Emflux Motors

Mechanical Internship Assessment Test

Note:

1. All questions are compulsory.
2. Write down all the assumptions, formulae used and comments along with the solution.
3. Answers to be submitted in a '.pdf' format. Handwritten answers will not be accepted (diagrams or representations can be drawn by hand).
4. The test will be used to evaluate the your approach to the problem, theoretical knowledge and assumptions they make to solve the practical problems related to automotive engineering.

   All the best!

[Question 1]

A motorcycle with the following specification was tested on an inertial dynamometer with a steel hollow cylinder 0.75m long, Outer diameter of 0.4m and an inner diameter of 0.3m.

Vehicle specifications:

Vehicle Mass = 169kg
Rider Mass = 75kg
Cd*A = 0.35m^2
Rolling resistance of tires = 0.01

The Cd*A value given above is of the rider in the prone position and this value is found to be 22% higher for the rider in the upright position.

The test revealed that the motorcycle was capable of rotating the roller with an angular acceleration of:

$$\alpha(x) = \begin{cases} 47 & ,0 \leq x \leq 1300 \\ (2 \times 10^6).x^{-1.49} & ,1300 < x \leq 2940 \end{cases}$$

Where, ‘x’ is the rpm of roller

Determine the following:

1. The motorcycle’s torque and power at the wheel across the operating speed range specified.

2. The time taken for the motorcycle to reach 100 Kmph on road and the top speed of the vehicle on the road.

3. Find the battery pack capacity when the vehicle reaches its top speed assuming it started with a pack capacity of 10 kWh. Now, consider that this vehicle is capable of reaching a stand still position from its top speed in 10 seconds using the regenerative braking system alone. Determine the final battery pack capacity after the vehicle reaches stand still condition.

4. Mention the assumptions (if any) you have made during the calculations for the above question

Note: Any software (Excel, MATLAB, Python, etc.) can be used to solve the above problem however only relevant graphs and calculations/formulae used should be submitted in pdf.

[Question 2]

A motorcycle has the following specifications-

Kerb weight	=	170 kg
Rider weight	=	80 kg
Pillion weight	=	70 kg
Front shock absorber stroke	=	130 mm
Front shock absorber stiffness	=	25 N/mm
Rear shock absorber stroke	=	60 mm
Rear shock absorber stiffness	=	80 N/mm
Rake angle	=	22°
Rear motion ratio	=	0.65
Wheelbase	=	1350 mm
Rear Tyre Size	=	140/70 R17
Front Tyre Size	=	120/70 R17

Under static loading (with rider and pillion),

Front axle load	=	1650 N
Rear axle load	=	1550 N
CG height	=	450mm

Assuming g=10m/s^2

a) Find the front and rear shock absorber displacement if -
   I) The bike is statically loaded
   II) The bike goes under a deceleration of 0.75g

b) For the normal loads acting on tyres in the 0.75g deceleration scenario above, design a braking system and find the following -
   I) Effective rotor diameters and corresponding clamping forces for front and rear
   II) For the given caliper and master cylinder specifications, find the lever and pedal ratio required at front and rear respectively.

Front caliper piston diameter	=	35mm
No of pistons in front caliper	=	2
Rear caliper piston diameter	=	35mm
No of pistons in rear caliper	=	1
Front Master cylinder bore diameter	=	16mm
Rear Master Cylinder bore diameter	=	18mm

Assume the coefficient of friction between rotor and pad to be 0.6, and between road and tyre be 0.8.

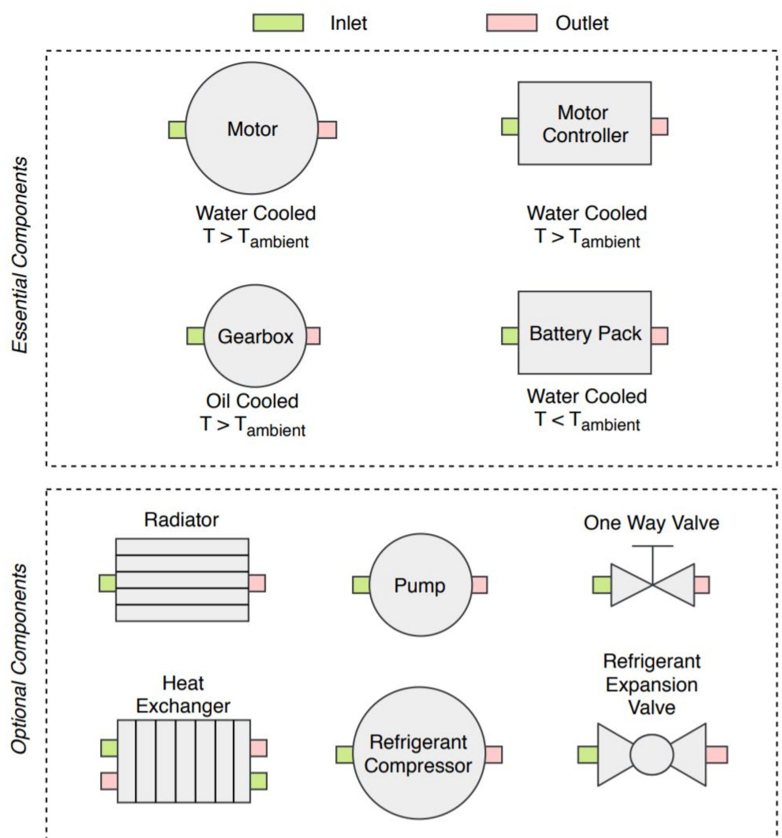
Note : Assumptions are allowed as far as the design is under ergonomic limits and practically feasible.

[Question 3]

An electric vehicle comprises essential components like Motor, Gearbox, Motor Controller, and Battery Pack. The Lithium Ion Battery Pack should operate within a certain temperature range, for example, 20 to 40 deg. Celsius for longevity of life. Thus there is a need of thermal management system which can cool the battery pack in hot climate and heat the battery pack in extremely cold climates so that optimum temperature is maintained.

Temperature of Motor, Motor Controller & Gearbox must be maintained above ambient and the temperature of the Battery Pack must be maintained below the ambient air temperature.

Question A:
Design a thermal management system which can provide cooling of Motor, Gearbox & Motor Controller and provide both cooling and heating of Battery Pack. You can use the optional components provided in the diagram below. There is no limit on quantity of optional components to be used, except for the radiator, <u>only one radiator can be used</u>. The goal is to design effective and compact thermal system. You are supposed to draw a schematic diagram using essential and optional components. The Motor, Motor Controller & Battery Pack are water cooled while the <u>Gearbox is oil cooled</u>. Coolant Inlets and outlets are mentioned in the diagram. Heat Exchanger is a liquid to liquid heat exchanger.



Question B:
Justify the sequence of components in respective thermal loops in previous answer.

[Question 4]

Design a speed reducer for the following requirements.

Power to be transferred = 20 kW
Input speed = 8000 rpm
Output Speed = 750 rpm to 850 rpm
Pressure angle for gears = 20 degrees
Maximum Gearbox Size = 300 mm x 200 mm x 100 mm

Gear Material:
Modulus of Elasticity = 200 GPa
Poisson Ratio = 0.3

Q1. Calculate wear and bending stresses on all the gears by choosing appropriate gearbox configuration and gear parameters.
Q2. Calculate reaction forces on shafts and estimate suitable shaft diameters and material providing sufficient fatigue and static stress capacity for infinite life of shaft, with minimum factor of safety factors of 1.5.
Q3. Also select appropriate bearings for the shafts.

Consider all shafts to be straddle mounted. Gear and Bearing Life > 12000 hours.
For gear weight use volume approximation to be (PCD*Face Width)

EMFLUX MECHANICAL INTERNSHIP

NAME : R.N. PREM KUMAR

Mail id : r.n.prem5051@gmail.com

Mobile : 7708087069

[Question 1]

Vehicle specifications:

Vehicle Mass = 169kg

Rider Mass = 75kg

Cd*A = 0.35m2

Rolling resistance of tires = 0.01

A steel hollow cylinder 0.75m long,

Outer diameter of 0.4m

Inner diameter of 0.3m.

Now,

Torque = I * α

Where, I = moment of inertia of hollow cylinder, (kg m^2)

α = angular acceleration (rad/s^2)

$$I = \frac{M}{2}\left[R_I^{\,2} + R_O^{\,2}\right]$$

We know that, Mass = Density * volume

$V = \pi\left[R_0^{\,2} - R_I^{\,2}\right]h$ = 0.75*3.14(0.2^2-0.15^2)

V = 41.212 * 10^6 mm^3

Mass = 41.212*10^6 x 7850(kg/m^3) = 323.49 kg

Moment of inertia, I = 10.1090 kg m^2

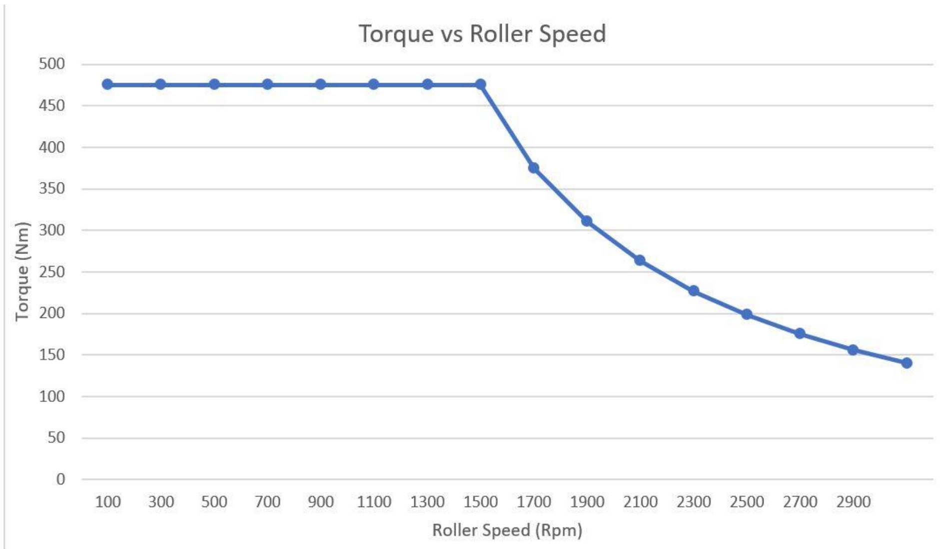
Given that for 0 to 1300rpm, α **= 47 rad/s^2**

- **T = 475.123 Nm at dynamometer**

For 1300 to 2940 rpm,

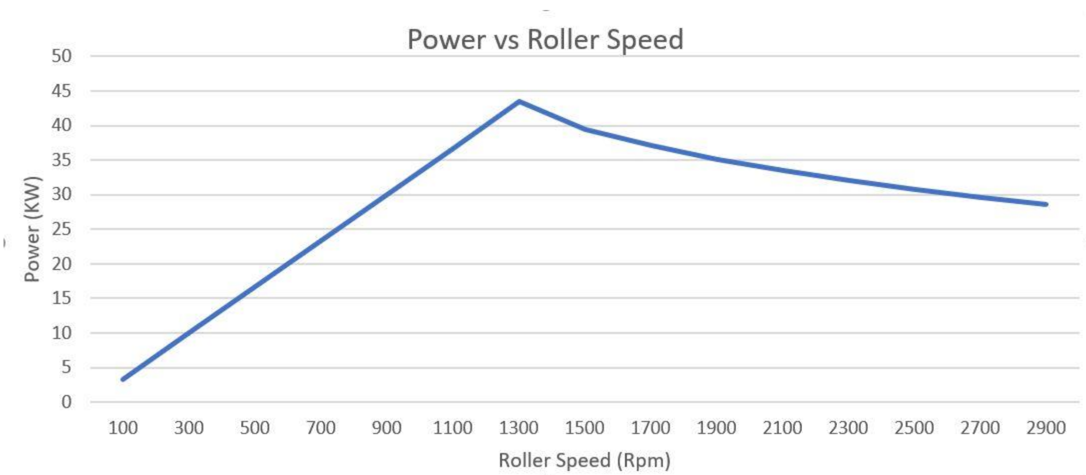
$$\alpha(x) = (2*10^6).x^{-1.49}$$

Power at dynamometer = $\dfrac{2\pi NT}{60}$, N = Roller rpm



The above graph shows the **torque at the dynamometer** vs roller speed

Maximum torque = **475.123 Nm**



The above graph shows the **power at the dynamometer** vs roller speed.

