If the length of each side of an equilateral triangle were increased by 50 percent, what would be the percent increase in the area?

(A) 75%

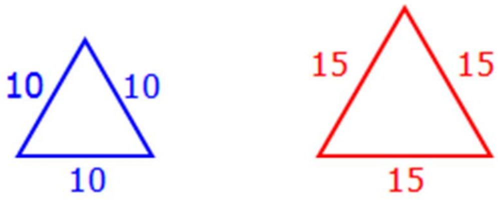
(B) 100%

(C) 125%

(D) 150%

(E) 225%

$$\boxed{\text{Area of equilateral } \Delta = \frac{\sqrt{3}}{4}\left(side^2\right)}$$



$$\text{Area} = \frac{\sqrt{3}}{4} \times 10^2 = 100 \times \frac{\sqrt{3}}{4} \qquad \text{Area} = \frac{\sqrt{3}}{4} \times 15^2 = 225 \times \frac{\sqrt{3}}{4}$$

$$100 \rightarrow 225$$

$$\frac{225 - 100}{100} = \frac{125}{100} = 125\%$$

What is the median number of televisions per household?

(A) Cannot be determined

(B) 1

(C) 2

(D) 2.5

(E) 3

$$\frac{n_{50} + n_{51}}{2} = \frac{3+3}{2} = 3$$

$$\underbrace{0,0,\ldots 0}_{16},\underbrace{1,1,\ldots \overset{n_{24}}{1}}_{8},\underbrace{2,2,\ldots \overset{n_{48}}{2}}_{24},\underbrace{\overset{n_{50}}{3},\overset{n_{51}}{3},3,\ldots 3}_{20}$$

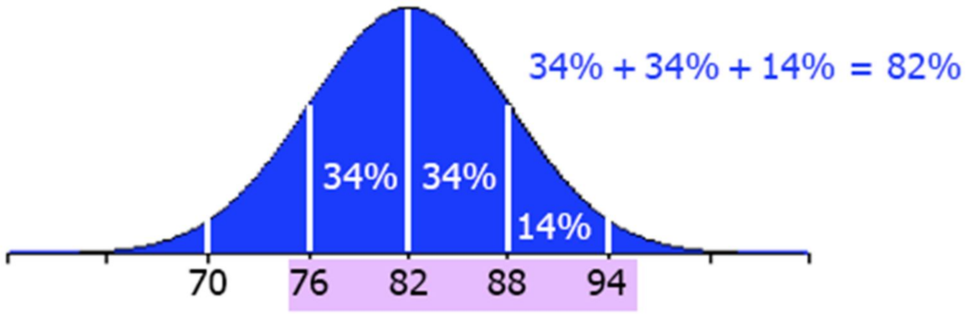
Number of televisions	Percent of households
0	16%
1	8%
2	24%
3	20%
4	20%
5 or more	12%

The table identifies the percentage of households in Townville that have a certain number of televisions.

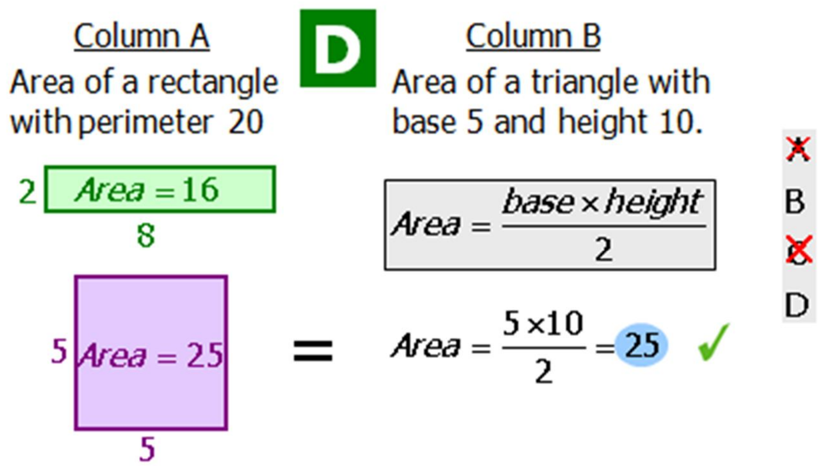
If a set of numbers is normally distributed with a mean of 82 and a standard deviation of 6, approximately what percent of the numbers are greater than 76 and less than 94?

Give your answer to the nearest whole percent.

82 %



FAQ: Where did the 34% come from? What is "the normal distribution"?
http://magoosh.com/gre/2012/normal-distribution-on-the-gre/



FAQ: How can all sides be 5 for the figure in Column A, wouldn't that be a square not a rectangle?

This question is tricky and is testing your knowledge of rectangles and squares. A square is actually a type of rectangle. The requirement for a rectangle is that opposite sides are parallel and equal and all angles are 90 degrees. A square is just a special type of rectangle where all sides happen to be equal.

For more information on squares, rectangles, and other quadrilaterals, we recommend watching the Related Lesson on Quadrilaterals.

If $6\left|\frac{-k}{3}+4\right|>12$, which of the following could be the value of k?

Indicate <u>all</u> values

[A] -15
[B] -10
[C] -5
[D] 0
[E] 5
[F] 10
[G] 15
[I] 20

$$6\left|\frac{-k}{3}+4\right|>12$$

$$\left|\frac{-k}{3}+4\right|>2$$

$$\frac{-k}{3}+4>2 \quad \text{or} \quad \frac{-k}{3}+4<-2$$

$$\frac{-k}{3}>-2 \quad \text{or} \quad \frac{-k}{3}<-6$$

$$k<6 \quad \text{or} \quad k>18$$

$\left(\sqrt{5+\sqrt{5}}-\sqrt{5-\sqrt{5}}\right)^2 =$

(A) $10-4\sqrt{5}$

(B) $10-2\sqrt{5}$

(C) $20-8\sqrt{5}$

(D) $20-4\sqrt{5}$

(E) $20-2\sqrt{5}$

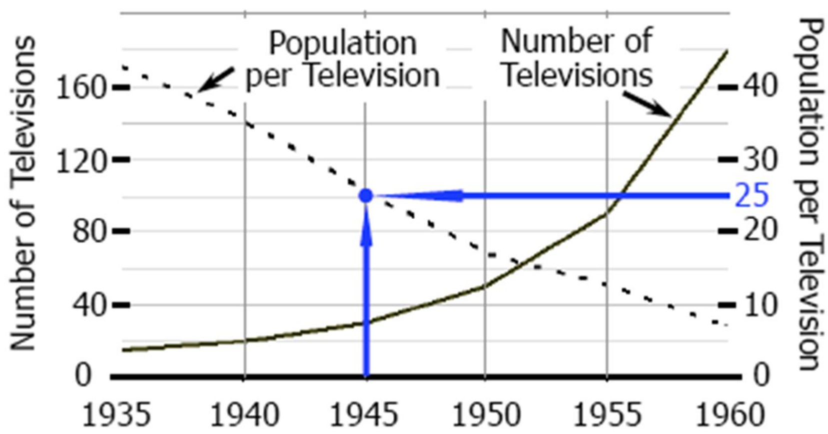
$$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$$

$$\left(\sqrt{5+\sqrt{5}}-\sqrt{5-\sqrt{5}}\right)^2 =$$

$$\boxed{\text{let } u = \sqrt{5}}$$

$$\begin{aligned}
\left(\sqrt{5+u}-\sqrt{5-u}\right)^2 &= \left(\sqrt{5+u}\right)^2 - 2\sqrt{5+u}\times\sqrt{5-u} + \left(\sqrt{5-u}\right)^2 \\
&= 5+u-2\sqrt{25-u^2}+5-u \\
&= 10-2\sqrt{25-u^2} \\
&= 10-2\sqrt{25-\sqrt{5}^2} \\
&= 10-2\sqrt{25-5} \\
&= 10-2\sqrt{20} \\
&= 10-4\sqrt{5}
\end{aligned}$$

TELEVISIONS IN TOWN X AND POPULATION PER TELEVISION



What was the approximate population of Town X in 1945?

