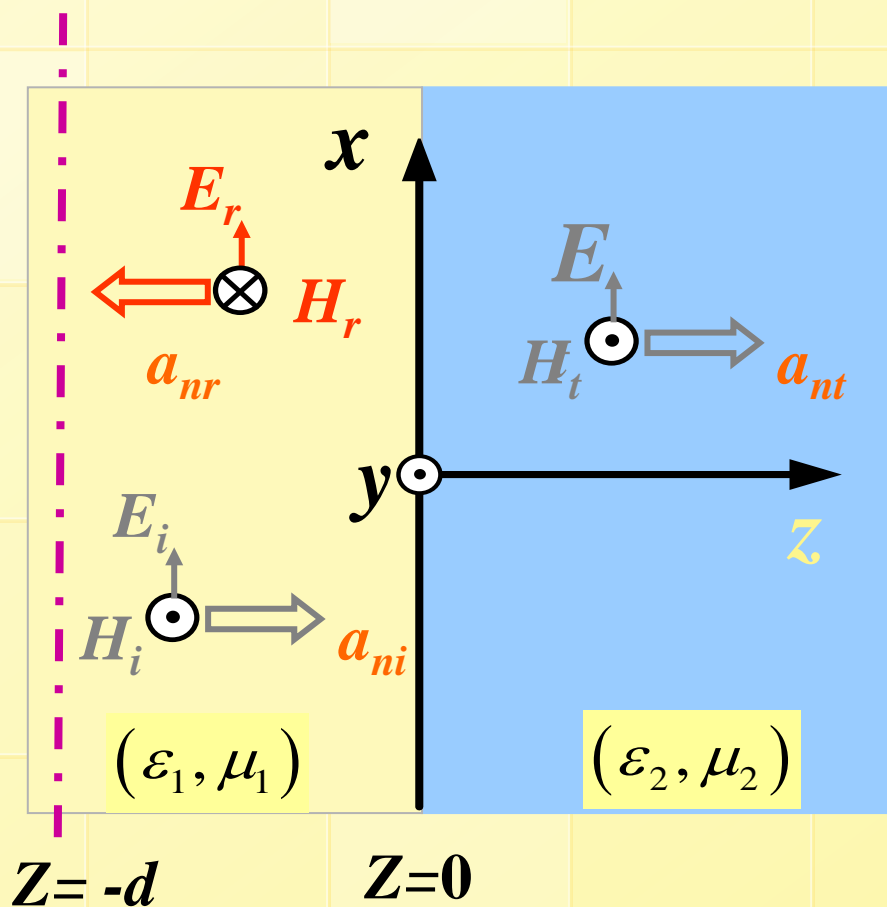
$$\eta = \frac{E_x^+(z)}{H_y^+(z)} = -\frac{E_x^-(z)}{H_y^-(z)}$$

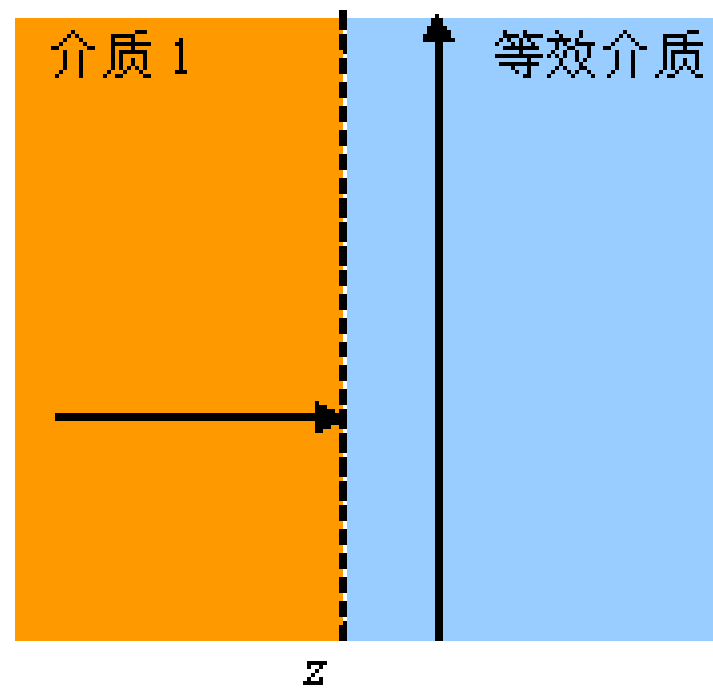
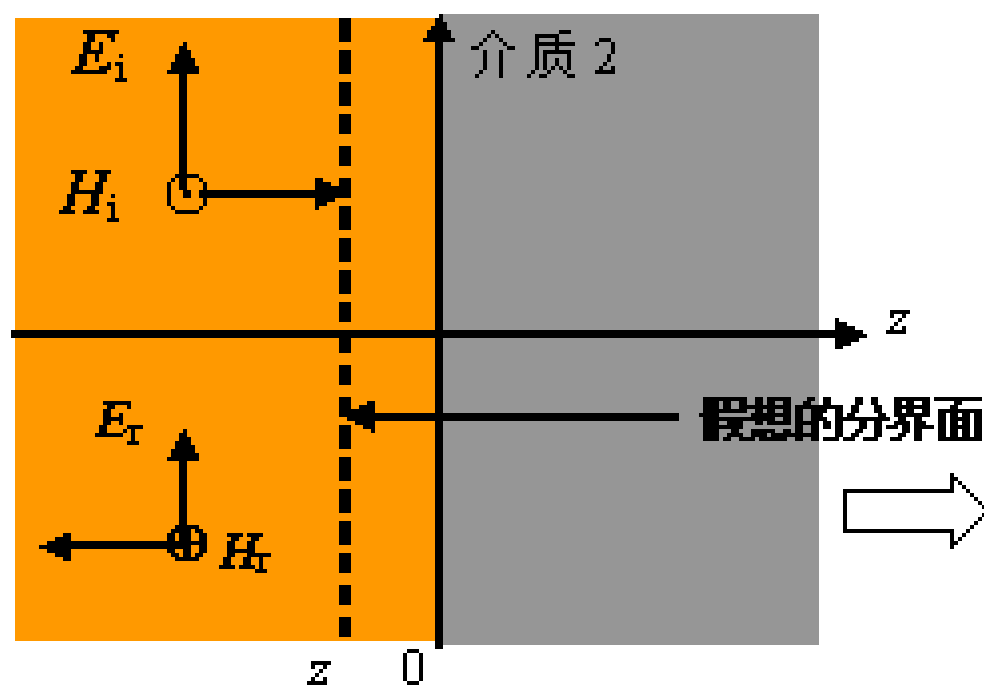
➤ 两种有界媒质的波阻抗

等效（入端、总场）波阻抗

$$Z(z) = \eta_1 \frac{e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}}{e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}}$$