Max power obtained at dynamometer = 42.285 KW @ 1300 rpm

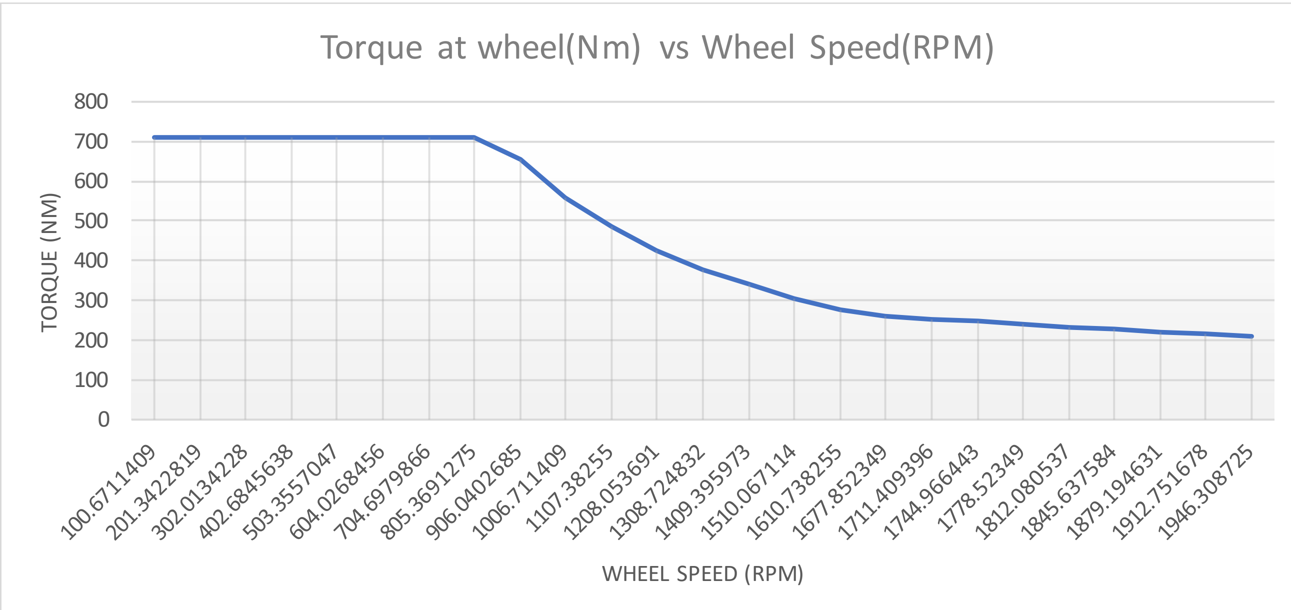
Now ,

To calculate the power and torque characteristics at motorcycle wheels:

Assuming rear tyre to be 140/70 r17, which is having diameter of 0.598m is coupled with the hollow cylinder of diameter of 0.4.

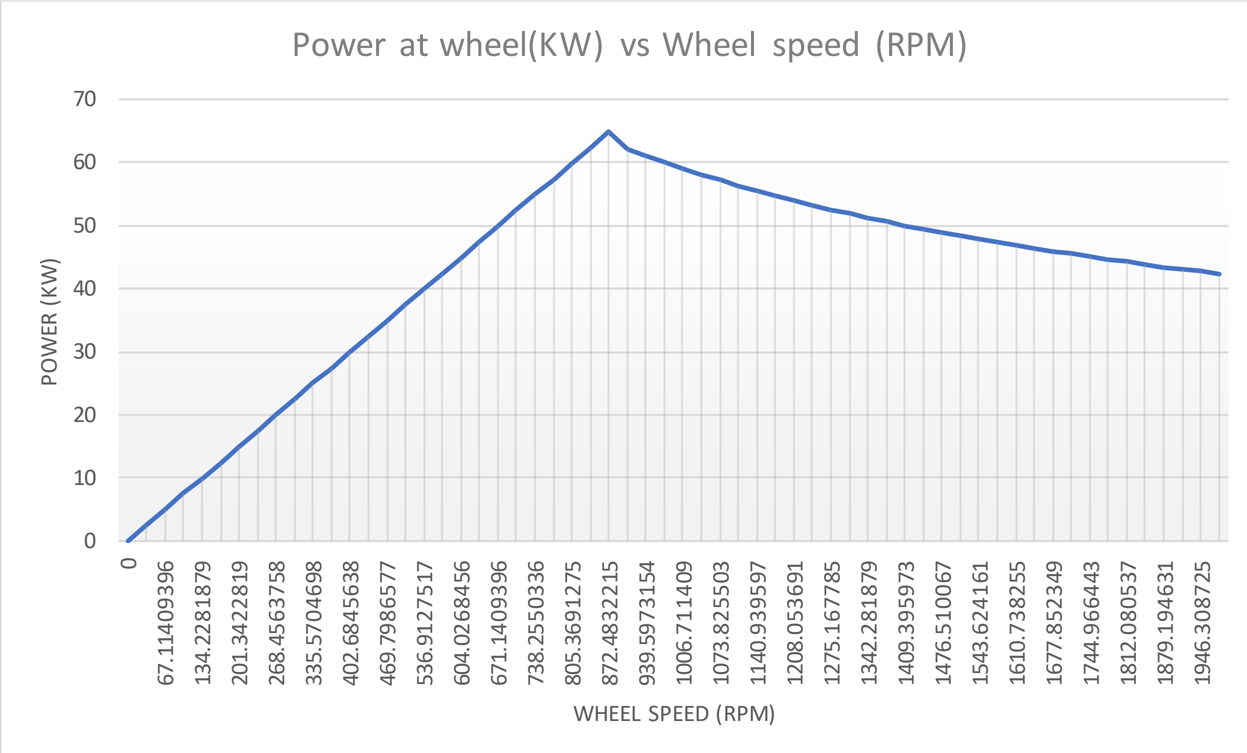
Speed ratio = 0.598/0.4 = 1.495.

- Therefore**, Toque at wheels = torque at dynamometer * speed ratio**
- **Power at wheels** = $\frac{2\pi(N/1.495)T}{60}$, N/1.495 = wheel speed
- **Wheel speed = Dynamometer speed / speed ratio**



The above graph shows the motorcycle torque at wheel vs wheel speed.

Maximum torque at wheels = 710.309 Nm



The above graph shows the motorcycle power at wheel vs wheel speed.

Maximum power at wheels = 64.886 KW (87 HP) @ 872.48 RPM

(ii) The time taken for the motorcycle to reach 100 Kmph on road and the top speed of the vehicle on the road.

At 100kmph, the wheel rpm = 890 wheel rpm and in the curve it coves mostly with linear acceleration itself. Hence to calculate linear acceleration,

- Rolling resistance

  $R_r = C_r * N$

  $= 0.01 * 2440$

  $\mathbf{R_r = 24.4\ N}$
- Starting torque = 710.3 Nm
- Tractive force = $T_f = \dfrac{T * \eta_t * \eta_f}{r}$,

  Where, η_t = transmission efficiency = 0.99

  η_f = chain drive efficiency = 0.98

  Tractive force = 2303.8 N
- Excess driving force = $[T_r – (R_r+R_a)]$, at low speed air resistance is zero
- EDF = 2303.8 – 24.4 = 2279.4

  Acceleration at low speed = 2279.4/m = 2279.4/244

  Acceleration at low speed = 9.34 m/s²

- Air resistance, $R_a = 0.5\ \rho C_d A V^2$, ρ = 1.225 Kg/m^3

Assuming driver at prone position,

At 100 kmph, **R_a=165.4 N**

- Excess driving force = [T_r – (R_r+R_a)]
  Ma = 2115.01,
  Acceleration = 2115.01/244 = 8.668 m/s^2

Therefore, average acceleration = 9.004 m/s^2

$v = u + at$

$t = \frac{\text{v-u}}{\text{a}}$ = 27.77/9.004

t = 3.084 sec

Top speed calculation :

By gear ratio:

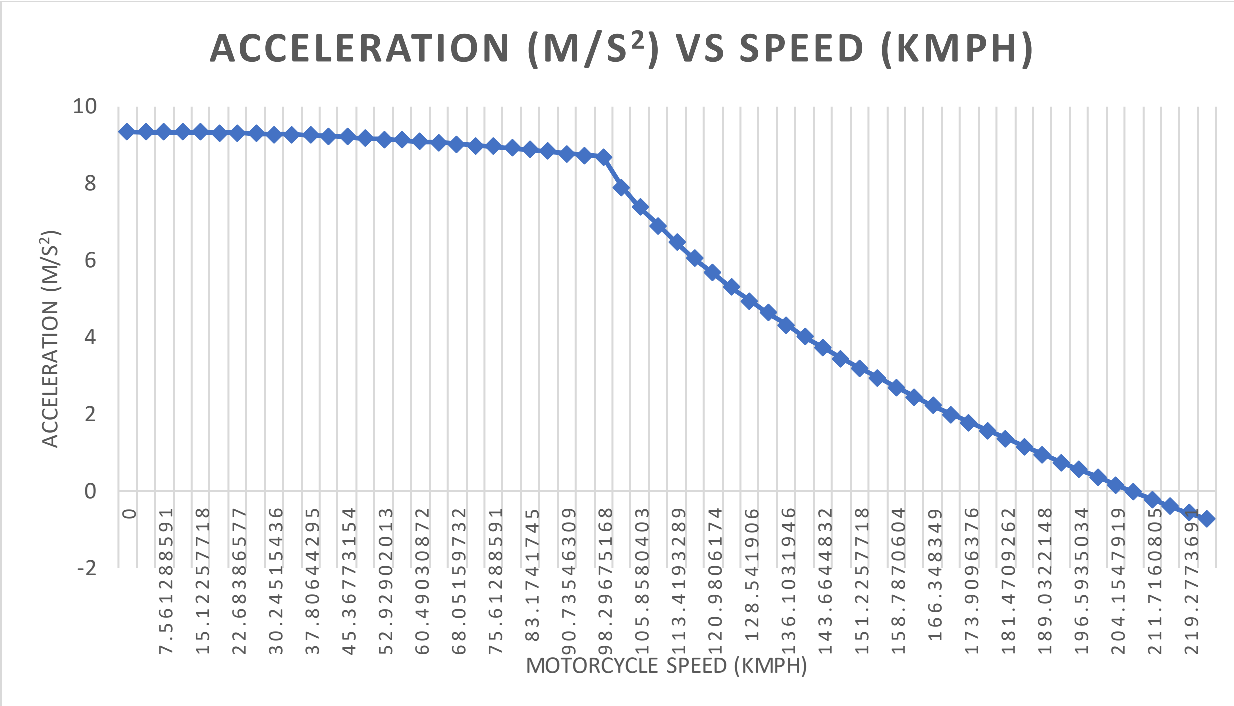
$v = \frac{2\pi \text{NR}}{60}$,

N = maximum wheel speed.

R = Radius of tyre = 0.299m

V = 222.3 kmph, it is only achieved if the wheel speed reaches the 1973.15 rpm, which is calculated from dynamometer.

- **But in actual case, the aerodynamic resistance dominates over the speed of motorcycle, and after reaching certain speed each kmph takes lot of time to overcome.**



- From the curve above shows that, the acceleration becomes negative at certain point, where the **resistance forces dominates the available tractive force** and hence the vehicle cannot be able to reach the kinametic top speed as obtained in above formula.

- So from the report, the maximum top speed it can be able to reach is 204.155 kmph and the time taken to reach that speed is also too high which is 323.53 sec, where the speed of 200 kmph having time taken about 151 sec.

