Spatial exploratory analysis

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Overview

- **Defining geostatistical problems.**
- **Spatial exploratory analysis based on the variogram.**

Epidemiological data

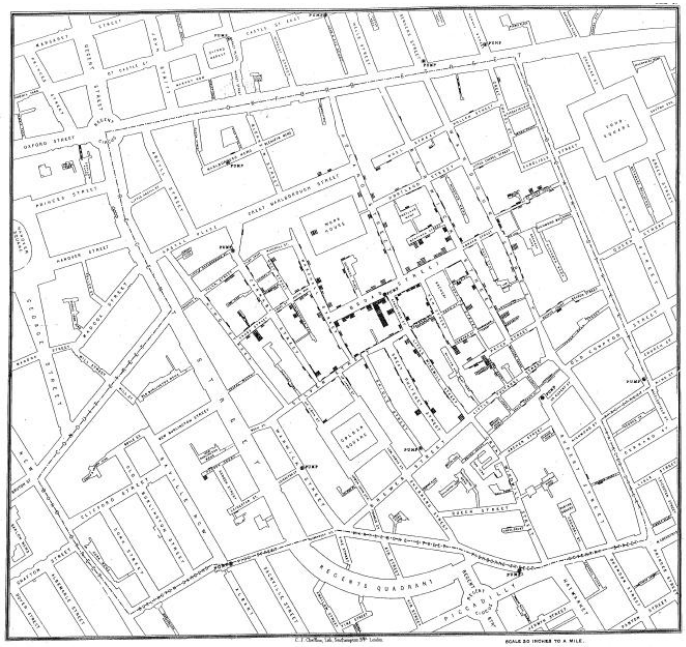
- incidence: number of new cases per unit time per unit population
- prevalence: number of existing cases per unit population
- risk: probability that a person will contract the disease (per unit time or per life-time)

General objective is to understand spatial variation in disease incidence and/or prevalence and/or risk according to context

Relevant books include

Elliott et al (2000); Gelfand et al (2010); Rothman (1986); Waller and Gotway (2004); Woodward (1999);

In the beginning: Cholera in Victorian London, 1854



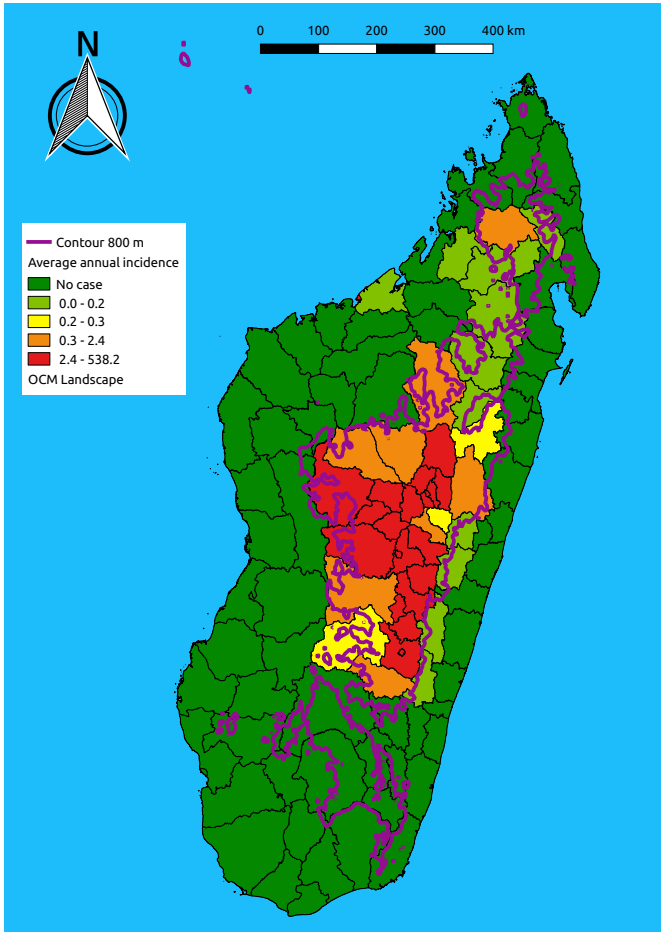
The physician John Snow famously removed the handle of the Broad Street water-pump, having concluded (correctly) that infected water was the source of the disease contrary to conventional wisdom at the time.

