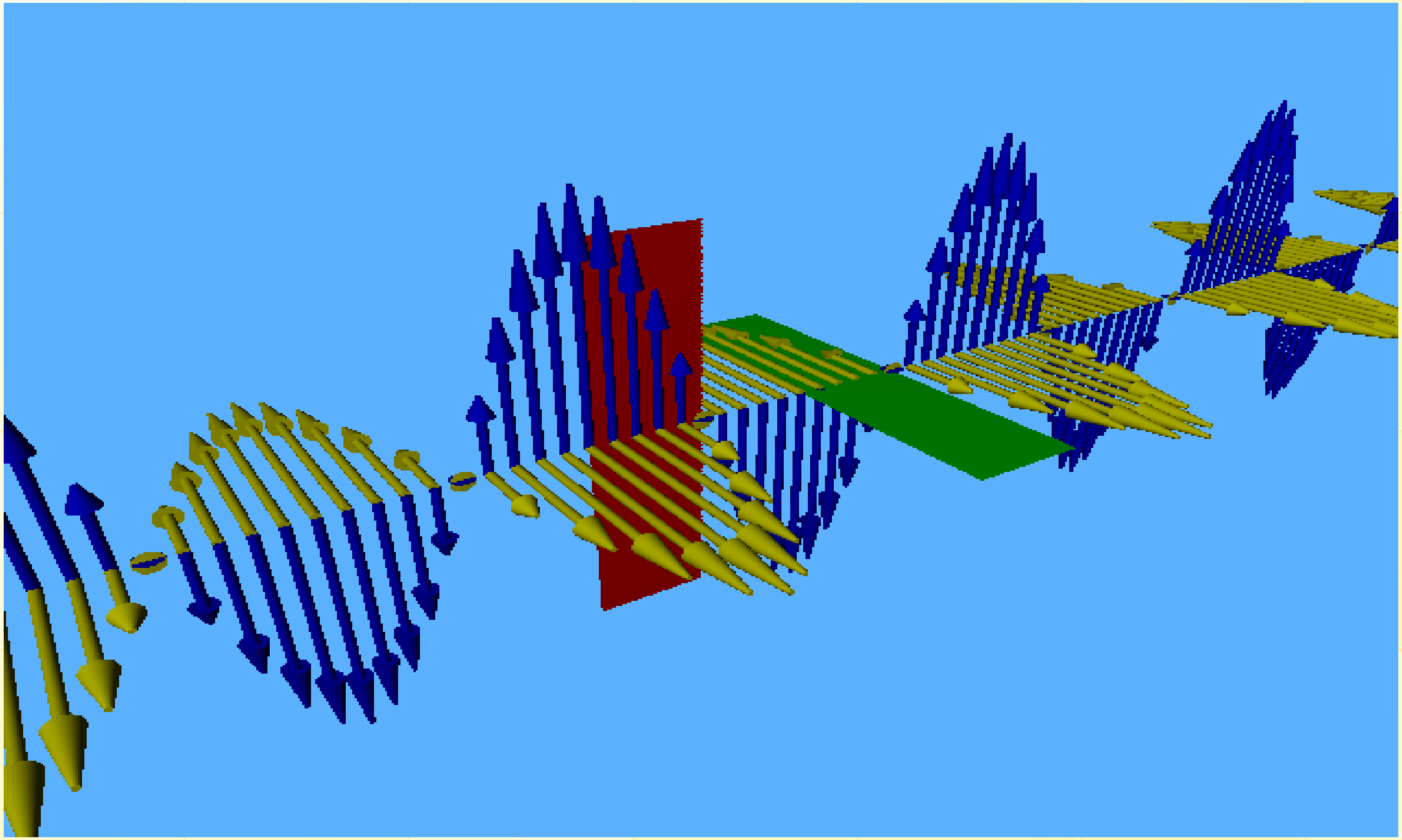
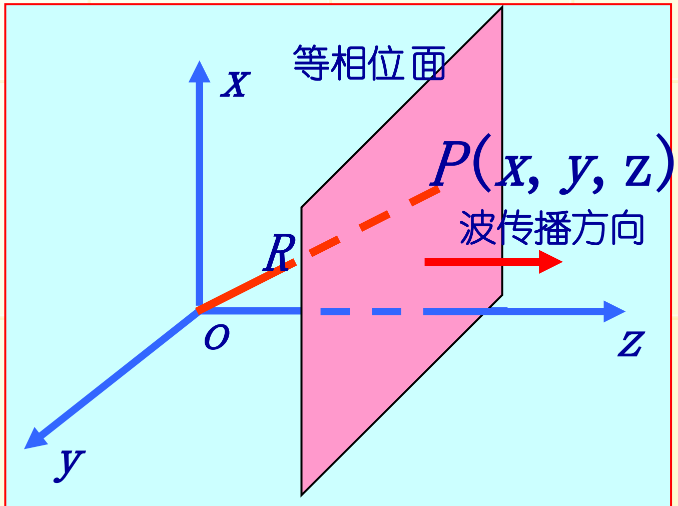
4.3 横电磁波（TEM波）

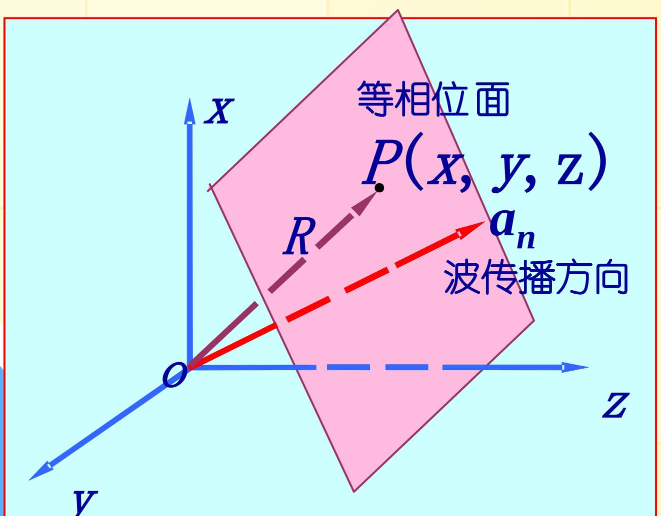
（Polarization of plane wave）



沿任意方向传播的均匀平面波：



沿+z方向传播的均匀平面波



沿任意方向传播的均匀平面波

定义波矢：$\boldsymbol{k} = k\boldsymbol{a}_n = \boldsymbol{a}_x k_x + \boldsymbol{a}_y k_y + \boldsymbol{a}_z k_z$

