Feedback

Effects of Poles & Zeros on Frequency Response(1)

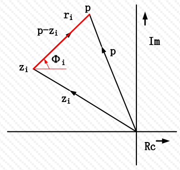
- Consider a general system transfer function:

$$H(s)=\frac{P(s)}{Q(s)} = b_0\frac{(s-z_1)(s-z_2)\cdots(s-z_N)}{(s-\lambda_1)(s-\lambda_2)\cdots(s-\lambda_N)}$$

zeros at $z_1\ z_2 \cdots z_N$

poles at $\lambda_1\ \lambda_2 \cdots \lambda_N$

- The value of the transfer function at some complex frequency s=p is:



$$H(s)|_{s=p} = b_0\frac{(p-z_1)(p-z_2)\cdots(p-zN)}{(p-\lambda_1)(p-\lambda_2)\cdots(p-\lambda_N)}$$

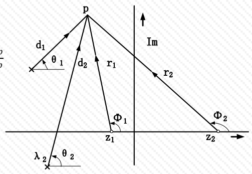
$$= b_0\frac{(r_1e^{j\phi_1})(r_2e^{j\phi_2})\cdots(rNej^{\phi_N})}{(d_1e^{j\theta_1})(d_2e^{j\theta_2})\cdots(dNej^{\theta_N})}$$

Effects of Poles & Zeros on Frequency Response(2)

- Therefore the magnitude and phase at s=p are given by:

$$H(s)|_{s=p} = b_0\frac{r_1r_2\cdots r_N}{d_1d_2\cdots dN}$$

$$= b_0\frac{\textit{product of the distances of zeros to p}}{\textit{product of the distances of poles to p}}$$



$\angle H(s)|_{s=p}=(\phi_1+\phi_2+\cdots+\phi_N)-(\theta_1+\theta_2+\cdots+\theta_N)$
=sum of zero angles to p – sum of pole angles to p

Effects of Poles & Zeros on Frequency Response(3)

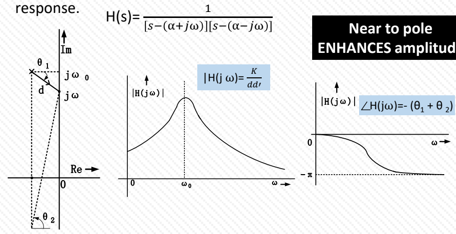
- Frequency Response of a system is obtained by **evaluating H(s) along the y-axis**(i.e. taking all value of s=jω).
- Consider the effect of two complex poles on the frequency response.

$$H(s)=\frac{1}{[s-(\alpha+j\omega)][s-(\alpha-j\omega)]}$$

Near to pole ENHANCES amplitude

$|H(j\omega)|=\frac{K}{dd'}$

$\angle H(j\omega)=-(\theta_1+\theta_2)$



Effects of Poles & Zeros on Frequency Response(4)

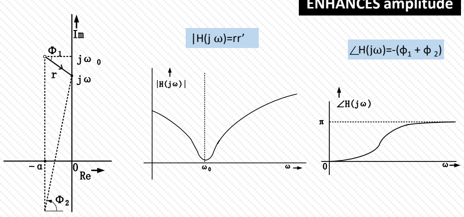
- Consider the effect of two complex zeros on the frequency response.

$H(s)=[s-(\alpha+j\omega)][s-(\alpha-j\omega)]$

Near to zeros ENHANCES amplitude

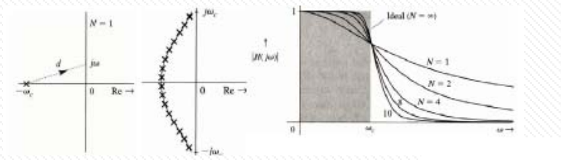
$|H(j\omega)|=rr'$

$\angle H(j\omega)=(\phi_1+\phi_2)$



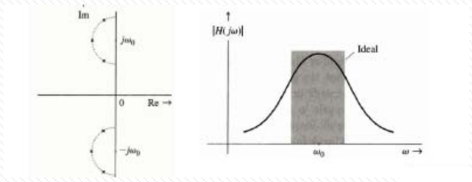
Poles & Low-pass filter

- Use the enhancement and suppression properties of poles & zeros to design filters
- Low-pass filter (LPF) has maximum gain at $\omega = 0$, and the gain decreases with ω
- Simplest LPF has a single pole on real axis, say at $(\omega = -\omega_c)$. Then $H(s) = \frac{1}{s+\omega_c}$ and $|H(j\omega)| = \frac{\omega_c}{d}$
- To have a "brickwall" type of LPF(i.e. very sharp cut-off), we need a WALL OF POLE as shown, the more poles we get, the sharper the cut-off



