Computer Science 384 Version S22.1
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Problem Set 1: Uncertainty

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1. Po is living his dream as the Dragon Warrior, protecting the Valley of Peace when he discovers that someone ate his bowl of noodles. He knows that it was one of the Furious Five; Tigress, Monkey, Viper, Crane, or Mantis.

   (a) With no other information, how confident can Po be that Tigress ate the noodles?

   20 %

   (b) Po finds a single paw print on the empty bowl. It belongs to one of the Furious Five. Po concludes that whoever this print belongs to must have ate the noodles[1]. Po compares the print on the bowl with Tigress' actual print and determines that it is a match. The method Po uses has a sensitivity (i.e., the true-positive rate) of 90% and a specificity of 70% (i.e., the true-negative rate). With this new information, how confident can Po be that Tigress ate the noodles? *Use your result from* (a) *as the prior probability. Round your answer down to the nearest percent.*

   42 %

   (c) To be extra sure, Po compares the print on the bowl with the actual prints of the other members of the Furious Five. It turns out that only Tigress' print is a match. The test results are mutually conditionally independent of each other given knowledge of who ate the noodles. How confident can Po be that Tigress ate the noodles now? *Use your result from* (b) *as the prior probability. Round your answer down to the nearest percent.*

   84 %

[1] Strictly speaking, Po is making an assumption here. Just because someone's print is on the bowl, does not mean that they ate the noodles. However, for this problem, you can assume that Po's assumption is correct.

2. Bayes' Burrito Bowls is a food truck that serves burrito bowls (obviously). You can choose:

   - a type of rice, which can be; white, brown
   - a type of bean, which can be; black, pinto, lima, or kidney
   - any of the following toppings; guacamole, corn, salsa, sour cream, onion, lettuce, or cheese

   We model each type of burrito as a triple (R,B,T), where R and B are the types of rice and bean, and T is any subset of the toppings. The type of burrito a customer picks depends on his/her preferences. We can model such preferences using a probability distribution over (R,B,T).

   (a) How many different types of burrito bowls are there?

   1024

   (b) Eshan builds a burrito bowl as follows: He first chooses the type of rice and bean independently of each other. Based on both these choices, he chooses the toppings (the toppings are not chosen independently of each other). Provide an appropriate factorization of $P_{\text{Eshan}}(R,B,T)$ and the number of values that must be known to compute it for any R,B,T.

   $P_{\text{Eshan}}(R,B,T) =$ $P(R)P(B)P(T|B,R)$ | 1024

   (c) Zafeer builds a book as follows: He first chooses type of rice. Based on this choice, he chooses a type of bean. He chooses the toppings independently of the other choices (toppings are not chosen independently of each other). Provide an appropriate factorization of $P_{\text{Zafeer}}(R,B,T)$ and the number of values that must be known to compute it for any R,B,T.

   $P_{\text{Zafeer}}(R,B,T) =$ $P(R)P(B|R)P(T)$ | 128

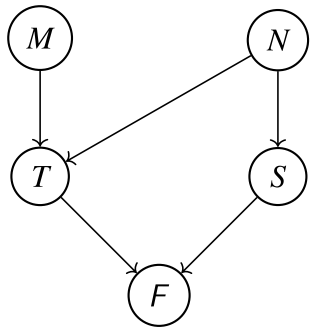
   (d) Would the number of values that must be known to compute P_{Eshan} or P_{Zafeer} get smaller, larger, or stay the same if some toppings are chosen independently of other toppings?

   Smaller ✓ Larger ☐ Same ☐

3. You and your friend are on the Amazing Race (see Wikipedia article for details) and the next leg is in a few hours. Once the leg begins, you will need to make your way to the airport. You want to use this time to decide how you are going to get there, and how many bags you need to pack.