$$\boldsymbol{E}(z) = \boldsymbol{E}_0 e^{-jkz} = \boldsymbol{E}_0 e^{-jk\boldsymbol{a}_z \cdot \boldsymbol{R}} = \boldsymbol{E}_0 e^{-j\boldsymbol{k} \cdot \boldsymbol{R}}$$

$$\boldsymbol{k} = \boldsymbol{a}_z k$$

$$\boldsymbol{a}_z \cdot \boldsymbol{E}_0 = 0$$

$$\boldsymbol{H}(z) = \frac{1}{\eta} \boldsymbol{a}_z \times \boldsymbol{E}(z)$$

$$\boldsymbol{E}(\boldsymbol{R}) = \boldsymbol{E}_0 e^{-j\boldsymbol{k} \cdot \boldsymbol{R}} = \boldsymbol{E}_0 e^{-jk\boldsymbol{a}_n \cdot \boldsymbol{R}} = \boldsymbol{E}_0 e^{-j(k_x x + k_y y + k_z z)}$$

$$\boldsymbol{k} = k\boldsymbol{a}_n = \boldsymbol{a}_x k_x + \boldsymbol{a}_y k_y + \boldsymbol{a}_z k_z$$

$$\boldsymbol{a}_n \cdot \boldsymbol{E}_0 = 0$$

$$\boldsymbol{H}(\boldsymbol{R}) = \frac{1}{\eta} \boldsymbol{a}_n \times \boldsymbol{E}(\boldsymbol{R})$$

【例4.3】证明均匀平面电磁波是TEM波

4.4 平面电磁波的极化

表征在空间给定点上电场强度矢量随时间变化的特性

用固定点的电场矢量末端点在与波矢***k***垂直的平面内投影随时间运动的轨迹来描述

电磁波的发射与接收，必须考虑电磁波电场矢量方向（极化方向）与天线形式相匹配

- 线极化 —— 电场矢量末端点轨迹为直线
- 圆极化 —— 电场矢量末端点轨迹为圆
- 椭圆极化 —— 电场矢量末端点轨迹为椭圆

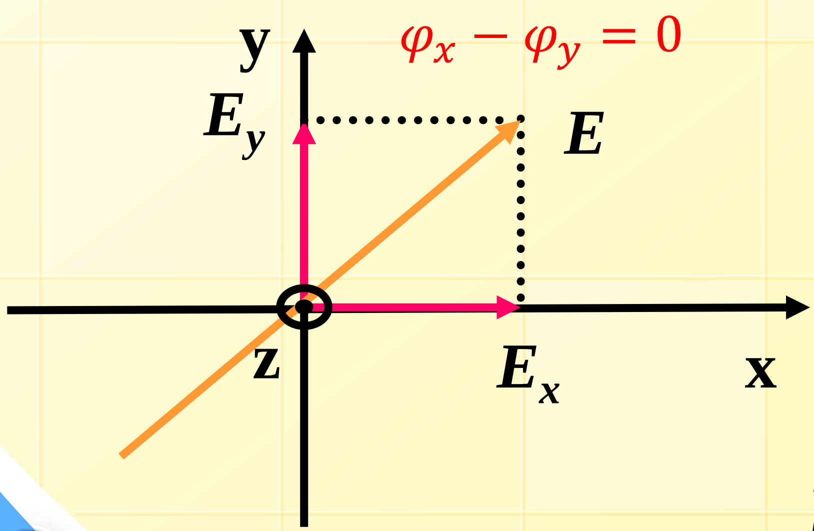
➢ 线极化（Linear Polarization）

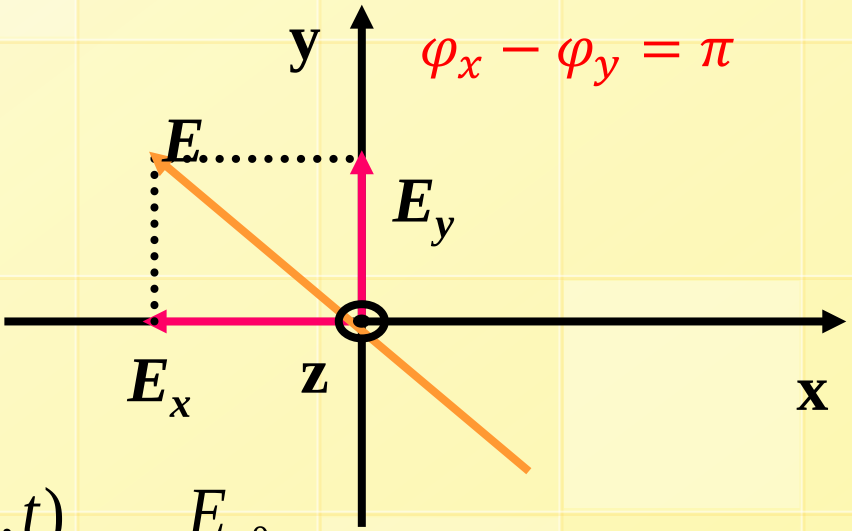
考虑如下两个平面波的叠加：

$$\boldsymbol{E}_x(z)=\boldsymbol{a}_x E_{x0}e^{j\phi_x}e^{-jkz} \qquad \boldsymbol{E}_y(z)=\boldsymbol{a}_y E_{y0}e^{j\phi_y}e^{-jkz}$$

$$\boldsymbol{E}(z)=\boldsymbol{E}_x(z)+\boldsymbol{E}_y(z)=\boldsymbol{a}_x E_{x0}e^{j\phi_x}e^{-jkz}+\boldsymbol{a}_y E_{y0}e^{j\phi_y}e^{-jkz}$$

$$\boldsymbol{E}(z,t)=\boldsymbol{a}_x E_x(z,t)+\boldsymbol{a}_y E_y(z,t)=\boldsymbol{a}_x E_{x0}\cos(\omega t-kz+\phi_x)+\boldsymbol{a}_y E_{y0}\cos(\omega t-kz+\phi_y)$$