- And the main assumption here is the **constant aerodynamic drag.** The time to reach the top speed may be high is the opposite wind force happens and becomes low if the motorcycle gets a tow from the vehicle at front.

(iii)Find the battery pack capacity when the vehicle reaches its top speed assuming it started with a pack capacity of 10 kWh. Now, consider that this vehicle is capable of reaching a stand still position from its top speed in 10 seconds using the regenerative braking system alone. Determine the final battery pack capacity after the vehicle reaches stand still condition.

Initial battery pack capacity = 10KWhr.

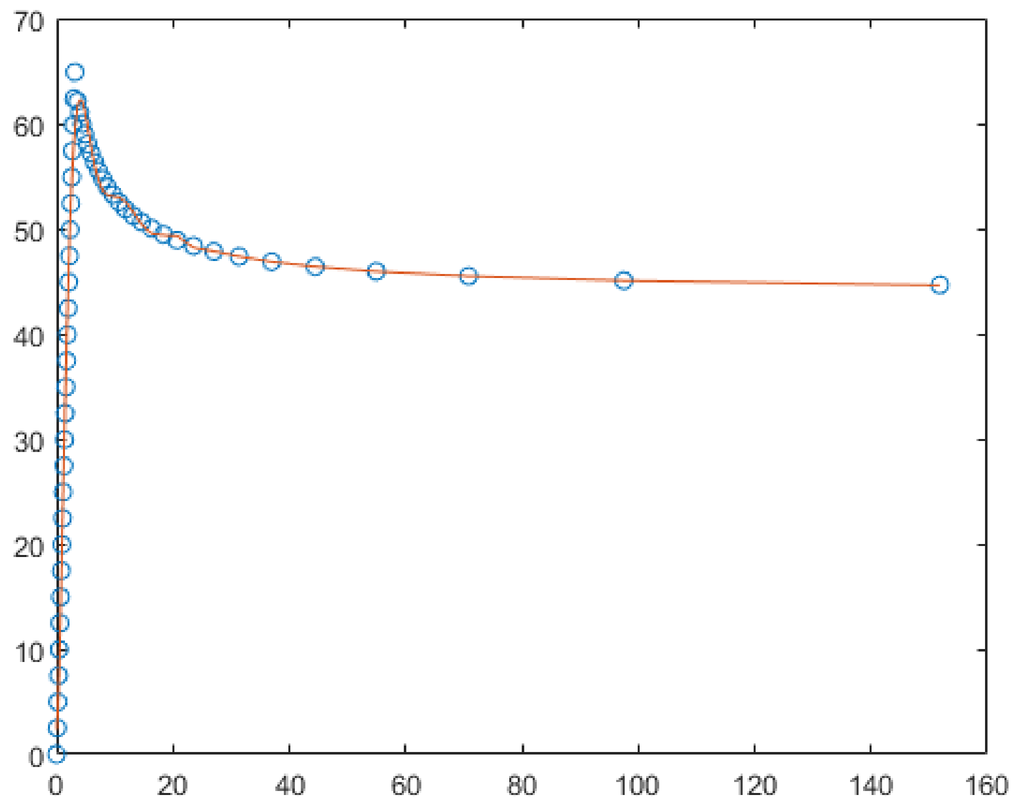
The vehicle reaches its top speed from its stand still position.

The **energy spent would be the area under the curve P-T**

The **time taken by vehicle = velocity / acceleration.**

Therefore the time data for each speed of the motorcycle is obtained.

By plotting the power vs time to reach the top speed curve,



X axis = time in sec

Y axis = power in KW.

The area under the curve would be the energy spent by the motorcycle.

By importing the curve in MATLAB and by doing the curve fitting by using polynomials, the equation for the curve can be generated.

Here 15 degree polynomial equation curve is fitted and their corresponding coefficents of the equation is generated.

-1.2409e+03
1.7617e+04
-9.6127e+04
2.5675e+05
-3.3901e+05
1.5201e+05
1.1648e+05
-1.4586e+05
2.2354e+04
2.6570e+04
-8.7580e+03
-1.3917e+03
626.7414
31.9964
-22.2378
50.9380

The coefficients of 15 degree polynomial equation is generated above.

On integrating the equation, energy can be obtained.

$$E = \int_0^t P.dt$$

$$y = \int_0^{0.0421} [-1240.9x^{15} + 17617x^{14} - 96127x^{13} + 256750x^{12} - 339010x^{11} + 152010x^{10} - 116480x^{9} - 145860x^{8} + 22354x^{7} + 26570x^{6} - 8758x^{5} - 1391.7x^{4} + 626.74x^{3} + 31.9964x^{2} - 22.237x + 50.938]$$

The maximum speed is chosen as 200.37kmph and the corresponding time taken to reach that top speed= 151.86 sec = 0.0421hrs

On solving , energy spent is obtained to be 2.13kwh.

Battery pack capacity after reaching its top speed = 10-2.13 = 7.87 kwh.

Now the motorcycle reaches to zero from 200kmph in 10 seconds, then the deceleration is considered as linear,

$$a = \frac{\text{v-u}}{t},$$

a = 5.56m/s^2

F = ma,

$$Fdx = m.a.dx$$

$$Fdx = m\frac{dv}{dt}dx$$

$$Fdx = m.v.dv$$

$$v^2 - u^2 = 2as$$

S = 55.64^2-0/2(5.56)=278.23m

Energy recovered = 244 * 5.56 *278.23 = 377.729 KJ/S

ENERGY RECOVERED = 104.9 Whr

After the motorcycle stops, battery capacity = 7.9749 kwhr

[QUESTION 2]

ANSWER:

- Kerb weight = 170 kg
- Rider weight = 80 kg
- Pillion weight = 70 kg
- Front shock absorber stroke = 130 mm
- Front shock absorber stiffness = 25 N/mm
- Rear shock absorber stroke = 60 mm
- Rear shock absorber stiffness = 80 N/mm
- Rake angle = 22° Rear motion ratio = 0.65
- Wheelbase = 1350 mm
- Rear Tyre Size = 140/70 R17
- Front Tyre Size = 120/70 R17
- Under static loading (with rider and pillion),
- Front axle load = 1650 N
- Rear axle load = 1550 N
- CG height = 450mm

To Find the front and rear shock absorber displacement if -

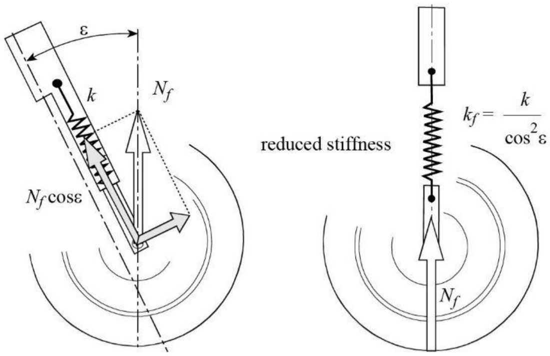
I) The bike is statically loaded,

FRONT:

REDUCED SUSPENSION STIFFNESS :

Since the motorcycle is provided with the rake angle the stiffness is reduced as the vertical load N_f resolved into components. Hence the reduced stiffness is considered for the calculation of displacement of the shock absorber.

$$K_f = \frac{K}{\cos^2 \delta}$$



Where ,

K_f = reduced suspension stiffness due to rake angle.

K = Front shock absorber stiffness

δ = Steering rake angle

K_f **= 29.08 N/mm**

Given front axle load = 1650 N

$$K_f = \frac{load}{deflection}$$

- Now the front wheel is on the ground, and the spring is mounted over the front wheel fork.
- The sprung mass is placed over the spring and so the stiffness is to be calculated without considering the unsprung mass load.
  Therefore,
  $K_f =$(front axle load – front unsprung mass)/ deflection
- Unsprung mass would be Tyre, rim, disc brake rotor, calipers and part of fork.

UNSPRUNG MASS ASSUMPTIONS :

1. Tyre = 120/70 r17,Pirelli diablo rosso ii, weight = 5.59 kg
2. Disc brake rotor = 1.59 kg (KTM 640 LC4 rotor)
3. Brembo caliper (M50) = 800 grm with brake pads
4. Rim = 5 kg
5. Part of fork = 1 kg

Total unsprung mass = 14 kgs

Therefore, 29.08 = (1650 – 140)/ deflection

Deflection of front spring under static loading = 51.92 mm

REAR:

Load at rear = 1550 N

- **Given motion ratio = Shock travel / wheel travel**

- **Wheel rate = spring rate * $(MR)^2$ * Spring angle**

- **Wheel rate = wheel load/ displacement**

Assuming, Spring at rear is mounted 20 degrees to vertical plane.

Wheel rate = 80N/mm * $(0.65)^2$ * cos 20

Wheel rate = 31.76 N/mm.

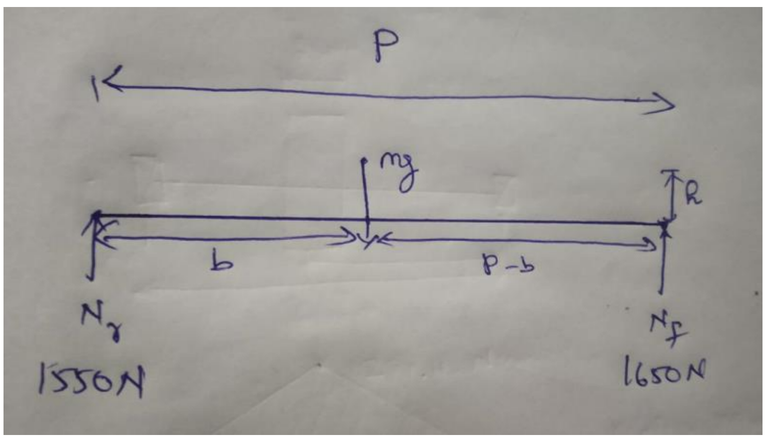
Wheel displacement = (wheel load / wheel rate)

Wheel displacement = 1550/31.76 = 48.8 mm

Therefore, shock displacement = Motion ratio * wheel displacement

Rear shock displacement under static loading = 31.72mm.

(ii) Under 0.75g Decleration:



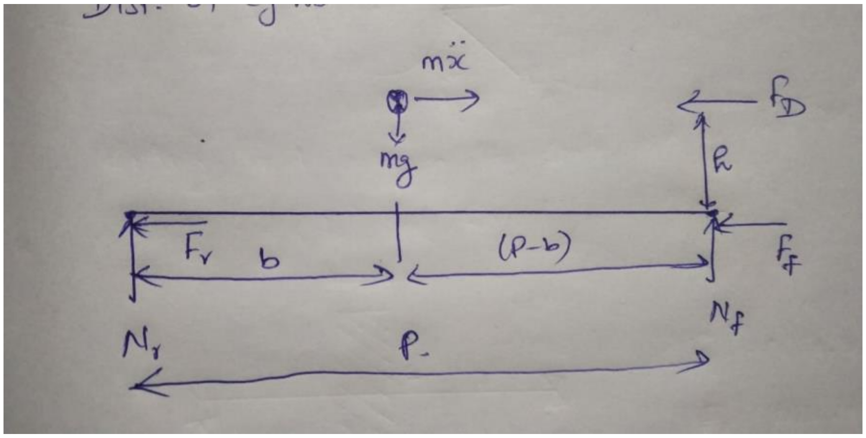
- $N_F + N_R = 3200$ N
- Taking moment about N_r,

1650(1350) – (mg)(b) = 0

b = 696.09 N

(p-b) = 653.90 N

Distance of cg from rear axle = 653.90 N



On resolving,

$-mx = F_f + F_r$

$mg - N_r - N_f = 0$

$-Fh - N_r b + N_f(p-b) = 0$

$\mathbf{N_F = mg(b/p)+F(h/p)}$

$\mathbf{N_R = mg(p-b/p)-F(h/p)}$

Therefore, load transfer towards front,

$N_F = mg(b/p)+F(h/p)$; and $F = mx$

$N_f = 3200[696.09/1350] + [450/1350](3200*0.75g)$

$N_f = 1649.99 + 800$

$\mathbf{N_f = 2450\ N}$

Reduced Load at rear wheel:

$N_R = mg(p-b/p)-F(h/p)$

$\mathbf{N_r = 750\ N}$

Now,

$K_f =$ (front axle load – front unsprung mass)/ deflection

29.08 = (2450 – 140)/ deflection

Front shock absorber displacement under 0.75g deceleration = 79.43 mm

REAR :

Wheel rate = 31.76 N/mm

Wheel displacement = (wheel load / wheel rate) = (750/31.76)

Wheel displacement = 23.61 mm

Shock displacement = MR * Wheel displacement

Shock displacement = 0.65 * 23.61 =15.34 mm

For the normal loads acting on tyres in the 0.75g deceleration scenario above, to design a braking system

I) Effective rotor diameters and corresponding clamping forces for front and rear

II) For the given caliper and master cylinder specifications, find the lever and pedal ratio required at front and rear respectively.