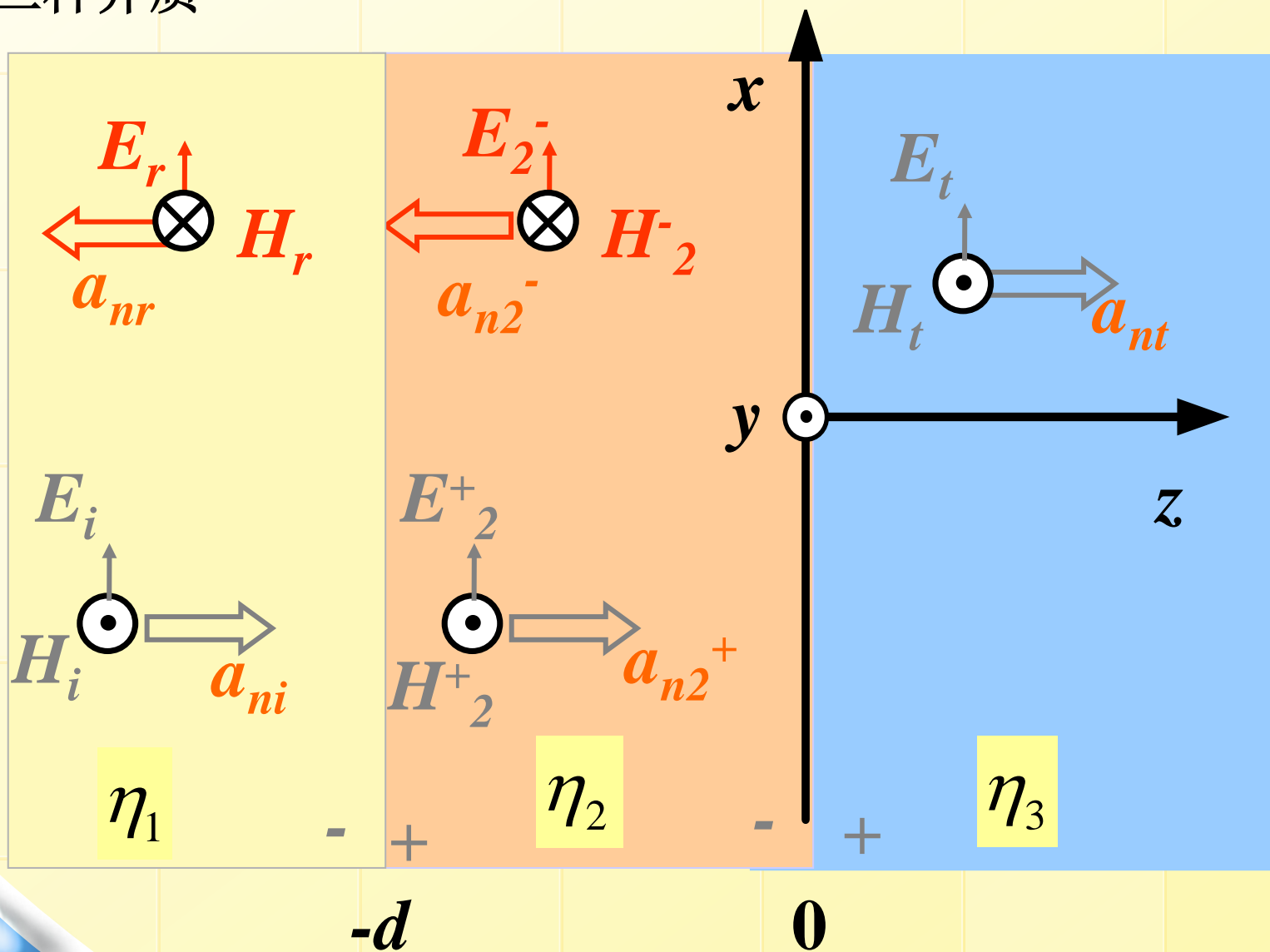
$$Z(-d) = \eta_1 \frac{\eta_2 \cos \beta_1 d + j\eta_1 \sin \beta_1 d}{\eta_1 \cos \beta_1 d + j\eta_2 \sin \beta_1 d}$$



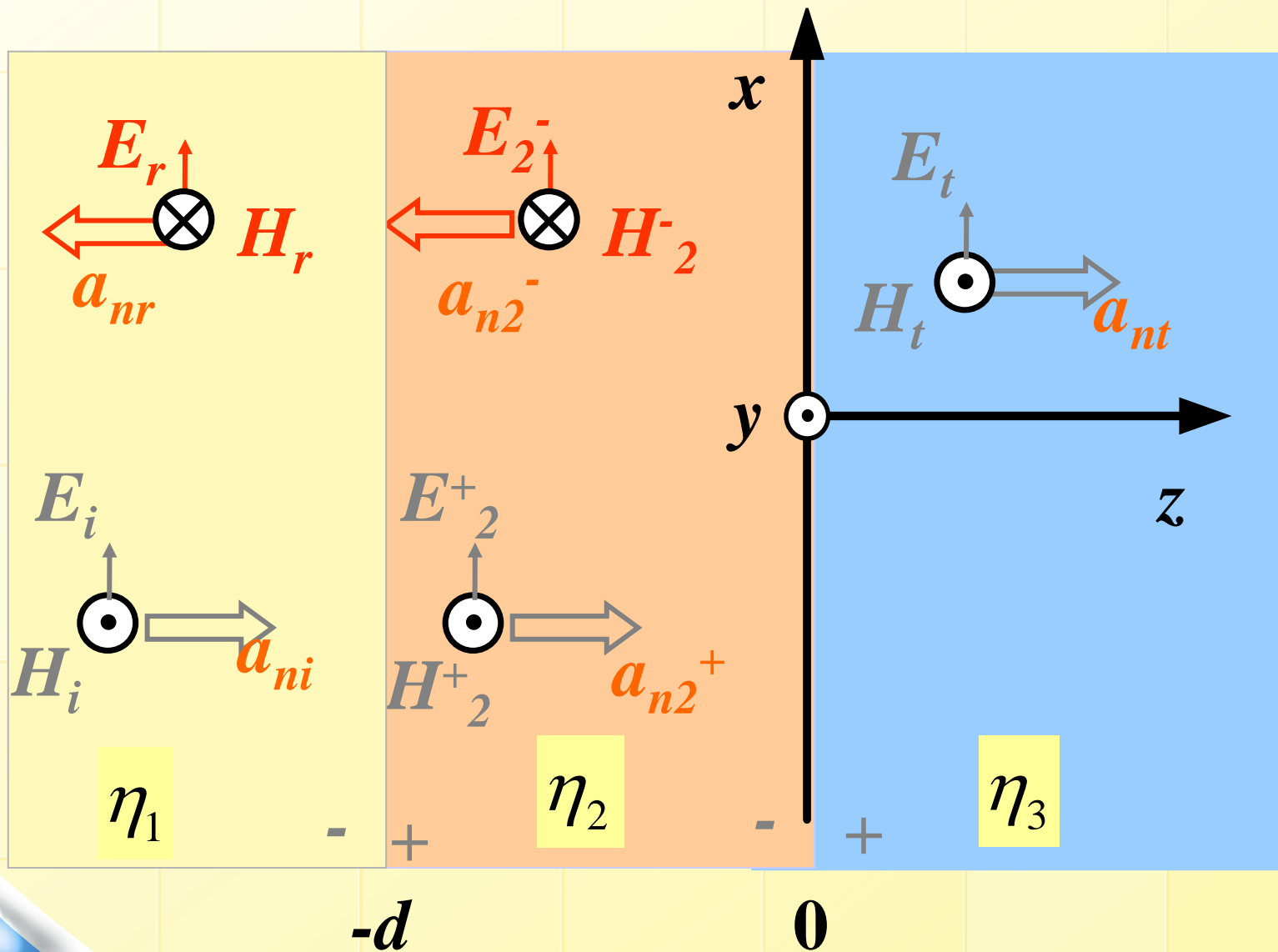


波阻抗等效

三种介质



【例4.9】：已知波阻抗 $\eta_1$ 和 $\eta_3$ ，求当介质1中的均匀平面波正入射到介质2的界面时，不发生反射时的 $d$ 及 $\eta_2$ 。



$$Z(-d) = \eta_2 \frac{\eta_3 \cos \beta_2 d + j \eta_2 \sin \beta_2 d}{\eta_2 \cos \beta_2 d + j \eta_3 \sin \beta_2 d} = \eta_1$$

$$\Rightarrow \begin{cases} \eta_3 \cos \beta_2 d = \eta_1 \cos \beta_2 d \\ \eta_2^2 \sin \beta_2 d = \eta_1 \eta_3 \sin \beta_2 d \end{cases}$$

1) 当  $\eta_1 = \eta_3$  时,  $\sin \beta_2 d = 0$ , 因此  $d = \frac{n}{2} \lambda_2$  ( $n = 1, 2, 3 \dots$ )