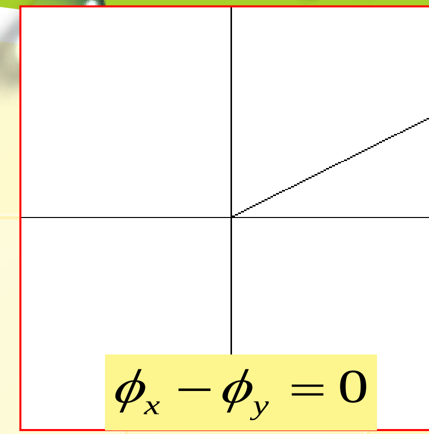


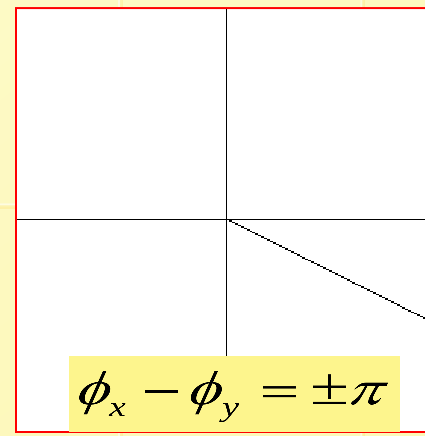


$$\tan\alpha=\frac{E_y(0,t)}{E_x(0,t)}=\pm\frac{E_{y0}}{E_{x0}}$$

合成波电场的模

$$E=\sqrt{E_x^2(0,t)+E_y^2(0,t)}=\sqrt{E_{x0}^2+E_{y0}^2}\cos(\omega t+\varphi_x)$$





- 特点：**合成波电场的大小随时间变化但其矢端，轨 迹与*x*轴的夹角始终保持不变。**

- 结论：**任何两个同频率、同传播方向且极化方向互相垂直的线极化波，当它们的相位相同或相差为$\pm\pi$时，其合成波为线极化波。**

圆极化（Circular Polarization）

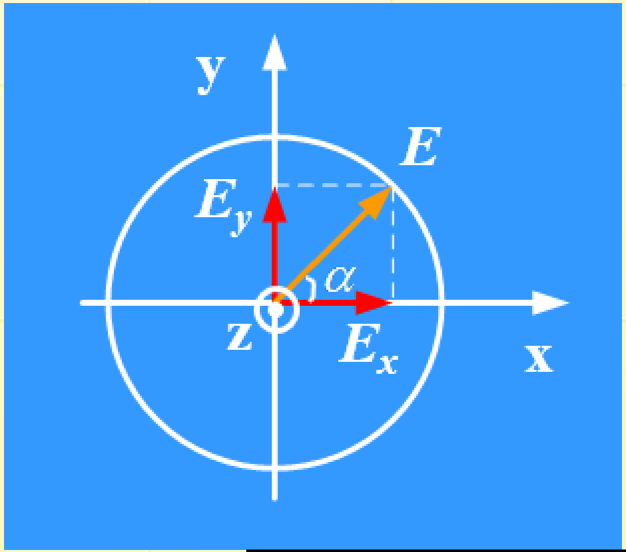
$$\boldsymbol{E}(z,t)=\boldsymbol{a}_xE_x(z,t)+\boldsymbol{a}_yE_y(z,t)$$

$$=\boldsymbol{a}_xE_{x0}\cos(\omega t-kz+\phi_x)+\boldsymbol{a}_yE_{y0}\cos(\omega t-kz+\phi_y)$$

思考：圆极化的相量形式？

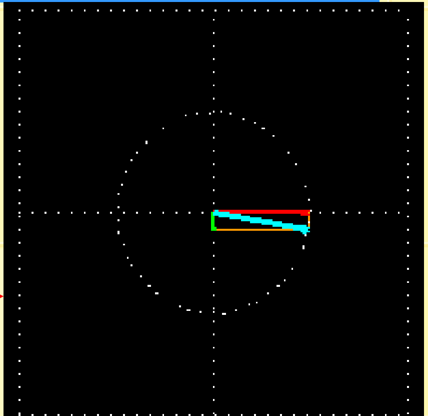
$$\left|\varphi_x-\varphi_y\right|=\frac{\pi}{2}\qquad E_{x0}=E_{y0}=E_0$$

$$[E_x(0,t)]^2+[E_y(0,t)]^2=E_0^{\ 2}$$



$$\varphi_x-\varphi_y=\frac{\pi}{2}$$ 右旋圆极化波

E旋转方向和波传播方向满足右手螺旋关系



$$\varphi_x-\varphi_y=-\frac{\pi}{2}$$ 左旋圆极化波

$$\boldsymbol{E}(z,t)=\boldsymbol{a}_x E_{x0}\cos(\omega t-kz+\phi_x)+\boldsymbol{a}_y E_{y0}\cos(\omega t-kz+\phi_y)$$

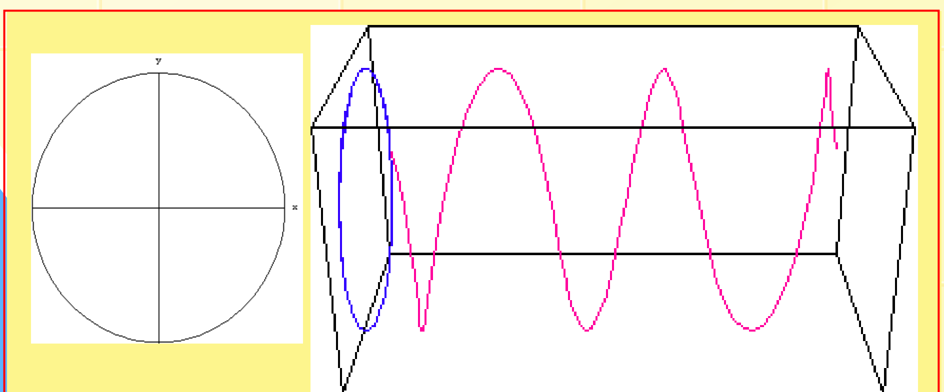
- **合成波电场的模** $E=\sqrt{E_x^2(0,t)+E_y^2(0,t)}=E_0$

- **合成波电场与**$+x$ **轴的夹角** $\alpha=\arctan[\pm\tan(\omega t+\phi_x)]=\pm(\omega t+\phi_x)$

