Chapter 1: ALGEBRAIC TRANSLATION

- **Relationships**:
  - Total Cost ($) =Unit Price ($ per unit) * Quantity purchased (units)
  - Total Sales or Revenue = Unit Price * Quantity sold
  - Profit = Revenue - Cost (all in $)
  - Sale Price = Unit Cost + Markup
  - Total Earnings ($) = Wage Rate ($ per hour) * Hours worked
- **Translating Words Correctly**: Insert simple test numbers to make sure that your translation is correct.
  Example: Jane bought twice as many apples as bananas → A=2B
- **Using Charts to Organize Variables**: Organize the information in a table when the problem involves several quantities and multiple relationships.

> 8 years ago, George was half as old as Sarah. Sarah is now 20 years older than George. How old will George be 10 years from now?

	8 years ago	Now	10 years from now
George	$G-8$	G	$G+10=?$
Sarah	$G+12$	$G+20$	$G+30$

- **Price and Quantities**: The quantities sum to a total, and the monetary values sum to a total.

> Paul has twenty-five transit cards, each worth either $5, $3, or $1.50. What is the total monetary value of all of Paul's transit cards?

	Unit Price	×	Quantity	=	Total Value
$5 cards	5	×	x	=	$5x$
$3 cards	3	×	y	=	$3y$
$1.50 cards	1.5	×	z	=	$1.5z$
Total	—		25		?

- **Hidden Constraints**: When the variable must be an integer / - / +

> If Kelly received 1/3 more votes than Mike in a student election, which of the following could have been the total number of votes cast for the two candidates?
>
> (A) 12 (B) 13 (C) 14 (D) 15 (E) 16

→ M + 4/3 M must be a whole number – Only possible when M = 3. Hence, the sum must be a multiple of 7.

> A store sells erasers for $0.23 each and pencils for $0.11 each. If Jessica buys both erasers and pencils from the store for a total of $1.70, what total number of erasers and pencils did she buy?

E	$P=\frac{170-23E}{11}$	Works?
0	$P=170/11$	No
1	$P=147/11$	No
2	$P=124/11$	No
3	$P=101/11$	No
4	$P=78/11$	No
5	$P=55/11=5$	Yes

In this problem, the possibilities for E and P are constrained not only to integer values but in fact to positive values.

Chapter 2: RATES & WORK

- **Matching Units in the RTD Chart**: All the units in your RTD chart must match up with one another.

> It takes an elevator four seconds to go up one floor. How many floors will the elevator rise in two minutes?

- **Multiple RTD Problems**: When you have more than one trip or traveler make a row in your RTD chart for each.
  Rate Relations, Time Relations, and Sample Situations:

"Car A and Car B are driving directly toward each other."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Car A	a	×		=	
Car B	b	×		=	
Shrinking Distance Between	$a+b$				

"Car A is chasing Car B and catching up."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Car A	a	×		=	
Car B	b	×		=	
Shrinking Distance Between	$a-b$				

"Sue left her office at the same time as Tara left hers. They met some time later."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Sue		×	t	=	
Tara		×	t	=	

"Sue left the office 1 hour after Tara, but they met on the road."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Sue		×	$t-1$	=	
Tara		×	t	=	

The Kiss (or Crash):

"Car A and Car B start driving toward each other at the same time. Eventually they crash into each other."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Car A	a	×	t	=	A's distance
Car B	b	×	t	=	B's distance
Total	$a+b$		t		Total distance covered
	ADD		SAME		ADD

(unless one car starts earlier than the other)

The Chase:

"Car A is chasing Car B. How long does it take for Car A to catch up to Car B?"

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Car A	a	×	t	=	A's distance
Car B	b	×	t	=	B's distance
Relative Position	$a-b$		t		Change in the gap between the cars
	SUBTRACT		SAME		SUBTRACT

(unless one car starts earlier than the other)

Following footsteps:

"Jan drives from home to the store along the same route as Bill."

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Jan		×		=	d
Bill		×		=	d
	VARIES		VARIES		SAME

Second-guessing:

"Jan drove home from work. If she had driven home along the same route 10 miles per hour faster…"

	Rate (miles/hour)	×	Time (hour)	=	Distance (miles)
Actual	r	×		=	d
Hypothetical	$r + 10$	×		=	d
	VARIES		VARIES		SAME

Example:

Stacy and Heather are 20 miles apart and walk towards each other along the same route. Stacy walks at a constant rate that is 1 mile per hour faster than Heather's constant rate of 5 miles/hour. If Heather starts her journey 24 minutes after Stacy, how far from her original destination has Heather walked when the two meet?