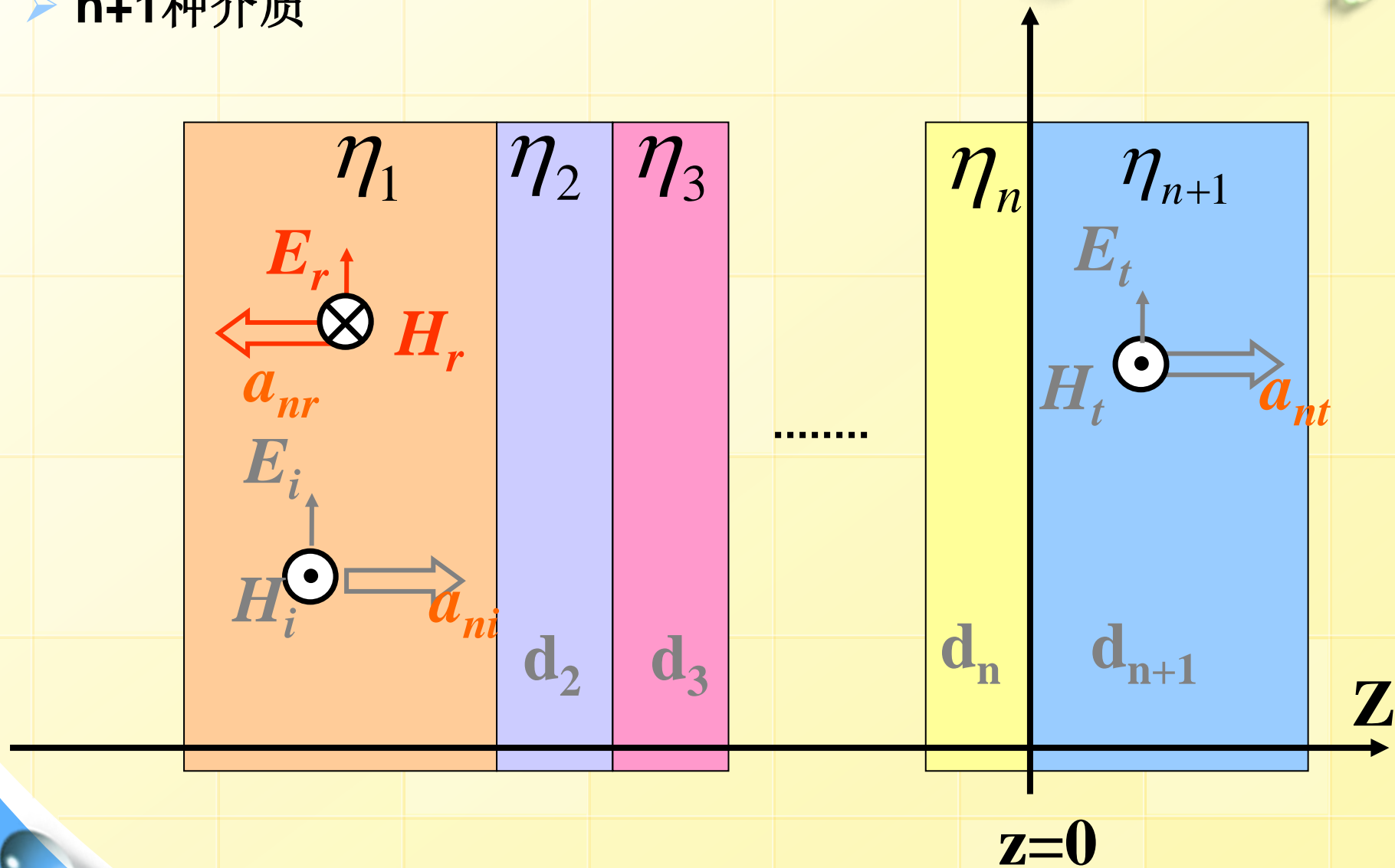
对给定的工作频率, 当介质层厚度为**介质中半波长**的整数倍时, 无反射发生, 因此这种介质层称为**半波介质窗**

2) 当  $\eta_1 \neq \eta_3$  时,  $\cos \beta_2 d = 0$ , 且  $\eta_2 = \sqrt{\eta_1 \eta_3}$

因此,  $d = \frac{2n+1}{4} \lambda_2$  ( $n = 0, 1, 2 \dots$ )

**四分之一波长阻抗变换器**

➤  $n+1$ 种介质



## ➤应用

如何克服分界面对电磁波反射而使得电磁波全部或者大部分能量透射，在实际中具有广泛的应用





## 4.10 在电介质平面边界上的倾斜入射

入射波:

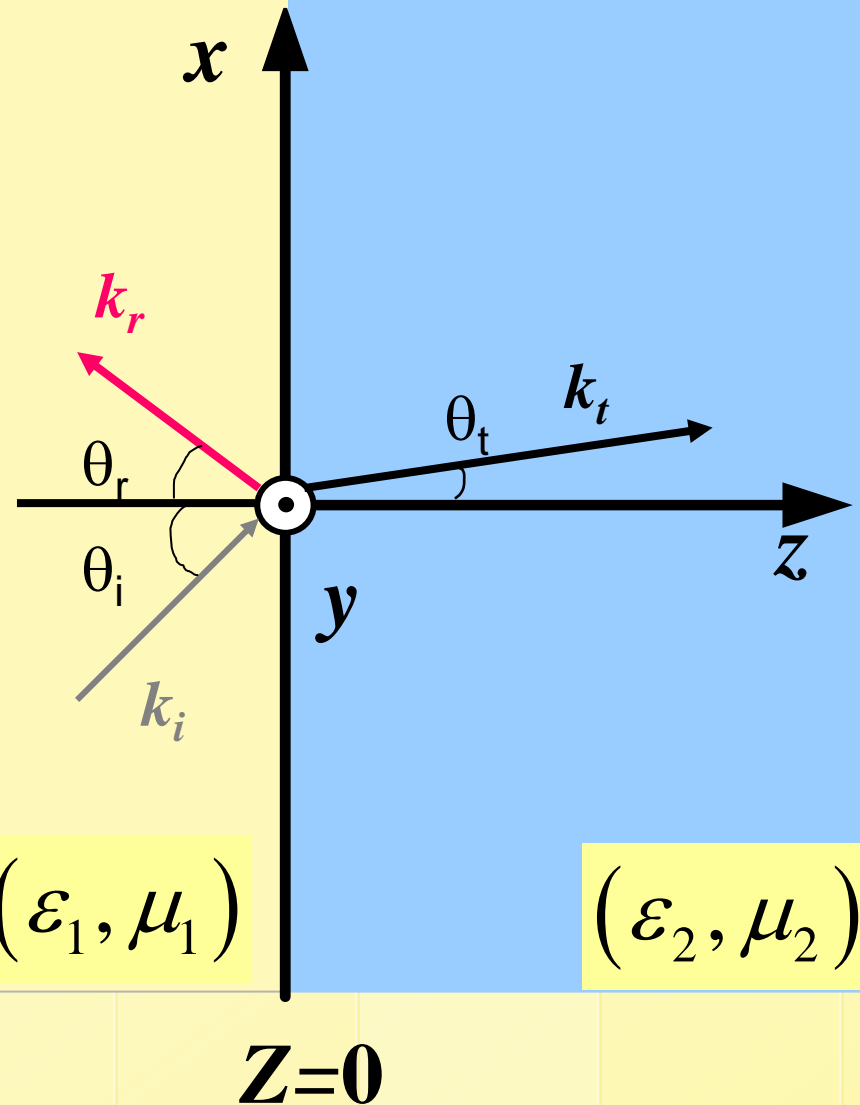
$$\vec{E}_i(\vec{r}) = \vec{E}_{i0} e^{-jk_1(x \sin \theta_i + z \cos \theta_i)}$$

反射波:

$$\vec{E}_r(\vec{r}) = \vec{E}_{r0} e^{-jk_1(x \sin \theta_r - z \cos \theta_r)}$$

透射波:

$$\vec{E}_t(\vec{r}) = \vec{E}_{t0} e^{-jk_2(x \sin \theta_t + z \cos \theta_t)}$$



### 4.10.1 倾斜入射的一般规律（相位匹配和斯涅尔定律）

由于分界面（ $z = 0$ ）上电场切向分量连续，得

$$\vec{e}_z \times [\vec{E}_{i0} e^{-jk_1 x \sin \theta_i} + \vec{E}_{r0} e^{-jk_1 x \sin \theta_r}] = \vec{e}_z \times \vec{E}_{t0} e^{-jk_2 x \sin \theta_t}$$

上述等式对于任意  $x$  均应成立，因此各项指数中对应的系数应该相等，即

$$k_1 \sin \theta_i = k_1 \sin \theta_r = k_2 \sin \theta_t$$

此式表明反射波及透射波的相位沿分界面的变化始终与入射波保持一致。因此，该式又称为分界面上的相位匹配条件。

由  $k_1 \sin \theta_i = k_1 \sin \theta_r$  , 得

$$\theta_r = \theta_i$$

—— 反射角  $\theta_r$  等于入射角  $\theta_i$   
(**Snell**反射定律)