- **Here either the pedal/lever ratios or rotor diameters to be assumed to get the solutions. Both of them is not able to calculate since the clamping force calculation is the function of pedal/lever ratios.**
- **So I have calculated the rotor diameters by assuming the pedal and lever ratios**

N_f = 2450 N

N_r = 750 N

Given that coefficient of frictiob between road and tyre = 0.8.

We know that the maximum braking force offered by tyre = μ ***N**

Therefore, front braking force = μ ***N_f = 0.8*2450**

- **Front braking force = 1960 N**

Similarly,

- **rear braking force = μ *N_r = 600 N.**
- **Front braking force = (torque on rotor)/(radius of tyre)**

Given the front wheel size as 120/70 r17, from this spec, diameter of front tyre = 599.8mm

Effective rolling of tyre = 0.95 ;

- **Radius of front tyre = 599.8 *0.95/2 =284.905 mm**
- **Torque req at rotor = 284.905*1960 = 558.4 Nm**
- Braking torque = clamping force * μ_r * effective radius;

  Where, μ_r = coefficient of friction between pad and rotor.

- **Clamping force = pressure in the brake line * area of caliper * no of piston**

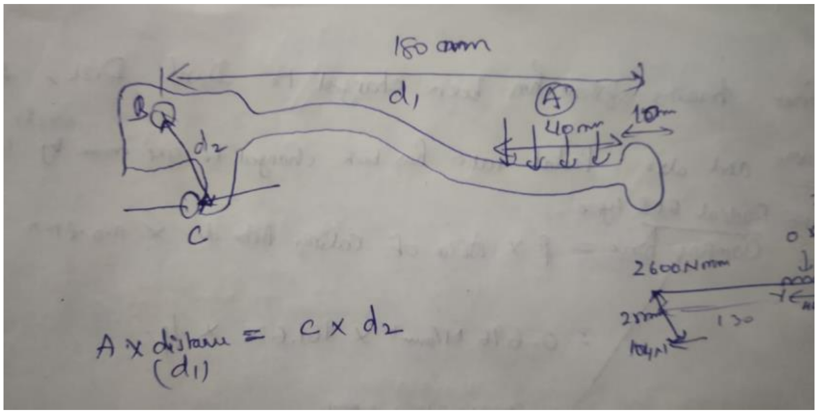
Given master cylinder dia = 16 mm

Area of master cylinder bore = **200.96 mm^2**

Given caliper piston diameter = 35mm

Area of caliper bore = **961.625mm²**

Assumption :



Lever dimension taken from Yamaha fazer 150

- Lever ratio = 175/25 = 7

Table 1
Maximum force levels for braking system controls during performance tests
(Clause 6.2& 6.3.2)

S. No	Type of control	Max. force to be applied N
1.	Finger grasp (flip levers and switches)	20
2.	Hand grasp	
3.	— upwards	400
4.	— downwards, sideways, fore-aft	300
5.	Foot treadle (ankle control)	350
6.	Foot pedal (leg control)	600

The above fig is from ARAI report

Assuming lever force to be 30 N under panic braking condition.

- Force on master cylinder = force on lever * lever ratio
  Force on master cylinder = 240 N
- Pressure = F/A = 1.044 Mpa
- Clamping force = 1.044 *961.625*2 = 1339.8 N
- Effective rotor radius = Torque /(clamping force * μ_r)
  Effective rotor radius = 463.5 mm
  Which is not possible size the rim diameter itself 432 mm.

Hence the front braking system has been changed to dual disc, 4 piston caliper and also lever ratio has been changed to get more pressure by using Brembo radial RCS master cylinder lever.

- Assuming length of the lever as 180mm and two settings of 18 mm and 20mm can be made with RCS RADIAL. 18 mm has been chosened
- Modified lever ratio = 180/18 = 10 : 1
- Force on master cylinder = force on lever * lever ratio = 30*10
  Force on master cylinder = 300 N

- Pressure on brake line = F/A = 1.4928 Mpa
- **Clamping force = 1.4928 *961.625*4 = 5742.18 N**
- Effective rotor radius = Torque /(clamping force * μ_r)

  Effective rotor radius = 162.07 mm

The effective rotor diameter at the front is 324.14 mm.

REAR:

- **Rear braking force = μ *N_r = 600 N**
- Torque req at rotor = braking force * effective radius of tyre

Rear tyre : 140/70 R17

Effective rolling radius of tyre **= 298.205mm**

- **Torque required = 600*298.205 = 178.923 Nm**

- Assumption: Pedal ratio is assumed to be (150/25) = 6 ,
  Where, dimensions were taken from fazer 150cc.
- Master cylinder force **=** force on pedal * pedal ratio.

Force on pedal is assumed to be 100 N (FROM ARAI REPORT)

- Master cylinder force **= 600 N**
- Master cylinder bore **=** 18mm (given)
- Area of master cylinder = **254.34 mm^2**
- Area of caliper bore dia = **961.625 mm^2**
- Pressure on rear brake line = 600/254.34 = **2.359 Mpa**
- **Clamping force = 2.359*961.625*1 = 2268.5 N**
- Effective rotor radius = 178.923/(2268.5*0.6) = 131.45mm

The effective rotor diameter at the rear is 262.9 mm.

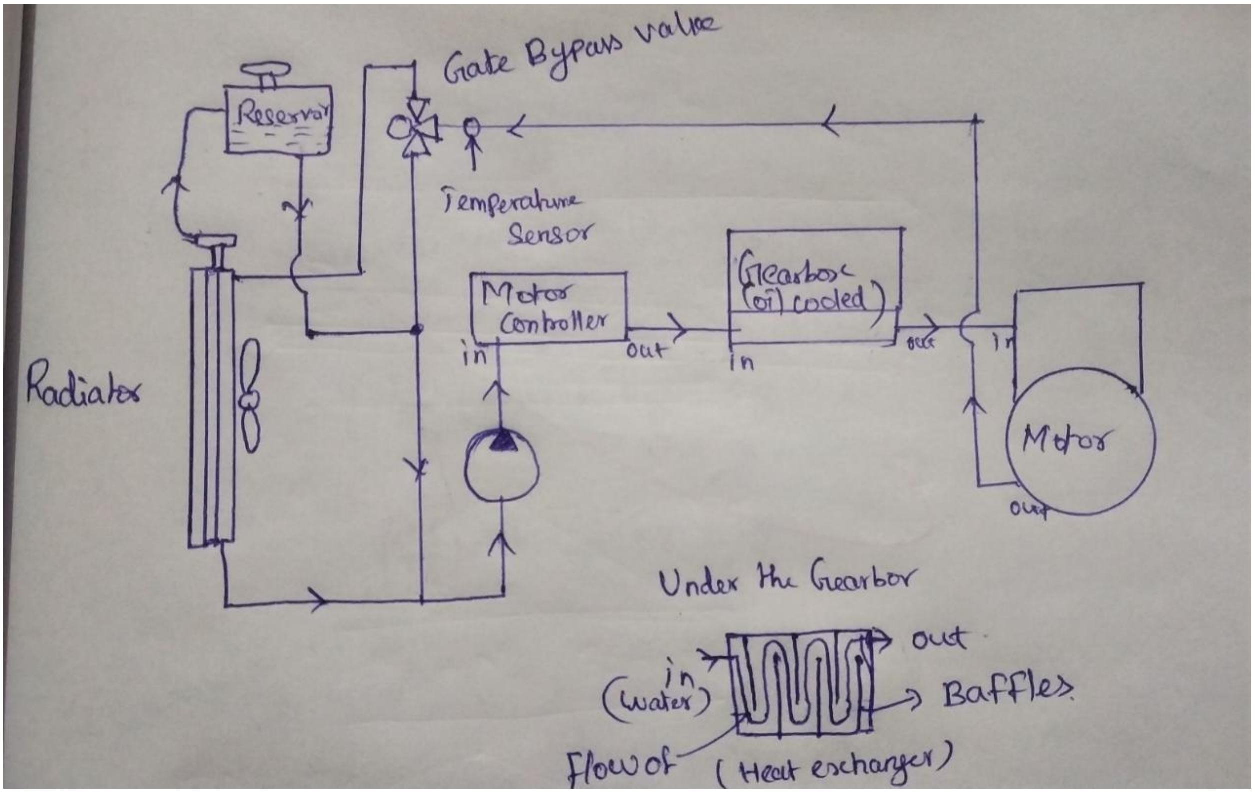
[Question 3]

- There are two cooling loops defined for the cooling of 4 components namely motor, motor controller, gear box and battery.
- The motor, motor controller and battery is mentioned as water cooled where as the gearbox is oil cooled.
- The first figure below explains the thermal management cooling loop of motor controller, gearbox and motor.

Sequence:

- The outlet of the cool coolent is connected by a hydraulic pump, which sends the cool coolent firstly into the motor controller then to the gearbox lower body and then into the motor.
- The sequence is arranged because motor controller creates heat less than gearbox in which the gearbox is already liquid cooled which produces less heat compared to motor.

- The coolent oil after reaches the gate bypass valve, where the temperature sensor senses the temperature and decides whether to bypass for recirculation into the circuit or to send it into the radiator for cooling.
- The gear box cooling is done by heat exchanger where the oil at the lower sump conducts heat to other side of the bay, where the water flows. The convective heat transfer takes place and carry away the heat from the oil to the water. Baffles are generally provided to provide good heat transfer.
- The motor cooling is done by passing the coolent on to the tube which is wounded around the circumference of the stator.

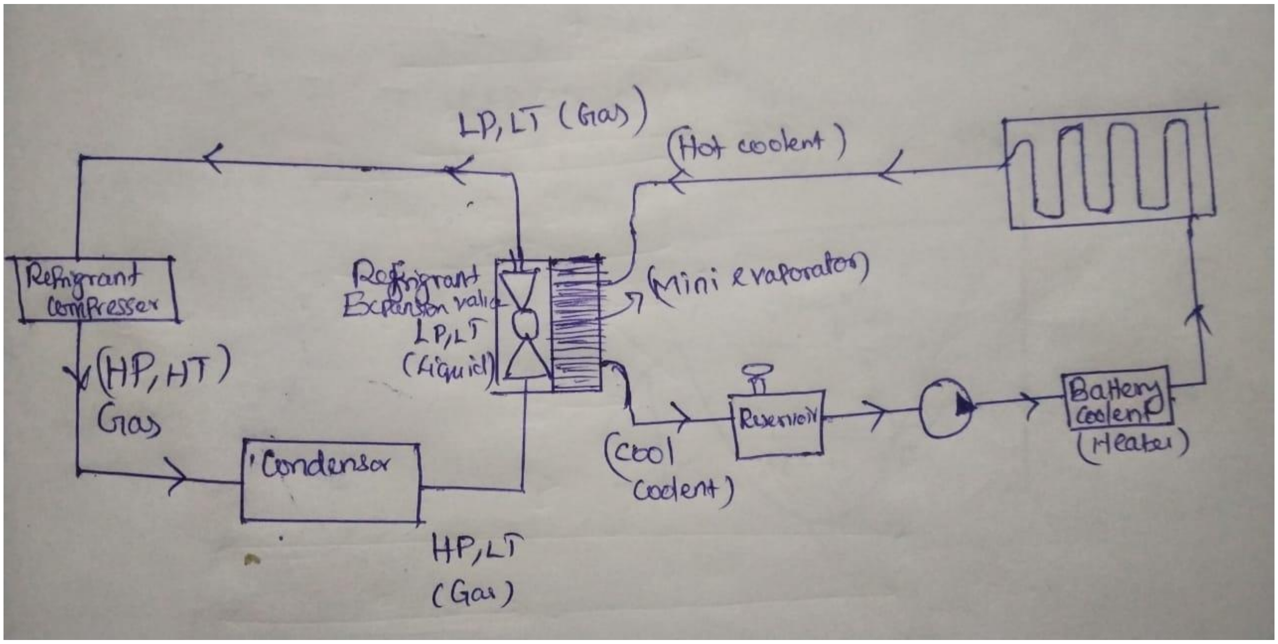


The above figure shows the cooling loop of motor controller, gear box and motor.