(A) 7 miles (B) 8 miles (C) 9 miles (D) 10 miles (E) 12 miles

	Rate (mi/h)	×	Time (hr)	=	Distance (mi)
Stacy	6	×	$t + 0.4$	=	
Heather	5	×	t	=	
Total					20 mi

- Be sure that whoever walks for more time has a larger time expression.
- You can only add the rates for the period during which the women are both walking.

- **Basic Work Problems**: Work Rate = Given # of Jobs / Given Amount of Time, OR 1 / Time to complete 1 Job
  - When two or more workers work together on a job, their rates add, not their times. The only exception to this rule comes in the rare case when one agent's work undoes the other agent's work. For example, one pump might put water into a tank, while another pump draws water out of that same tank.
- **Population Problem**: These can be solved with a Population Chart.

The population of a certain type of bacterium triples every 10 minutes. If the population of a colony 20 minutes ago was 100, in approximately how many minutes from now will the bacteria population reach 24,000?

Time Elapsed	Population
20 minutes ago	100
10 minutes ago	300
NOW	900
in 10 minutes	2,700
in 20 minutes	8,100
in 30 minutes	24,300

Chapter 3: RATIOS

- Ratios can express a part-part or a part-whole relationship.
  - A part-part relationship: The ratio of men to women in the office is 3:4.
  - A part-whole relationship: There are 3 men for every 7 employees.
- For more complicated ratio problems. use the Unknown Multiplier.
  - Ratio of men to women is 3x / 4x. So, 7x represents the number of employees.

A recipe calls for amounts of lemon juice, wine, and water in the ratio of 2 : 5 : 7. If all three combined yield 35 milliliters of liquid, how much wine was included?

Make a quick table: Lemon Juice + Wine + Water = Total
$2x + 5x + 7x = 14x$

  - You should never have two Unknown Multipliers in the same problem.
- Multiple Ratios: Make a common term

In a box containing action figures of the three Fates from Greek mythology, there are three figures of Clotho for every two figures of Atropos, and five figures of Clotho for every four figures of Lachesis.
(a) What is the least number of action figures that could be in the box?
(b) What is the ratio of Lachesis figures to Atropos figures?

C : A : L				C : A : L
3 : 2	→	Multiply by 5	→	15 : 10
5 : : 4	→	Multiply by 3	→	15 : : 12

This is the combined ratio: $\boxed{15 : 10 : 12}$

Chapter 4: COMBINATORICS (Advanced Counting: how many different ways you can arrange things.)

- **Fundamental Counting Principle**: To count all your options, multiply the choices you have for each separate option.
- **Slot Method**: draw empty slots corresponding to each of the choices you have to make.

An office manager must choose a five-digit lock code for the office door. The first and last digits of the code must be odd, and no repetition of digits is allowed. How many different lock codes are possible?

First set up the slots:	___	___	___	___	___
Next, fill in the restricted slots:	5				4
Then fill in the remaining slots:	5	8	7	6	4

- **Simple Factorial and Anagrams**:
  - When you have repeated items, divide the "total factorial" by each "repeat factorial" to count the different arrangements.

Ex: How many different anagrams (meaningful or nonsense) are possible for the word PIZZAZZ? → 7! / 4!

- **Anagram Grids**:
  - In a "pick a group" problem, we only care about who is in or out, not the internal order of the chosen group. You can't use the Slot Method for these problems since, by their nature, slots are distinguishable.
    Ex: If three of seven standby passengers are selected for a flight, how many different combinations of standby passengers can be selected? → FFFNNNN → 7! / (4! *3!)
- **Multiple Arrangements**: combines the Fundamental Counting Principle the anagram approach.
  - **RULE**: If a GMAT problem requires you to choose two or more sets of items from separate pools, count the arrangements separately-perhaps using a different anagram grid each time. Then multiply the numbers of possibilities for each step.

The I Eta Pi fraternity must choose a delegation of three senior members and two junior members for an annual interfraternity conference. If I Eta Pi has 12 senior members and 11 junior members, how many different delegations are possible?