`https://en.wikipedia.org/wiki/1854_Broad_Street_cholera_outbreak`

Study-designs

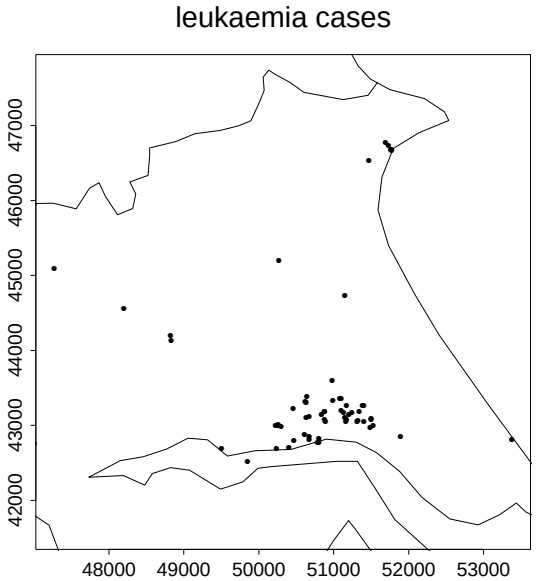
- Registry
  - case-counts in sub-regions to partition study-region (numerators)
  - population size in each sub-region (denominators)
  - collateral information from national census (covariates)
- Case-control
  - cases: all known cases within study region
  - controls: probability sample of non-cases within study-region
- Survey
  - sample of locations within study-region
  - collect data from each location
  - commonly used in developing country settings

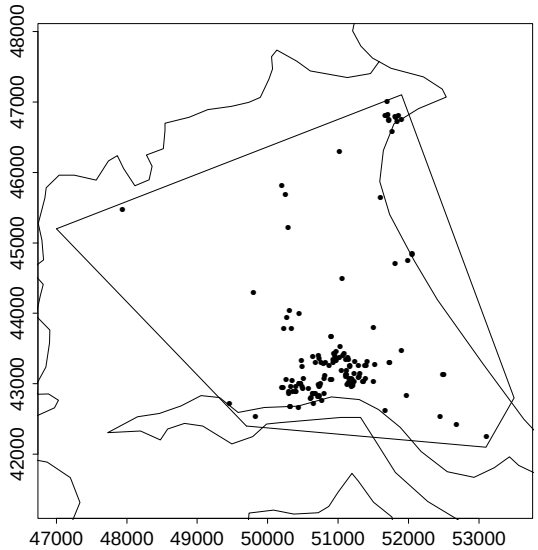
Registry example. Plague in Madagascar



How much does risk in plague infection increase in areas above 800 m? Giorgi, E. et al. (2016). *Modelling of spatio-temporal variation in plague incidence in Madagascar from 1980 to 2007.* Spatial and Spatio-temporal Epidemiology. 19:125-135

Case-control example Childhood leukaemia in Humberside





- residential locations of all known cases of childhood leukaemia in Humberside, England, over the period 1974-82;
- residential locations of a random sample of births

Cuzick and Edwards (1990); Diggle and Chetwynd (1991).