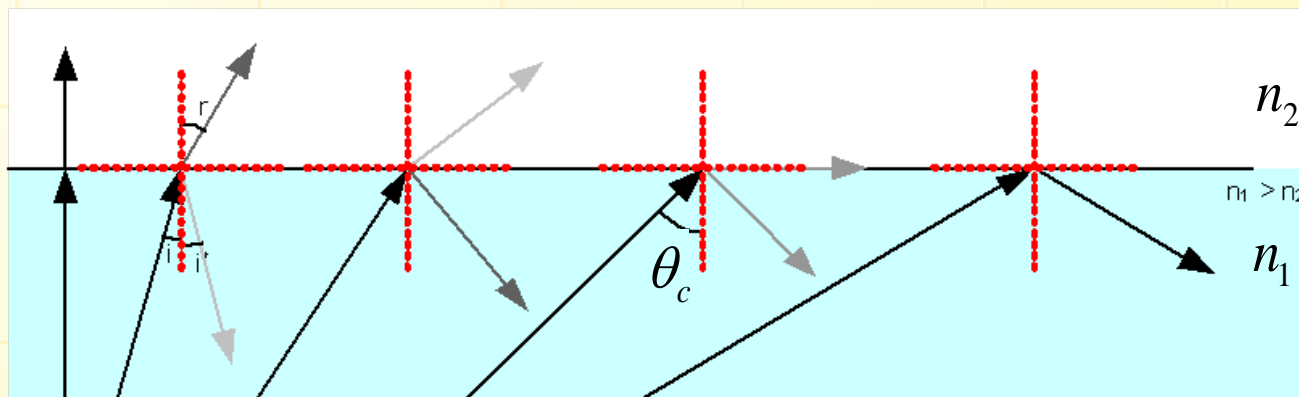
由  $k_1 \sin \theta_i = k_2 \sin \theta_t$  , 得

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{k_2}{k_1} = \frac{\omega \sqrt{\mu_2 \epsilon_2}}{\omega \sqrt{\mu_1 \epsilon_1}} = \frac{n_2}{n_1}$$

—— 折射角  $\theta_t$  与入射角  $\theta_i$  的关系  
(**Snell**折射定律)

上述两条结论总称为Snell定律，在电磁波反射和折射现象中广泛应用。

## 全反射



### 临界角

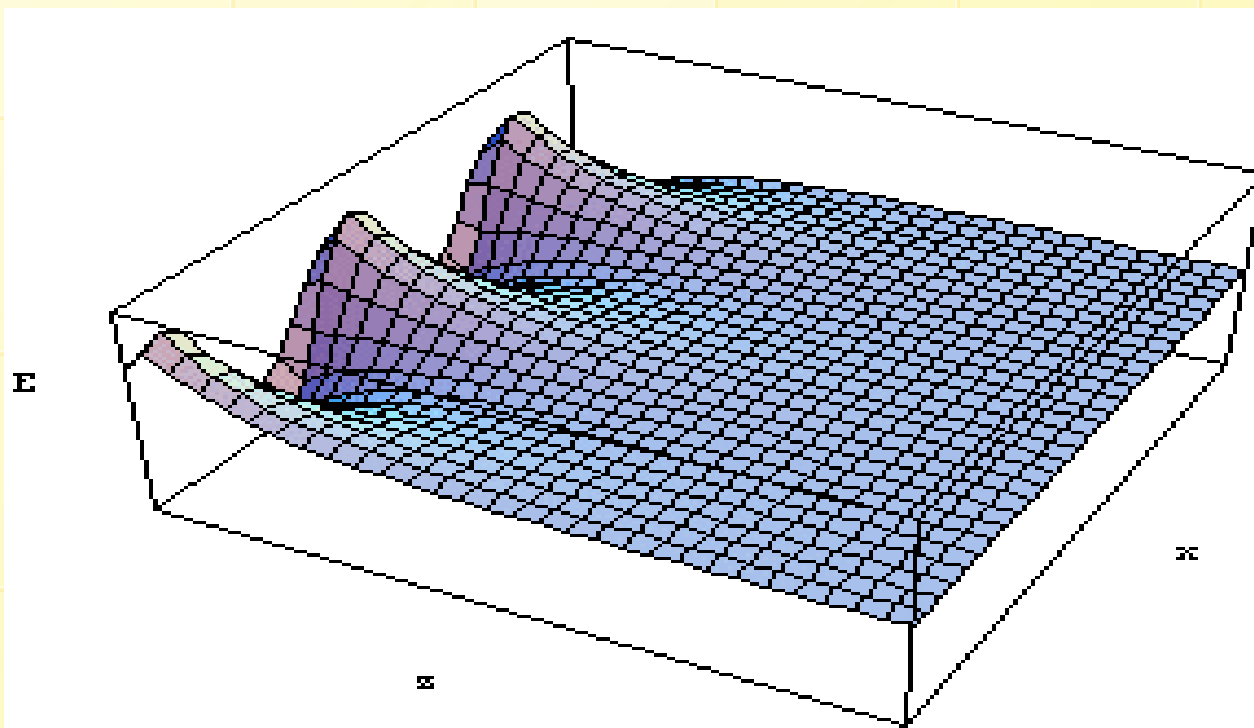
使透射角等于  $\frac{\pi}{2}$  的入射角称为临界角。

$$\theta_c = \arcsin \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \arcsin \frac{n_2}{n_1}$$

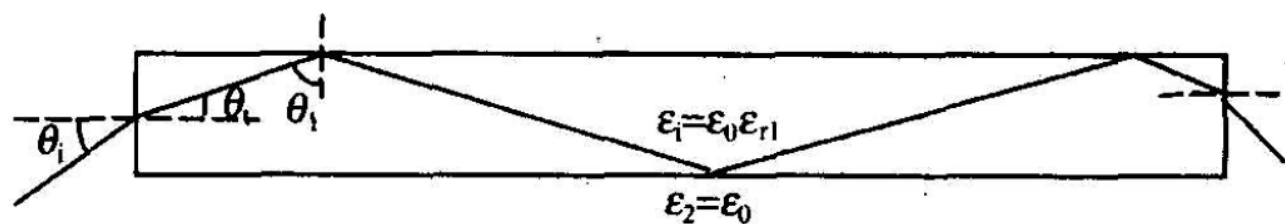
## ➤ 全反射

### 表面波

透射电磁波是沿表面传播的行波，其振幅沿法线方向衰减，场量主要集中在表面附近，故称为表面波。



**【例 4.10】** 图示的一根介质棒或透明材料的纤维可在全内反射的条件下用来导引光或电磁波。要求波无论从任何角度入射到它的一端以后,都全部被约束在介质棒内,直至传输到另外一端。这种导波媒质的介电常数最小应为多少?



**解:**

全内反射,无论  $\theta_i$  为何值,  $\theta_1$  都应大于全反射角  $\theta_c$ 。所以  $\sin\theta_1 \geq \sin\theta_c$ 。