    → (12! / 3! * 9!) * (11! / 2! * 9!)
  - Distinguish problems-which require choices from separate pools from complex problems that are still single arrangements: For instance, a problem requiring the choice of a treasurer, a secretary, and three more representatives from one class of 20 students may seem like two or more separate problems, but it is just one: an anagram of one T, one S, three R's, and fifteen N's in one 20-letter "word."
- **Arrangement with Constraints**: If the problem has unusual constraints, try counting arrangements without constraints first. Then subtract the forbidden arrangements.

Greg, Marcia, Peter, Jan, Bobby, and Cindy go to a movie and sit next to each other in 6 adjacent seats in the front row of the theater. If Marcia and Jan will not sit next to each other, in how many different arrangements can the six people sit?

  → 6! – (2 * 5!)
  - For problems in which items or people must be next to each other, pretend that the items "stuck together" are actually one larger item. The "stuck together" moviegoers could be in order either as J-M or as M-J.

Chapter 5: PROBABILITY P (A or B) = P(A) + P(B) – P (A and B)

- **Probability**: For the probability fraction to be meaningful, all the outcomes must be equally likely.

If a fair coin is tossed three times, what is the probability that it will turn up heads exactly twice?

  use a counting tree to find the solution: 3/8
- **More than One Event: "AND" vs, "OR"**
  - AND means multiply the probabilities. You will wind up with a smaller number, which indicates a lower probability of success.
  - OR means add the probabilities. You will wind up with a larger number, which indicates a larger probability of success.
    - If an "OR" problem features events that cannot occur together, then you can find the "OR" probability by adding the probabilities of the individual events, as illustrated above.
    - If an "OR" problem features events that can occur together, then use the following formula to find the "OR" probability: P (A or B) = P(A) + P(B) – P (A and B)
      Ex: P (H1 or H2) = P(H1) + P(H2) – P (H1 and H2) = 1/2 + 1/2 - (1/2) * (1/2) = ¾
      3/4 or 75% is the true chance of getting heads at least once in two fair coin flips.

Molly is playing a game that requires her to roll a fair die repeatedly until she first rolls a 1, at which point she must stop rolling the die. What is the probability that Molly will roll the die less than four times before stopping?	$P(\text{one roll}) + P(\text{two rolls}) + P(\text{three rolls}) = \frac{1}{6}+\left(\frac{5}{6}\cdot\frac{1}{6}\right)+\left(\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}\right)=\frac{91}{216}$
A fair die is rolled once and a fair coin is flipped once. What is the probability that either the die will land on 3 or that the coin will land on heads?	the probability of either event occurring is $\left(\frac{1}{6}+\frac{1}{2}\right)-\frac{1}{12}=\frac{7}{12}$.

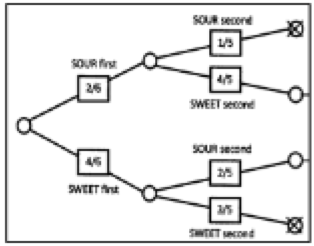
  These outcomes are not mutually exclusive, since both can occur together.
- **The 1-x Probability Trick**: Sometimes it is easier to calculate the probability that an event will NOT happen than the probability that the event WILL happen.
  - Probability of SUCCESS + Probability of FAILURE = 1

What is the probability that, on three rolls of a single fair die, AT LEAST ONE of the rolls will be a six?

  1 – Probability that NONE of the rolls will yield a 6

- **The Domino Effect**: Be careful of situations in which the outcome of the first event affects the outcome of the second event.
  Ex: In a box with 10 blocks, 3 of which are red, what is the probability of picking out a red block on each of your first two tries? Assume that you do NOT replace the first block after you have picked it.
- **Probability Trees**: When you use a probability tree, multiply down the branches and add across the results. Avoid setting up complicated trees, which GMAT problems almost never require.

> Renee has a bag of 6 candies, 4 of which are sweet and 2 of which are sour. Jack picks two candies simultaneously and at random. What is the chance that exactly 1 of the candies he has picked is sour?



Chapter 6: STATISTICS

- **Averages**: Sum / # of Terms → A * n = S
  The first thing to do for any GMAT average problem is to write down the average formula. Then, fill in any of the 3 variables S, n, and A.

> Sam earned a $2,000 commission on a big sale, raising his average commission by $100. If Sam's new average commission is $900, how many sales has he made?

