Background Notes

greyhypotheses

Seasonal Patterns

In pure mathematics, one of the angle sum identity theorems states that

Theorem 1 (Angle Sum Identity) *If α & ϕ are each acute angles of two distinct right-angled triangles, and α & ϕ are adjacent, then*

$$A\,sin(\alpha + \phi) \;\equiv\; a\,sin\alpha\,cos\phi \;+\; b\,cos\alpha\,sin\phi \tag{1}$$

Hence, and for seasonal pattern modelling purposes, the expression

$$Asin(2\pi ft + \phi) \tag{2}$$

is expressible as

$$A\,sin(2\pi ft + \phi) \;\equiv\; a\,sin(2\pi ft) + b\,cos(2\pi ft) \tag{3}$$

if A & ϕ values exist for the expression

$$Asin(2\pi ft)\,cos\phi \;+\; Acos(2\pi ft)\,sin\phi \;\equiv\; a\,sin(2\pi ft) \;+\; b\,cos(2\pi ft) \tag{4}$$

Proof

By the coefficients of *Eq.* 4

$$a = Acos\phi \tag{5}$$
$$b = Asin\phi \tag{6}$$

The quotient of *Eq.* 5 & *Eq.* 6 is

$$\frac{a}{b} = \frac{cos\phi}{sin\phi} = tan\phi \tag{7}$$

Eq. 7 is in line with a right-angled triangle with acute angle ϕ, i.e., *Fig. 1*. By the Pythagoras theorem, the length of the hypotenuse side of the acute right-angled triangle is

$$\sqrt{a^2 + b^2} \tag{8}$$

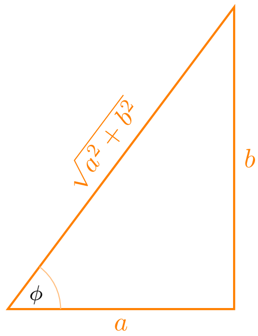

Figure 1: An acute right-angled triangle

Eq. 8 is derivable from *Eq.* 5 & *Eq.* 6, i.e.,

$$a^2 + b^2 = A^2 cos^2\phi + A^2 sin^2\phi = A^2 \tag{9}$$

$\Rightarrow$

$$A = \sqrt{a^2 + b^2} \tag{10}$$

Therefore, A exists; it is the length of the hypotenuse of the acute right-angled triangle (*Fig. 1*). Altogether, A & ϕ exists.

Sum & Product Rules

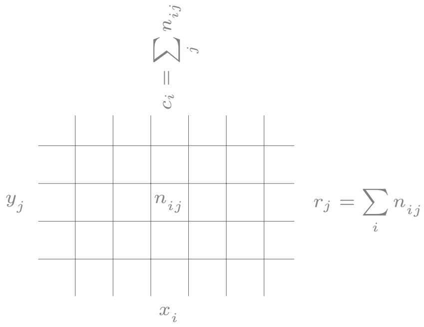

Figure 2: Events

The marginal; sum rule

$$p(X = x_i) = \frac{c_i}{N} \tag{11}$$

but

$$c_i = \sum_j n_{ij} \tag{12}$$

and

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N} \tag{13}$$

$\Rightarrow$

$$n_{ij} = Np(X = x_i, Y = y_j) \tag{14}$$

Hence

$$
\begin{aligned}
p(X = x_i) &= \frac{c_i}{N} && (15)\\
&= \frac{1}{N}\sum_j n_{ij} && (16)\\
&= \frac{1}{N}\sum_j N p(X = x_i, Y = y_j) && (17)\\
&= \sum_j p(X = x_i, Y = y_j) && (18)
\end{aligned}
$$

The joint; product rule

$$
\begin{aligned}
p(X = x_i, Y = y_j) &= \frac{n_{ij}}{N} && (19)\\
&= \frac{n_{ij}}{c_i} \times \frac{c_i}{N} && (20)
\end{aligned}
$$

but

$$
p(Y = y_j | X = x_i) = \frac{n_{ij}}{c_i} \qquad (21)
$$

and

$$
p(X = x_i) = \frac{c_i}{N} \qquad (22)
$$

Hence

$$
\begin{aligned}
p(X = x_i, Y = y_j) &= \frac{n_{ij}}{N} && (23)\\
&= \frac{n_{ij}}{c_i} \times \frac{c_i}{N} && (24)\\
&= p(Y = y_j | X = x_i) p(X = x_i) && (25)
\end{aligned}
$$