$P(M = \text{train})$
0.6

M	N	$P(T = \text{early} \mid M, N)$
train	0	0.9
train	1	0.85
train	2	0.60
car	0	0.75
car	1	0.75
car	2	0.75



$P(N = 0)$	$P(N = 1)$
0.4	0.5

N	$P(S = \text{fast} \mid N)$
0	0.9
1	0.8
2	0.7

T	S	$P(F = \text{yes} \mid T, S)$
early	fast	0.95
early	slow	0.7
late	fast	0.7
late	slow	0.35

Both of these decisions influence whether you will make your flight or miss it by influencing when you get to the airport and how long it takes you to get through security. This scenario is modelled using the Bayesian network above:

- $M \in \{\text{train}, \text{car}\}$ is how you will get to the airport
- $N \in \{0, 1, 2\}$ is the number of bags you pack
- $T \in \{\text{early}, \text{late}\}$ is when you get to the airport
- $S \in \{\text{fast}, \text{slow}\}$ is how long it takes to get through security
- $F \in \{\text{yes}, \text{no}\}$ is whether you make the flight or not

(a) Compute $P(F = \text{yes} \mid N, M)$ for all M and N and fill out the table below.

M/N	0	1	2
train	0.899	0.8595	0.763
car	0.86	0.8325	0.805

(b) How should you get to the airport, and how many bags should you bring to maximize your chances of making the flight? Circle the appropriate choices.

$M =$ train **or** car

$N =$ 0 **or** 1 **or** 2

CSC 384 Problem Set 1

1. a) $P(\text{tigress ate}) = 20\%$

b) $P(\text{tigress ate} \mid \text{tigress positive})$

$$= \frac{P(\text{tigress positive} \mid \text{tigress ate})P(\text{tigress ate})}{P(\text{tigress positive})}$$

$$= \frac{P(\text{tigress positive} \mid \text{tigress ate})P(\text{tigress ate})}{P(\text{tigress positive} \mid \text{tigress ate})P(\text{tigress ate}) + P(\text{tigress positive} \mid \text{tigress not ate})P(\text{tigress not ate})}$$

$$= \frac{(90\%)(20\%)}{(90\%)(20\%) + (1-70\%)(1-20\%)}$$

$$\simeq 42\%$$

c) $P(\text{tigress ate} \mid \text{tigress positive, monkey negative, viper negative, crane negative, mantis negative})$

$$= \frac{P(\text{ti pos, mo neg, vi neg, cr neg, ma neg} \mid \text{ti ate})P(\text{ti ate})}{P(\text{ti pos, mo neg, vi neg, cr neg, ma neg})}$$

$$= \frac{P(\text{ti pos} \mid \text{ti ate})P(\text{mo neg} \mid \text{ti ate})P(\text{vi neg} \mid \text{ti ate})P(\text{cr neg} \mid \text{ti ate})P(\text{ma neg} \mid \text{ti ate})P(\text{ti ate})}{\begin{array}{l} P(\text{ti pos, mo neg, vi neg, cr neg, ma neg} \mid \text{ti ate})P(\text{ti ate}) + \\ P(\text{ti pos, mo neg, vi neg, cr neg, ma neg} \mid \text{mo ate})P(\text{mo ate}) + \\ P(\text{ti pos, mo neg, vi neg, cr neg, ma neg} \mid \text{vi ate})P(\text{vi ate}) + \\ P(\text{ti pos, mo neg, vi neg, cr neg, ma neg} \mid \text{cr ate})P(\text{cr ate}) + \\ P(\text{ti pos, mo neg, vi neg, cr neg, ma neg} \mid \text{ma ate})P(\text{ma ate}) \end{array}}$$