Survey example Loa loa prevalence in Cameroon

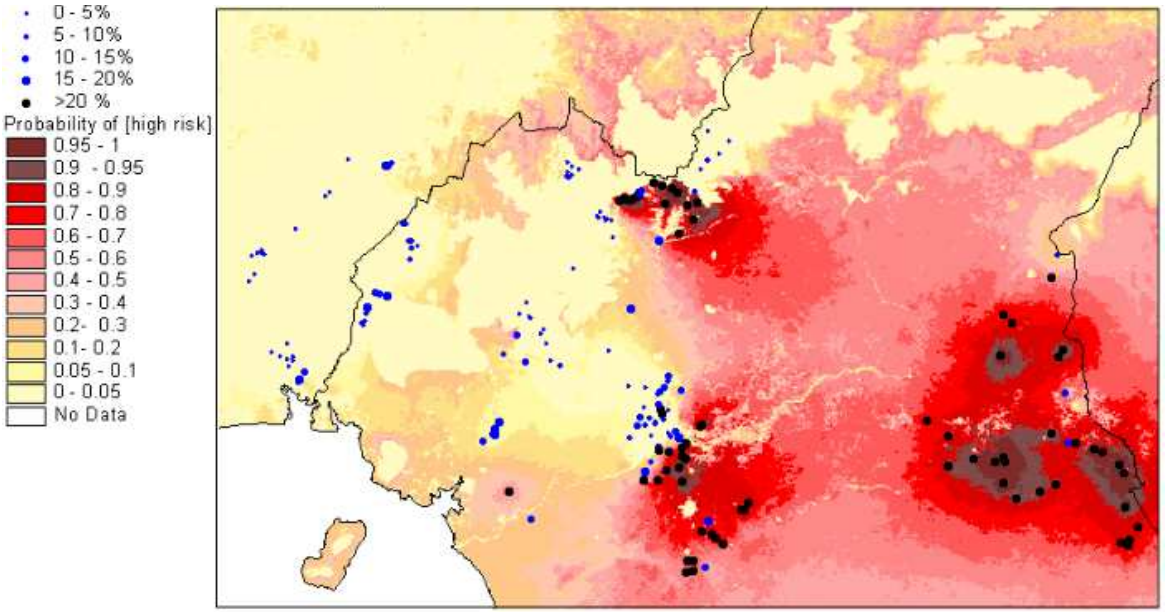


Figure 6: PCM for [high risk] in Cameroon based on LSMr with ground truth data.

Data are empirical prevalences in surveyed villages

Map shows predictive probabilities of exceeding 20% prevalence threshold

Diggle et al (2007)

What is the research question?

1. Plague in Madagascar
   - Is elevation an important risk factor for plague infection?
   - And if so, why?

2. Childhood leukaemia in Humberside
   - Do cases show a surprising tendency to cluster together?

3. Loa loa in Cameroon
   - What environmental characteristics affect the risk of disease?
   - Can we predict where the prevalence of the disease exceeds a policy-based intervention threshold?

Epidemic vs endemic patterns of incidence

- Foot-and-mouth in Cumbria (the 2001 epidemic)

Diggle (2006)

- Gastro-enteric disease in Hampshire (AEGISS)

Diggle, Rowlingson and Su (2005)

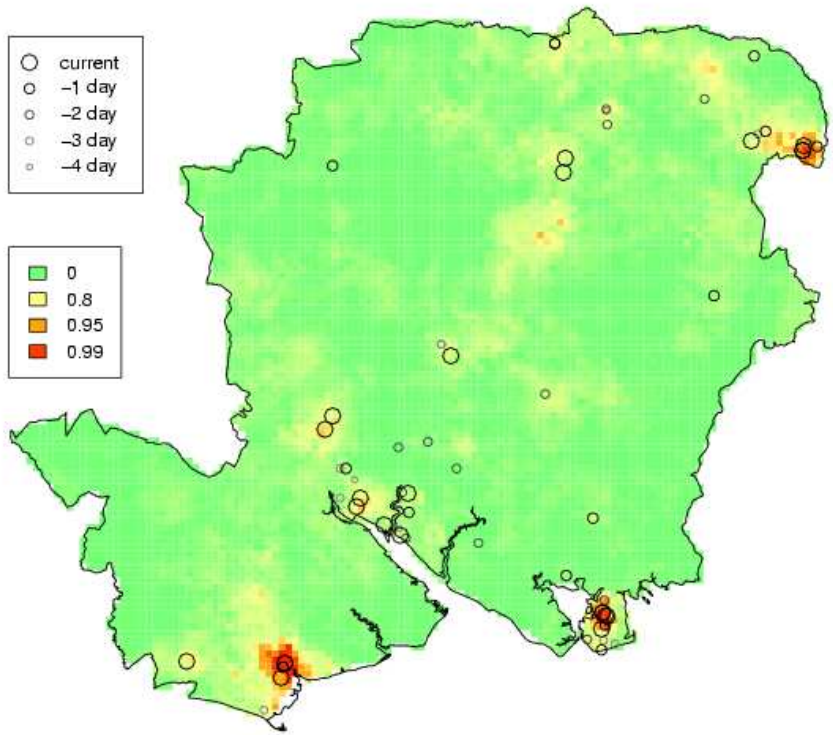
Animations at:

- `https://www.lancaster.ac.uk/staff/diggle/FMD/`
- `https://www.lancaster.ac.uk/staff/diggle/aegiss/`

