Computer Science 384
St. George Campus

Version S22.1
University of Toronto

Problem Set 2: Knowledge

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1. Someone stole Chandra's laptop to try to find the final exam. He has hired a detective to solve the case. The detective determines that:

   S1: The laptop was stolen in the library at 12 p.m.

   S2: Anyone who was in the library at 12 p.m had an opportunity to steal the laptop.

   S3: Abel was in the library at 12 p.m, and had the motive to steal the laptop.

   S4: Anyone who stole the laptop must have had the motive and opportunity to do so.

   (a) Formulate each of the statements, **S1** to **S4** in first-order logic using the following vocabulary:

   - **Constants**:
     - **Abel**; refers to Abel
     - **laptop**; refers to Chandra's laptop.
     - **library**; refers to the library.
     - **12**; refers to 12 p.m.
   - **Variables**: x, y, z
   - **Functions**:
     - location(x,t); the location of x at time, t
   - **Predicates**:
     - stole(x, y, l, t); x stole y at the location l at time t
     - motive(x); x had the motive to steal the laptop.
     - opportunity(x); x had the opportunity to steal the laptop.

S1	$\exists x$ (stole (x, laptop, library, 12))
S2	$\forall x$ (location (x, 12) = library $\rightarrow$ opportunity (x))
S3	(location (Abel, 12) = library) $\wedge$ motive (Abel)
S4	$\forall x$ (stole (x, laptop, library, 12) $\rightarrow$ motive (x) $\wedge$ opportunity (x))

(b) The detective claims makes the following claim:

Q: Abel stole the laptop.

You are a part of Abel's defense team. Show that the detective's claim may be incorrect by finding a model of $\{\mathbf{S1},\ldots,\mathbf{S4}\}$ that is not a model of $\{\mathbf{S1},\ldots,\mathbf{S4},\mathbf{Q}\}$.

$D =$ $\{a, c, l, t, r\}$

$\text{stole}^{\mathcal{M}}$	$\{\langle r, c, l, t\rangle\}$
$\text{motive}^{\mathcal{M}}$	$\{\langle a\rangle, \langle r\rangle\}$
$\text{opportunity}^{\mathcal{M}}$	$\{\langle a\rangle, \langle r\rangle\}$

$$\text{location}^{\mathcal{M}}(d) = \begin{cases} \text{location}^{\mathcal{M}}(a,t) = l \\ \text{location}^{\mathcal{M}}(r,t) = l \\ \text{location}^{\mathcal{M}}(c,t) = l \\ \text{location}^{\mathcal{M}}(l,t) = l \end{cases}$$

$\text{Abel}^{\mathcal{M}} = a$

$\text{laptop}^{\mathcal{M}} = c$

$\text{library}^{\mathcal{M}} = l$

$12^{\mathcal{M}} = t$

2. Given predicates P, Q, R and variables x, y, z; convert the following sentences to clausal form: **Clearly indicate any Skolem functions or constants used in the conversion**. You need not fill every row.

   (a) $\forall x\,(R(x) \rightarrow \exists y P(x,y))$

C1	$\forall x\ (\neg R(x) \vee \exists y P(x,y))$
C2	$\forall x\ (\neg R(x) \vee P(x, g(x))$
C3	$\neg R(x) \vee P(x, g(x))$
C4	
C5	

   Skolem constants $= \{$ $\}$

   Skolem functions $= \{$ $g(x)$, $\}$

   (b) $\exists x \forall y\,((Q(x,y) \wedge Q(y,x)) \vee \neg R(y))$

C1	$\forall y\,[(Q(c,y) \wedge Q(y,c)) \vee \neg R(y)]$
C2	$\forall y\,[(Q(c,y) \vee \neg R(y)) \wedge (Q(y,c) \vee \neg R(y))]$
C3	$[Q(c,y) \vee \neg R(y)] \wedge [Q(y,c) \vee \neg R(y)]$
C4	$Q(c,y) \vee \neg R(y)$, $Q(y,c) \vee \neg R(y)$
C5	

   Skolem constants $= \{$ c $\}$

   Skolem functions $= \{$ $\}$

3. Let move be a 3-ary predicate name. Assume functions and constants are lower case and variables are upper case. For each of the pairs below, give the most general unifier (MGU) or write $\emptyset$ to denote that no MGU exists.

   (a) move$(f(X), f(h(Y)), g(h(Y)))$ and move$(Z, f(h(a)), g(X))$

   $\delta =$ $\{Z = f(h(a)), Y = a, X = h(a)\}$

   (b) move$(X, h(Y), f(X))$ and move$(f(Y), h(b), f(g(a)))$

   $\delta =$ $\emptyset$

3. a)

Iteration	D_k	S_k	δ_k
0	$\{f(X), Z\}$	$\{move(f(X), f(h(Y)), g(h(Y))), move(Z, f(h(a)), g(X))\}$	$\{\}$
1	$\{f(h(Y)), f(h(a))\}$	$\{move(f(X), f(h(Y)), g(h(Y))), move(f(X), f(h(a)), g(X))\}$	$\{Z = f(X)\}$
2	$\{g(h(a)), g(X)\}$	$\{move(f(X), f(h(a)), g(h(a))), move(f(X), f(h(a)), g(X))\}$	$\{Z = f(X), Y = a\}$
3	$\{\}$	$\{move(f(h(a)), f(h(a)), g(h(a))), move(f(h(a)), f(h(a)), g(h(a)))\}$	$\{Z = f(h(a)), Y = a, X = h(a)\}$

$\delta = \{Z = f(h(a)), Y = a, X = h(a)\}$

b)

Iteration	D_k	S_k	δ_k
0	$\{X, f(Y)\}$	$\{move(X, h(Y), f(X)), move(f(Y), h(b), f(g(a)))\}$	$\{\}$
1	$\{h(Y), h(b)\}$	$\{move(f(Y), h(Y), f(f(Y))), move(f(Y), h(b), f(g(a)))\}$	$\{X = f(Y)\}$
2	$\{f(f(b)), f(g(a))\}$	$\{move(f(b), h(b), f(f(b))), move(f(b), h(b), f(g(a)))\}$	$\{X = f(b), Y = b\}$

$\{f(f(b)), f(g(a))\}$ not unifiable $\Rightarrow$ no MGU $\Rightarrow$ $\delta = \emptyset$