(A) 150

(B) 750

(C) 1500

(D) 3000

(E) 6000

\# of televisions ≈ 30

population per television ≈ 25

➡ population ≈ 30 × 25 ≈ 750

If $\sqrt{\sqrt{\sqrt{3x}}} = \sqrt[4]{2x}$, what is the greatest possible value of x?

$\boxed{0.75}$

$$\sqrt{\sqrt{\sqrt{3x}}} = \sqrt[4]{2x}$$

$$\left(\left[(3x)^{\frac{1}{2}}\right]^{\frac{1}{2}}\right)^{\frac{1}{2}} = (2x)^{\frac{1}{4}}$$

$$(3x)^{\frac{1}{8}} = (2x)^{\frac{1}{4}}$$

$$\left[(3x)^{\frac{1}{8}}\right]^{8} = \left[(2x)^{\frac{1}{4}}\right]^{8}$$

$$(3x)^{1} = (2x)^{2}$$

$$3x = 4x^2$$

$$3x = 4x^2$$

$$4x^2 - 3x = 0$$

$$x(4x-3) = 0$$

$$x = 0$$

$$4x - 3 = 0$$

$$4x = 3$$

$$x = \frac{3}{4} = 0.75$$

For numbers p, q, and r, $(p*q*r) < 0$ and $\frac{(p*q)^2}{r} < 0.$

Column A	Column B
p*q	0

First consider the inequality

$$\frac{(p*q)^2}{r} < 0$$

We know that whatever that numerator equals, it must be positive, because squaring any non-zero number makes it positive. Since we are guaranteed that the numerator is positive, we know that the only way the whole fraction can be negative is if the denominator is negative. Thus, we know: r is negative, i.e. r < 0.

Now, look at the first expression, which we are going to re-write a little:

(p*q*r) = (p*q)*r < 0

We used the **associative law** to re-group the factors. Essentially what we have in that expression is that the product of (p*q) and r is negative. Well, we know r is negative, and we know the only way we'll get a negative product is by multiply negative by positive. If (p*q) were also negative, the product of (p*q) and r would be positive, because negative time negative is positive. Since we to get a negative product, that must mean (p*q) is positive, i.e. (p*q) > 0. Therefore, Column A is greater than Column B. Answer = **A**

If $\dfrac{5x^2+65x+60}{x^2+10x-24}=\dfrac{5x+5}{x-2}$, then which of the following are possible values of x?

Indicate <u>all</u> such values.

This question is about ***factoring***: this topic is treated in depth in the Related Lesson beneath this . Factoring is a very important skill to master for the GRE Quantitative section.

The first thing we have to do is factor both the numerator and denominator of the fraction on the left. To factor the quadratic in the numerator, the easiest first step is to factor out the GCF of 5.

$$\frac{5x^2+65x+60}{x^2+10x-24}=\frac{5(x^2+13x+12)}{x^2+10x-24}$$

$$=\frac{5(x+12)(x+1)}{(x+12)(x-2)}$$

Now, set this equal to the equation on the other side, and factor out the 5 in the numerator of that fraction as well.

$$\frac{5(x+12)(x+1)}{(x+12)(x-2)}=\frac{5x+5}{x-2}$$

$$\frac{5(x+12)(x+1)}{(x+12)(x-2)}=\frac{5(x+1)}{x-2}$$

Now, here comes a crucial question. Can we cancel the factor of (x + 12) in the numerator and the denominator? Well, canceling is division, so the underlying question is: can you divide by a variable or by a variable expression? The blanket answer is: **NO**, because you may be dividing by zero. The more nuanced answer is: consider two cases, one in which the variable expression equals zero and one in which it does not equal zero, and find the consequences in either case.

Here, if x = −12, which would make (x + 12) = 0, the fraction on the left becomes 0/0, which is a mathematical obscenity. That is absolutely not allowed. Therefore, we have determined that x = −12 is a totally illegal value of x.

If x ≠ −12, then (x + 12) ≠ 0, which would mean we could divide by it and cancel. That would lead us to:

$$\frac{5(x+1)}{x-2} = \frac{5(x+1)}{x-2}$$

Here, we have the exact same expression on both sides. These two expressions would be equal for every value of x for which they are defined. The only other value that is problematic is x = 2, which makes both denominators equal zero. Dividing by zero is one of the all-time big no-no's in mathematics, so x = 2 is an absolutely illegal value. Notice, though, if x equals anything other than 2 or −12, those two expressions would be equal to each other for the whole continuous infinity of real numbers.

Thus, the answer is ---- the possible values are x are ***absolutely everything except***x = 2 and x = −12. Here, that would be choices **(A)** −60, **(C)** −1, **(D)** 1, **(F)** 5.

If x and y are positive integers, and 1 is the greatest common divisor of x and y, what is the greatest common divisor of $2x$ and $3y$?

(A) Cannot be determined
(B) 1
(C) 2
(D) 5
(E) 6

The GCD of x and y is 1

Case 1: $x=1$, $y=1$ → $\left.\begin{array}{l} 2x = 2 \\ 3y = 3 \end{array}\right\}$ GCD = 1

Case 2: $x=3$, $y=2$ → $\left.\begin{array}{l} 2x = 6 \\ 3y = 6 \end{array}\right\}$ GCD = 6

Andy drove from Townville to Villageton at an average speed of 40 miles per hour. He then drove from Villageton to Townville at an average speed of 60 miles per hour.

A

Column A	Column B
50 ✓	The average speed of Andy's entire trip in miles per hour.

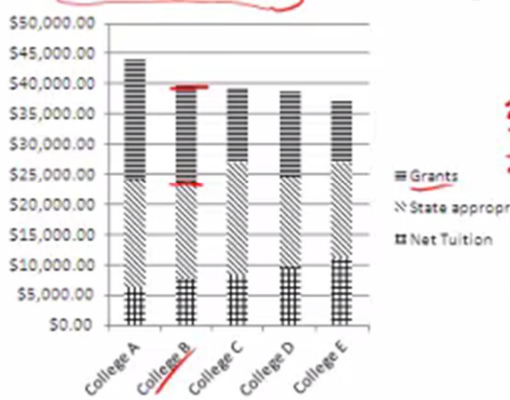
$$\boxed{\text{time} = \frac{\text{distance}}{\text{speed}}}$$

$$\frac{d}{40} + \frac{d}{60} = \frac{3d}{120} + \frac{2d}{120} = \frac{5d}{120}$$

$$\boxed{\text{Ave speed} = \frac{\text{total distance}}{\text{total time}}}$$

$$\text{Ave speed} = \frac{2d}{5d/120} = 48$$

Revenue per student at five state colleges



2006 Enrollments

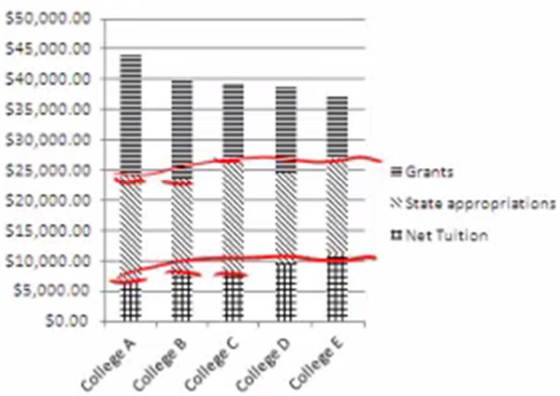
College A	25,000
College B	12,000
College C	33,000
College D	36,000
College E	85,000

39,000
23 000
16000 · 12000
16000 · 10000 = 160,000,000

2) What is the total dollar amount that College B received in grants?

(A) $16,000,000
(B) $48,000,000
(C) $96,000,000
(D) $160,000,000
(E) $192,000,000

Revenue per student at five state colleges



2006 Enrollments

College A	25,000
College B	12,000
College C	33,000
College D	36,000
College E	85,000

141,000

$$\frac{85000}{141{,}000} \rightarrow 95000$$

4) The total dollar amount that College E receives in state appropriations is what percent of the total state appropriations received by these five colleges?