BATTERY COOLING SYSTEM :

- Battery cooling can be done by a separate loop which consists of refrigerant compressor, refrigerant expansion valve, heat exchanger(mini evaporator), pump and coolent heater.
- There are 2 circuits that provides heating and cooling of battery.
- Starting with reservoir, the coolent is pumpued to the battery coolent heater, where is measures the temperature and if the temperature is low, the heater heats up the coolent and sends away to the battery.
- The battery is packed in between the tubes, where the coolent oil flows, and if the battery heats up the coolent receives the heat by conduction to the pipes and then convective heat transfer to the coolent.
- The coolent then carry away the heat from the battery and goes into the mini evaporator.

- The refrigerant compressor compresses the Low pressure, low temperature gas into high pressure high temperature vapour. Then it travels to condenser where it condenses the heat to the atmosphere and becomes High pressure low temperature vapour.
- The vapour now enters the refrigerant expansion valve and produces Low pressure low temperature liquid and enters the mini evaporater chamber.
- Now the heat transfer takes place between hot coolent and refrigerant liquid and the refrigerant liquid carries the heat from the hot coolent and changes its phase to low temperature low pressure vapour and enters the next cycle.



The above figure shows the cooling and heating system of a battery.

[Question 4]

Design a speed reducer for the following requirements.

Power to be transferred = 20 kW.

Input speed = 8000; Rpm Output Speed = 750 rpm to 850 rpm

Pressure angle for gears = 20 degrees

Maximum Gearbox Size = 300 mm x 200 mm x 100 mm

Gear Material: Modulus of Elasticity = 200 GPa ; Poisson Ratio = 0.3

Q1. Calculate wear and bending stresses on all the gears by choosing appropriate gearbox configuration and gear parameters.

Q2. Calculate reaction forces on shafts and estimate suitable shaft diameters and material providing sufficient fatigue and static stress capacity for infinite life of shaft, with minimum factor of safety factors of 1.5.

Q3. Also select appropriate bearings for the shafts. Consider all shafts to be straddle mounted. Gear and Bearing Life > 12000 hours. For gear weight use volume approximation to be (PCD*Face Width)

Answer:

Power to be transferred = 20KW.

Speed reducer with Input speed = 8000 ; Output speed is taken as 800rpm.

Speed ratio = Drive gear / driven gear =8000/800 = 10 : 1;

Gear ratio : 10 : 1.

2 Stage reduction can be done for the 10 : 1 reduction.

1st stage = 3 : 1 and 2nd stage 3.33 : 1

Pressure angle = 20 degrees.

Selection of gear type:

- If it is in case of electric vehicle, noise is a major factor hence **Helical gear** is chosen since it provides a noiseless smooth operation, good contact ratio, also having high load bearing capacity since it is having point to line contact upon meshing. Whereas spur is having line contact upon meshing, produces shock effect which reduces the load capacity.

Helix angle : 15° is chosen. More helix angle increases load capacity but leads to more axial forces which may requires large bearings to eliminate them.

Calculation Method : AGMA 2001 D04 (American Gear Manufacturing Assosiation) standards.

The fundamental formulas for bending stress and contact stress on the gear tooth is gives as,

- $\sigma_{bend} = \frac{W_t \times P_d}{F\ J} K_o K_v K_s K_m K_b$
- $\sigma_c = C_p \sqrt{\left(\frac{W_t}{\mathrm{F\ d}}\right) . K_o K_v K_s K_m (\frac{C_f}{I})}$

CALCULATIONS:

1st STAGE REDUCTION, 3 : 1

No. of teeth in pinion (Z_1) is taken as 18, to avoid interference.

No. of teeth on gear(Z_2) = 3 x 18 = 54.

1) **Transverse Pressure angle,**

$$\tan \varphi_t = \tan \varphi_n / \cos \theta$$

$$\varphi_t = 20.64^\circ$$

Where,

φ_t = Transverse pressure angle

φ_n = Normal pressure angle, 20˚

θ = Helix angle, 15˚

2) Transverse module,

$$m_t = m_n / \cos\theta$$

m_t = 2.0705 mm

Where,

m_t = Transverse Module.

m_n = Normal module, 2mm is chosen to provide compact gearbox

3) Transverse pinion pitch diameter,

$$d_p = m_t * Z_1 \qquad d_g = m_t * Z_2$$

$$d_p = 1.467\ inch \qquad d_g = 4.401\ inch$$

Where,

d_p = Pinoin Pitch circle diameter.

d_g = Gear pitch circle diameter.

Z_1 = Number of teeth in pinion, 18

Z_2 = Number of teeth in gear, 54

4) Transvere diametral pitch,

$$P_d = Z / d$$

P_d = 12.269 / in

Where,

P_d = Diametral pitch

Z = No. of teeth of pinion/gear.

d = pitch circle diameter of pinon/gear

5) Transmitted load,

$W_t = 33000\ P/V$

$V = \pi d_p n_p / 12$

V = 3072.47 ft/min.

W_t = 288.061 lbf

Where,

P = Power in HP, 26.82 HP.

V = Pitch line velocity, ft/min

n_p = Speed of pinion rotation, 8000 rpm

W_t = Transmitted tangential load, lbf

6) Factors,

- Overload factor, $\boldsymbol{K_o}$ **= 1**, Since speed reducer, no gear shifting, assuming power source as uniform loading.
- Size factor, K_s

$$K_s = 1.192\left[\frac{F\sqrt{Y}}{P_d}\right]^{0.0535} = 1/K_b \qquad ....(6.1)$$

From lewis form factor table,

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

*Lewis form factor values for 20˚ pressure angle, AGMA - Shigley

Z_1 = 18, Y = 0.309

Z_2 = 54, Y = 0.4142

F = Facewidth

- $8/P_d$ < F < $16/P_d$ = 0.6297 < F < 1.2596
- Facewidth is chosen as 0.9842 in (25 mm)

By substitution of values on eq. 6.1,

For Pinion, $\boldsymbol{K_s}$ **= 1.0092**

For gear, $\boldsymbol{K_s}$ **= 1.0172**

- Rim thickness factor, K_b = 1.
- Dynamic factor, K_v

$$K_v = \left[\frac{A+\sqrt{V}}{A}\right]^B$$; A = 50+56(1-B)

B = $0.25(1-Q_v)^{2/3}$

Gear quality 3 – 7 (commercial quality gears)

Gear quality 8 – 12 (Precision quality gears)

Assuming gear quality = 8 (Precision) values of A and B are found to be,

A = 0.6299; B = 70.72

From the above formula, $\boldsymbol{K_v}$ **= 1.439**.

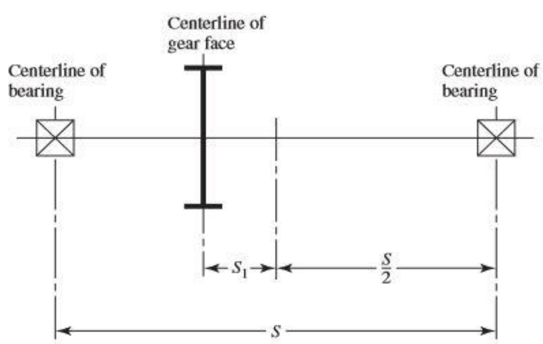
- Load distribution factor, K_m

$$K_m = C_m F = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \qquad \text{....(6.2)}$$

Where,

C_{mc} = 1, for uncrowned teeth.

C_{pm} = 1, for straddle-mounted pinion with S1/S < 0.175



$$C_{pf} = \frac{F}{10d} - 0.025$$ Facewidth, F <= 1 in; d = PCD of pinion = 1.417 in

C_{pf} = 0.0694.

$$C_{ma} = A + BF + CF^2$$

For Precision, closed units,

A = 0.0675; B = 0.0128; C = $-0.926(10^{-4})$

C_{ma} = 0.799

On substitution of values in eq 6.2,

$$K_m = \mathbf{1.149}$$

- Geometry factor, J

From table, **AGMA 908 B89** datasheet, For pressure angle of 20˚ and helix angle of 15˚, the geometry factor values for the corresponding teeth of pinion and gears are found to be,

I AND *J* FACTORS FOR:[1]

20.0 DEG. PRESSURE ANGLE
15.0 DEG. HELIX ANGLE
0.250 TOOL EDGE RADIUS
EQUAL ADDENDUM ($x_1 = x_2 = 0$)

2.250 WHOLE DEPTH FACTOR
0.024 TOOTH THINNING FOR BACKLASH
LOADED AT TIP

GEAR		PINION TEETH 12		14		17		21		26		35		55		135	
TEETH		P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	I																
	J	U	U														
14	I																
	J	U	U	U	U												
17	I					0.124											
	J	U	U	U	U	0.43	0.43										
21	I					0.139		0.128									
	J	U	U	U	U	0.44	0.46	0.47	0.47								
26	I					0.154		0.143		0.132							
	J	U	U	U	U	0.45	0.49	0.48	0.50	0.50	0.50						
35	I					0.175		0.165		0.154		0.137					
	J	U	U	U	U	0.46	0.52	0.49	0.53	0.51	0.53	0.54	0.54				
55	I					0.204		0.196		0.187		0.171		0.143			
	J	U	U	U	U	0.47	0.55	0.50	0.56	0.53	0.57	0.56	0.58	0.59	0.59		
135	I					0.244		0.241		0.237		0.229		0.209		0.151	
	J	U	U	U	U	0.48	0.59	0.51	0.60	0.54	0.61	0.57	0.62	0.61	0.64	0.65	0.65

1 The letter "U" indicates a gear tooth combination which produces an undercut tooth form in one or both components and should be avoided. See Section 7 and Fig 7–1.

For pinion, **J = 0.47**

For gear, **J = 0.55**

- Geometry factor, I

From table, AGMA 908 B89 datasheet, For pressure angle of 20˚ and helix angle of 15˚, the geometry factor values for the corresponding teeth of pinion and gears are found to be,

For both pinion and gear, **I = 0.204**

- Surface condition factor, C_f

Standard surface conditions for gear teeth have not yet been established. When a detrimental surface finish effect is known to exist, AGMA specifies a value C_f of greater than unity.

Assuming good surface finish of gears, C_f = 1.

7) Elastic Coefficient, C_p

$$C_P = \left[\frac{1}{\pi(\frac{1-\mu_P{}^2}{E_P}) + (\frac{1-\mu_G{}^2}{E_G})} \right]^{0.5}$$

Where,

C_P = Elastic coefficient, $[lb/in]^2$

$\mu_P = \mu_G$ = poisson ratio = 0.3

E_P = E_G = Modulus of elasticity = 200Gpa

8) Gear life > 12000 hrs

Pinion life = 8000*12000*60 = 5.76 x 10^9 cycles

Assume life to design = 6 x 10^9 cycles

Bending Stress Equation,

$$\sigma_{\text{bend}} = \frac{W_t \text{ x } P_d}{F\ J} K_o K_v K_s K_m K_b$$

On substitution,

For pinion,

$$\sigma_{bend} = \frac{(288.06*1*1.439*1.0092*12.269*1.149*1)}{0.9842*0.47}$$

σ_{bend} = 12,748.77 psi , for pinion

For Gear,

$$\sigma_{bend} = \frac{(288.06*1*1.439*1.0172*12.269*1.149*1)}{0.9842*0.55}$$

σ_{bend} = 10,980.78 psi , for gear

Design Bending Strenght:

Since Agma specifies a wide range of allowable strength values and has a generalized strength values for steel and no specific material strength has been specified, allowable bending strength equation has been taken from PSG databook, given as,

$$\sigma_{allowable} = \frac{1.4K_{bl}}{nK_{\sigma}}.\sigma_{-1}$$

Where, $\sigma_{allowable}$ = Allowable bending strength of material.

