Experiment 1: Linear Polarization Measurement of a Lab Source

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1 Aim

To measure the degree p and angle θ of linear polarisation of a polarised and an unpolarised source.

2 Theory - Physics

Linear polarisation can be computed from normalised stokes parameters q and u.

$$p = \sqrt{q^2 + u^2} \tag{1}$$

$$\theta = \frac{1}{2}\arctan\left(\frac{u}{q}\right) = \frac{1}{2}\text{arctan2}\left(u, q\right) \tag{2}$$

These stokes parameters can be computed as

$$q = \frac{I_0 - I_1}{I_0 + I_1} \qquad u = \frac{I_2 - I_3}{I_2 + I_3} \tag{3}$$

I_0 = Output beam intensity when polarizer axis is along x-axis(0°).
I_1 = Output beam intensity when polarizer axis is along y-axis(90°).
I_2 = Output beam intensity when polarizer axis is along xy-axis(45°).
I_3 = Output beam intensity when polarizer axis is along -xy-axis(135°).

We can get these intensities from the number of detected photons N_i and the exposure time T_i.

$$I_i = \frac{N_i}{T_i} \tag{4}$$

Hence

$$q = \frac{N_0 - N_1}{N_0 + N_1} \qquad u = \frac{N_2 - N_3}{N_2 + N_3} \tag{5}$$

3 Theory - Data Analysis

3.1 Poisson noise in the photon detection

The variance in the photon counts is

$$\sigma^2(N) = N \tag{6}$$

3.2 Error propagation

3.2.1 General formula

When we try to compute any quantity using measured data, the uncertainty is passed on to that quantity as well. If know that the mean and variance of x is $\bar{x}, \sigma_x$ and that of y is $\bar{y}, \sigma_y$, then

$$\text{the mean of } f(x,y), \quad \bar{f} = f(\bar{x}, \bar{y}) \tag{7}$$

$$\text{the variance of } f(x,y), \quad \sigma_f^2 = \left(\left.\frac{\partial f}{\partial x}\right|_{\bar{x},\bar{y}} \sigma_x \right)^2 + \left(\left.\frac{\partial f}{\partial y}\right|_{\bar{x},\bar{y}} \sigma_y \right)^2 \tag{8}$$

if the function can be linearized in a few sigma intervals around the mean.

Proof: This can be easily proved by taking taylor expansion of $f(x,y)$ around $(\bar{x}, \bar{y})$

$$f(x,y) - f(\bar{x},\bar{y}) = \left.\frac{\partial f}{\partial x}\right|_{\bar{x},\bar{y}} (x - \bar{x}) \; + \; \left.\frac{\partial f}{\partial y}\right|_{\bar{x},\bar{y}} (y - \bar{y}) \tag{9}$$

Hence the mean $\bar{f} = f(\bar{x}, \bar{y})$.

The variance of the sum of two random variables is simply the sum of respective variances.
When multiplying a random variable by a constant, then the standard deviation gets scaled by the same constant.
Hence proved.

3.2.2 Error in p and θ

Using the equations 1 and 2, the mean and variance of p and θ can be related to those of q and u.
Mean of p is

$$\bar{p} = \sqrt{\bar{q}^2 + \bar{u}^2} \tag{10}$$

$$\left.\frac{\partial p}{\partial q}\right|_{\bar{q},\bar{u}} = \frac{2\bar{q}}{2\sqrt{\bar{q}^2 + \bar{u}^2}} = \frac{\bar{q}}{\sqrt{\bar{q}^2 + \bar{u}^2}} \tag{11}$$

$$\left.\frac{\partial p}{\partial u}\right|_{\bar{q},\bar{u}} = \frac{2\bar{u}}{2\sqrt{\bar{q}^2 + \bar{u}^2}} = \frac{\bar{u}}{\sqrt{\bar{q}^2 + \bar{u}^2}} \tag{12}$$