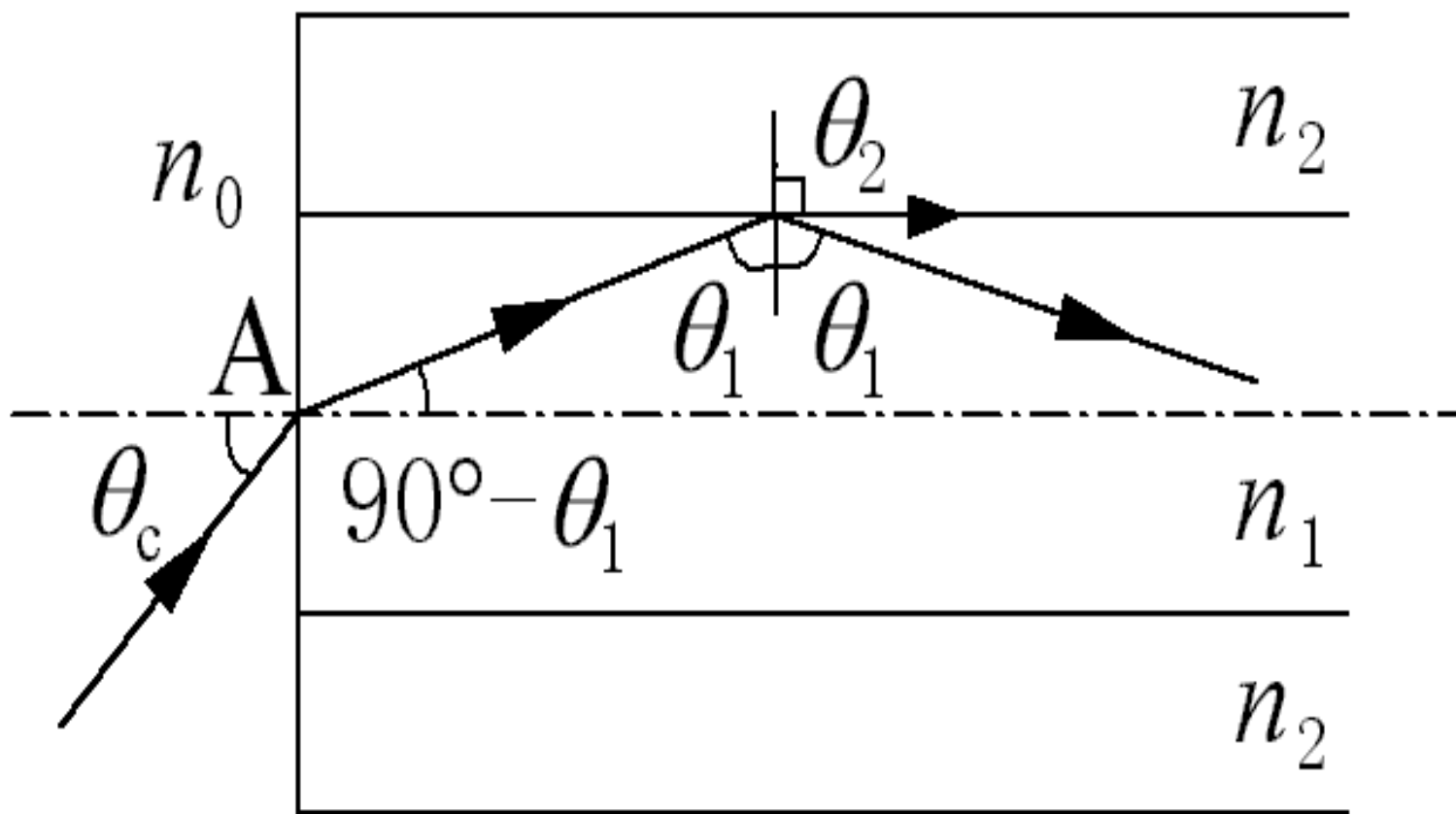
根据折射定律 
$$\sin\theta_t = \frac{1}{\sqrt{\epsilon_{r1}}} \sin\theta_i$$

有  $\theta_1 = \frac{\pi}{2} - \theta_t$ , 则 
$$\sqrt{1 - \frac{1}{\epsilon_{r1}} \sin^2 \theta_i} \geq \frac{1}{\sqrt{\epsilon_{r1}}}$$

由此可得 
$$\epsilon_{r1} \geq 1 + \sin^2 \theta_i$$

当  $\theta_i = \frac{\pi}{2}$  时,上式右方达到最大值,所以要求介质的介电常数至少等于 2,相应的折射率  $n_1 = \sqrt{2}$ 。玻璃和石英一般都能满足上述要求。

**练习：**光纤是芯径极细，外涂包层的二氧化硅介质棒，其沿轴线的子午面如图所示。 $n_1$ 为光纤芯的折射率， $n_2$ 为包层的折射率。 $n_1$ 略大于 $n_2$ 。试求：光在光纤中持续传输的最大入射角 $\theta_c$ 。(已知在空气中 $n_0 = 1$ )



## 4.10.2 垂直极化波倾斜入射的反射与透射

媒质1中的入射波:

由于  $\vec{k}_i = \hat{a}_i k_1, \quad k_1 = \omega \sqrt{\mu_1 \epsilon_1}$

$$\hat{a}_i = \hat{a}_x \sin \theta_i + \hat{a}_z \cos \theta_i$$

$$\vec{r} = \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$$

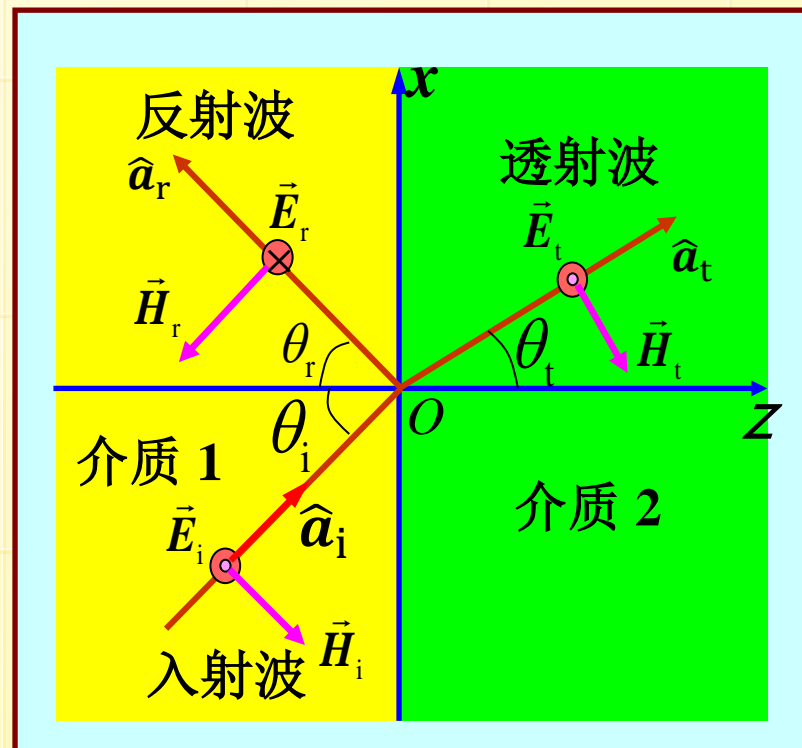
故

$$\vec{E}_i(\vec{r}) = \hat{a}_y E_{i0} e^{-jk_1(x \sin \theta_i + z \cos \theta_i)}$$

$$\vec{H}_i(\vec{r}) = \frac{1}{\eta_1} \hat{a}_i \times \vec{E}_i(\vec{r})$$

$$= \frac{1}{\eta_1} (\hat{a}_x \sin \theta_i + \hat{a}_z \cos \theta_i) \times \hat{a}_y E_{i0} e^{-jk_1(x \sin \theta_i + z \cos \theta_i)}$$

$$= \frac{E_{i0}}{\eta_1} (-\hat{a}_x \cos \theta_i + \hat{a}_z \sin \theta_i) e^{-jk_1(x \sin \theta_i + z \cos \theta_i)}$$





媒质1中的反射波:

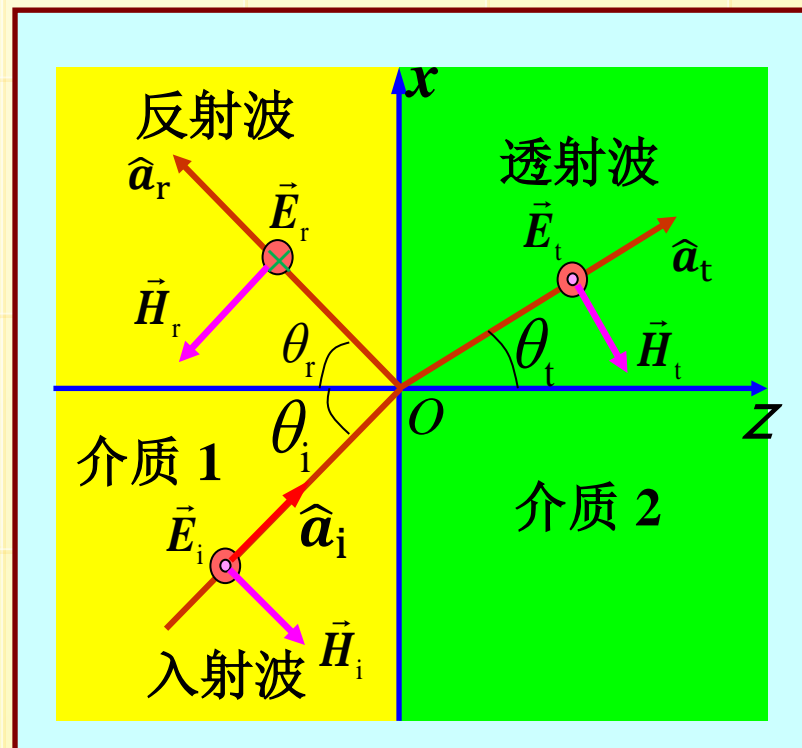
由于  $\vec{k}_r = \hat{a}_r k_1, \quad k_1 = \omega \sqrt{\mu_1 \epsilon_1}$