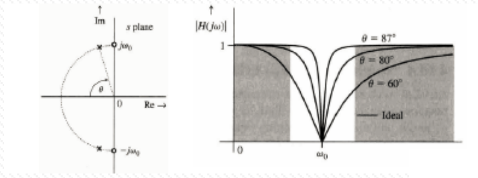
Poles & Band-pass filter

- Band-pass filter has gain enhanced over the entire passband, but suppressed elsewhere
- For a passband centred around ω_0, we need lots of poles opposite the imaginary axis in front of the passband center at ω_0

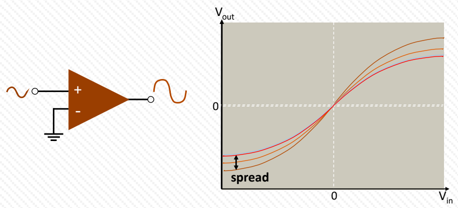


Notch filter

- Notch filter could in theory be realised with two zeros placed at $\pm j\omega_0$. However, such a filter would not have unity gain at zero frequency, and the notch will not be sharp
- To obtain a good notch filter, put two poles close to the two zeros on the semicircle as shown. Since the both pole/zero pair are equal-distance to the origin, the gain at zero frequency is exactly one. Same for $\omega = \infty$.

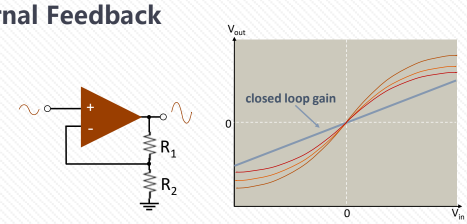


Exernal Feedback



Open loop amplifiers are typically nonlinear and have process spread.

External Feedback



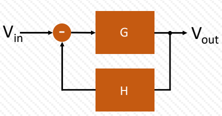
Apply feedback for linearization:

$$V_{out} = A \cdot \left(V_{in} - \frac{R_2}{R_1 + R_2} V_{out}\right) \Rightarrow V_{out} = \frac{A \cdot V_{in}}{1 + A \cdot \frac{R_2}{R_1 + R_2}} \overset{A>>1}{\approx} \left(1 + \frac{R_1}{R_2}\right) \cdot V_{in}$$

gain is only defined by external resistors ➡ very linear & deterministic

Black' s Formula

Let's generalize:



$$A_{cl} = \frac{V_{out}}{V_{in}} = \frac{G}{1 + G \cdot H}$$

$G \gg \frac{1}{H}$ ➡ $A_{cl} \approx \frac{1}{H}$

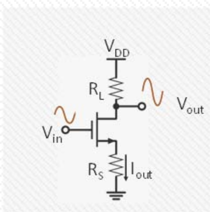
$G \ll \frac{1}{H}$ ➡ $A_{cl} \approx G$

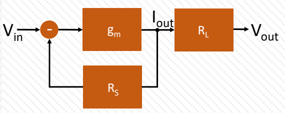
In our example: $G = A \qquad H = \frac{R_2}{R_2 + R_1}$

➡ $A_{cl} \approx \frac{1}{H} = 1 + \frac{R_1}{R_2}$ for $G \gg \frac{1}{H}$

➡ Build amplifiers with high open loop gain to satisfy G >> 1/H.
Typically: **1000 < A < 100000**