	Average	×	Number	=	Sum
Old Total	800	×	n	=	$800n$
This Sale	2000	×	1	=	2000
New Total	900	×	$n+1$	=	$900(n+1)$

- **Evenly Spaced Sets: Take the Middle**: add the first and last terms and divide that sum by 2.
  Ex: The average of the set {101, 111, 121…581, 591, 601} is equal to 351, which is (101 + 601 = 702) divided by 2.
- **Weighted Average**: The average of x 20s and y 30s is: $\frac{x}{x+y}(20)+\frac{y}{x+y}(30)$
  If you know the ratio of the weights, you know the weighted average- and vice versa.

> A mixture of "lean" ground beef (10% fat) and "super-lean" ground beef (4% fat) contains twice as much lean beef as super-lean beef. What is the percentage of fat in the mixture?

$$\frac{(10\%)(2)+(4\%)(1)}{2+1}=\frac{24\%}{3}=8\%$$

- **Using Units to Determine Weights**:
  - To find the average speed for Joe's whole trip, you can simply divide the total distance by the total time.
- **Standard Deviation**: indicates how far from the average (mean) the data points typically fall.
  - A small SD indicates that a set is clustered closely around the average (mean) value.
  - A large SD indicates that the set is spread out widely, with some points appearing far from the mean.

Chapter 7: OVERLAPPING SETS

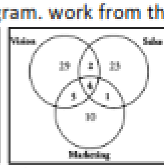
- **Double-Set Matrix**: The most efficient tool for problems involving only two categorizations or decisions.
  - Make sure that the column labels represent opposite situations. Do the same for the row labels.
- **Overlapping Sets and Algebraic Representation**: Pay attention to the wording of the problem.

> Santa estimates that 10% of the children in the world have been good this year but do not celebrate Christmas, and that 50% of the children who celebrate Christmas have been good this year. If 40% of the children in the world have been good, what percentage of children in the world are not good and do not celebrate Christmas?

	Good	Not Good	TOTAL
X-mas	$0.5x$		x
No X-mas	10		
TOTAL	40		100

- **2 Sets, 3 Choices: Still Double-Set Matrix**. Ex: If respondents can answer "Yes," "No," or "Maybe".
- **3 Sets Problems: Venn Diagrams:** When you use a Venn Diagram. work from the INSIDE OUT.

> Workers are grouped by their areas of expertise and are placed on at least one team. 20 workers are on the Marketing team, 30 are on the Sales team, and 40 are on the Vision team. 5 workers are on both the Marketing and Sales teams, 6 workers are on both the Sales and Vision teams, 9 workers are on both the Marketing and Vision teams, and 4 workers are on all three teams. How many workers are there in total?



  - Workers on 2 teams: Here we must remember to subtract those workers who are on all 3 teams. For example, the problem says that there are 5 workers on the Marketing and Sales teams. However, this includes the 4 workers who are on all three teams.

Chapter 8: MINOR PROBLEM TYPES

The GMAT occasionally contains problems that fall under one of three umbrellas: Optimization, Grouping, and Scheduling. Just like other Word Translation problems. these minor types require an organized approach. The general approach is to focus on extreme scenarios.

- **Optimization**: You are asked to maximize or minimize some quantity, given constraints on other quantities.

> The guests at a football banquet consumed a total of 401 pounds of food. If no individual guest consumed more than 2.5 pounds of food, what is the minimum number of guests that could have attended the banquet?

Pounds of food per guest	×	Guests	=	Total pounds of food
At MOST 2.5 maximize	×	At LEAST ??? minimize	=	EXACTLY 401 constant

Note that you may need to round up or down depending on the situation. Ex: Here, 160.4 should be rounded up.

- **Grouping**: Putting people or items into different groups to maximize or minimize some characteristic. One approach is to determine the limiting factor on the number of complete groups.

> Orange Computers is breaking up its conference attendees into groups. Each group must have exactly one person from Division A, two people from Division B, and three people from Division C. There are 20 people from Division A, 30 people from Division B, and 40 people from Division C at the conference. What is the smallest number of people who will not be able to be assigned to a group?

There are enough Division A people for 20 groups, but only enough Division B people for 15 groups, and only enough Division C people for 13 groups. So the limiting factor is Division C.

- **Scheduling**: Focus on the extreme possibilities to solve scheduling problems.

> How many days after the purchase of Product X does its standard warranty expire? (1997 is not a leap year.)
> (1) When Mark purchased Product X in January 1997, the warranty did not expire until March 1997.
> (2) When Santos purchased Product X in May 1997, the warranty expired in May 1997.