$$\hat{a}_r = \hat{a}_x \sin \theta_i - \hat{a}_z \cos \theta_i$$

故

$$\begin{aligned} \vec{E}_r(\vec{r}) &= -\hat{a}_y E_{r0} e^{-j\vec{k}_r \cdot \vec{r}} \\ &= -\hat{a}_y E_{r0} e^{-jk_1(x \sin \theta_i - z \cos \theta_i)} \end{aligned}$$

$$\begin{aligned} \vec{H}_r(\vec{r}) &= \frac{1}{\eta_1} \hat{a}_r \times \vec{E}_r(\vec{r}) \\ &= -\frac{1}{\eta_1} (\hat{a}_x \sin \theta_r - \hat{a}_z \cos \theta_r) \times \hat{a}_y E_{r0} e^{-jk_1(x \sin \theta_r - z \cos \theta_r)} \\ &= \frac{E_{r0}}{\eta_1} (-\hat{a}_z \sin \theta_r - \hat{a}_x \cos \theta_r) e^{-jk_1(x \sin \theta_r - z \cos \theta_r)} \end{aligned}$$



## 媒质2中的透射波:

由于  $\vec{k}_t = k_2 \hat{a}_t, \quad k_2 = \omega \sqrt{\mu_2 \epsilon_2}$

$$\hat{a}_t = \hat{a}_x \sin \theta_t + \hat{a}_z \cos \theta_t$$

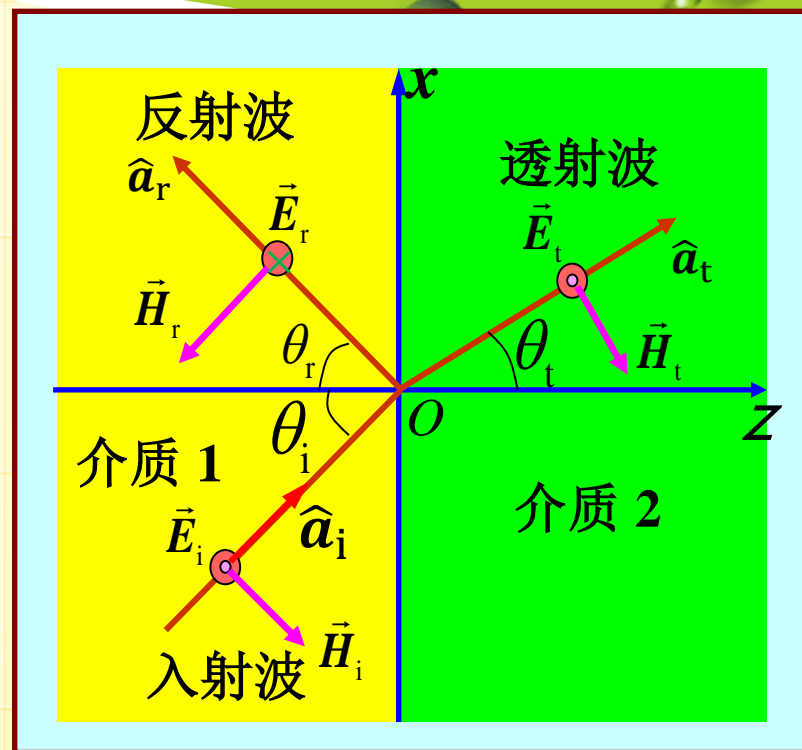
$$\vec{r} = \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$$

故  $\vec{E}_t(\vec{r}) = \hat{a}_y E_{t0} e^{-jk_2(x \sin \theta_t + z \cos \theta_t)}$

$$\vec{H}_t(\vec{r}) = \frac{1}{\eta_2} \hat{a}_t \times \vec{E}_t(\vec{r})$$

$$= \frac{1}{\eta_2} (\hat{a}_x \sin \theta_t + \hat{a}_z \cos \theta_t) \times \hat{a}_y E_{t0} e^{-jk_2(x \sin \theta_t + z \cos \theta_t)}$$

$$= \frac{E_{t0}}{\eta_2} (-\hat{a}_x \cos \theta_t + \hat{a}_z \sin \theta_t) e^{-jk_2(x \sin \theta_t + z \cos \theta_t)}$$



其中, 由反射/折射定律:  $\theta_i = \theta_r, \quad k_1 \sin \theta_i = k_2 \sin \theta_t$

分界面上电场强度和磁场强度的切向分量连续，有

$$\mathbf{E}_{iy}(x, 0) + \mathbf{E}_{ry}(x, 0) = \mathbf{E}_{ty}(x, 0) \quad \Rightarrow \quad 1 - \Gamma_{\perp} = \tau_{\perp}$$

$$\mathbf{H}_{ix}(x, 0) + \mathbf{H}_{rx}(x, 0) = \mathbf{H}_{tx}(x, 0) \quad \Rightarrow \quad \frac{\cos \theta_i}{\eta_1} (1 + \Gamma_{\perp}) = \frac{\cos \theta_t}{\eta_2} \tau_{\perp}$$

$$\Rightarrow \begin{cases} \Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = -\frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \\ \tau_{\perp} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \end{cases}$$

菲涅尔公式

对于非磁性介质， $\mu_1 = \mu_2 = \mu_0$ ，则

$$\frac{\eta_1}{\eta_2} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}, \quad \sin \theta_t = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin \theta_i \quad \Rightarrow \quad \begin{cases} \Gamma_{\perp} = \frac{\cos \theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2 \theta_i}} \\ \tau_{\perp} = \frac{2 \cos \theta_i}{\cos \theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2 \theta_i}} \end{cases}$$

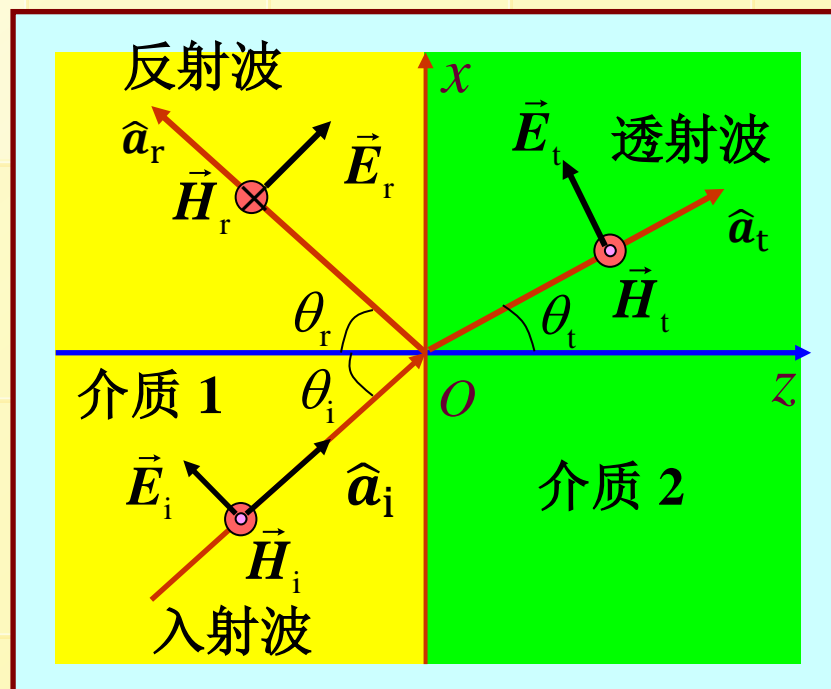
### 4.10.3 平行极化波倾斜入射的反射与透射

#### ■ 媒质1中的入射波

由于  $\vec{k}_i = \hat{a}_i k_1 = \hat{a}_i \omega \sqrt{\mu_1 \epsilon_1}$ ,  $\hat{a}_i = \hat{a}_x \sin \theta_i + \hat{a}_z \cos \theta_i$