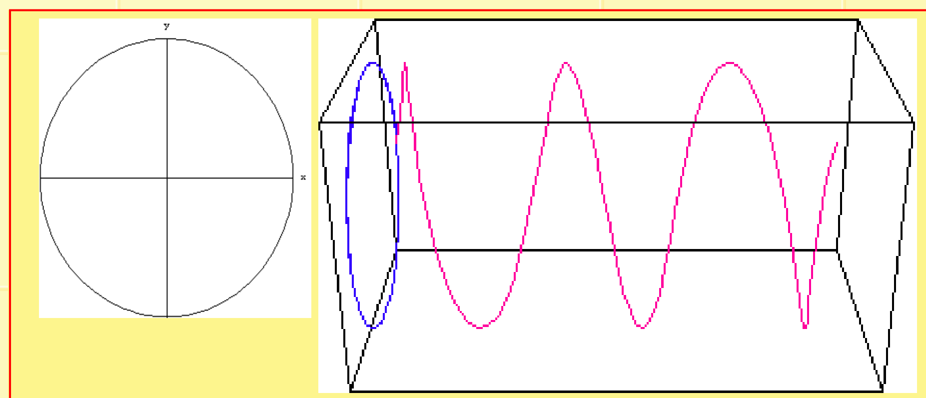
- 特点：**合成波电场的大小不随时间改变，但方向却随时间变化，电场的矢端在一个圆上并以角速度**ω **旋转。**

- 结论：**同频率、同振幅、同传播方向、相互垂直的线极化波相位差为**$\pm\pi/2$ **时，其合成波为圆极化波。**

右旋圆极化波



左旋圆极化波



➢ 椭圆极化（Elliptical Polarization）

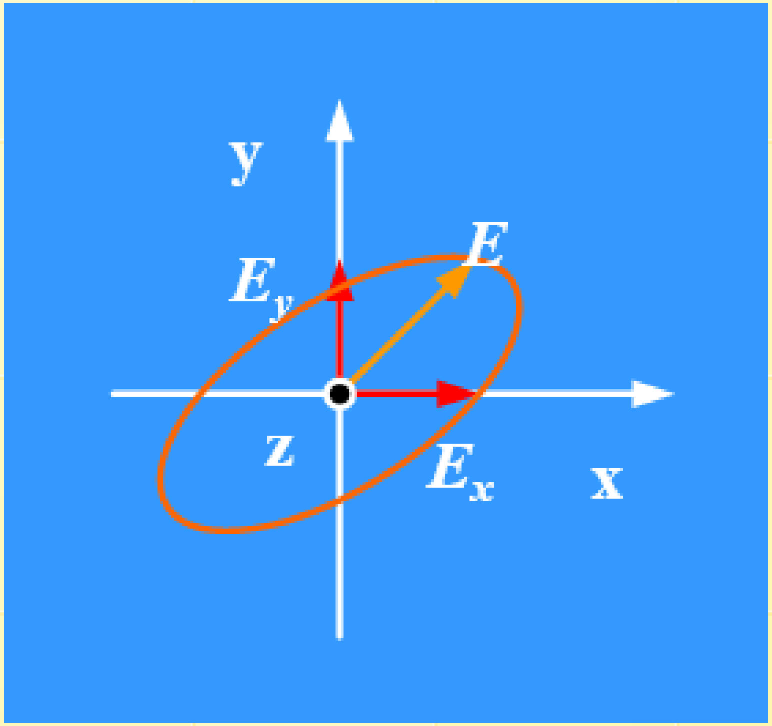
$$\boldsymbol{E}(z,t)=\boldsymbol{a}_x E_x(z,t)+\boldsymbol{a}_y E_y(z,t)$$
$$=\boldsymbol{a}_x E_{x0}\cos(\omega t-kz+\phi_x)+\boldsymbol{a}_y E_{y0}\cos(\omega t-kz+\phi_y)$$

E_{x0}、E_{y0} 任意

ϕ_x 、ϕ_y 任意

$$\frac{E_x^2}{E_{x0}^2}+\frac{E_y^2}{E_{y0}^2}-\frac{2E_xE_y}{E_{x0}E_{y0}}\cos\triangle\varphi=\sin^2\triangle\varphi$$

$$(\triangle\varphi=\varphi_x-\varphi_y;\ -\pi\le\triangle\varphi\le\pi)$$



极化的工程应用

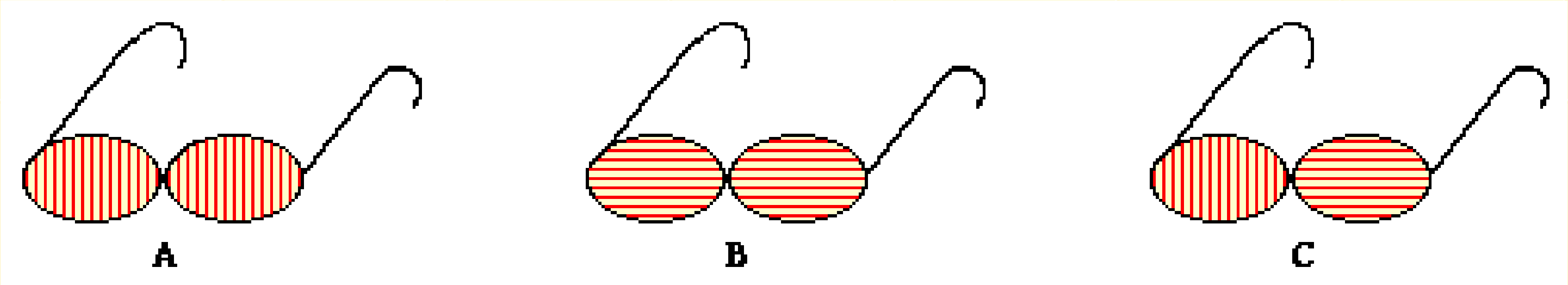
- **在雷达目标探测的技术中，利用目标对电磁波散射过程中改变极化的特性实现目标的识别**

- **无线电技术中，利用天线发射和接收电磁波的极化特性，实现最佳无线电信号的发射和接收。**

- **在光学工程中利用材料对于不同极化波的传播特性设计光学偏振片等等**


Problem:

Consider the three pairs of sunglasses below. Which pair of glasses is capable of eliminating the glare from a road surface? Explain. (The polarization axes are shown by the straight lines.)