(A) 8.6%
(B) 19.0%
(C) 26.4%
(D) 34.8%
(E) 42.9%

The average (arithmetic mean) of y numbers is x. If 30 is added to the set of numbers, then the average will be $x-5$. What is the value of y in terms of x?

(A) $\frac{x}{7}-5$

(B) $\frac{x}{6}-6$

(C) $\frac{x}{6}-5$

(D) $\frac{x}{5}-7$

(E) $\frac{x}{5}-6$

> If ***m*** is the mean of ***n*** numbers, then the sum of the numbers is ***nm***

➡ sum of y numbers $= xy$

Add 30 to existing sum

➡ $$\frac{xy+30}{y+1} = x-5$$

$$xy+30 = (x-5)(y+1)$$
$$xy+30 = xy + x - 5y - 5$$
$$xy+35 = xy + x - 5y$$
$$35 = x - 5y$$
$$5y = x - 35$$
$$y = \frac{x-35}{5} = \frac{x}{5} - 7$$

Slacka Head
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Anne pays 150 percent more for a wholesale widget than Bart pays.

Anne's retail price per widget is 15 percent greater than the wholesale price she paid.

Bart's retail price per widget is 185 percent greater than the wholesale price he paid.

Column A		Column B
Anne's retail price.	A	Bart's retail price.
\$287.50 ✓		\$285

Bart's wholsale price = \$100

Bart's retail price = \$100 + (185% of \$100)

= \$100 + (\$185) = \$285

Anne's wholsale price = \$250

Anne's retail price = \$250 + (15% of \$250)

= \$250 + (\$37.50) = \$287.50

If $f(x) = 5 - 2x$ and $f(3k) = f(k+1)$, then $f(k) =$

(A) 0.5
(B) 1
(C) 3
(D) 4
(E) 6

$$f(3k) = f(k+1)$$
$$5 - 2(3k) = 5 - 2(k+1)$$
$$5 - 6k = 5 - 2k - 2$$
$$5 - 6k = 3 - 2k$$
$$5 = 3 + 4k$$
$$2 = 4k$$
$$\frac{1}{2} = k \Rightarrow f(k) = f\left(\frac{1}{2}\right) = 5 - 2\left(\frac{1}{2}\right)$$
$$= 5 - 1$$
$$= 4$$

Slacka Head
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If x and y are positive numbers and $\sqrt{x^2 - y^2} = 3y - x$, what is the value of $\frac{x}{y}$?

Give your answer to the nearest 0.1 $\boxed{1.7}$

$$\sqrt{x^2 - y^2} = 3y - x$$
$$\left(\sqrt{x^2 - y^2}\right)^2 = (3y - x)^2$$
$$x^2 - y^2 = 9y^2 - 6xy + x^2$$
$$-y^2 = 9y^2 - 6xy$$
$$0 = 10y^2 - 6xy$$
$$6xy = 10y^2$$
$$6x = 10y$$

$$6x = 10y$$
$$\frac{6x}{y} = 10$$
$$\frac{x}{y} = \frac{10}{6}$$
$$\frac{x}{y} = \frac{5}{3}$$
$$\frac{x}{y} \approx 1.66$$

If x and y are integers, and $w = x^2y + x + 3y$, which of the following statements must be true?

Indicate all such statements.

[A] If w is even, then x must be even.
[B] If x is odd, then w must be odd.
[C] If y is odd, then w must be odd.
[D] If w is odd, then y must be odd.

x	y	$x^2y+x+3y = w$
E	E	$(0)^2(0)+0+3(0)=0 \rightarrow$ even
E	O	$(0)^2(1)+0+3(1)=3 \rightarrow$ odd
O	E	$(1)^2(0)+1+3(0)=1 \rightarrow$ odd
O	O	$(1)^2(1)+1+3(1)=5 \rightarrow$ odd

When a certain coin is flipped, the probability of heads is 0.5
If the coin is flipped 6 times, what is the probability that there are exactly 3 heads?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{5}{16}$

(D) $\frac{31}{64}$

(E) $\frac{1}{2}$

$$\text{Probability} = \frac{\#\text{ outcomes with 3 heads}}{\text{total }\#\text{ of outcomes}} = \frac{20}{64} = \frac{5}{16}$$

$$\underset{\#1}{\underline{2}} \times \underset{\#2}{\underline{2}} \times \underset{\#3}{\underline{2}} \times \underset{\#4}{\underline{2}} \times \underset{\#5}{\underline{2}} \times \underset{\#6}{\underline{2}} = 64$$

$$\underset{\#1}{\underline{\quad}} \; \underset{\#2}{\underline{\quad}} \; \underset{\#3}{\underline{\quad}} \; \underset{\#4}{\underline{\quad}} \; \underset{\#5}{\underline{\quad}} \; \underset{\#6}{\underline{\quad}}$$

Ways to select 3 out of 6 tosses $= {}_6C_3 = 20$

$$\boxed{{}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

---

x is a positive integer. k is the remainder when $x^3 - x$ is divided by 3.

Column A		Column B
k	B	1 ✓
0		

$$x^3 - x = x(x^2 - 1)$$
$$= x(x-1)(x+1)$$
$$= (x-1)x(x+1)$$

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 . . .

➡ $\boxed{x^3 - x \text{ is divisible by 3}}$

If 8 tigers were added to the zoo, the new ratio of lions to tigers would be 4 to 3. How many bobcats are at the zoo?

(A) 4
(B) 8
(C) 12
(D) 24
(E) 48

Let T = Current # of tigers
Let L = Current # of lions

$$\frac{L}{T}=\frac{32}{8} \Rightarrow 32T=8L \Rightarrow 4T=L$$

$$\frac{L}{T+8}=\frac{4}{3} \Rightarrow 4(T+8)=3L$$

$$4(T+8)=3(4T)$$

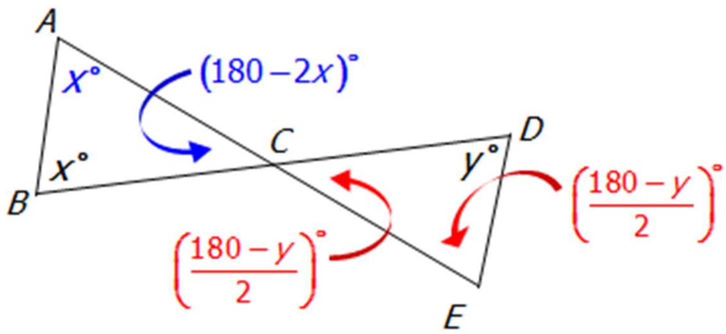
$$4T+32=12T$$

$$32=8T$$

$$4=T$$

ANIMAL DISTRIBUTION AT THE ZOO

Animal	Percent	
Lions	32%	
Leopards	16%	
Ocelots	20%	
Tigers	8%	4
Bobcats	24%	12



If $AC=BC$ and $CD=DE$ then, in terms of x, the value of y is

(A) x
(B) $180-2x$
(C) $90-2x$
(D) $4x-180$
(E) $45+\frac{x}{4}$

$$180-2x=\frac{180-y}{2}$$

$$360-4x=180-y$$

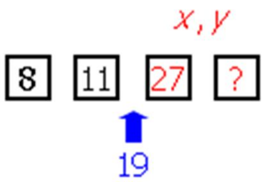
$$180-4x=-y$$

$$-180+4x=y$$

The median of x, y, 8 and 11 is 19.

Column A		Column B
x	A	23

27 or greater ✓



In the coordinate plane, rectangular region R has vertices at (0,0), (0,3), (4,3) and (4,0). If a point in region R is randomly selected, what is the probability that the point's y-coordinate will be greater than its x-coordinate?