Variance in p is

$$\sigma_p^2 = \left(\frac{\bar{q}}{\sqrt{\bar{q}^2+\bar{u}^2}}\sigma_q\right)^2 + \left(\frac{\bar{u}}{\sqrt{\bar{q}^2+\bar{u}^2}}\sigma_u\right)^2 \tag{13}$$

$$= \left(\frac{\bar{q}^2}{\bar{q}^2+\bar{u}^2}\sigma_q^2\right) + \left(\frac{\bar{u}^2}{\bar{q}^2+\bar{u}^2}\sigma_u^2\right) \tag{14}$$

$$= \frac{\bar{q}^2\sigma_q^2+\bar{u}^2\sigma_u^2}{\bar{q}^2+\bar{u}^2} \tag{15}$$

Error in p is

$$\sigma_p = \sqrt{\frac{\bar{q}^2\sigma_q^2+\bar{u}^2\sigma_u^2}{\bar{q}^2+\bar{u}^2}} \tag{16}$$

Mean of θ is

$$\theta = \frac{1}{2}\arctan\left(\frac{u}{q}\right) \tag{17}$$

$$\left.\frac{\partial\theta}{\partial q}\right|_{\bar{q},\bar{u}} = \left[\frac{1}{2}\right]\frac{1}{1+(\bar{u}/\bar{q})^2}\left(-\frac{\bar{u}}{\bar{q}^2}\right) = -\left[\frac{1}{2}\right]\frac{\bar{u}}{\bar{q}^2+\bar{u}^2} \tag{18}$$

$$\left.\frac{\partial\theta}{\partial u}\right|_{\bar{q},\bar{u}} = \left[\frac{1}{2}\right]\frac{1}{1+(\bar{u}/\bar{q})^2}\left(\frac{1}{\bar{q}}\right) = \left[\frac{1}{2}\right]\frac{\bar{q}}{\bar{q}^2+\bar{u}^2} \tag{19}$$

Variance in θ is

$$\sigma_\theta^2 = \left(-\left[\frac{1}{2}\right]\frac{\bar{u}}{\bar{q}^2+\bar{u}^2}\sigma_q\right)^2 + \left(\left[\frac{1}{2}\right]\frac{\bar{q}}{\bar{q}^2+\bar{u}^2}\sigma_u\right)^2 \tag{20}$$

$$= \left[\frac{1}{4}\right]\left(\frac{\bar{u}^2}{(\bar{q}^2+\bar{u}^2)^2}\sigma_q^2 + \frac{\bar{q}^2}{(\bar{q}^2+\bar{u}^2)^2}\sigma_u^2\right) \tag{21}$$

$$= \left[\frac{1}{4}\right]\frac{\bar{u}^2\sigma_q^2+\bar{q}^2\sigma_u^2}{(\bar{q}^2+\bar{u}^2)^2} \tag{22}$$

Error in θ is

$$\sigma_\theta = \left[\frac{1}{2}\right]\sqrt{\frac{\bar{u}^2\sigma_q^2+\bar{q}^2\sigma_u^2}{(\bar{q}^2+\bar{u}^2)^2}} \tag{23}$$

3.2.3 Error in q and u

Similarly we can get the mean and variance of q and u from those of N_0, N_1, N_2 and N_3

$$q = \frac{N_0-N_1}{N_0+N_1} \qquad u = \frac{N_2-N_3}{N_2+N_3} \tag{24}$$

$$\frac{\partial q}{\partial N_0} = \frac{(N_0 + N_1) - (N_0 - N_1)}{(N_0 + N_1)^2} \tag{25}$$

$$= \frac{2N_1}{(N_0 + N_1)^2} \tag{26}$$

$$\frac{\partial q}{\partial N_1} = \frac{-(N_0 + N_1) - (N_0 - N_1)}{(N_0 + N_1)^2} \tag{27}$$

$$= \frac{-2N_0}{(N_0 + N_1)^2} \tag{28}$$

Variance in q is

$$\sigma_q^2 = \left(\frac{2N_1}{(N_0 + N_1)^2}\right)^2 \sigma_{N_0}^2 + \left(\frac{-2N_0}{(N_0 + N_1)^2}\right)^2 \sigma_{N_1}^2 \tag{29}$$

$$= \left(\frac{4N_1^2}{(N_0 + N_1)^4}\right) N_0 + \left(\frac{-4N_0^2}{(N_0 + N_1)^4}\right) N_1 \tag{30}$$

$$= \left(\frac{4N_0N_1}{(N_0 + N_1)^3}\right) \tag{31}$$

Error in q is

$$\sigma_q = \sqrt{\frac{4N_0N_1}{(N_0 + N_1)^3}} \tag{32}$$

Similarly error in u is

$$\sigma_u = \sqrt{\frac{4N_2N_3}{(N_2 + N_3)^3}} \tag{33}$$

4 Procedure

There were two sources, one is called unpolarised source and the other is called polarised source. The whole procedure is repeated for each source separately.