例题

自由空间中均匀平面波的电场为：

$$\boldsymbol{E}(\mathrm{z}) = (a_{\mathrm{x}}\mathrm{j} + a_{\mathrm{y}}\sqrt{5} + a_{\mathrm{z}}2\mathrm{j})\mathrm{e}^{\mathrm{j}(2\mathrm{x}-\mathrm{z})}$$

求传播方向、波长、角频率和极化状态。

4.5 在导电媒质中传播的平面电磁波

4.5.1 导电媒质中电磁波的传播特性

无源、无界、有损媒质

$$\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}$$

$$\nabla \times \boldsymbol{H} = \sigma\boldsymbol{E} + j\omega\varepsilon\boldsymbol{E}$$

$$\nabla \cdot \boldsymbol{E} = 0$$

$$\nabla \cdot \boldsymbol{H} = 0$$

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon_c\boldsymbol{E}$$

复介电常数

$$(\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega})$$

$$\nabla^2\boldsymbol{E} + k_c^2\boldsymbol{E} = 0$$

$$(k_c = \omega\sqrt{\mu\varepsilon_c})$$

定义传播常数 γ

$$\gamma = jk_c = j\omega\sqrt{\mu\varepsilon_c}$$

导电媒质中的平面电磁波

$$\nabla^2\boldsymbol{E} - \gamma^2\boldsymbol{E} = 0$$

$$\nabla^2\boldsymbol{H} - \gamma^2\boldsymbol{H} = 0$$

$$\gamma = jk_c = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{\frac{1}{2}}$$

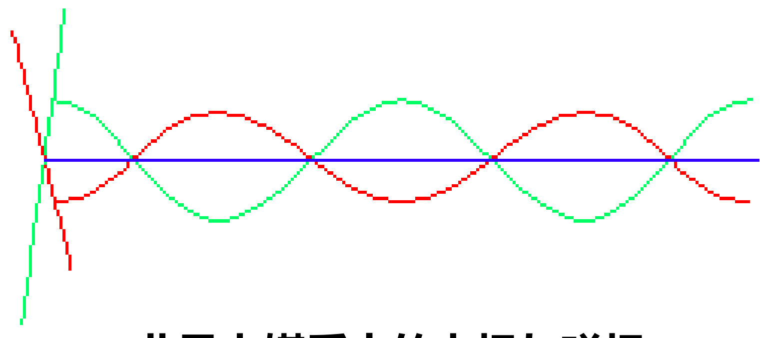
$$\gamma = \alpha + j\beta$$

$$\nabla^2 \boldsymbol{E} - \gamma^2 \boldsymbol{E} = 0 \Rightarrow \boldsymbol{E}(z) = \boldsymbol{a}_x E_x(z) = \boldsymbol{a}_x E_0 \boldsymbol{e}^{-\gamma z} = \boldsymbol{a}_x E_0 \boldsymbol{e}^{-\alpha z} \boldsymbol{e}^{-j\beta z}$$

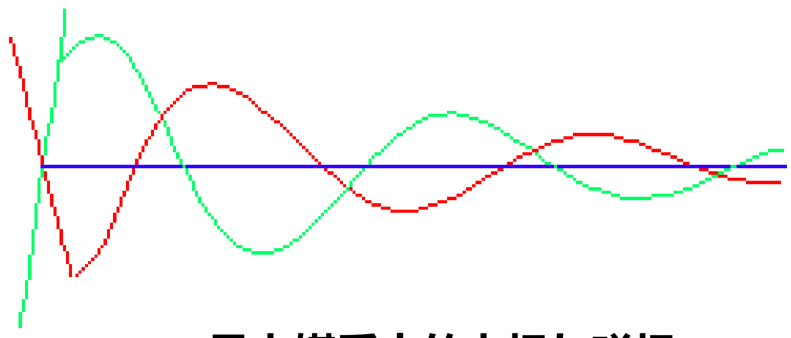
瞬时值：$\boldsymbol{E}(z,t) = \boldsymbol{a}_x E_m \boldsymbol{e}^{-\alpha z} \cos(\omega t - \beta z)$

$e^{-\alpha z}$ 是衰减因子，α 是衰减常数，单位是奈培每米 (Np/m)

$e^{-j\beta z}$是相位因子，β 是相位常数，单位是弧度每米 (rad/m)



非导电媒质中的电场与磁场



导电媒质中的电场与磁场

4.5.2 导电媒质的分类

损耗角正切

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega} = \varepsilon' - j\varepsilon''$$

$$\Rightarrow \begin{cases} \varepsilon' = \varepsilon \\ \varepsilon'' = \dfrac{\sigma}{\omega} \end{cases}$$

$$\boxed{\tan\delta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon}}$$

（传导电流与位移电流的幅度之比）

导电媒质的分类

根据 $\dfrac{\sigma}{\omega\varepsilon}$ 分类

媒质种类	损耗正切 $\dfrac{\sigma}{\omega\varepsilon}$
理想导体	$\to \infty$
良导体	$>>1$
不良导体	≈ 1
低损耗电介质	$<<1$
理想电介质	$=0$

1）不良导体 $\dfrac{\sigma}{\omega\varepsilon}\approx 1$

$$\gamma = jk_c = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{\frac{1}{2}} = \alpha + j\beta$$

$$\alpha = \omega\sqrt{\mu\varepsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]\right\}^{\frac{1}{2}} \qquad \beta = \omega\sqrt{\mu\varepsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]\right\}^{\frac{1}{2}}$$

$$\lambda = \frac{2\pi}{\beta} = 1\Bigg/\left\{f\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2}+1\right]}\right\}$$

$$v = \frac{\omega}{\beta} = 1\Bigg/\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+(\frac{\sigma}{\omega\varepsilon})^2}+1\right]}$$

相速不仅与媒质参数有关，而与电磁波的频率有关

➢2）低损耗电介质 $\dfrac{\sigma}{\omega\varepsilon} << 1$

$$\gamma = jk_c = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{\frac{1}{2}} = \alpha + j\beta$$

$$\Rightarrow \alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \quad (Np/m) \qquad \beta = \omega\sqrt{\mu\varepsilon}\left[1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right] (rad/m)$$

本征阻抗 $\eta_c = \sqrt{\dfrac{\mu}{\varepsilon}}\left(1+\dfrac{\sigma}{j\omega\varepsilon}\right)^{-\frac{1}{2}} \approx \sqrt{\dfrac{\mu}{\varepsilon}}\left(1+j\dfrac{\sigma}{2\omega\varepsilon}\right)$

低损耗电介质中，衰减常数与电导率成正比（较小），

相位常数与理想无损媒质的相位常数 $\omega\sqrt{\mu\varepsilon}$ 差别极小，

电场强度与磁场强度不再同相，存在很小的相位差。

3）良导体 $\dfrac{\sigma}{\omega\varepsilon} >> 1$

$$\gamma = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{\frac{1}{2}} \approx \frac{1+j}{\sqrt{2}}\sqrt{\omega\mu\sigma}$$

$$\boxed{\alpha = \beta = \sqrt{\pi f\mu\sigma}}$$

相速 $v_p = \dfrac{\omega}{\beta} = \sqrt{\dfrac{2\omega}{\mu\sigma}}$ $\quad \lambda = \dfrac{2\pi}{\beta} = 2\sqrt{\dfrac{\pi}{f\mu\sigma}}$