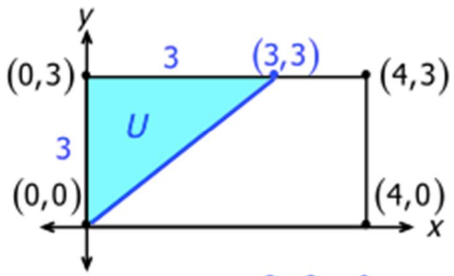
(A) $\frac{7}{12}$

(B) $\frac{5}{12}$

(C) $\frac{3}{8}$

(D) $\frac{1}{3}$

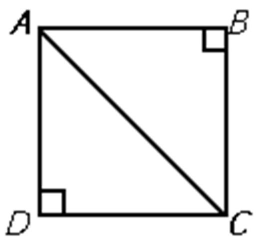
(E) $\frac{1}{4}$



Area of region U $= \frac{3 \times 3}{2} = \frac{9}{2}$

Area of region R $= 3 \times 4 = 12$

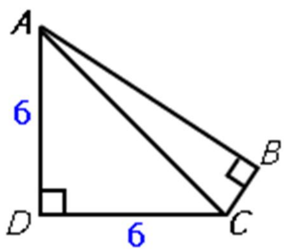
Probability $= \frac{\frac{9}{2}}{12} = \frac{3}{8}$



$AD = DC = 6$

Column A	Column B
AB	BC

ext Explanation

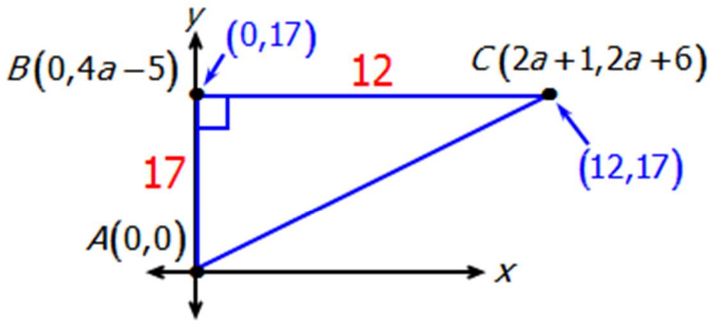


$AD = DC = 6$

Column A	D	Column B
AB ✓	=	BC

The points $A(0,0)$, $B(0,4a-5)$ and $C(2a+1,2a+6)$ form a triangle. If $\angle ABC = 90°$, what is the area of triangle ABC?

(A) 102
(B) 120
(C) 132
(D) 144
(E) 156



$$\begin{aligned} 4a-5 &= 2a+6 \\ 2a-5 &= 6 \\ 2a &= 11 \\ a &= 5.5 \end{aligned}$$

$$\text{Area} = \frac{\text{base} \times \text{height}}{2} = \frac{12 \times 17}{2} = 102$$

Events A and B are independent.
The probability that events A and B both occur is 0.6

Column A		Column B
The probability that event A occurs	A	0.3

0.6 or greater ✓

If events A and B are independent then
$P(A\&B) = P(A) \times P(B)$

$P(A\&B) = P(A) \times P(B)$
$0.6 = P(A) \times P(B)$ ➡ $P(A) \geq 0.6$, $P(B) \geq 0.6$

---

x and y are positive integers such that $x < y$. If $6\sqrt{6} = x\sqrt{y}$, then xy could equal

(A) 36
(B) 48
(C) 54
(D) 96
(E) 108

$6\sqrt{6} = x\sqrt{y}$

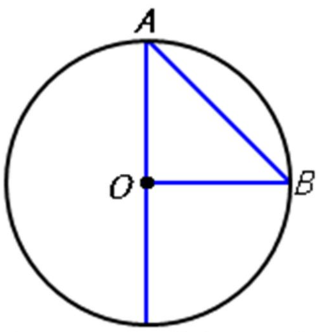
$6\sqrt{6} = 2 \times 3 \times \sqrt{6}$
$= \sqrt{4} \times 3 \times \sqrt{6}$
$= 3 \times \sqrt{24}$

$x = 3,\ y = 24$
$\rightarrow xy = 72$

$6\sqrt{6} = x\sqrt{y}$

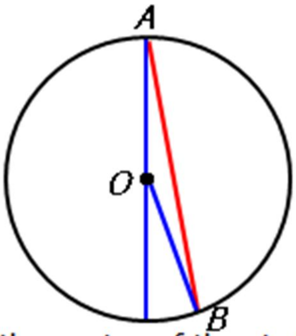
$6\sqrt{6} = 2 \times 3 \times \sqrt{6}$
$= 2 \times \sqrt{9} \times \sqrt{6}$
$= 2 \times \sqrt{54}$

$x = 2,\ y = 54$
$\rightarrow xy = 108$



O is the center of the circle.

Column A	Column B
Length of AO	Length of AB

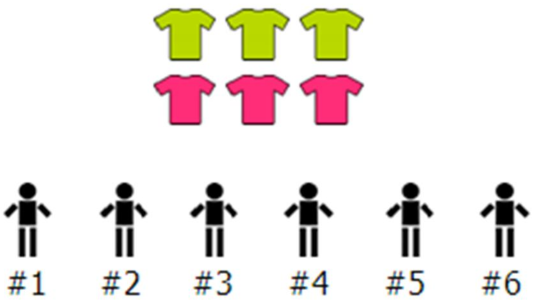


O is the center of the circle.

Column A		Column B
Length of *AO*	D	Length of *AB*

In how many different ways can 3 identical green shirts and 3 identical red shirts be distributed among 6 children such that each child receives a shirt?

(A) 20
(B) 40
(C) 216
(D) 720
(E) 729



$$\boxed{{}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}} \quad {}_6C_3 = 20$$

FAQ: This is hard! Can you explain it further?

According to the problem, we have "3 identical green shirts and 3 identical red shirts" --- suppose A & B & C are the green shirts and D & E & F are the red shirt --- then the two configurations above would no longer be different but rather the same identical outcome. Counting as if all six shirts were different leads to major overlap and redundancies, giving 720, and is A LOT bigger than the OA, 20.

The factorial rule for permutations, n!, only works if all n items are unique and different one another. If some of the items are the same, we need to use the MISSISSIPPI rule (explained in the lesson video of that title) to remove the redundancies.

Using the MISSISSIPPI rule, we have 6 items total, and two sets of three identical items, so the total number of cases is

# of outcomes = (6!)/[(3!)*(3!)] = 20

That's one way to approach the problem using the MISSISSIPPI rule.

Here's a second, entirely different way to think about problem. I have 3 identical green shirts and 3 identical red shirts to distribute to six children. If I simply designate the three who will get green shirts, that determines everything, because once I know which three get the green shirts, I automatically know the other three get the red shirts. So, the number of possible outcomes is just the number of ways we could choose a combination of 3 from the set of six. That's the combination number nCr

6C3 = 20

Here's the tricky thing about this problem. We have no guarantee that the fraction on the left is written in simplest form. When a fraction equals 2/3, the numerator and denominator could be both very big or both very small. For example:

$$\frac{2}{3} = \frac{2000}{3000} = \frac{0.0002}{0.0003}$$

The algebraic fraction given in the prompt has a numerator of (p – q) and a denominator of (p + q). Notice that Column A is the denominator. Well, as we see above, the denominator could equal 3000, or it could equal 0.0003, so for different choices, it could be much greater than 5 or much less than 5. We can't determine. Answer = **D**.

In how many ways can Ann, Bob, Chuck, Don and Ed be seated in a row such that Ann and Bob are not seated next to each other?