Example: Source Degeneration



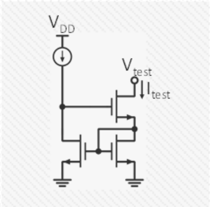


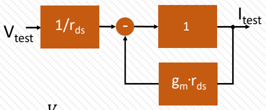
$$I_{out} = g_m (V_{in} - I_{out} R_S)$$

$$\Rightarrow I_{out} = \frac{g_m}{1 + g_m R_S} \cdot V_{in}$$

Black's formula: $g_m \gg \frac{1}{R_S}$ ➡ $\frac{I_{out}}{V_{in}} \approx \frac{1}{R_S}$

Example: Wilson Current Mirror



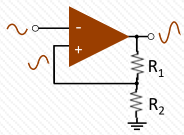


$$I_{test} = \frac{V_{test}}{r_{ds}} - I_{test} \cdot g_m\, r_{ds}$$

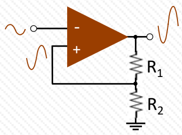
$$I_{test} \approx \frac{V_{test}}{(1 + g_m r_{ds}) \cdot r_{ds}}$$

Black's formula: $1 \gg \frac{1}{g_m r_{ds}} \Rightarrow \frac{I_{out}}{V_{test}} \approx \frac{1}{r_{ds}} \cdot \frac{1}{g_m r_{ds}}$

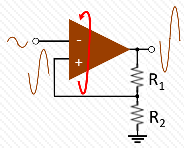
Stability



Stability

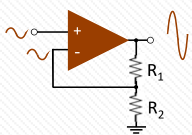


Stability

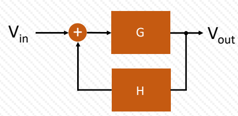


What is wrong?

Stability

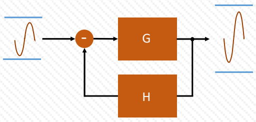


Stability



Positive feedback makes the loop unstable.

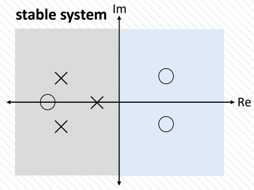
BIBO



A general stability criteria is that for a bounded input there must be a bounded output (**BIBO**).

Stability Criteria

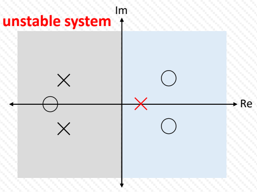
$$A_{cl} = \frac{(s-z_1)(s-z_1)\cdots(s-z_n)}{(s-p_1)(s-p_1)\cdots(s-p_n)}$$



A system is stable if all poles of the closed loop response lie on the left half plane (LHP).
Any pole in the right half plane (RHP) results in instability

Stability Criteria

$$A_{cl} = \frac{(s-z_1)(s-z_1)\cdots(s-z_n)}{(s-p_1)(s-p_1)\cdots(s-p_n)}$$



A system is stable if all poles of the closed loop response lie on the left half plane (LHP).
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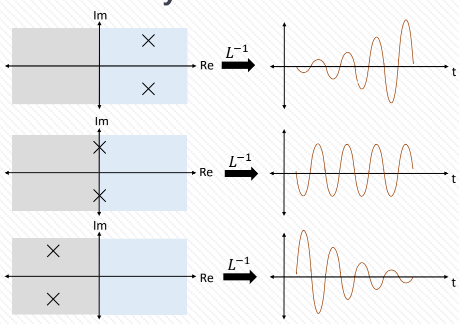
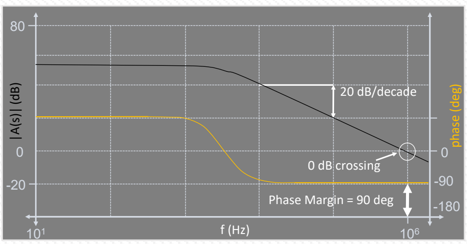
Intution on Stability

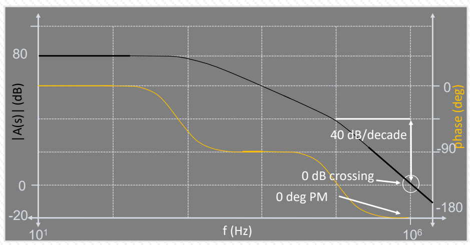


Bode Plot



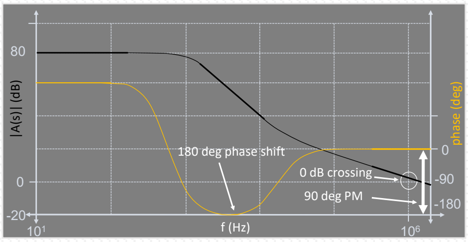
There is a more intuitive way to check on the Bode Plot of the **open loop** response:
phase margin > 0 @ 0 dB crossing

Bode Plot



2nd order roll-off creates low phase margin.