电磁波在良导体中的传播速度比在自由空间中慢得多

波阻抗 $$\eta_c = \sqrt{\frac{\mu}{\varepsilon}}\left(1+\frac{\sigma}{j\omega\varepsilon}\right)^{-\frac{1}{2}} \approx (1+j)\sqrt{\frac{\pi f\mu}{\sigma}}$$

良导体的本征阻抗的相位角为45°，电磁波磁场强度*H*滞后于电场强度 45°。

集肤效应

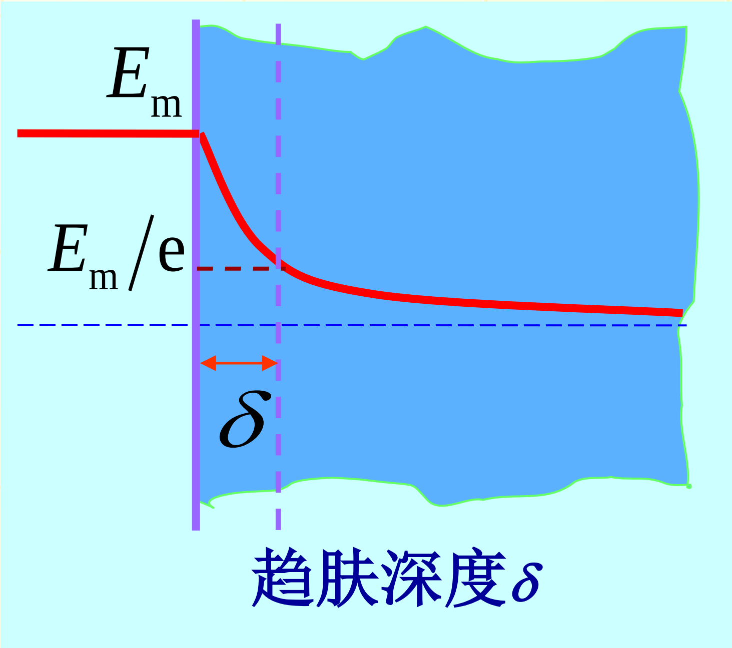
电磁波频率越高，衰减系数越大。高频下，电磁波和电流只能存在于良导体的表面层内，这称为集肤效应。