(A) 24
(B) 48
(C) 56
(D) 72
(E) 96

without restrictions − # with A&B together

A, B, C, D, E

$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$

$\boxed{AB}, C, D, E$

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$

$\boxed{BA}, C, D, E$

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$

$120 - 24 - 24 = 72$

---

x and y are integers greater than 5.
x is y percent of x^2

Column A		Column B
x	C	10
10		

x is y percent of x^2

$$x = \frac{y}{100}x^2 \Rightarrow 1 = \frac{y}{100}x \Rightarrow 100 = xy$$

$x=1,\ y=100$	$x=5,\ y=20$	$x=25,\ y=4$
$x=2,\ y=50$	$x=10,\ y=10$	$x=50,\ y=2$
$x=4,\ y=25$	$x=20,\ y=5$	$x=100,\ y=1$

$$\begin{array}{r} A\ A\ 6 \\ \times \quad\ \ 6 \\ \hline C\ 6\ 5\ 6 \end{array} \xrightarrow{A=7} \begin{array}{r} 7\ 7\ 6 \\ \times \quad\ \ 6 \\ \hline 4\ 6\ 5\ 6 \end{array}$$

If A, B and C represent different digits in the above multiplication, then $A+B+C=$

(A) 9
(B) 12
(C) 14
(D) 15
(E) 17

$B = \cancel{0}, \cancel{1}, \cancel{5}$ or $6 \Rightarrow B = 6$

$A = 2$ or $7 \Rightarrow A = 7$

$C = 4$

$$A+B+C = 7+6+4$$
$$= 17$$

---

x is a positive integer.
When x is divided by 2, 4, 6 and 8, the remainder is 1.

Column A	D	Column B
x		24
$x = 25 \Rightarrow 25$ ✓		24
$x = 1 \Rightarrow 1$		24 ✓

FAQ: How can I divide 1 by 2, 4, or 6 and have a remainder?
When you divide 1 by 2, 4, or 6, the remainder is 1 because the quotient is 0 (2 goes into 1 zero times). The "remainder" is what's left after you multiply the 0 by 2 and subtract from 1.

Basically, just like 14/3:
quotient = 4
remainder = 14 - (4 x 3) = 2

1/2:
quotient = 0
remainder = 1 - (0 x 2) = 1

Pump A can empty a pool in A minutes, and pump B can empty the same pool in B minutes. Pump A begins emptying the pool for 1 minute before pump B joins. Beginning from the time pump A starts, how many minutes will take to empty the pool?

(A) $\frac{A+B-1}{2}$

(B) $\frac{A(B+1)}{A+B}$

(C) $\frac{AB}{A+B}$

(D) $\frac{AB}{A+B}-1$

(E) $\frac{A(B-1)}{A+B}$

In 1 minute

Pump A: $\frac{1}{A}$ Pump B: $\frac{1}{B}$

Pumps A & B: $\frac{1}{A}+\frac{1}{B}=\frac{A+B}{AB}$

➡ Empty ENTIRE pool in $\frac{AB}{A+B}$ minutes

After 1 minute

$\frac{1}{A}$ pumped out ➡ $\frac{A-1}{A}$ remaining

$$\boxed{Time = 1+\frac{AB}{A+B}\left(\frac{A-1}{A}\right)=\frac{A(B+1)}{A+B}}$$

---

a, b, c and d are different positive numbers.
The average (arithmetic mean) of a and b is 30.
The average of a, b, c and d is 40.

Column A		Column B
The greatest possible value of d	**A**	99

99.9+ ✓

If m is the mean of n numbers, then the sum of the numbers is nm

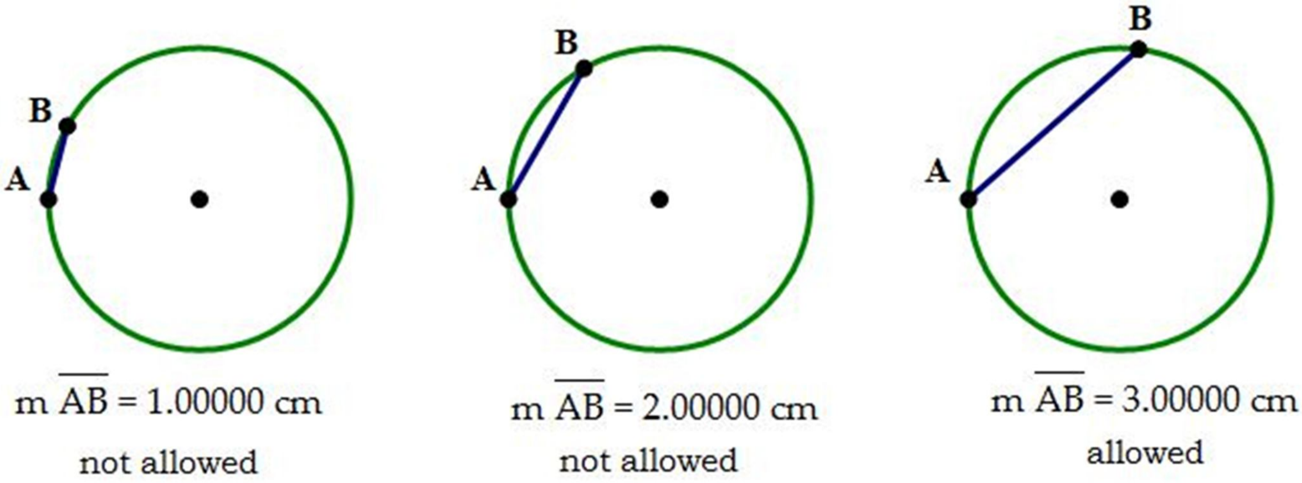
$a+b+c+d=160$

$a+b=60$ ➡ $c+d=100$

$0.1+99.9=100$

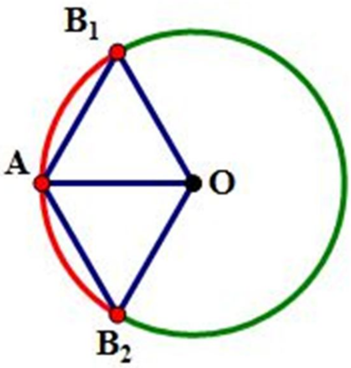
If points A and B are randomly placed on the circumference of a circle with radius 2, what is the probability that the length of chord AB is greater than 2?

First of all, the circle has what is called rotational symmetry, and we will take advantage of this. Because of rotational symmetry, we can pick, at random, any location we want for point A, and just consider how far away randomly chosen point B locations would be. If the locations of points A & B that are, say, 3 cm apart, it will not make any difference to the problem where A and B are individually on the circle --- the only thing that matters to the problem is the distance between them. That's why we can simply select an arbitration location for point A and consider only the random possibilities for the location of point B.



Because we are interested in an inequality (chord > 2 cm), we will employ a standard mathematical strategy of ***solving the equation first*** (chord = 2 cm). Even though this particular chord would not satisfy the condition, this chord or location on the circle will form, as it were, a "boundary" between the "allowed" region and the "not allowed" region. Again, this is a standard mathematic strategy, often used in algebraic inequalities for example: turn it into an equation and solve the equation first --- the solutions to the equation form what mathematicians call the "boundary conditions" for the solution to the inequality.

Also, keep in mind ---- because point B could be randomly located anywhere on the circle, it could be on either side of point A --- clockwise from point A, or counterclockwise from point A. Therefore, we have to consider the (chord = 2) case on both sides of the circle.

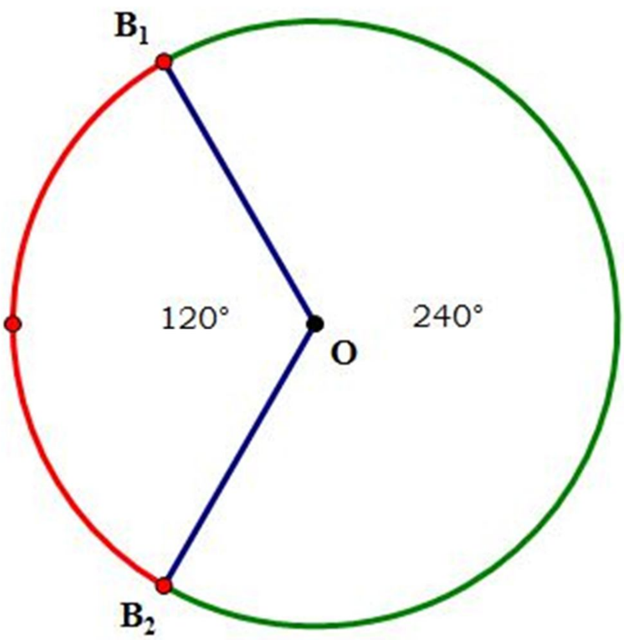


In that diagram, chords AB_1 and AB_2 both have a length of exactly 2 cm, so those points, and anything closer to point A than those points would be a "not allowed" location, a place with a chord less than or equal to 2 cm. This is the red region on the circle. The places where the chord would be greater than 2 cm is the green region of the circle. The probability question really reduces to a geometric question: what percent of the whole circle is the green arc?

