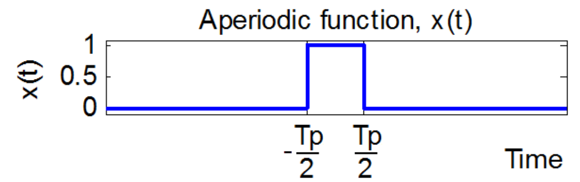
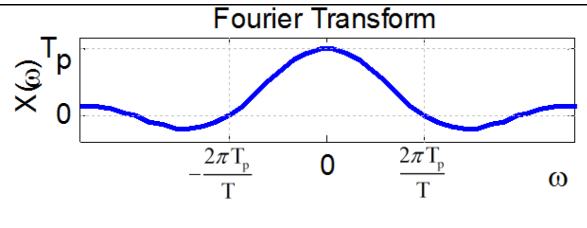
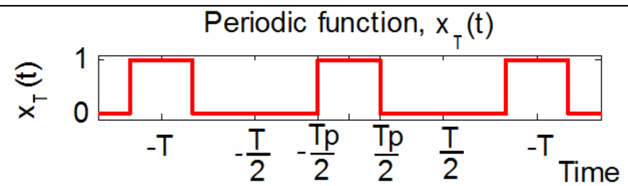
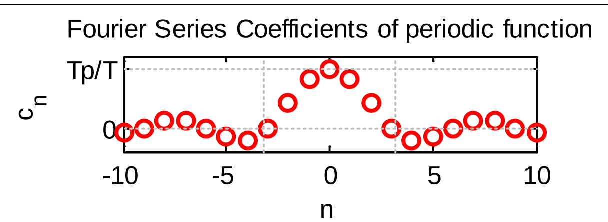
# Relationship between Fourier Series and Transforms for Periodic & Aperiodic Functions

Note: In this document, $X(\omega)$ and $c_n$ are real for ease of plotting. In general they are complex.

For an aperiodic function, $x(t)$. $\int_{-\infty}^{+\infty}|x(t)|dt < \infty$. Use Fourier Transform.

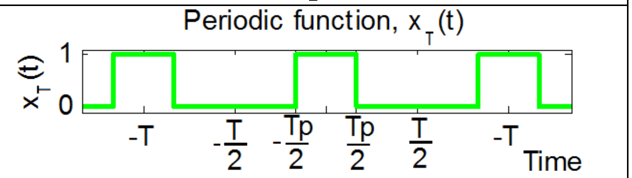
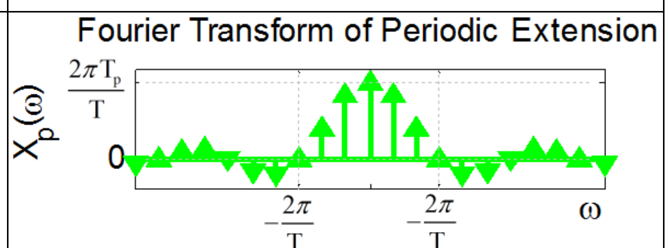
| **Transform** (Aperiodic) | **Example function** | **Graph** |
|---|---|---|
| **Synthesis** $x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}d\omega$ | $x(t) = \Pi\left(\frac{t}{T_p}\right) = \begin{cases} 1, & t < \frac{T_p}{2} \\ 0, & \text{otherwise} \end{cases}$ |  |
| **Analysis** $X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$ | $X(\omega) = T_p \operatorname{sinc}\left(\frac{\omega T_p}{2\pi}\right)$ |  |

For a periodic function, $x_T(t)$, that is a periodic extension of $x(t)$, with period $T=2\pi/\omega_0$. Use Fourier Series.

| **Series** (Periodic function) | **Example function** | **Graph** |
|---|---|---|
| **Synthesis:** $x_T(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}$ | $x_T(t) = \Pi_T\left(\frac{t}{T_p}\right)$ (periodic extension of $\Pi(t/T_p)$) |  |
| **Analysis:** $c_n = \frac{1}{T}\int_T x_T(t)e^{-jn\omega_0 t}dt = \frac{1}{T}X(n\omega_0)$ | $c_n = \frac{T_p}{T}\operatorname{sinc}\left(n\frac{T_p}{T}\right)$ |  |

If $x_T(t)$ is a periodic version of $x(t)$ with $T=2\pi/\omega_0$, then $Tc_n=X(n\omega_0)$.

For a periodic function, $x_T(t)$ $\left(\text{with } \int_{-\infty}^{+\infty}|x_T(t)|dt = \infty\right)$. Use Fourier Transform, with impulses in frequency.

| **Transform** (Periodic) | **Example function** | **Graph** |
|---|---|---|
| **Synthesis** $x_T(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}d\omega$ | $x_T(t) = \Pi_T\left(\frac{t}{T_p}\right)$ (periodic extension of $\Pi(t/T_p)$) |  |
| **Analysis** $X_T(\omega) = 2\pi\sum_{-\infty}^{+\infty} c_n\delta(\omega - n\omega_0)$ $c_n = \frac{1}{T}\int_T x_p(t)e^{-jn\omega_0 t}dt$ $c_n$ = Fourier Series coefficient | $X_T(\omega) = 2\pi\sum_{n=-\infty}^{+\infty} c_n\delta(\omega - n\omega_0)$ $c_n = \frac{T_p}{T}\operatorname{sinc}\left(n\frac{T_p}{T}\right)$ |  |