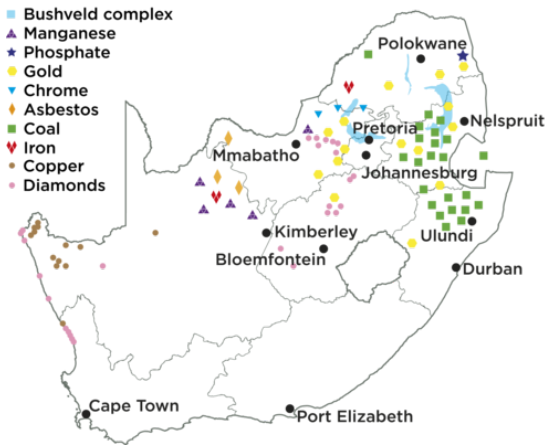
What are the similarities and differences between the two phenomena?

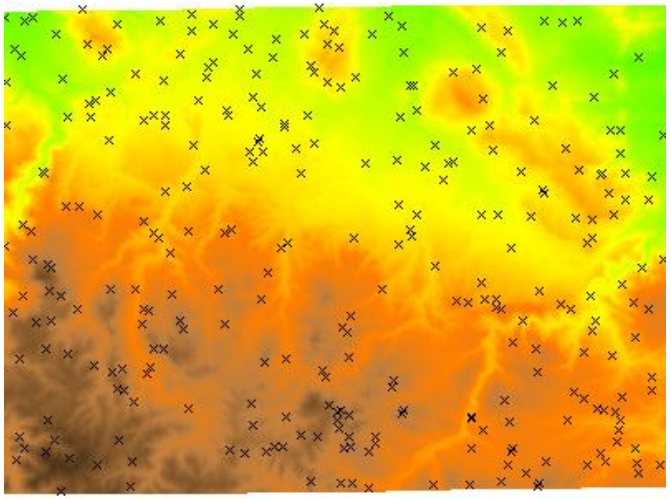
Empirical modelling: The AEGISS project (Diggle, Rowlingson and Su, 2005)

- early detection of anomalies in local incidence
- data on 3374 consecutive reports of non-specific gastro-intestinal illness
- log-Gaussian Cox process, space-time correlation $\rho(u, v)$



Geostatistics





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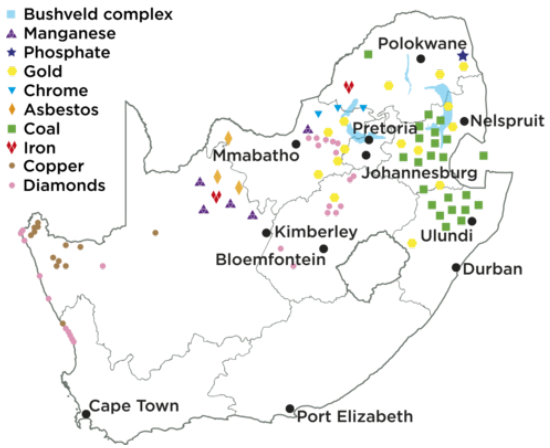

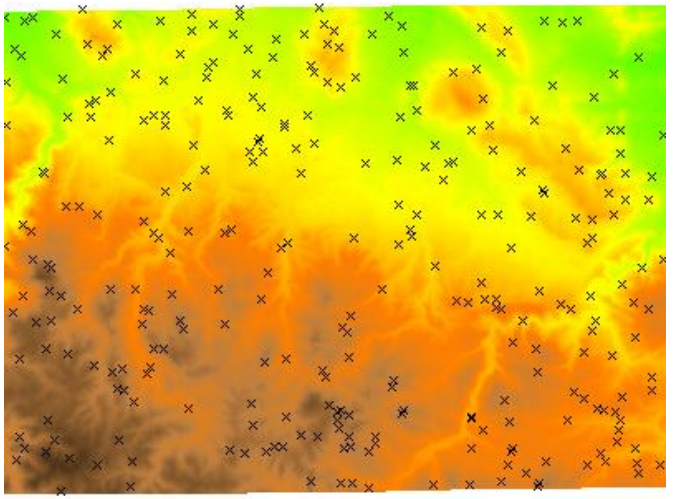

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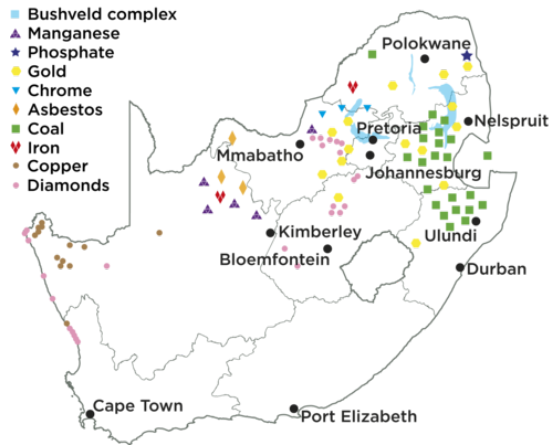