Notice there's a most fortuitous set of connections in that diagram. There are five chord ---- AO, AB_1, AB_2, OB_1, & OB_2 ---- that all have the length of 2 cm. The segments AB_1 & AB_2, were chosen to have this length, and the other three are all radii of the circle, and we are told that the length of a radius is 2 cm.

This means the two triangles --- triangle OAB_1 & triangle OAB_2 ---- are both**equilateral triangles**. (In fact, any time you have chord = radius, and you connect radii to the endpoints of the chord, you will get an equilateral triangle). This means that each of angles is 60°. Since angle AOB_1 = 60° and angle AOB_2 = 60°, we know that angle B_1OB_2 = 120°. This is the central angle associated with the red arc (see the lesson on "Pieces of Pi" below).

There are 360° in a circle altogether. Since there are 120° in the angle associated with the red arc, the green arc must have an angle of 360° – 120° = 240°:



If point B lands anywhere on the green arc, it meets the condition (chord > 2 cm), and if point B lands anywhere on the red arc, it fails to meet the condition (chord ≤ 2 cm). The green arc takes up 240° of the full 360° of the circle.

$$\text{Probability} = \frac{240}{360} = \frac{24}{36} = \frac{2}{3}$$

Answer = **D**

In a group of 200 workers, 10 percent of the males smoke, and 49 percent of the females smoke.

Column A		Column B
Total number of workers who smoke	**C**	59

59

let M = number of males let F = number of females

$$\text{Total smokers} = \frac{10}{100}M + \frac{49}{100}F$$

$$M = 50,\ F = 150 \Rightarrow \frac{10}{100}(50) + \frac{49}{100}(150) = 5 + 73.5$$

$$M = 100,\ F = 100 \Rightarrow \frac{10}{100}(100) + \frac{49}{100}(100) = 10 + 49 = 59$$

FAQ: Why not have F = 200, and then we would have 98 smokers?

Technically, that would be correct - if there are 0 males, having 98 smokers is a possibility. However, on the same principle that the GRE wouldn't have you answer that 73.5 females smoke, it also wouldn't list that 10% of male workers smoke if there are 0 male workers.

If $n = 2\times3\times5\times7\times11\times13\times17$, then which of the following statements must be true?

not true I. n^2 is divisible by 600

not true II. $n + 19$ is divisible by 19

not true III. $\frac{n+4}{2}$ is even

(A) I only
(B) II only
(C) III only
(D) I and III
(E) None of the above

III. $\frac{n+4}{2} = \frac{(2\times3\times5\times7\times11\times13\times17)+4}{2}$

$= \frac{(\cancel{2}\times3\times5\times7\times11\times13\times17)}{\cancel{2}} + \frac{4}{2}$

$= \underbrace{3\times5\times7\times11\times13\times17}_{\text{odd}} + \underbrace{2}_{+\text{even}}$

$= \text{odd}$

Column A	D	Column B
22 percent of x		$\frac{2}{9}$ of x
22 percent of x		$22.\bar{2}$ percent of x ✓
$x = 0$ ➡ 0	=	0

$$\boxed{\frac{1}{9} = 11.\bar{1}\% \Rightarrow \frac{2}{9} = 22.\bar{2}\%}$$

A sum of money was distributed among Lyle, Bob and Chloe. First, Lyle received 4 dollars plus one-half of what remained. Next, Bob received 4 dollars plus one-third of what remained. Finally, Chloe received the remaining $32. How many dollars did Bob receive?

(A) 10
(B) 20
(C) 26
(D) 40
(E) 52

Let $A = \$$ remaining AFTER Lyle received his amount

Let $B = \$$ Bob received

$$B = 4 + \frac{1}{3}(A-4) \Rightarrow 3B = 12 + A - 4$$

$$3B = A + 8$$

$$A - 3B = -8$$

$$-\quad A - B = 32$$

$$-2B = -40$$

$$B = 20$$

If a and b are integers and $\left(\sqrt[3]{a} \times \sqrt{b}\right)^6 = 500$, then $a+b$ could equal

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

$$\left(\sqrt[3]{a} \times \sqrt{b}\right)^6 = 500$$

$$\left(a^{\frac{1}{3}} \times b^{\frac{1}{2}}\right)^6 = 500 \qquad \boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$$

$$a^2 \times b^3 = 500 \qquad \boxed{\left(x^a y^b\right)^c = x^{ac} y^{bc}}$$

$$a^2 \times b^3 = 2 \times 2 \times 5 \times 5 \times 5$$

$$a^2 \times b^3 = 2^2 \times 5^3$$

$$\Rightarrow b = 5, \; a = \pm 2 \Rightarrow a + b = 5 + 2 = 7$$

$$a + b = 5 + (-2) = 3$$

If k is an integer and $121 < k^2 < 225$, then k can have at most how many values?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 8

$$121 < k^2 < 225$$

$11^2 \quad k = 12 \quad k = 13 \quad k = 14 \quad 15^2$

$$121 < 144 \quad 169 \quad 196 < 225$$

$k = -12 \quad k = -13 \quad k = -14$

How many positive integers less than 100 have a remainder of 2 when divided by 13?

(A) 6
(B) 7
(C) 8
(D) 9
(E) 10

Remainder 2 when divided by 13

$$\underbrace{2,\ 15,\ 28,\ 41,\ 54,\ 67,\ 80,\ 93}_{\text{8 altogether}}$$

What is the sum of all roots of the equation $|x+4|^2 - 10|x+4| = 24$?

(A) -16
(B) -14
(C) -12
(D) -8
(E) -6

$$|x+4|^2 - 10|x+4| = 24$$

let $u = |x+4| \Rightarrow u^2 - 10u = 24$

$$u^2 - 10u - 24 = 0$$

$$(u-12)(u+2) = 0$$

$$\Rightarrow u = -2 \Rightarrow |x+4| = -2$$

$$\Rightarrow u = 12 \Rightarrow |x+4| = 12$$

$|x+4| = -2 \Rightarrow$ no solution

$$|x+4| = 12 \Rightarrow \begin{cases} x+4 = 12 \rightarrow x = 8 \\ x+4 = -12 \rightarrow x = -16 \end{cases}$$

$$\text{sum of roots} = 8 + (-16) = -8$$

At a certain university, 60% of the professors are women, and 70% of the professors are tenured. If 90% of the professors are women, tenured or both, then what percent of the men are tenured?

(A) 25
(B) 37.5
(C) 50
(D) 62.5
(E) 75

100	Ten	~Ten	
Women	40	20	→ 60
Men	30	10	→ 40
	↓ 70	↓ 30	

$$\frac{30}{40}=\frac{3}{4}=75\%$$

If $x+|x|+y=7$ and $x+|y|-y=6$, then $x+y=$

(A) -1
(B) 1
(C) 3
(D) 5
(E) 13

$\boxed{y \text{ is } positive?}$

$x+|y|-y=6 \rightarrow x+|\cancel{pos}|-(\cancel{pos})=6 \rightarrow x=6$

$x+|x|+y=7 \rightarrow 6+|6|+y=7 \rightarrow y=-5$ (contradiction!!)

➡ y must be *negative*

$\boxed{x \text{ is } negative?}$

$x+|x|+y=7 \rightarrow (\cancel{neg})+|\cancel{neg}|+y=7 \rightarrow y=7$ (contradiction)

➡ x must be *positive*

$$\left.\begin{array}{l} x \text{ is } positive \rightarrow x+|x|+y=7 \rightarrow 2x+y=7 \\ y \text{ is } negative \rightarrow x+|y|-y=6 \rightarrow x-2y=6 \end{array}\right\} x=4,\ \ y=-1$$

➡ $x+y=4+(-1)=3$

How many integers between 1 and 10^{21} are such that the sum of their digits is 2?