Rephrase the two statements in terms of extreme possibilities:
(1) Shortest possible warranty period: Jan. 31 to Mar. 1 (29 days later)
Longest possible warranty period: Jan. 1 to Mar. 31 (89 days later)
Note that 1997 was not a leap year.
(2) Shortest possible warranty period: May 1 to May 2, or similar (1 day later)
Longest possible warranty period: May 1 to May 31 (30 days later)

- **Computation Problems**: Very occasionally, the GMAT features problems centered on computation-problems that contain no variables at all, and in principle require nothing more than "plug and chug" techniques.

> Five identical pieces of wire are soldered together to form a longer wire, with the pieces overlapping by 4 cm at each join. If the wire thus made is exactly 1 meter long, how long is each of the identical pieces? (1 meter = 100 cm)

→ $5x = 100+4*4$

- **Graphing Problems**: Study the graph/table/chart carefully both before and after reading the problem.
  - Pay attention to labels, units, scales, and patterns in the shape of the graph.

Chapter 9: STRATEGIES FOR DATA SUFFICIENCY

- **Rephrasing: Translating Words into Algebra**: Be sure to keep track of the variables you assign to each unknown.

> A hot dog vendor who sells only hot dogs and soft drinks charges $3 for a hot dog. If the vendor collected $1,000 in total revenue last month, how much does he charge for a soft drink?
> (1) The vendor sold twice as many hot dogs as soft drinks last month.
> (2) The revenue from hot dog sales last month was 3/4 of the total monthly revenue.

Given that $3H + xS = 1{,}000$, what is x?
(1) $H = 2S$ → $6S+xS = 1{,}000$
(2) $3H = 750$ → $750 +xS = 1{,}000$ → $xS =250$
Answer C: SUFFICIENT TOGETHER

Chapter 10: OFFICIAL GUIDE PROBLEM SETS: PART I

Chapter 11: RATES & WORK: ADVANCED

- Do not be intimidated by RTD or RTW algebra; it generally looks scarier than it actually is – Problems can generally be solved by the Multiple RTD or RTW chart.

> Liam is pulled over for speeding just as he is arriving at work. He explains to the police officer that he could not afford to be late today, and has arrived at work only four minutes before he is to start. The officer explains that if Liam had driven 5 mph slower for his whole commute, he would have arrived at work exactly on time. If Liam's commute is 30 miles long, how fast was he actually driving? (Assume that Liam drove at a constant speed for the duration of his commute.)

	Rate (mi/h)	×	Time (h)	=	Distance (mi)
Actual	$r + 5$	×	$\frac{30}{r+5}$	=	30
Hypothetical	r	×	30/[illegible]	=	30

- In the example above, the equation is $\frac{30}{r} = \frac{30}{r+5} + \frac{1}{15}$. At this point, you could plug in answers.

> Wendy begins sanding a kitchen floor by herself and works for 4 hours. She is then joined by Bruce, and together the two of them finish sanding the floor in 2 hours. If Bruce can sand the floor by himself in 20 hours, how long would it take Wendy to sand the floor by herself?

	R	×	T	=	W
Wendy actual	r	×	4	=	
Both actual	$1/20 + r$	×	2	=	
Wendy theoretical	r	×	?	=	1

- Be ready to break rate or work problems into natural stage.
  Likewise, combine workers or travelers into a single row when the work or travel together.

• **Equations for Exponential Growth/Decay**: Few population problems may call for an exponential formula to represent population growth.
  - If the population doubles in the interval, then we have this variation: Population = $S*2^{t/I}$
  - Likewise, if the population is cut in half in the interval, the formula Population = $S*0.5^{t/I}$

<u>NOTE</u>: In an exponential growth formula, make sure all your time units are the same.

Chapter 12: COMBINATORICS, PROBABILI1Y, & STATISTICS: ADVANCED

• **Disguised Combinatorics**:

> Alicia lives in a town whose streets are on a grid system, with all streets running east–west or north–south without breaks. Her school, located on a corner, lies three blocks south and three blocks east of his home, also located on a corner. If Alicia is equally likely to choose any possible path from home to school, and if she only walks south or east, what is the probability that she will walk south for the first two blocks?

$$Probability = \frac{\#\ of\ desired\ possibilites}{Total\ \#\ of\ possibilities} = \frac{\frac{4!}{3!\,1!}}{\frac{6!}{3!\,3!}} \rightarrow \frac{EEES}{EEESSS}\ where\ order\ does\ not\ matter$$

To get home, Alicia must walk exactly three blocks South and three blocks East; the only issue is the order in which she does so. Therefore, the problem involves anagramming three S's and three E's in the denominator. For the numerator, since you cannot change the first two letters, just ignore them.