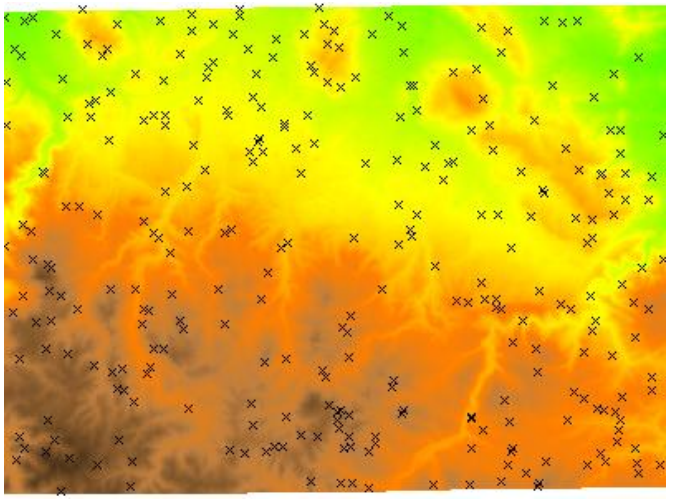

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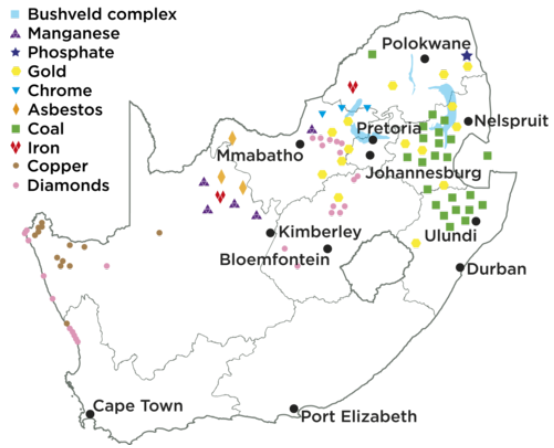