(A) 190
(B) 210
(C) 211
(D) 230
(E) 231

$10^{21} = 1{,}000{,}000{,}000{,}000{,}000{,}000{,}000$

$000{,}000{,}000{,}000{,}010{,}100{,}000 = 10{,}100{,}000$

$000{,}000{,}000{,}000{,}000{,}020{,}000 = 20{,}000$

Case I : two 1's ➡ ${}_{21}C_2$ ways ➡ 210 ways

$$\boxed{{}_nC_r = \frac{n!}{r!(n-r)!}}$$

___,___,___,___,___,___,___

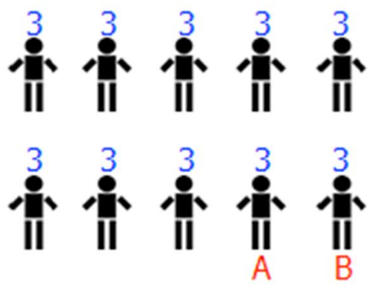
Case II : one 2 ➡ 21 ways

___,___,___,___,___,___,___

$$\text{Total} = 210 + 21 = 231$$

There are 10 people in a room. If each person shakes hands with exactly 3 other people, what is the total number of handshakes?

(A) 15
(B) 30
(C) 45
(D) 60
(E) 120



$$\frac{3+3+3+3+3+3+3+3+3+3}{2} = \frac{30}{2} = 15$$

FAQ: Why do we divide by 2?

Each handshake necessarily involves two people. Let's call the first person A, the second person B, etc. We don't know with which three people A shook hands, but we do know: whoever those three people were, the handshake with A counts as one of A's three handshakes and also counts as one of that other person's three handshakes. Say A shook hands with, for example, E - that handshake would be one of A's three handshakes and one of E's three handshakes. Thereby, regardless of who is shaking whose hand, each handshake is counted twice. That's why we have to divide by two.

How many integers from 1 to 900 inclusive have exactly 3 positive divisors?

(A) 10
(B) 14
(C) 15
(D) 29
(E) 30

$\boxed{\text{Only the squares of integers have an odd number of positive divisors.}}$

Divisors of 16 : 1, 2, 4, 8, 16
Divisors of 25 : 1, 5, 25
Divisors of 49 : 1, 7, 49

3 divisors $\rightarrow$ square of a prime number

$$\underbrace{2^2, 3^2, 5^2, 7^2, 11^2, 13^2, 17^2, 19^2, 23^2, 29^2}_{10}$$

If x and y are positive odd integers, the which of the following must also be an odd integer?

✓ I. x^{y+1}
✗ II. $x(y+1)$
✗ III. $(y+1)^{x-1}+1$

(A) I only
(B) II only
(C) III only
(D) I and III
(E) None of the above

I. $x^{y+1} = (odd)^{even}$
$= (odd) \times \cdots \times (odd)$
$= odd$

II. $x(y+1) = (odd)(even)$
$= even$

III. $(y+1)^{x-1} + 1 = (even)^{even} + 1$
$\Rightarrow (2)^2 + 1 = 4 + 1 = 5$
$\Rightarrow (2)^0 + 1 = 1 + 1 = 2$

In the game of Dubblefud, red chips, blue chips and green chips are each worth 2, 4 and 5 points respectively. In a certain selection of chips, the product of the point values of the chips is 16,000. If the number of blue chips in this selection equals the number of green chips, how many red chips are in the selection?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

$$16{,}000 = 2\times2\times2\times2\times2\times2\times2\times5\times5\times5$$
$$= 2\times2\times2\times2\times2\times2\times2\times5\times5\times5$$
$$= 2\times(2\times2)\times(2\times2)\times(2\times2)\times5\times5\times5$$
$$= 2\times4\times4\times4\times5\times5\times5$$
$$= 2\times4\times4\times4\times5\times5\times5$$

If $\sqrt{17+\sqrt{264}}$ can be written in the form $\sqrt{a}+\sqrt{b}$, where a and b are integers and $b < a$, then $a-b=$

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

$$\sqrt{17+\sqrt{264}} = \sqrt{a}+\sqrt{b}$$
$$\left(\sqrt{17+\sqrt{264}}\right)^2 = \left(\sqrt{a}+\sqrt{b}\right)^2$$
$$17+\sqrt{264} = a+2\sqrt{ab}+b$$
$$17+\sqrt{264} = (a+b)+2\sqrt{ab}$$
$$17+2\sqrt{66} = (a+b)+2\sqrt{ab}$$
$$\left.\begin{aligned} \Rightarrow a+b&=17 \\ \Rightarrow ab&=66 \end{aligned}\right\} a=11,\quad b=6$$

$$a-b=11-6=5$$

If x is an odd negative integer and y is an even integer, which of the following statements must be true?

I. $(3x - 2y)$ is odd ✔

~~II.~~ xy^2 is an even negative integer

~~III.~~ $(y^2 - x)$ is an odd negative integer

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III

I. $3x - 2y$ is odd

$(odd)(odd) - (even)(even)$

$(odd) - (even) = odd$

II. xy^2 is an even negative integer

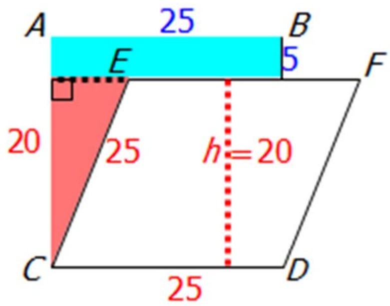
$(-3)(0)^2 = 0$ (not negative)

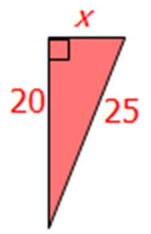
III. $(y^2 - x)$ is an odd negative integer

$(2)^2 - (-3) = 7$ (not negative)

If $ABCD$ is a square with area 625, and $CEFD$ is a rhombus with area 500, then the area of the shaded region is

(A) 125
(B) 175
(C) 200
(D) 250
(E) 275





$x^2 + 20^2 = 25^2$

$x^2 + 400 = 625$

$x^2 = 225$

$x = 15$

Area of rectangle $= l \times w$

$= 25 \times 5 = 125$

Area of triangle $= \dfrac{b \times h}{2}$

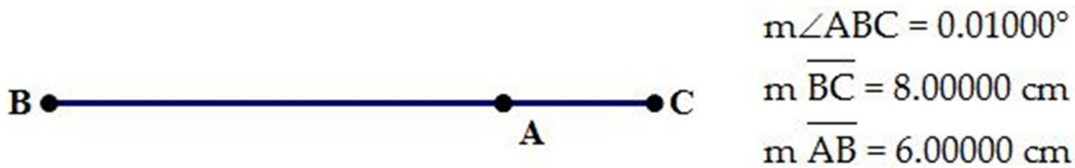
$= \dfrac{15 \times 20}{2} = 150$

Total area $= 125 + 150$

$= 275$

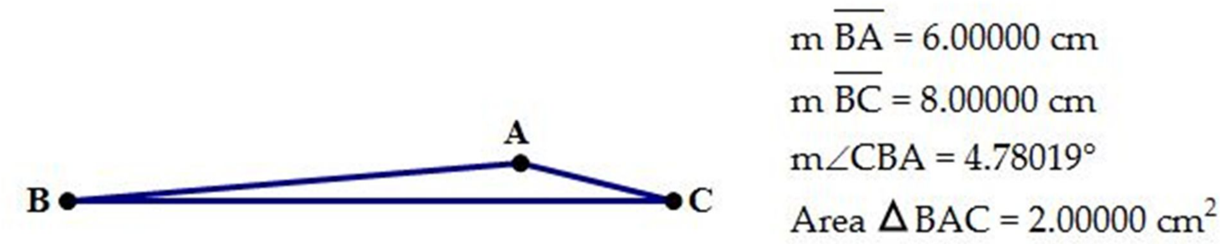
This requires a picture explanation. Consider the leg of 6 and the leg of 8 attached at a "hinged" joint at

B.