$$= \frac{P(\text{ti pos} \mid \text{ti ate})P(\text{mo neg} \mid \text{ti ate})P(\text{vi neg} \mid \text{ti ate})P(\text{cr neg} \mid \text{ti ate})P(\text{ma neg} \mid \text{ti ate})P(\text{ti ate})}{\begin{array}{l} P(\text{ti pos} \mid \text{ti ate})P(\text{mo neg} \mid \text{ti ate})P(\text{vi neg} \mid \text{ti ate})P(\text{cr neg} \mid \text{ti ate})P(\text{ma neg} \mid \text{ti ate})P(\text{ti ate}) + \\ P(\text{ti pos} \mid \text{mo ate})P(\text{mo neg} \mid \text{mo ate})P(\text{vi neg} \mid \text{mo ate})P(\text{cr neg} \mid \text{mo ate})P(\text{ma neg} \mid \text{mo ate})P(\text{mo ate}) + \\ P(\text{ti pos} \mid \text{vi ate})P(\text{mo neg} \mid \text{vi ate})P(\text{vi neg} \mid \text{vi ate})P(\text{cr neg} \mid \text{vi ate})P(\text{ma neg} \mid \text{vi ate})P(\text{vi ate}) + \\ P(\text{ti pos} \mid \text{cr ate})P(\text{mo neg} \mid \text{cr ate})P(\text{vi neg} \mid \text{cr ate})P(\text{cr neg} \mid \text{cr ate})P(\text{ma neg} \mid \text{cr ate})P(\text{cr ate}) + \\ P(\text{ti pos} \mid \text{ma ate})P(\text{mo neg} \mid \text{ma ate})P(\text{vi neg} \mid \text{ma ate})P(\text{cr neg} \mid \text{ma ate})P(\text{ma neg} \mid \text{ma ate})P(\text{ma ate}) \end{array}}$$

$$= \frac{(90\%)(70\%)^4(20\%)}{(90\%)(70\%)^4(20\%) + (4)(1-70\%)(1-90\%)(70\%)^3(20\%)}$$

$$= 84\%$$

2. a) $2 \cdot 4 \cdot \sum_{i=0}^{7} \binom{7}{i} = 2 \cdot 4 \cdot 2^7 = 1024$ Combinations

b) $P_{Eshan}(R, B, T) = P(R)\, P(B)\, P(T \mid B, R)$

$P(R)$: $|dom(R)| = 2 \Rightarrow 2$
$P(B)$: $|dom(B)| = 4 \Rightarrow 4$
$P(T \mid B, R)$: $|dom(T)| = 2^7 \Rightarrow 2^7 \cdot 4 \cdot 2 = 1024$

$\Rightarrow$ 1024 values

c) $P_{Zafeer}(R, B, T) = P(R)\, P(B \mid R)\, P(T)$

$P(R)$: 2
$P(B \mid R)$: $4 \cdot 2 = 8$
$P(T)$: $2^7 = 128$

$\Rightarrow$ 128 values

d) smaller

3. a) $P(f = yes \mid N, M) = \dfrac{P(f = yes, N, M)}{\sum_{\forall f} P(f, N, M)}$

$= \dfrac{P(f = yes, N, M)}{\sum_{\forall f} \sum_{\forall T, S} P(f, N, M, T, S)}$

$= \dfrac{P(F = yes, N, M)}{\sum_{\forall F} \sum_{\forall T} \sum_{\forall S} P(M)P(N)P(T \mid M, N)P(S \mid N)P(F \mid T, S)}$

$= \dfrac{P(F = yes, N, M)}{P(M)P(N) \sum_{\forall F} \sum_{\forall T} P(T \mid M, N) \sum_{\forall S} P(S \mid N)P(F \mid T, S)}$

$g_1(N, F, T) = \sum_{\forall S} P(S \mid N)P(F \mid T, S)$

$= P(S = fast \mid N)P(F \mid T, S = fast) + P(S = slow \mid N)P(F \mid T, S = slow)$