趋肤深度

电磁波穿入良导体中，当波的振幅下降为表面处振幅的**1/e**时所传播的距离：

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{\lambda}{2\pi}$$

例：电磁波在银中的趋肤深度：



$$\sigma = 6.15\times10^{7}(S/m)$$

$$\mu_0 = 4\pi\times10^{-7}(H/m)$$

$$\Downarrow$$

$$\delta = \frac{0.0642}{\sqrt{f}}(m)$$

当 f=3GHz 时， δ=1.17μm

4.5.3 电磁波的色散和群速

- 色散

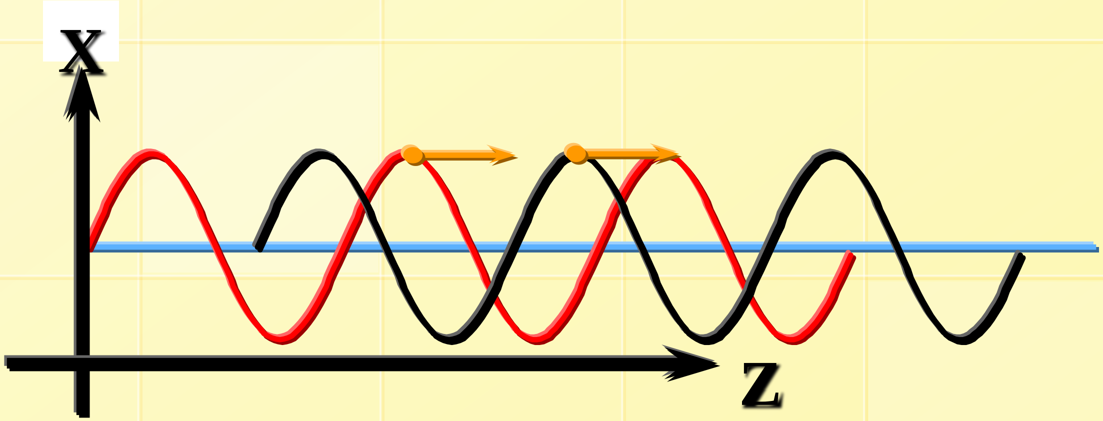
光学色散的本质：

折射率不同：

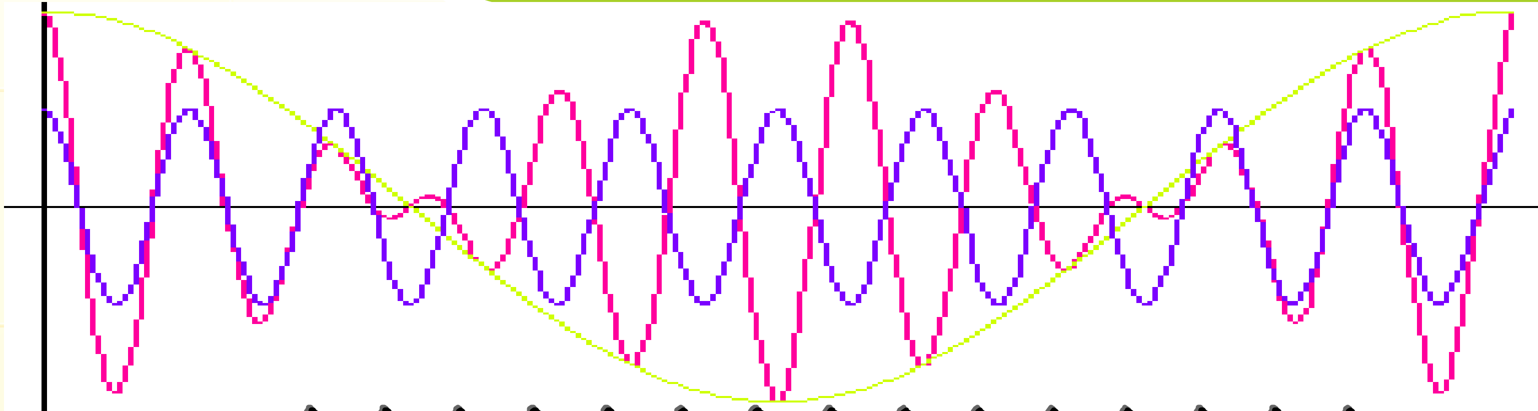
$$n = \frac{c}{v_p}$$

电磁波的相速随频率而变，从而引起信号失真的现象。

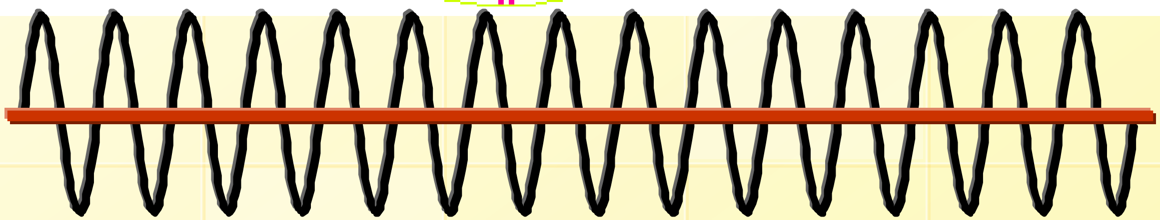
- 相速　　电磁波恒定相位点的推进速度



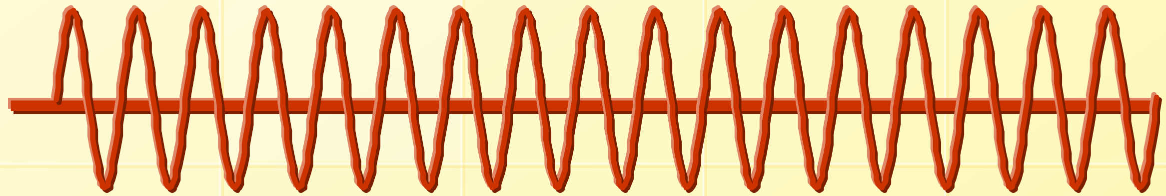
➢群速



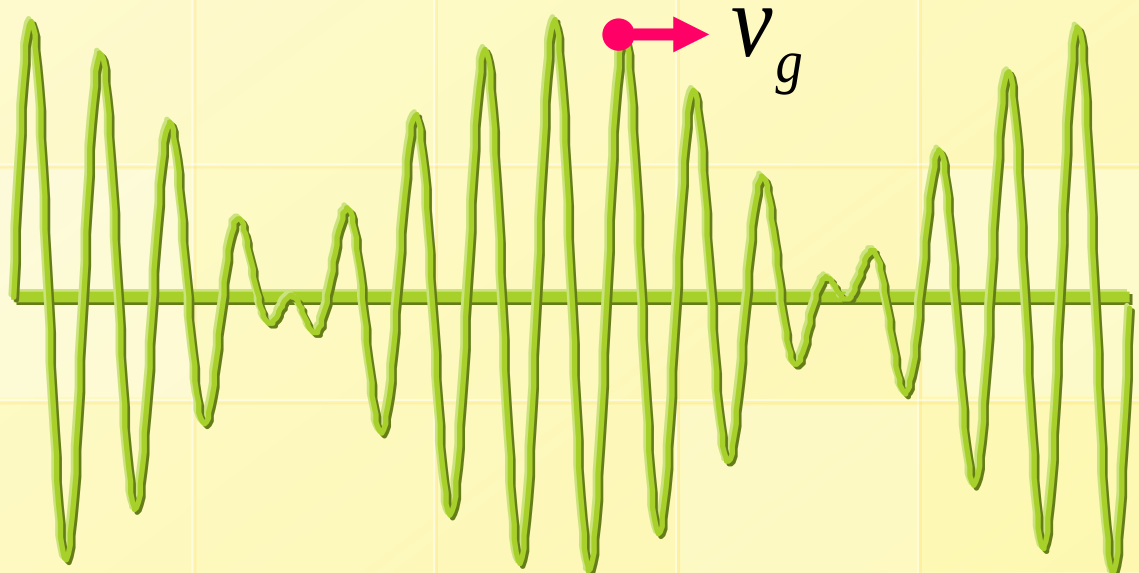
$$\omega_1 = \omega_0 - \Delta\omega$$



$$\omega_2 = \omega_0 + \Delta\omega$$



Sum →



$$\boldsymbol{E}_1(z,t)=\boldsymbol{a}_x E_m \cos[(\omega_0-\Delta\omega)t-(\beta_0-\Delta\beta)z]$$

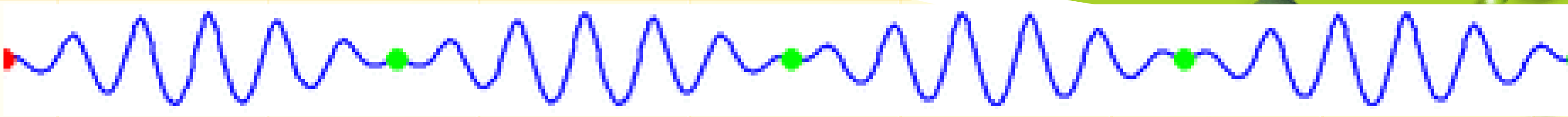
$$\boldsymbol{E}_2(z,t)=\boldsymbol{a}_x E_m \cos[(\omega_0+\Delta\omega)t-(\beta_0+\Delta\beta)z]$$