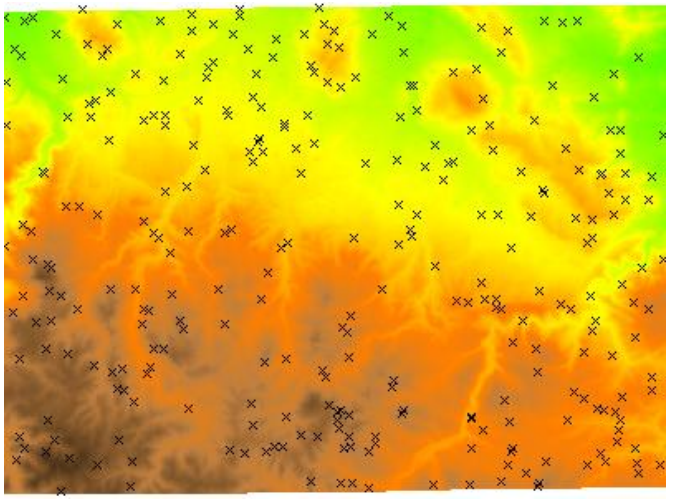

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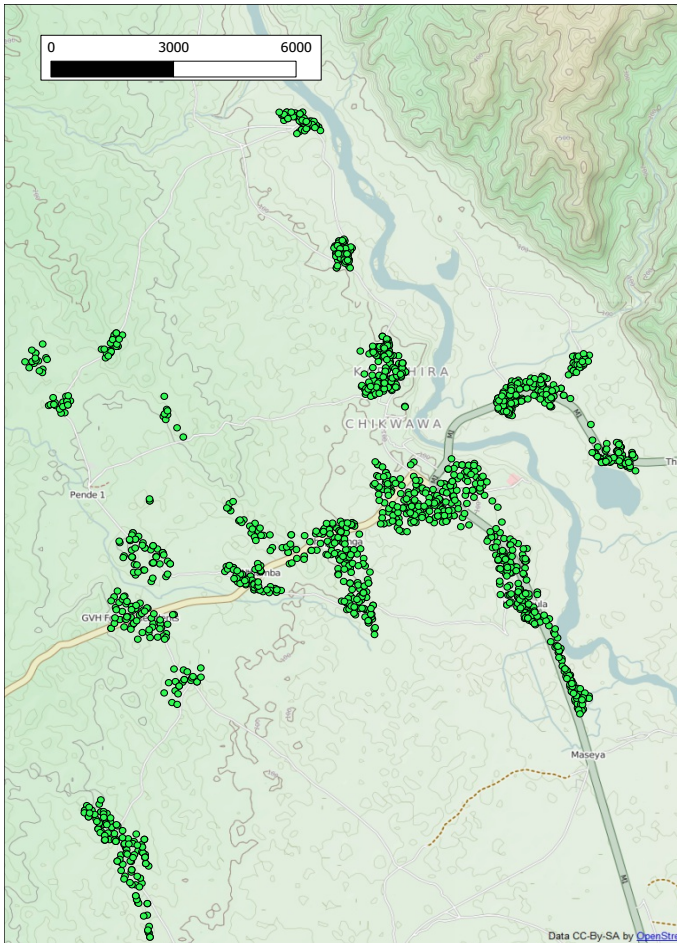
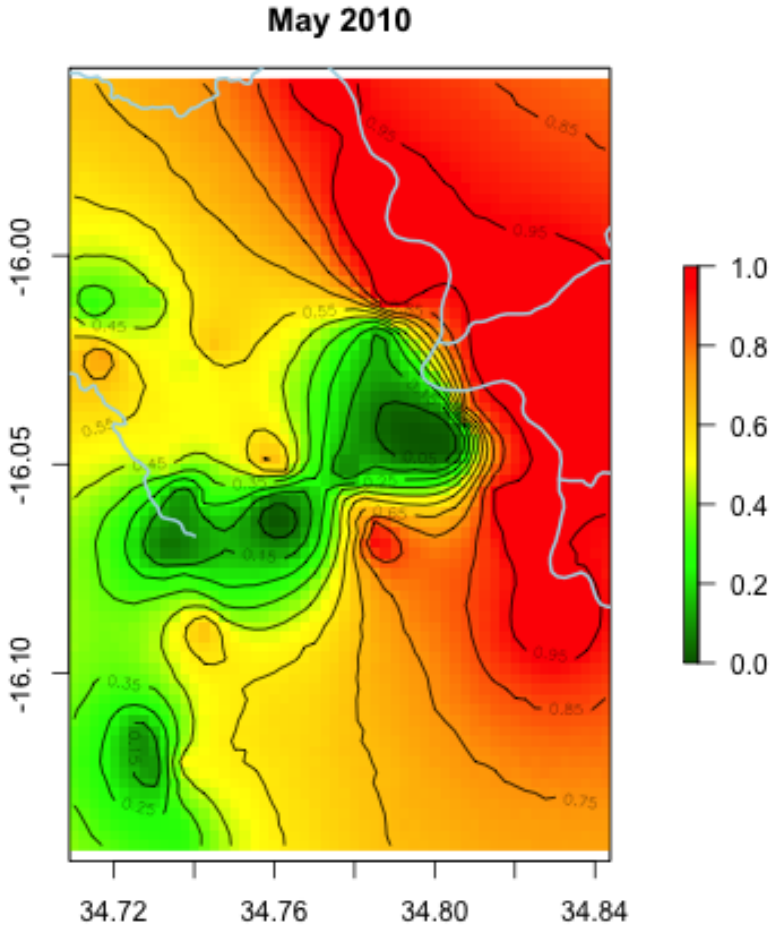
- Objective: $\int_A S(x)\, dx$ (yielding of a mining operation).