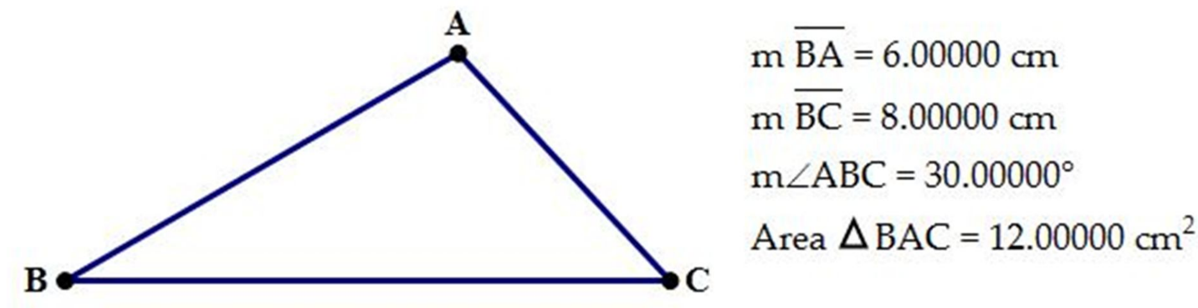


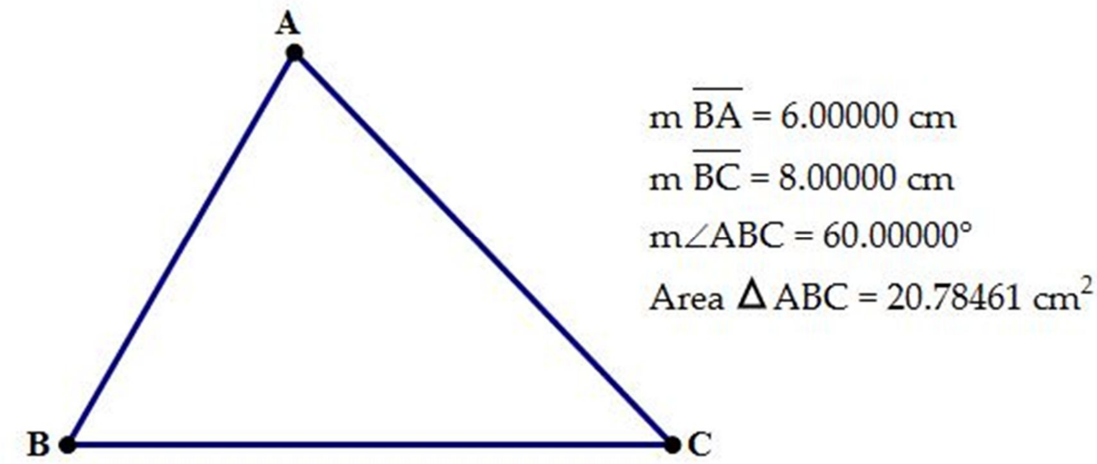
Here, the angle is made very narrow, only 1/100 of a degree. The area of this triangle would be 0.00419 --- you don't need to be able to calculate something like this for the test. The point is -- if the angle were one millionth, or one billionth, the area could be really really small --- greater than zero, but a really tiny decimal. So, clearly, the area can be less than two.

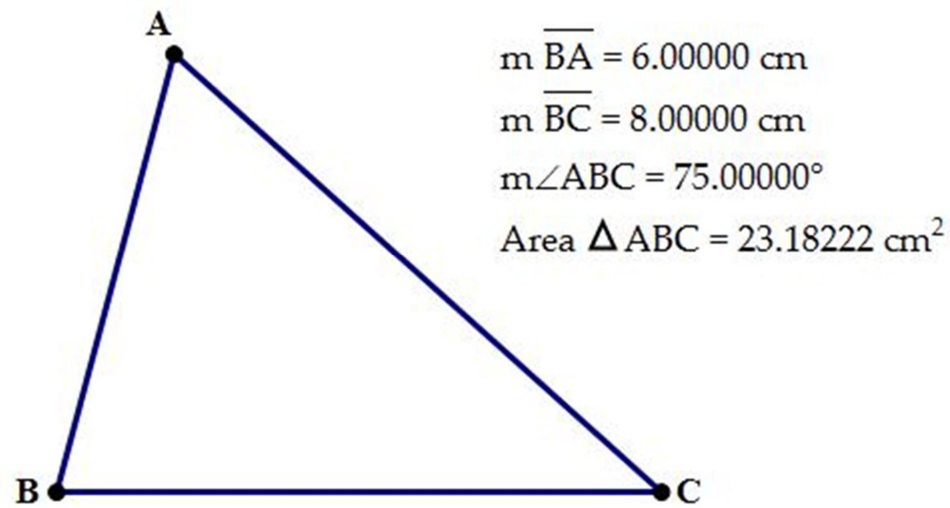
Here, I moved the legs of 6 and 8 apart a little, and produced a triangle with an area of exactly two:



Again, you don't have to know how to build a triangle like this: this is just to show you that it is, indeed possible.
As we increase the angle, we get triangles with more and more area. You do not need to know how to find these areas --- this is just to demonstrate that the area would increase.







Now, there's a geometry theorem that states, when the angle is 90°, the area of the triangle will be a maximum. At that point, the length of 8 will be the base, and the length of 6 will be the height, so the area = 0.5*bh = 24, the maximum possible area.

Thus, the triangle can have all three areas: I, II, and III.

Answer = **E**

What is the sum of all possible solutions to the equation $\sqrt{2x^2 - x - 9} = x + 1$?

(A) -2
(B) 2
(C) 3
(D) 5
(E) 6

$$\sqrt{2x^2 - x - 9} = x + 1$$
$$\left(\sqrt{2x^2 - x - 9}\right)^2 = (x+1)^2$$
$$2x^2 - x - 9 = x^2 + 2x + 1$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0 \Rightarrow x = 5, \cancel{-2}$$

$$\sqrt{2(5)^2 - (5) - 9} = (5) + 1 \qquad \sqrt{2(-2)^2 - (-2) - 9} = (-2) + 1$$
$$\sqrt{36} = 6 \qquad \sqrt{1} = -1$$
$$6 = 6 \checkmark \qquad 1 = -1 \times$$

How many positive integers less than 10,000 are such that the product of their digits is 210?

(A) 24
(B) 30
(C) 48
(D) 54
(E) 72

$$210 = 2 \times 3 \times 5 \times 7$$

Case i: 4-digit number with 2, 3, 5, and 7
➡ 4! (24) possibilities

Case ii: 3-digit number with 5, 6, and 7
➡ 3! (6) possibilities

Case iii: 4-digit number with 1, 5, 6, and 7
➡ 4! (24) possibilities

Total = 24 + 6 + 24 = **<u>54</u>**

FAQ: Shouldn't we also have a 4-digit number that includes 5, 6, 7, and 0?

The question does not specify the number of digits in the number-- just that it be less than 10,000, which means all numbers from 9,999 and down are fair game. That means the number could be 4 digits, 3 digits, 2 digits, or even 1 digit. However, we know that 2 and 1 aren't possible because the "product of their digits" needs to be 210, and we can't make that happen if we only have 2 numbers or 1 number to work with. However, both 3-digit and 4-digit numbers are completely valid, as long as the digits produce a product of 210, so if the number is 567 (or 765, 756, 657, 675, 576), it does not need another 0 to fill up a missing spot, or anything like that, since it's fine on its own. The addition of a zero would also make the product equal to 0, and not 210.

FAQ: Why do we only combine 2 and 3, not 2 and 5, 3 and 5, etc.?

Now the reason we only combine the 2 and 3, and not 5 and 3 is because 2 and 3 give us 6, which is still a digit. But 5 and 3 gives us 15, which is not a digit. For instance if we had the number 2715, the product of the digits would be:

2 * 7 * 1 * 5 = 70; not 2 * 7 * 15 = 210.