K_{bl} = life factor for bending

K_{σ} = fillet stress concentration factor

σ_{-1} = Endurance limit stress for bending, kgf/cm^2

Material selection : 20MnCr5

The material is selected because of widely used having good strength, low cost and easy availability having ultimate strength of 1000 – 1300 Mpa.

Heat treatment after machining : Case hardening and tempered = 58 HRC

Case hardened depth = 0.35 to 0.55 mm

From PSG data book,

K_{bl} = 0.7 (from table life factor bending for steel > 350 HB, > 25 x 10^7 cycles)

K_{σ} = 1.2 (steel case hardened, 0<x<0.1, addendum modification coefficient)

For alloy steel, $\sigma_{-1} = 0.35\sigma_u + 1200$, kgf/$cm^2$

σ_{-1}=0.35(10197.2)+1200 = 67732.62 psi

On substitution,

$\sigma_{allowable}$ **= 55314.97 psi**

Safety factors,

For pinion, n = 4.33

For gear, n = 5.037

Contact Stress Equation,

$$\sigma_c = C_p \sqrt{\left(\frac{W_t}{\text{F d}}\right) . K_o K_v K_s K_m (\frac{C_f}{I})}$$

On substitution,

For pinion,

$$\sigma_c = 2252.67\sqrt{\frac{288.06*1*1.439*1.0092*1.149*1}{1.467*0.9842*0.204}}$$

σ_c = 91,001.02 psi , for pinion

For gear,

$$\sigma_c = 2252.67\sqrt{\frac{288.06*1*1.439*1.0172*1.149*1}{1.467*0.9842*0.204}}$$

σ_c = 91,360.98 psi , for gear

Design Contact Strenght:

Since Agma specifies a wide range of allowable strength values and has a generalized strength values for steel and no specific material strength has been specified, allowable bending strength equation has been taken from PSG databook, given as,

$$\sigma_{allowablecontact} = C_R * HRC * K_{cl}$$

Where, $\sigma_{allowablecontact}$ = Allowable contact strength of material.

K_{cl} = life factor for pitting

C_R = Coefficient depending on surface hardness

HRC = Rockwell hardness C number

Material selection : 20MnCr5

Ultimate tensile strength = 1000-1300 Mpa

HRC = 58.

From PSG data book,

For alloy steel, case hardened, HRC = 58, C_R = 280 Kgf/cm^2

$\sigma_{allowablecontact}$ = 280 * 58 * 0.585 = 9500.4 Kgf/cm^2

$\sigma_{allowablecontact}$ = 135,127.45 psi

Safety factors,

For pinion, n = 1.484

For gear, n = 1.479.

2nd STAGE REDUCTION: 3.33 : 1

No. of teeth in pinion (Z_1) is taken as 18, to avoid interference.

No. of teeth on gear (Z_2) = 3.33 x 18 = 60.

Input speed = 8000/3 = 2666.67 rpm

For the second stage reduction **module** is chosen as **2mm** along with **facewidth** as **25mm** and found the stress values as,

- Bending stress , for pinion = 32,270.92 psi

  For gear = 27,457 psi

Comparing with allowable bending strength value of 55,314.97 psi, FOS are found to be,

For pinion, n = 1.7140

For gear, n = 2.01.

- Contact stress is found as, For pinion = 141,735.17 psi

  For gear = 142,330.77 psi

Comparing with allowable contact stress(HRC = 58) = 135,127.45 psi, **DESIGN FAILED.**

Hence the **module** is increased to **2.5mm, HRC** increased to **63** and **facewidth** also increased to **30mm.**

The 2nd stage reduction parameters and their corresponding values were tabulated in the following and the formulae used were the same as the formulae used in the 1st stage reduction.

Parameter	**Value**	**Description**
Z_1	18	No. of teeth on pinion
Z_2	60	No. of teeth on gear
m_n (mm)	2.5	Normal module
m_t (mm)	2.588	Transverse module
φ_t	20.64	Transverse pressure angle
d_p (in)	1.833	Transverse pcd of pinion
dg (in)	6.113	Transverse pcd of gear
P_d (/in)	9.819	Transverse diametral pitch
*W_t (lbf)	691.99	Transmitting tangential load at tooth interface
T (lb in)	634.209	Torque at second stage pinion
F (in)	1.181	Face Width
V (ft/min)	1024.16	Pitch line velocity
K_o	1	Overload Factor
K_m	1.127	Load Distribution Factor
K_s	1.0313	Size Factor, pinion
K_s	1.0400	Size factor, gear
K_v	1.2934	Dynamic Factor
J	0.4725	Geometry Factor(bending), pinion
J	0.56	Geometry factor(bending), gear
I	0.214	Geometry factor (pitting)
C_p [lb in]$^{0.5}$	2252.67	Elastic coefficient
C_f	1	Surface Condition Factor

***Transmitted tangential load (W_t),**

$$T = \frac{P*60}{2\pi N_p}$$

N_P = speed of pinion, rpm = 2666.67.

T = 71.655 Nm = 634.209 lb in.

$W_t = 2T/P_d$ = 691.99 lb

Gear life > 12000 hrs

Pinion life = 2666.67*12000*60 = 1.9 x 10^9 cycles

Assume life to design = 2 x 10^9 cycles

Bending Stress Equation,

$$\sigma_{\text{bend}} = \frac{W_t \text{x} P_d}{F\ J} K_o K_v K_s K_m K_b$$

On substitution,

For pinion,

$$\sigma_{bend} = \frac{(691.99*1*1.29*1.0313*9.819*1.127*1)}{1.181*0.4725}$$

σ_{bend} = 18,256.35 psi , for pinion

For Gear,

$$\sigma_{bend} = \frac{(691.99*1*1.29*1.04*9.819*1.127*1)}{1.181*0.56}$$

σ_{bend} = 15,533.74 psi , for gear.

Design Bending Strenght:

Since Agma specifies a wide range of allowable strength values and has a generalized strength values for steel and no specific material strength has been specified, allowable bending strength equation has been taken from PSG databook, given as,

$$\sigma_{allowable} = \frac{1.4K_{bl}}{nK_\sigma}.\sigma_{-1}$$

Where, $\sigma_{allowable}$ = Allowable bending strength of material.

K_{bl} = life factor for bending

K_σ = fillet stress concentration factor

σ_{-1} = Endurance limit stress for bending, kgf/cm^2

Material selection : 20MnCr5

The material is selected because of widely used having good strength, low cost and easy availability having ultimate strength of 1000 – 1300 Mpa.

Heat treatment after machining : Case hardening and tempered = **63** HRC

Case hardened depth = 0.35 to 0.55 mm

From PSG data book,

K_{bl} = 0.7 (from table life factor bending for steel > 350 HB, > 25 x 10^7 cycles)

K_σ = 1.2 (steel case hardened, 0<x<0.1, addendum modification coefficient)

For alloy steel, $\sigma_{-1} = 0.35\sigma_u + 1200$, kgf/$cm^2$

σ_{-1}=0.35(10197.2)+1200 = 67732.62 psi

On substitution,

$\sigma_{allowable}$ **= 55314.97 psi**

Safety factors,

For pinion, n = 3.029

For gear, n = 3.561

Contact Stress Equation,

$$\sigma_c = C_p \sqrt{\left(\frac{W_t}{\mathrm{F\ d}}\right).K_o K_v K_s K_m (\frac{C_f}{I})}$$

On substitution,

For pinion,

$$\sigma_c = 2252.67\sqrt{\frac{691.99*1*1.29*1.0313*1.127*1}{1.833*1.181*0.214}}$$

σ_c = 106,606.5 psi , for pinion

For gear,

$$\sigma_c = 2252.67\sqrt{\frac{691.99*1*1.29*1.04*1.127*1}{1.833*1.181*0.214}}$$

σ_c = 107,055.2 psi , for gear

Design Contact Strenght:

Since Agma specifies a wide range of allowable strength values and has a generalized strength values for steel and no specific material strength has been specified, allowable bending strength equation has been taken from PSG databook, given as,

$$\sigma_{allowablecontact} = C_R * HRC * K_{cl}$$

Where, $\sigma_{allowablecontact}$ = Allowable contact strength of material.

K_{cl} = life factor for pitting

C_R = Coefficient depending on surface hardness

HRC = Rockwell hardness C number

Material selection : 20MnCr5

Ultimate tensile strength = 1000-1300 Mpa

HRC = 63.

From PSG data book,

For alloy steel, case hardened, **HRC = 63**, C_R = 280 Kgf/cm^2

$\sigma_{allowablecontact}$ = 280 * 63 * 0.585 = 10319.4 Kgf/cm^2

$\sigma_{allowablecontact}$ = 146,776.36 psi

Safety factors,

For pinion, n = 1.38

For gear, n = 1.37.

SHAFT DESIGN

- The 4 gears were accommodated in 3 shafts in the gearbox.
- Input shafts holds pinion and 2 bearings
- Intermediate shaft holds a gear from first stage reduction and pinon for second stage reduction and 2 bearings.
- Output shaft holds gear from second stage reduction and 2 bearings.

The diameter of shafts were determined in each stages by using ASME deisgn code, which is given below,

$$\tau_{allowable} = \frac{16}{(1-k^4)\pi d^3}\sqrt{(C_b * M + \frac{\alpha f d(1+k)^2}{8})^\wedge 2) + (C_t * T)^\wedge 2)}$$

Where,

$\tau_{allowable}$ = Allowable shear stress of shaft material.

d = diameter of shaft, m

k = Ratio of inner to outer diameters of the shaft

(k = 0 for a solid shaft)

F = Axial force (tensile or compressive)

α = Column-action factor(1.0 for tensile load)

M = Bending moment acting on shaft, Nm

T = Twisting moment acting on shaft, Nm

C_b = Bending factor

C_t = Torsion factor

Load acting on the input shaft,

- Torque acting on input shaft,

$$T = \frac{P * 60}{2\pi N}$$

Where,

P = Power (20 kw)

N = Speed of input shaft, 8000 RPM

T = 23.885 Nm

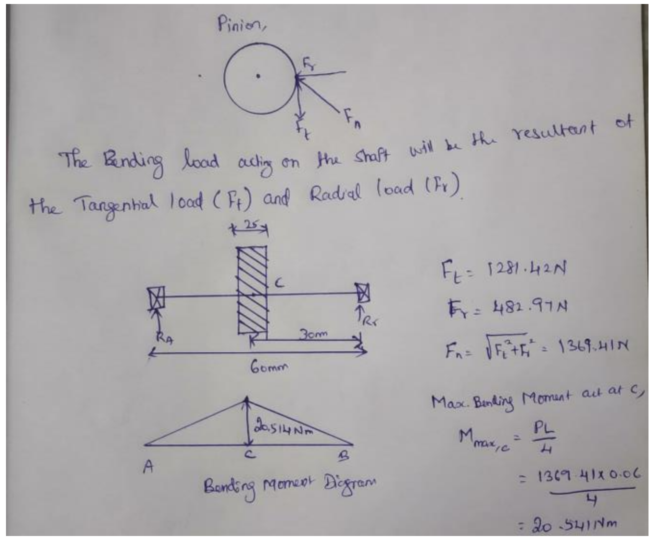
- Bending Moment acting on shaft,

The resultant of the radial and tangential forces acting on gear would be the bending load acting on the shaft.