故  $\vec{E}_i(\vec{r}) = E_{i0}(\hat{a}_x \cos \theta_i - \hat{a}_z \sin \theta_i) e^{-jk_1(x \sin \theta_i + z \cos \theta_i)}$

$$\begin{aligned}\vec{H}_i(\vec{r}) &= \frac{1}{\eta_1} \hat{a}_i \times \vec{E}_i(\vec{r}) \\ &= \hat{a}_y \frac{E_{i0}}{\eta_1} e^{-jk_1(x \sin \theta_i + z \cos \theta_i)}\end{aligned}$$

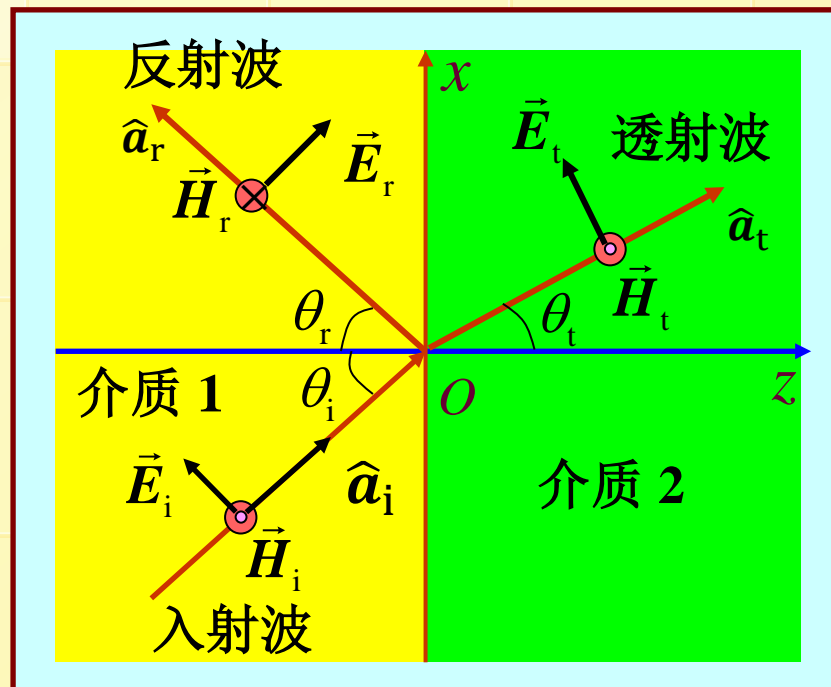


## ■ 媒质1中的反射波

由于  $\vec{k}_r = \hat{a}_r k_1$ ,  $k_1 = \omega \sqrt{\mu_1 \epsilon_1}$ ,  $\hat{a}_r = \hat{a}_x \sin \theta_i - \hat{a}_z \cos \theta_i$

故  $\vec{E}_r(\vec{r}) = (\hat{a}_z \sin \theta_i + \hat{a}_x \cos \theta_i) E_{r0} e^{-jk_1(x \sin \theta_i - z \cos \theta_i)}$

$$\vec{H}_r(\vec{r}) = \frac{1}{\eta_1} \hat{a}_r \times \vec{E}_r(\vec{r}) = -\hat{a}_y \frac{E_{r0}}{\eta_1} e^{-jk_1(x \sin \theta_r - z \cos \theta_r)}$$



## ■ 媒质2中的透射波

$$\vec{k}_t = k_2 \hat{a}_t, \quad k_2 = \omega \sqrt{\mu_2 \epsilon_2}, \quad \hat{a}_t = \hat{a}_x \sin \theta_t + \hat{a}_z \cos \theta_t$$

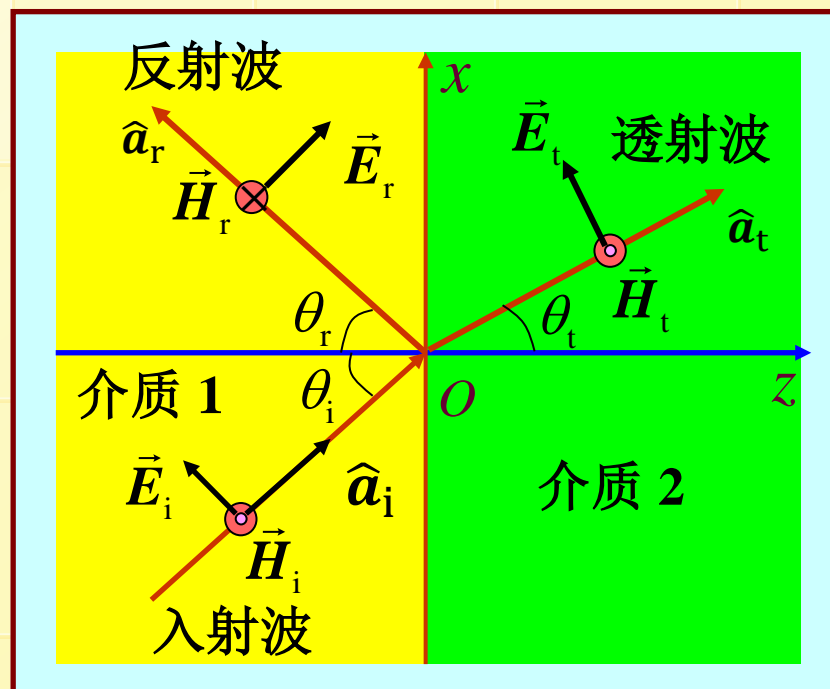
$$\vec{E}_t(\vec{r}) = E_{t0}(\hat{a}_x \cos \theta_t - \hat{a}_z \sin \theta_t) e^{-jk_2(x \sin \theta_t + z \cos \theta_t)}$$

$$\begin{aligned} \vec{H}_t(\vec{r}) &= \frac{1}{\eta_2} \hat{a}_t \times \vec{E}_t(\vec{r}) \\ &= \hat{a}_y \frac{E_{t0}}{\eta_2} e^{-jk_2(x \sin \theta_t + z \cos \theta_t)} \end{aligned}$$

其中，根据反射、折射定律：

$$\theta_i = \theta_r$$

$$k_1 \cos \theta_i = k_2 \cos \theta_t$$



**分界面上电场强度和磁场强度切向分量连续，即**

$$\mathbf{E}_{ix}(x, 0) + \mathbf{E}_{rx}(x, 0) = \mathbf{E}_{tx}(x, 0) \quad \Rightarrow \quad (1 + \Gamma_{//}) \cos \theta_i = \tau_{//} \cos \theta_t$$

$$\mathbf{H}_{iy}(x, 0) + \mathbf{H}_{ry}(x, 0) = \mathbf{H}_{ty}(x, 0) \quad \Rightarrow \quad \frac{1}{\eta_1} (1 + \Gamma_{//}) = \frac{1}{\eta_2} \tau_{//}$$

$$\Rightarrow \begin{cases} \Gamma_{//} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} \\ \tau_{//} = \frac{2\eta_2 \cos \theta_i}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t} \end{cases}$$

菲涅尔公式

**对于非磁性介质， $\mu_1 = \mu_2 = \mu_0$ ，则**  $\frac{\eta_1}{\eta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$ ,  $\sin \theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i$