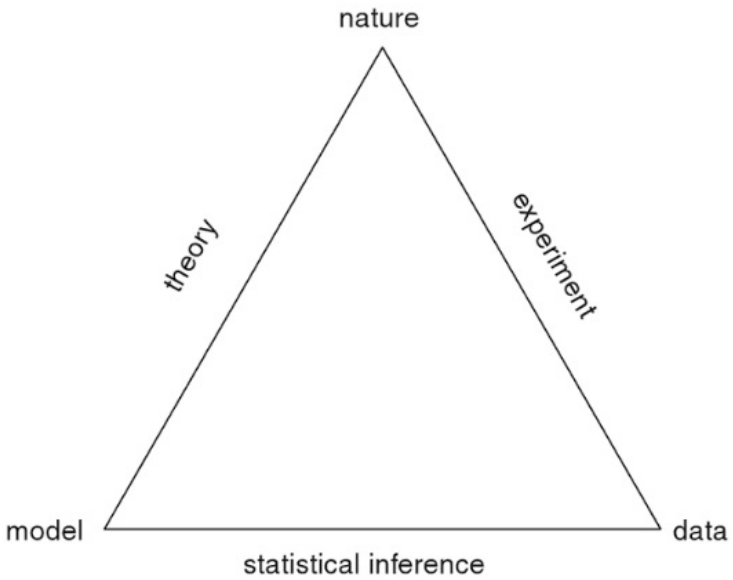
Model-based geostatistics





Animation at:
`https://www.lancaster.ac.uk/staff/giorgi/malaria/`

Science and statistics



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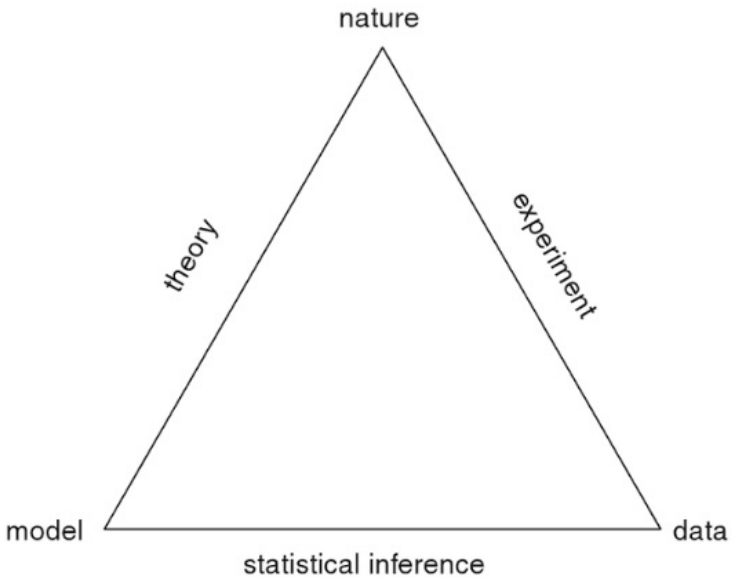

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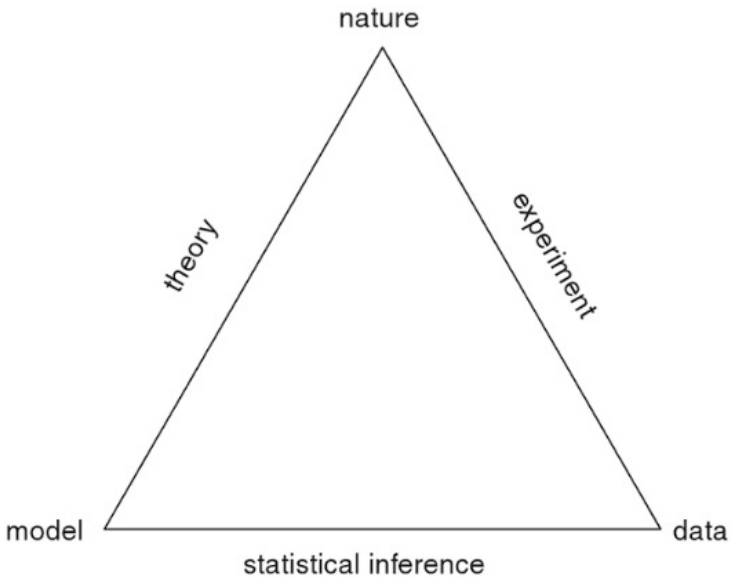

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$$[Y, S] = [S][Y|S]$$

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- Stationary and isotropic process: $Var[Y(x)] = \sigma^2$, $f(x_i, x_j) = \rho(u), u = \|h\|, h = x_i - x_j$