Tangential load on pinion teeth = F_t = 288.06 lb = 1281.42 N

Radial load on pinion teeth = $F_R = F_t \frac{\tan\varphi}{\cos\theta}$ = 482.97 N

Resultant load = $F_n = \sqrt{F_T^{\,2} + F_R^{\,2}}$ = 1369.41 N, acting on the shaft.



From the image above, length of shaft is initially assumed to be 60mm. The maximum bending moment of the shaft acting at center = PL/4

$M_{max,c}$= 20.541 Nm

The reaction loads acting on shaft = R_a=R_b= 1369.41/2 = 684.7 N.

- Axial load acting on the shaft due to helical gear pattern,

  $F_a = F_T \tan\theta$ = 1281.42 x tan 15

  F_a = 343.35 N

Column action factor, $\alpha = \dfrac{1}{1 - 0.0044(L/K)}$; L = length of shaft,

K = radius of gyration = $\sqrt{\dfrac{I}{A}}$ = 0.25d^2

On solving,

$$\alpha = \frac{d}{d - 0.0015}.$$

Permissible shear stress,

From design of machine elements book, V B Bhandari,

$$\begin{matrix} \tau = 0.3(\sigma_{yt}) \\ \tau = 0.18(\sigma_{ut}) \end{matrix}$$, whichever is minimum.

Material : 20MnCr5, σ_{ut} = 1000 Mpa

Therefore, $\tau_{allowable}$ = 180 Mpa.

Bending and torsion factors is taken for load applied suddenly and having minor shock,

$C_m = 1.5$

$C_T = 1$

For solid shaft, K = 0,

$$\tau_{allowable}/n = \frac{16}{\pi d^3}\sqrt{(C_b * M + \frac{\alpha f d^2}{8})^2) + (C_t * T)^2)}$$

On substituting the values on the above equation with **safety factor, n =1.5** and when simplified an 8 order equation is obtained

$5.546*10^{14}d^8 - 1.1646*10^{12}d^7 + 6.114*10^8 d^6 - 1841.95d^5 - 2644.7d^3 - 1516.8d^2 + 3.1911d - 1.6753*10^{-3}$ = 0.

On further solving, from 8 solutions the maximum diameter is chosen as the diameter of the shaft,

d = 11.87mm. In order to get Fos > 1.5, d is chosen as **13mm.**

Now, to determine the fos of the shaft for infinite life cycles.

For steels, $S_{e'}$ = 0.5 S_{ut}

- $S_e = K_a K_b K_c K_d S_{e'}$'

Where,

S_e = Endurance limit stress of a particular mechanical component subjected to reversible bending stress

$S_{e'}$ = Endurance limit stress of a rotating beam specimen subjected to reversible bending stress

K_a = Surface finish factor

K_b = Size factor

K_c = Reliability factor

K_d = Modifying factor to account for stress concentration.

Taken the diameter of shaft at gear portion to be 1mm more, hence 14mm

$$K_a = a\left(S_{ut}\right)^b$$

$$0.9S_{ut} = 900Mpa$$

$$\log_{10}(0.9S_{ut}) = 2.9452$$

$$\log_{10}(S_e) = \log_{10}(208.27) = 2.3186$$

$$\log_{10}(10^7) = 7 \qquad ; \; K_b = 1.26d^{-0.107}$$

$$\log_{10}(10^3) = 3$$

$$\log_{10}(10^6) = 6$$

$$\sigma_b = \frac{32(M_b)}{\pi d^3}$$

$$\log_{10}(S_f)' = 2.954 - \frac{(2.954 - 2.3186)}{(6-3)}(7-3)$$

Now, assuming surface finish as ground, from **VB Bhandari book**

Table 5.1 *Values of coefficients a and b in surface finish factor*

Surface finish	*a*	*b*
Ground	1.58	–0.085
Machined or cold-drawn	4.51	–0.265
Hot-rolled	57.7	–0.718
As forged	272	–0.995

,

a = 1.58 and b = -0.085.

K_a = 0.878

K_b = 1.24(14)$^{-0.107}$ = 0.934

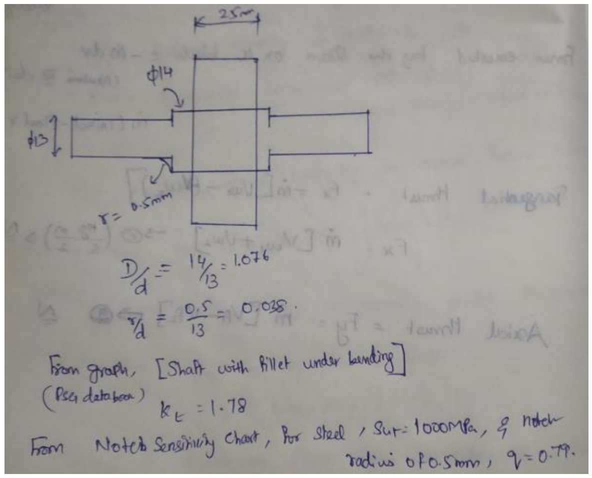
K_C = 0.814 for the reliability of 99%

Table 5.3 *Reliability factor*

Reliability R (%)	K_c
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

$K_d = 1/K_f$;

$K_f = 1 + q(K_t - 1)$



The above diagram shows the fillet portion of shaft that is taken under consideration.

D/d = 1.076

r/d = 0.038

From PSG databook for shaft with fillet radius under bending

$K_t = 1.78$.

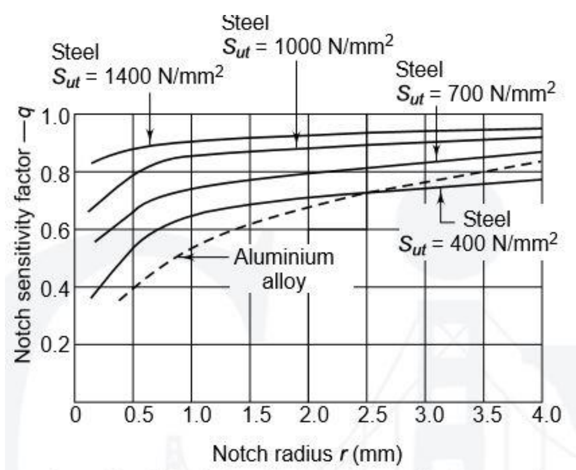


Fig. 5.22 *Notch Sensitivity Charts (for Reversed Bending and Reversed Axial Stresses)*

From the above graph, notch sensitivity factor for steel with 1000 Mpa and notch radius of 0.5mm is

$$q = 0.79$$

Therefore, $K_f = 1+0.79(1.78-1)$

$$\mathbf{K_f = 1.6162}$$

On substituting the values in S_e, it is found to be

S_e **= 208.27 Mpa**

Now to determine the fatigue strength and fos of shaft for infinite cycles.

Assume infinite cycles to be 1 x 10^7

$$0.9S_{ut} = 900Mpa$$
$$\log_{10}(0.9S_{ut}) = 2.9452$$
$$\log_{10}(S_e) = \log_{10}(208.27) = 2.3186$$
$$\log_{10}(10^7) = 7$$
$$\log_{10}(10^3) = 3$$
$$\log_{10}(10^6) = 6$$

Therefore, $\log_{10}(S_f)^{'} = 2.954 - \frac{(2.954 - 2.3186)}{(6-3)}(7-3)$

$(S_f)^{'}$ = 127.8 Mpa

S_f = σ_b = $\frac{32(M_b)}{\pi d^3}$ = 76.28 Mpa

FOS : $(S_f)^{'}$ **/** S_f **= 1.675 > 1.5**

SHAFT 2 : INTERMEDIATE SHAFT:

Loads acting on the intermediate shaft,

- LOAD ON GEAR ON FIRST STAGE DRIVEN
  1. Tangential load R_T = 1281.42 N
  2. Radial load R_A = 482.97 N
  3. Axial load, F_a = 343.35 N

Resultant load = 1369.41 N

- LOAD ON PINION ON SECOND STAGE DRIVER
  1. Tangential load R_T = 2T/d = (2*23873*3/46.58) = 3075.09 N; d = pinion dia of 2nd stage; T = torque on 2nd stage = (1st stage torque *3(gear ratio))
  2. Radial load R_A = 1158.7 N
  3. Axial load, F_a = 823.96 N

Resultant load = 3286.18 N

These two resultant loads are 90 degrees to each other.

The bending moment diagrams were drawn for the corresponding gears.

- **Vertical plane:**

  The reaction load offered by **driven gear of stage 1** to the shaft is

  R_b = 432.4 N

  R_a = 936.96 N

- **Horizontal plane:**

  The reaction loads offered by **pinion of 2nd stage drive** is

  R_a = 1095.38 N

  R_b = 2190.76 N

- **THE RESULATNT REACTION FORCES ON THE INTERMEDITE SHAFT IS**

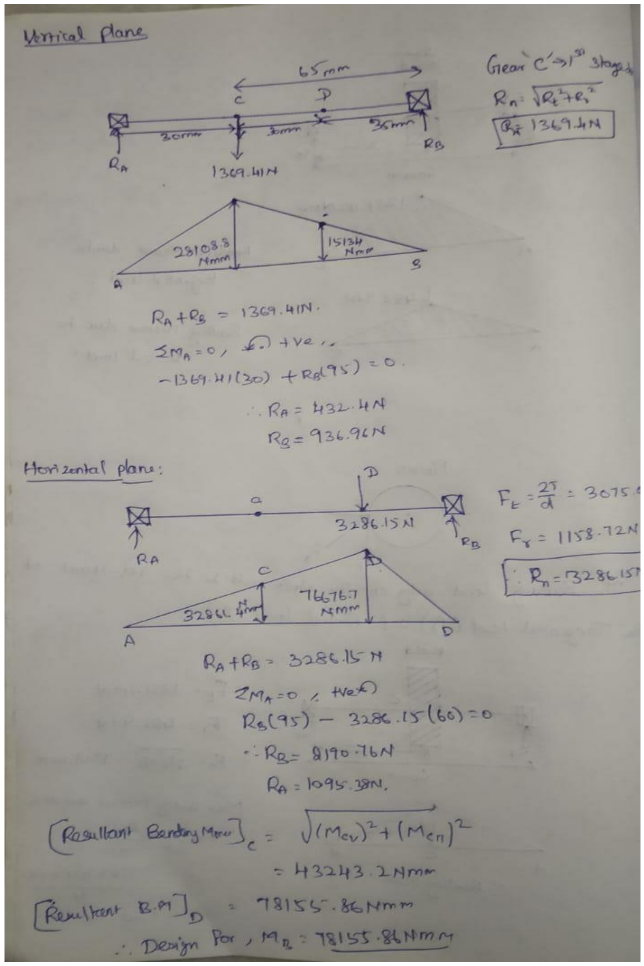
  R_A = 1441.44 N

  R_B = 2233.02 N.

The resultant bending moment acting on the intermediate shaft is

$R_{BM,C}$ = $\sqrt{(28108.8^2 + 32861.4^2)} = 43243.2Nmm$

$R_{BM,D}$ = $\sqrt{(15134^2 + 76676.7^2)} = 78155.86Nmm$



Therefore, the shaft is to be designed for higher bending moment i.e **M_B = 78155.86 Nmm**

The max twisting moment acting on the shaft**, T** = 3 * 23873 = **71619 Nmm**

As per ASME DESIGN CODE,

$$\tau_{allowable}/n = \frac{16}{\pi d^3}\sqrt{(C_b * M + \frac{\alpha f d^2}{8})^2) + (C_t * T)^2)}$$

On substitution of above values on the equation with factor of safety is taken as 1.5, the 8 order equation is obtained followed by chosen the highest diameter from the 8 solutions and the diameter of intermediate shaft is found to be

d = 18.078 mm

The diameter of intermediate shaft is taken as **19 mm** for fos > 1.5.

Now to calculate the fatigue strength of the material by assuming the life as infinite cycles as 10e7.