N	F	T	g_1
0	yes	early	$0.9 \times 0.95 + 0.1 \times 0.7 = 0.925$
0	yes	late	$0.9 \times 0.7 + 0.1 \times 0.35 = 0.665$
0	no	early	$0.9 \times 0.05 + 0.1 \times 0.3 = 0.075$
0	no	late	$0.9 \times 0.3 + 0.1 \times 0.65 = 0.335$
1	yes	early	$0.8 \times 0.95 + 0.2 \times 0.7 = 0.9$
1	yes	late	$0.8 \times 0.7 + 0.2 \times 0.35 = 0.63$
1	no	early	$0.8 \times 0.05 + 0.2 \times 0.3 = 0.1$
1	no	late	$0.8 \times 0.3 + 0.2 \times 0.65 = 0.37$
2	yes	early	$0.7 \times 0.95 + 0.3 \times 0.7 = 0.875$
2	yes	late	$0.7 \times 0.7 + 0.3 \times 0.35 = 0.595$
2	no	early	$0.7 \times 0.05 + 0.3 \times 0.3 = 0.125$
2	no	late	$0.7 \times 0.3 + 0.3 \times 0.65 = 0.405$

$$g_2(N,F,M) = \sum_{\forall T} P(T|M,N)\, g_1(N,F,T)$$
$$= P(T=\text{early}|M,N)\, g_1(N,F,T=\text{early}) + P(T=\text{late}|M,N)\, g_1(N,F,T=\text{late})$$

N	F	M	g_2
0	yes	train	$0.9 \times 0.925 + 0.1 \times 0.665 = 0.899$
0	yes	car	$0.75 \times 0.925 + 0.25 \times 0.665 = 0.86$
0	no	train	$0.9 \times 0.075 + 0.1 \times 0.335 = 0.101$
0	no	car	$0.75 \times 0.075 + 0.25 \times 0.335 = 0.14$
1	yes	train	$0.85 \times 0.9 + 0.15 \times 0.63 = 0.8595$
1	yes	car	$0.75 \times 0.9 + 0.25 \times 0.63 = 0.8325$
1	no	train	$0.85 \times 0.1 + 0.15 \times 0.37 = 0.1405$
1	no	car	$0.75 \times 0.1 + 0.25 \times 0.37 = 0.1675$
2	yes	train	$0.60 \times 0.875 + 0.4 \times 0.595 = 0.763$
2	yes	car	$0.75 \times 0.875 + 0.25 \times 0.595 = 0.805$
2	no	train	$0.60 \times 0.125 + 0.40 \times 0.405 = 0.237$
2	no	car	$0.75 \times 0.125 + 0.25 \times 0.405 = 0.195$

$$g_3(N,M) = \sum_{\forall F} g_2(N,F,M)$$
$$= g_2(N, F=\text{yes}, M) + g_2(N, F=\text{no}, M)$$

N	M	g_3
0	train	$0.899 + 0.101 = 1$
0	car	$0.86 + 0.14 = 1$
1	train	$0.8595 + 0.1405 = 1$
1	car	$0.8325 + 0.1675 = 1$
2	train	$0.763 + 0.237 = 1$
2	car	$0.805 + 0.195 = 1$

$$g_4(N,M) = P(M)P(N)g_3(N,M)$$

N	M	g_4
0	train	$0.6 \times 0.4 \times 1 = 0.24$
0	car	$0.4 \times 0.4 \times 1 = 0.16$
1	train	$0.6 \times 0.5 = 0.3$
1	car	$0.4 \times 0.5 = 0.2$
2	train	$0.6 \times 0.1 = 0.06$
2	car	$0.4 \times 0.1 = 0.04$

$$P(F = yes \mid N=0, M=train) = \frac{0.6 \times 0.4 \times 0.899}{0.24} = 0.899$$

$$P(F = yes \mid N=0, M=car) = \frac{0.4 \times 0.4 \times 0.86}{0.16} = 0.86$$

$$P(F = yes \mid N=1, M=train) = \frac{0.6 \times 0.5 \times 0.8595}{0.3} = 0.8595$$

$$P(F = yes \mid N=1, M=car) = \frac{0.4 \times 0.5 \times 0.8325}{0.2} = 0.8325$$

$$P(F = yes \mid N=2, M=train) = \frac{0.6 \times 0.1 \times 0.763}{0.06} = 0.763$$

$$P(F = yes \mid N=2, M=car) = \frac{0.4 \times 0.1 \times 0.805}{0.04} = 0.805$$

b) 0 bags, train