- Five sets of measurement were given. For each set, the following procedure is repeated in loop.
  - Each measurement consist of 4 images each for half wave plate position of $\alpha = 0°, 22.5°, 45°, 67.5°$. For each of the images, the following procedure is repeated.
    * Using the 'fits' module from 'astropy.io', the image in fits format is opened as header data units (hdu).
    * Number of pixels in x-axis and exposure time are extracted from the header of hdu and stored for further use.

        - The image data from the hdu is stored as numpy array of floats.
        - The median of the counts in the image is found using numpy and it is assumed to be the background.
        - The background is subtracted from every pixel in the image.
        - Median Absolute Deviation is computed using 'mad_std' from astropy and taken as the background σ.
        - By plotting the counts vs pixel along one axis, the FWHM is estimated to be 26 pixels and the aperture radius is taken to be 20 pixels.
        - Using 'DAOStarFinder' from photutils, sources with FWHM of 26 are found with a 5σ threshold.
        - Using 'CircularAperture' from photutils, apertures are created for all the sources found above, with radius of 20 pixels.
        - Using 'aperture_photometry' from photutils, electron counts for all detected sources is computed and then divided by gain of 2.5 to get photon counts.
        - More than one source were detected very close to each other, hence all the sources were classified based on whether it is in left or right side of the image.
        - The photon counts of brightest source from left side is taken as N_e and that from right side is taken as N_o.
        - Using exposure time, the intensity (photon rate) is computed and stored.
    - The correction factor K is computed using the stored intensity values.

$$K = \left[\frac{I_o(0°)I_o(22.5°)I_o(45°)I_o(67.5°)}{I_e(0°)I_e(22.5°)I_e(45°)I_e(67.5°)}\right]^{0.25} \tag{34}$$

    - For each value of alpha,
        - I_e is multiplied by the correction factor K.
        - R_α is computed using intensities I_e and I_o
        - Background is added to the total photon counts N_o and N_e and then error in R_α due to photon noise is computed.
    - $R_{0°}$ and $R_{22.5°}$ are taken to be q and u respectively.
    - Degree of polarisation, p and angle of polarisation, θ and are computed from q and u.
    - Error propagation is also computed using the formulas proved in theory section.

- Now that we variances for each measurement set, so inverse variance weighted mean of p and θ is computed.
- Error in those inverse variance weighted mean values are also computed.

5 Code

Python code for doing as per above procedure is in my github repo here. `https://github.com/premvijay/polarimetry-data-analysis/blob/master/polarimetry.py`

6 Results

For polarised source,

Set	Degree of polarisation, p	Angle of polarisation, θ
1	0.979 ± 0.00185	$115.1 \pm 0.0544°$
2	0.979 ± 0.00185	$115.3 \pm 0.0544°$
3	0.980 ± 0.00186	$115.2 \pm 0.0545°$
4	0.981 ± 0.00188	$115.3 \pm 0.550°$
5	0.984 ± 0.00185	$115.2 \pm 0.542°$

Inverse variance weighted mean degree of polarisation, $p = 0.980 \pm 0.00083$
Inverse variance weighted mean angle of polarisation, $\theta = 115.2 \pm 0.0244°$.

For unpolarised source,

Set	Degree of polarisation, p	Angle of polarisation, θ
1	0.00769 ± 0.00120	$93.5 \pm 4.37°$
2	0.00475 ± 0.00118	$71.8 \pm 7.15°$
3	0.00263 ± 0.00119	$81.3 \pm 12.9°$
4	0.00358 ± 0.00119	$135.9 \pm 9.54°$
5	0.00454 ± 0.00117	$101.6 \pm 7.4°$

Inverse variance weighted mean degree of polarisation, $p = 0.00462 \pm 0.00053$
Inverse variance weighted mean angle of polarisation, $\theta = 94.6 \pm 3.06°$.