Shaft material : 20MnCr5,

Now, the endurance limit of shaft is determined by

S_e = K_a K_b K_c K_d S_e'

By taken the values of 19mm minor diameter and at gear portion it is taken as 20mm diameter with fillet radius of 0.5mm is chosen. By substituting the values like the formula given above is given as

K_a = 0.878

K_b = 0.899

K_c = 0.814 (99% reliability)

K_d = 1/1.7505

S_e' = 0.5*1000 Mpa; (for steel 20mncr5, S_{ut} =1000 Mpa

S_e = (0.5 * 1000))0.878*0.899*0.814*1/1.7505

S_e = 183.70 Mpa

Now assuming a infinite life of 10e7 for the shaft and the fatigue strength is determines by log log method,

$$0.9S_{ut} = 900Mpa$$
$$\log_{10}(0.9S_{ut}) = 2.9452$$
$$\log_{10}(S_e) = \log_{10}(183.7) = 2.264$$
$$\log_{10}(10^7) = 7$$
$$\log_{10}(10^3) = 3$$
$$\log_{10}(10^6) = 6$$

$$\log_{10}(S_f)' = 2.954 - \frac{(2.954 - 2.264)}{(6-3)}(7-3)$$

S_f' = 108.126 Mpa

S_f = $\frac{32(M_b)}{\pi d^3}$ **= 99.561 Mpa**

FOS = S_f'/S_f = 1.08, which is less than 1.5.

Hence to get fos more than 1.5, diameter of shaft is chosen as 23mm

S_f = $\frac{32(M_b)}{\pi d^3}$ **= 65.463 Mpa**

Therefore, **FOS = 1.65**

SHAFT 3 : OUTPUT SHAFT

The load acting on the output shafts is given as,

1. Tangential load R_T = 3075.09 N
2. Radial load R_r = 1158.72 N
3. Axial load, F_a = 823.96 N

Resultant load acting on shaft = $\sqrt{R_t^2 + R_r^2}$ = 3286.15 N, in which the gear is placed at the centre of the 80mm length shaft.

Therefore the **reaction forces** are

R_a = R_b = 1643.075 N

The maximum bending moment acts at the center of the shaft which is calculated as **PL/4**.

Therefore, **M_b = 65723 Nmm**

The maximum twisting moment acting on output shaft = 23873 x 3. **T = 71619 Nmm**

$$\tau_{allowable}/n = \frac{16}{\pi d^3}\sqrt{(C_b * M + \frac{\alpha f d^2}{8})^2) + (C_t * T)^2)}$$

On substitution of above values on the equation with factor of safety is taken as 1.5, the 8 order equation is obtained followed by chosen the highest diameter from the 8 solutions and the diameter of intermediate shaft is found to be

d = 17.37 mm

The diameter of intermediate shaft is taken as **18 mm** for fos > 1.5.

Now to calculate the fatigue strength of the material by assuming the life as infinite cycles as 10e7.

Shaft material : 20MnCr5,

Now, the endurance limit of shaft is determined by

$S_e = K_a K_b K_c K_d S_e'$

By taken the values of 19mm minor diameter and at gear portion it is taken as 20mm diameter with fillet radius of 0.5mm is chosen. By substituting the values like the formula given above is given as

K_a = 0.878

K_b = 0.905

K_c = 0.814 (99% reliability)

K_d = 1/1.5925

S_e' = 0.5*1000 Mpa; (for steel 20mncr5, S_{ut} =1000 Mpa)

S_e = (0.5 * 1000))0.878*0.904*0.814*1/1.5925

S_e = 203.07 Mpa.

Now assuming a infinite life of 10e7 for the shaft and the fatigue strength is determines by log log method,

$$0.9S_{ut} = 900Mpa$$
$$\log_{10}(0.9S_{ut}) = 2.9452$$
$$\log_{10}(S_e) = \log_{10}(203.07) = 2.3076$$
$$\log_{10}(10^7) = 7$$
$$\log_{10}(10^3) = 3$$
$$\log_{10}(10^6) = 6$$

$$\log_{10}(S_f)' = 2.954 - \frac{(2.954 - 2.3076)}{(6-3)}(7-3)$$

S_f' = 123.61 Mpa

S_f = $\frac{32(M_b)}{\pi d^3}$ **= 97.65 Mpa**

FOS = S_f'/S_f = 1.26, which is less than 1.5.

Hence to get fos more than 1.5, diameter of shaft is chosen as 21mm

S_f = $\frac{32(M_b)}{\pi d^3}$ **= 72.32 Mpa**

Therefore, **FOS = 1.709.**

BEARING CALCULATION

SELECTION OF BEARING:

- The shaft that experiences the radial and axial loads (which is due to application of helical gear) is used to provide with either Ball bearings or taper roller bearings.
- Usually ball bearings can also be used in wide applications that can be able to withstand the axial loads but the size of the bearings would be larger or the life would be reduced.
- So here I've designed the shaft that utilizes taper roller bearing. Pre loading is to be done for taper roller bearings in order to make the bearing uniformly loaded.

BEARING SELECTION IN SHAFT 1:

The resultant force as already calculated in the shaft design will be the radial road acts on the bearing. The resultant loads are

Radial load on bearing, F_r **R_a=R_b= 1369.41/2 = 684.7 N**

Axial load on bearing, **F_a = 343.35 N**

$F_a/F_r = 0.501 > e$

From PSG data book,

The value for X and Y is taken for the taper bearings as,

X = 0.4

Y = 1.35;

Service factor, **S** is chosen as **1.1** (rotary m/c, with no impact), since speed reducer there won't have any impact load due to no gear shifting.

Equivalent load : **$P = (XF_r + YF_a)S$**

On substitution, **P = 811.14 N**

DYNAMIC CAPACITY OF BEARING:

Life should be > 12000 hrs. Say 12500 hrs.

12500*8000*60 **= 6 x 10^9 cycles.**

DYNAMIC CAPACITY, $C = \left[\frac{L}{L_{10}}\right]^{1/K} P$

L = design required life of bearing

L_{10} = 1 million revolution of bearing

K = 10/3, for roller bearings.

P = Equivalent load.

On substitution, **C = 11.028 KN**

The bearing is chosen from SKF website as,

SKF 30302; **D = 42mm (Major diameter)**

d = 15mm (Minor diameter)

w = 9.413 mm (width)

DYNAMIC CAPACITY, [C] = 27.7 KN > C , Design is safe.

But the design shaft diameter is only 13mm, but due to no availability of bearing shaft diameter is adjusted to **15mm** for input shaft.

BEARING DESIGN IN SHAFT 2:

The resultant force as already calculated in the shaft design will be the radial road acts on the bearing. The resultant loads are

Radial load on bearing $F_{r,}$

R_A = 1441.44 N …(i)

R_B = 2233.02 N …(ii)

Axial load on bearing, **F_a = 823.9 N**

$F_a/F_r = 0.267 < e$

From PSG data book,

The value for X and Y is taken for the taper bearings as,

X = 1

Y = 0;

Service factor, **S** is chosen as **1.1** (rotary m/c, with no impact), since speed reducer there won't have any impact load due to no gear shifting.

Equivalent load : **$P = (XF_r + YF_a)S$**

On substitution, **P = 1414.127 N**

DYNAMIC CAPACITY OF BEARING:

Life should be > 12000 hrs. Say 12500 hrs.

12500*2666.67*60 = **2 x 10^9 cycles.**

DYNAMIC CAPACITY, $C = \left[\frac{L}{L_{10}}\right]^{1/K} P$

L = design required life of bearing

L_{10} = 1 million revolution of bearing

K = 10/3, for roller bearings.

P = Equivalent load.

On substitution, **C = 13.829 KN**

The bearing is chosen from SKF website as,

SKF 320/22 X; D = 44mm (Major diameter)

d = 22mm (Minor diameter)

w = 10.68 mm (width)

DYNAMIC CAPACITY, [C] = 30.9 KN > C , Design is safe.

- The other side of the bearing with radial load, R_B = 2233.02 N is having an equivalent load of **2504.8 N**
- Dynamic capcity is found as **24.49 KN**
- Hence the dynamic capacity of above bearing is more, SKF 320/22 X is chosen for other side also.

BEARING DESIGN IN SHAFT 3 :

The resultant force as already calculated in the shaft design will be the radial road acts on the bearing. The resultant loads are

Radial load on bearing,F_r **R_a=R_b =1643.075 N**

Axial load on bearing, **F_a = 823.9 N**

$F_a/F_r = 0.501 > e$

From PSG data book,

The value for X and Y is taken for the taper bearings as,

X = 0.4

Y = 1.35;

Service factor, **S** is chosen as **1.1** (rotary m/c, with no impact), since speed reducer there won't have any impact load due to no gear shifting.

Equivalent load : **$P = (XF_r + YF_a)S$**

On substitution, **P = 1946.53 N**

DYNAMIC CAPACITY OF BEARING:

Life should be > 12000 hrs. Say 12500 hrs.

12500*800*60 **= 6 x 10^8 cycles.**

DYNAMIC CAPACITY, $C = \left[\dfrac{L}{L_{10}}\right]^{1/K} P$

L = design required life of bearing

L_{10} = 1 million revolution of bearing

K = 10/3, for roller bearings.

P = Equivalent load.

On substitution, **C = 13.264 KN**

The bearing is chosen from SKF website as,

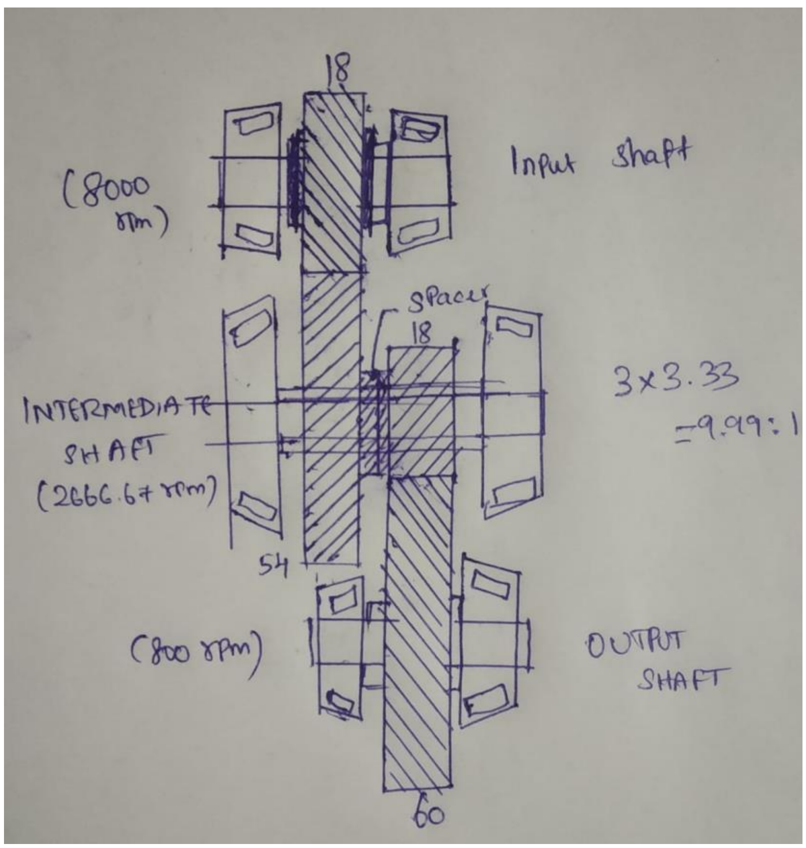
SKF 32004 X; D = 42mm (Major diameter)

d = 20mm (Minor diameter)

w = 10.184 mm (width)

DYNAMIC CAPACITY, [C] = 29.7 KN > C , Design is safe.

But the design shaft diameter increased as 21 mm to improve fatigue strength is with fos 1.709, but due to no availability of bearing shaft diameter is adjusted to **20mm**, grinding only on the bearing portion for output shaft.



The above diagram shows the rough sketch of gear box arrangement.

In case of making the gear box more compact and lighter, Epicyclic reduction can be used.