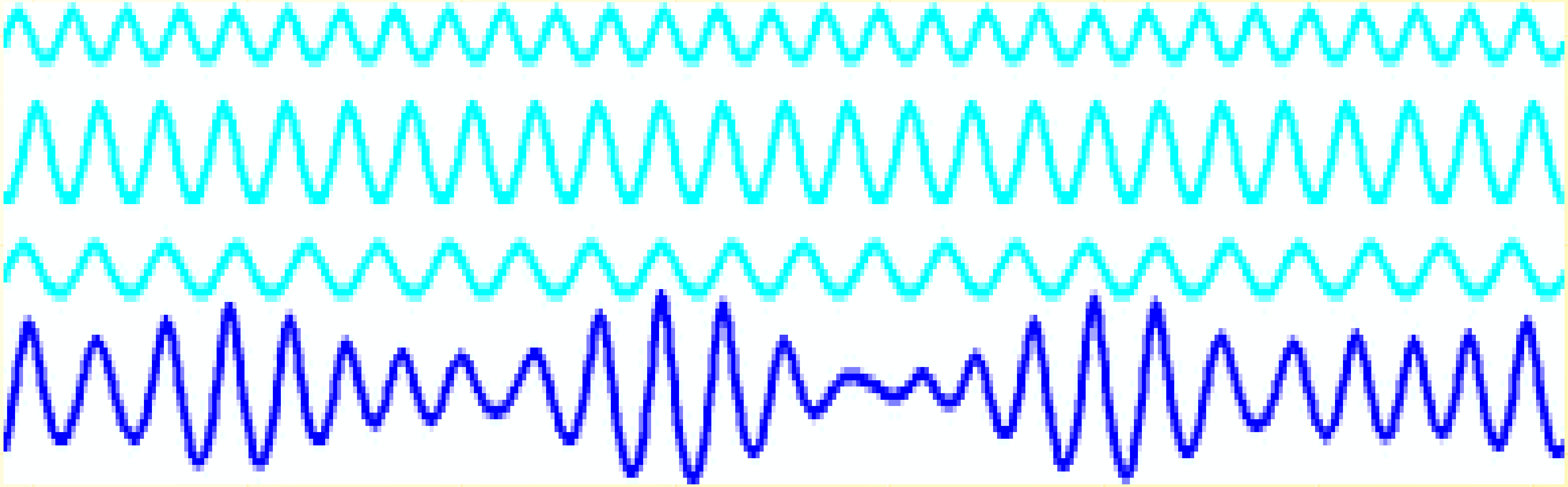
$$\boldsymbol{E}(z,t)=\boldsymbol{E}_1(z,t)+\boldsymbol{E}_2(z,t)=\boldsymbol{a}_x 2E_m\cos(\Delta\omega t-\Delta\beta z)\cos(\omega_0 t-\beta_0 z)$$

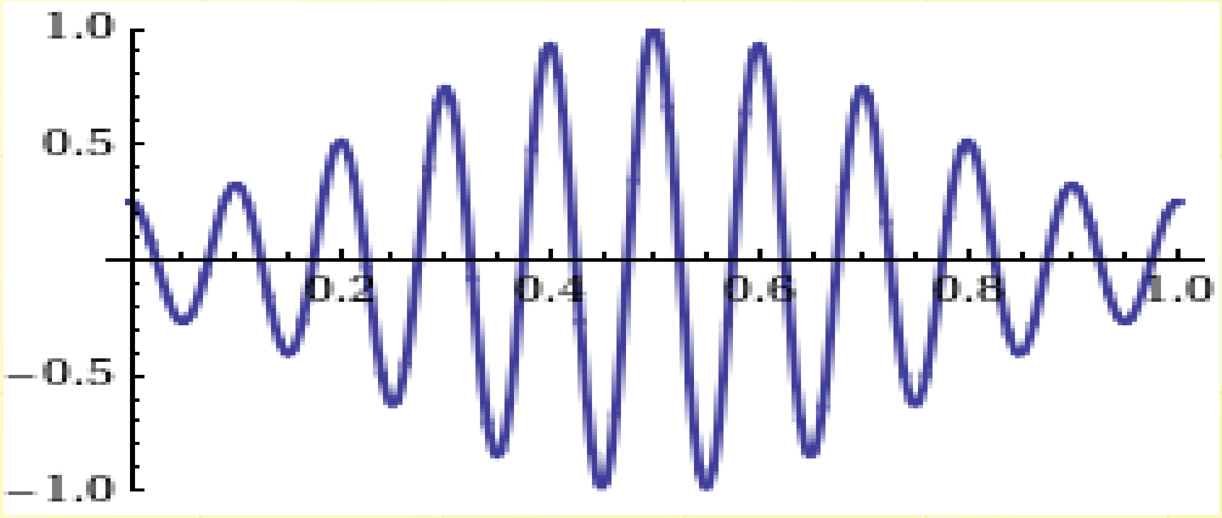
群速

$$v_g=\frac{\Delta\omega}{\Delta\beta}=\frac{d\omega}{d\beta} \qquad v_g=\frac{v_p}{1-\dfrac{\omega}{v_p}\dfrac{dv_p}{d\omega}}$$

$\frac{dv_p}{d\omega}=0$	$v_g=v_p$	无色散
$\frac{dv_p}{d\omega}<0$	$v_g<v_p$	正常色散
$\frac{dv_p}{d\omega}>0$	$v_g>v_p$	异常色散






1.0
0.5
−0.5
−1.0
0.2
0.4
0.6
0.8
1.0

【例 4.6】 有一个窄带信号在电介质中传播，信号的载频为 550 kHz，媒质的相对介电常数为 2.5，媒质的损耗正切 $\dfrac{\sigma}{\omega\varepsilon}$ 等于 0.2。求：

(a) 电介质的衰减常数 α 和相位常数 β；

(b) 相速度 v_p 和群速度 v_g，判断该媒质是否是色散媒质。

解：(a) 因为损耗正切 $\dfrac{\sigma}{\omega\varepsilon}$ 等于 0.2，所以该电介质可以近似认为是低损耗电介质。该电介质的电导率为 $\sigma = \dfrac{\sigma}{\omega\varepsilon}\cdot\omega\varepsilon = 1.53\times10^{-5}\,(\mathrm{S/m})$

因此 $\alpha = \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon}} = 1.82\times10^{-3}\,(\mathrm{Np/m})$

$$\beta = \omega\sqrt{\mu\varepsilon}\left[1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]$$

$$= 2\pi(550\times10^3)\frac{\sqrt{2.5}}{3\times10^8}\left[1+\frac{1}{8}(0.2)^2\right] = 0.0183\ (\mathrm{rad/m})$$

(b) 相速度 $v_p = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{\mu\varepsilon}\left[1+\dfrac{1}{8}\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2\right]} \approx 1.888\times10^8\,(\mathrm{m/s})$

群速度 $v_g = \dfrac{1}{\dfrac{d\beta}{d\omega}} = \dfrac{1}{\sqrt{\mu\varepsilon}}\left[1+\dfrac{1}{8}\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2\right] \approx 1.907\times10^8\,(\mathrm{m/s})$

因为在该媒质中相速度与群速度不相同，所以它是色散媒质。由于媒质的损耗正切不大，所以相速度与群速度也相差不大。