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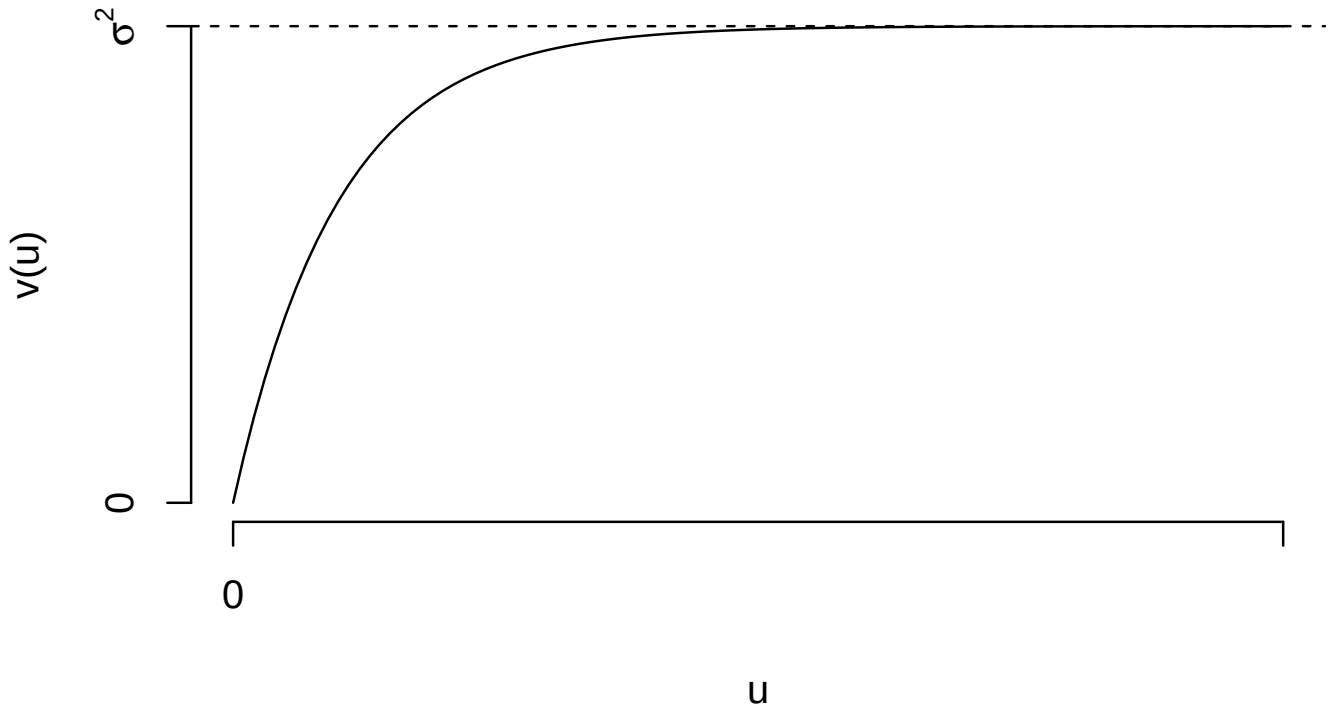

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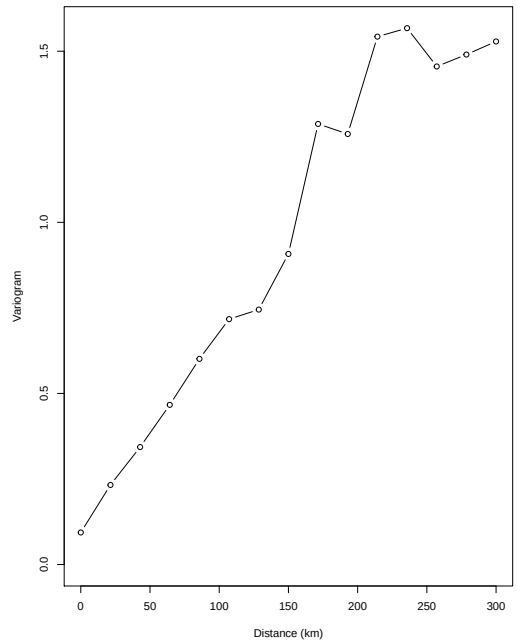
- How do we know if $\hat{v}(u)$ shows evidence of spatial correlation?



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