Bode Plot

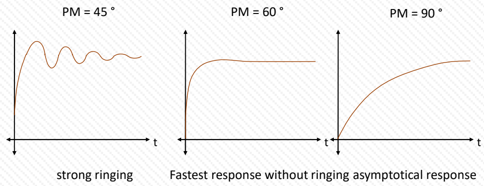


Is this stable? Suprisingly **yes**.

Nyquist is derived from time domain ➡ Start-Up transients are present
➡ Higher frequencies present, that drive loop into stability.

Phase Margin

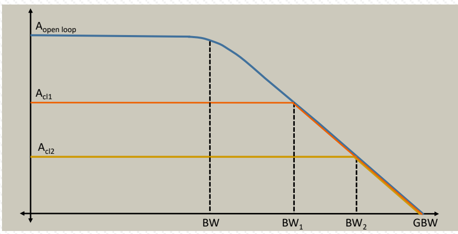
How much phase margin do we need?



The closer the loop is at instability the more oscillations will be present.

Over stabilization results is slow loop response.

Gain Bandwidth



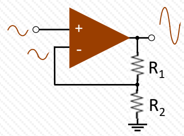
Gain Bandwidth (GBW) Product is a constant.

➡ Increasing closed loop gain decreases closed loop bandwidth.

Tradeoff: **gain ⟺ BW**

Commonly Used Feedback Configurations

Non Inverting Amplifier:

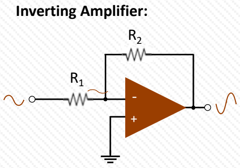


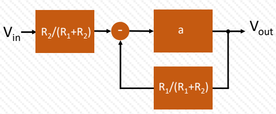
feedback factor β (H): $\beta = \frac{R_2}{R_2 + R_1}$

$$A \approx \frac{1}{\beta} = 1 + \frac{R_1}{R_2}$$

Commonly Used Feedback Configurations

Inverting Amplifier:



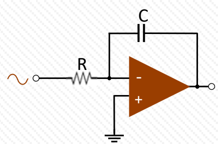


$$\beta = \frac{R_1}{R_2 + R_1}$$

$$A \approx -\frac{R_2}{R_2 + R_1} \cdot \frac{1}{\beta} = -\frac{R_2}{R_1}$$

Commonly Used Feedback Configurations

Inverting Integrator:

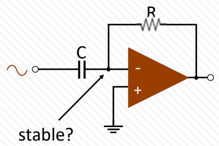


$$\beta = \frac{R_1}{1/sC + R_1}$$

$$A \approx -\frac{\frac{1}{sC}}{\frac{1}{sC} + R} \cdot \frac{1}{\beta} = -\frac{1}{sRC}$$

Commonly Used Feedback Configurations

Differentiator:



$$\beta = \frac{1/sC}{R + 1/sC}$$

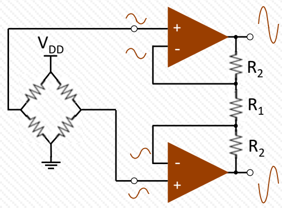
$$A \approx -\frac{R}{R+1/sC} \cdot \frac{1}{\beta} = -sRC$$

Additional pole added by external feedback.

➡ Take care of stability.

Commonly Used Feedback Configurations

Differential Bridge Readout Amplifier:



$$\beta = \frac{R_1/2}{\frac{R_1}{2} + R_2}$$

$$A \approx \frac{1}{\beta} = 1 + 2 \cdot \frac{R_2}{R_1}$$

Rejects Input CM and amplifies Input DM.
High Input Impedance.

Recall

Feedback helps for accurate gain and linearity.

Feedback loop is stable for positive phase margin of the open loop response.

Tradeoff between Gain and Bandwidth.