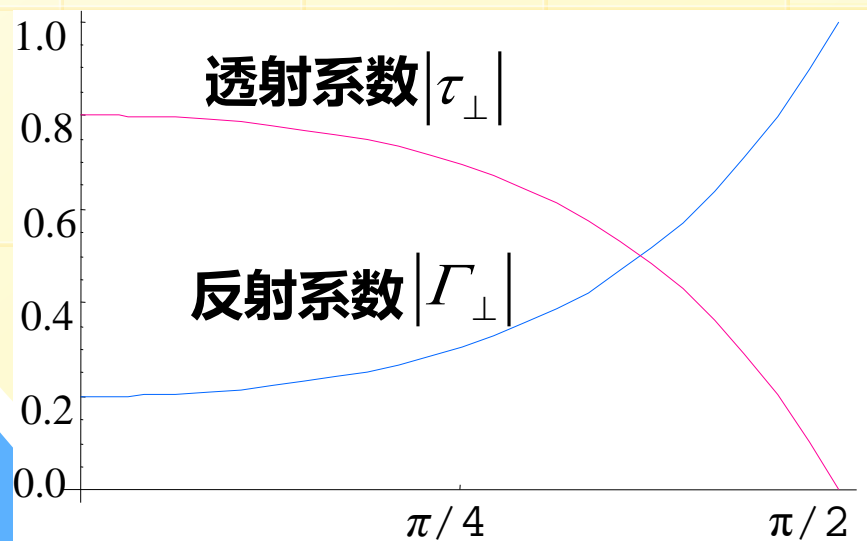
$$\Rightarrow \begin{cases} \Gamma_{//} = \frac{\sqrt{(\frac{\epsilon_2}{\epsilon_1}) - \sin^2 \theta_i} - (\frac{\epsilon_2}{\epsilon_1}) \cos \theta_i}{(\frac{\epsilon_2}{\epsilon_1}) \cos \theta_i + \sqrt{(\frac{\epsilon_2}{\epsilon_1}) - \sin^2 \theta_i}} \\ \tau_{//} = \frac{2\sqrt{(\frac{\epsilon_2}{\epsilon_1})} \cos \theta_i}{(\frac{\epsilon_2}{\epsilon_1}) \cos \theta_i + \sqrt{(\frac{\epsilon_2}{\epsilon_1}) - \sin^2 \theta_i}} \end{cases}$$

## 不同入射方向的反射系数和透射系数

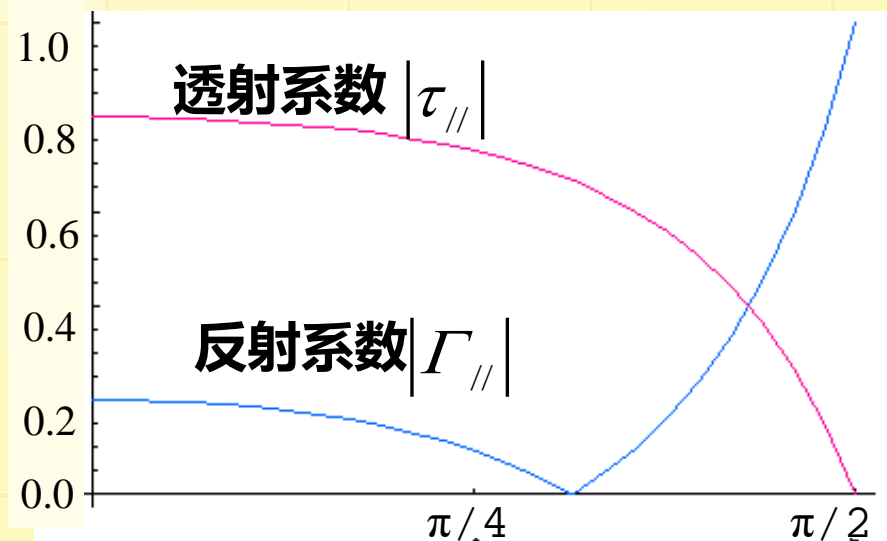
$$\begin{cases} \Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = -\frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \\ \tau_{\perp} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \end{cases}$$

$$\begin{cases} \Gamma_{//} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} \\ \tau_{//} = \frac{2\eta_2 \cos \theta_i}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t} \end{cases}$$

假设:  $\varepsilon_1 = \varepsilon_0$ ,  $\varepsilon_2 = 2.25\varepsilon_0$ ,  $\mu_1 = \mu_2 = \mu_0$



垂直极化波的入射角



平行极化波的入射角



➤ 布儒斯特角（全透射）：

1) 垂直极化波

$$\eta_2 \cos \theta_{B\perp} - \eta_1 \cos \theta_t = 0 \quad \Rightarrow \quad \sin^2 \theta_{B\perp} = \frac{1 - \frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}$$

非磁性介质：

垂直极化时不存在反射为零的布儒斯特角

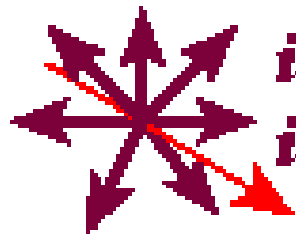
2) 平行极化波

$$\eta_2 \cos \theta_t - \eta_1 \cos \theta_{B//} = 0 \quad \Rightarrow \quad \sin^2 \theta_{B//} = \frac{1 - \frac{\mu_2 \varepsilon_1}{\mu_1 \varepsilon_2}}{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}$$

结论（非磁性介质）：

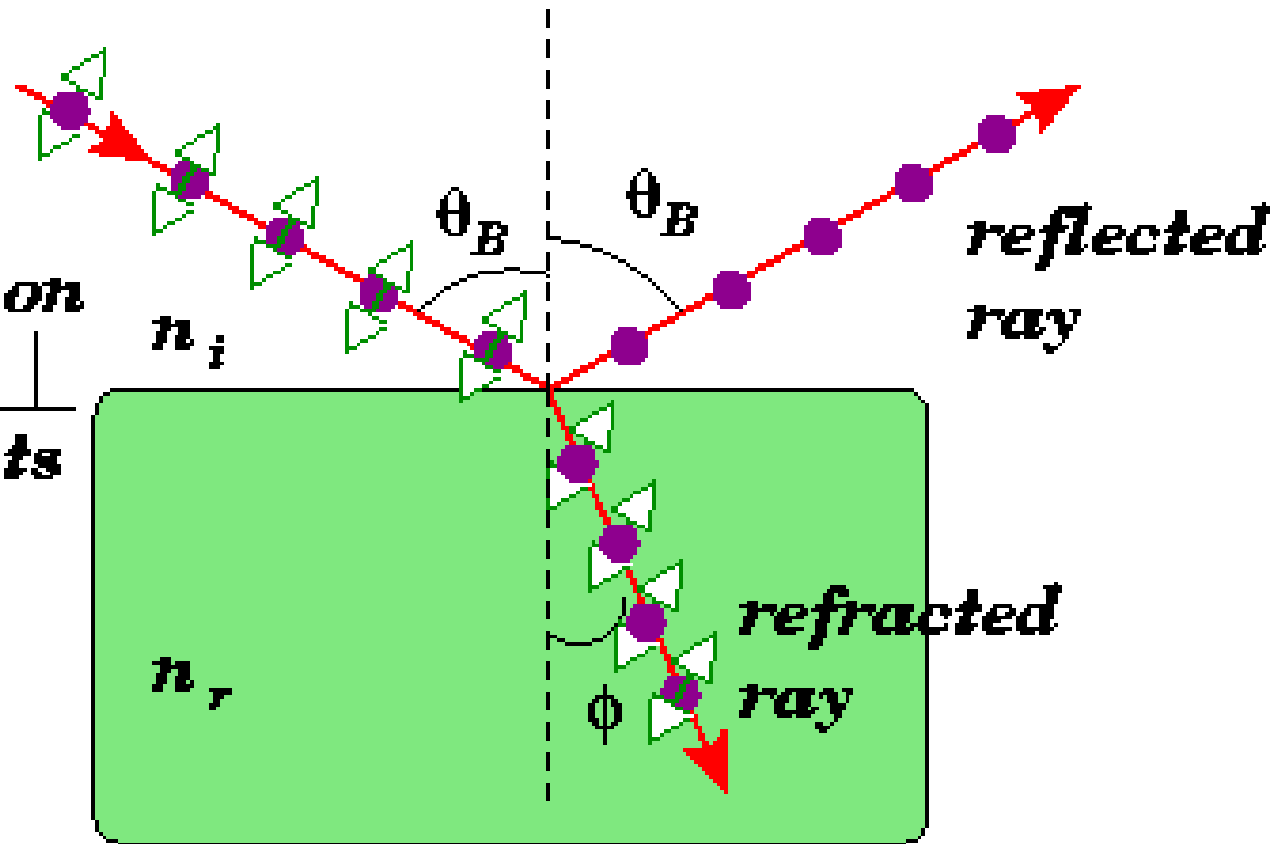
平行极化时可发生全透射现象，  
此时入射角称为布儒斯特角。

$$\theta_{B//} = \arctan \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$



*incident light  
is unpolarized*

*resolve  
polarization  
into  $\parallel$  and  $\perp$   
components*



Light incident on a reflecting surface of dielectric material is partially polarized. Light incident **at the Brewster angle** is fully polarized