• **Combination and Permutation Formulas**:

Combination: A selection of items from a larger pool in which the order of items does not matter.

The number of combinations of r items, chosen from a pool of n items, is $\frac{n!}{k!(n-k)!}$

Permutation: A selection of items from a larger pool in which the order of items does matter.

The number of permutation of r items, chosen from a pool of n items, is $\frac{n!}{(n-k)!}$

<u>NOTE</u>: If switching the elements in a chosen set creates a different set, it is a permutation. If not, it is a combination. Also, 0! = 1.

> Three small cruise ships, each carrying 10 passengers, will dock tomorrow. One ship will dock at Port A, another at Port B, and the third at Port C. At Port A, two passengers will be selected at random; each winner will receive one gift certificate worth $50. At Port B, one passenger will be selected at random to receive a gift certificate worth $35, and at Port C, one passenger will be selected at random to receive a gift certificate worth $25. How many different ways can the gift certificates be given out?

$$Number\ of\ possibilities = 3! * \frac{10!}{8!\,2!} * 10 * 10 \rightarrow\ Permut. * Comb. * Comb. * Comb.$$

• **Combinatorics and Probability**:

> Kate and her twin sister Amy want to be on the same relay-race team. There are 6 girls in the group, and only 4 of them will be placed at random on the team. What is the probability that Kate and Amy will both be on the team?

$$Probability = \frac{\#\ of\ desired\ possibilites}{Total\ \#\ of\ possibilities} = \frac{\frac{4!}{2!\,2!}}{\frac{6!}{4!\,2!}} \rightarrow \frac{YYNN}{YYYYNN}\ where\ order\ does\ not\ matter$$

Each of the winning scenarios will include Kate and Amy, plus 2 of the remaining 4 girls. Therefore, we Reduce the Pool by 2 people, and we find the number of winning scenarios by counting the ways we can choose 2 of the remaining 4 girls:

- **Combinatorics and the Domino Effect**: * P of events in a sequence, taking earlier events into account.

> A miniature gumball machine contains 7 blue, 5 green, and 4 red gumballs, which are identical except for their colors. If the machine dispenses three gumballs at random, what is the probability that it dispenses one gumball of each color?

$$\left(\frac{7}{16} * \frac{5}{15} * \frac{4}{14}\right) * 6$$

  - In general, when you have a symmetrical problem with multiple equivalent cases, calculate the probability of one case (often by using the domino-effect rule). Then multiply by the number of cases. Use combinatorics to calculate the number of cases, if necessary.

- **Reformulating Difficult Problems**: If a probability problem seems to require extensive calculation, try to reformulate it in a way that either takes advantage of symmetry in the problem or groups several individual cases together at once.

> A medical researcher must choose one of 14 patients to receive an experimental medicine called Progaine. The researcher must then choose one of the remaining 13 patients to receive another medicine, called Ropecia. Finally, the researcher administers a placebo to one of the remaining 12 patients. All choices are equally random. If Donald is one of the 14 patients, what is the probability that Donald receives either Progaine or Ropecia?

$$\frac{1}{14} + \frac{1}{14}$$

> A gambler rolls three fair six-sided dice. What is the probability that two of the dice show the same number, but the third shows a different number?

Let the result of the first die be any number-so that you do not need to include the first die in the probability calculation-and concentrate on whether the second and third dice match the first die.

  - Second matches first, third does not: 1/6 * 5/6 = 5/36
  - Third matches first, second does not: 5/6 * 1/6 = 5/36
  - Second and third match each other: 5/6 * 1/6 = 5/36

- **Two Time-Saving Shortcuts for Averages**:

Changes in Mean: $\frac{New\ term - Old\ mean}{New\ Number\ of\ terms}$

  Ex: Change in mean is $100, and New term - Old mean is $1200. The formula yields at once the fact that Sam has made 12 sales (New number of terms).

Residual: Date Point - Mean

  Ex: If the class average on a test is 85 points, then a score of 91 has a residual of +6. Likewise, a score of 15 has a residual of -10.

  NOTE: For any set, the residuals sum to zero. Alternatively, the positive residuals ("overs") and negative residuals ("unders") for any set will cancel out.