EC4304: Economic and Financial Forecasting

Project: Exchange Rate Forecasting

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Contents

1 Introduction

The international foreign exchange market is the largest financial market in the world, with a daily transaction volume in excess of US$5 trillion per day. It is used by a variety of stakeholders such as corporations, institutional investors, and retail investors, for purposes such as hedging (for currency and interest rate risk), portfolio diversification, and speculation. The size and importance of the FX market led us to base this forecasting project on it.

The main goal of this project is to use existing macroeconomic data and economic theory to forecast exchange rates. The overarching macroeconomic theory central to our project is the Taylor Rule, derived by Dr. John Taylor in his 1993 paper *Discretion versus Policy Rules in Practice*. Using the Taylor Rule Differentials Model as derived by Ince, Moldotsova and Papell (2013), we attempt to estimate different parameter coefficients for the model. We also consider adding Volatility Index (VIX), which tracks expected volatility in the stock market, as an additional variable by studying the relationship between the VIX and exchange rate fluctuations. We use these variables to forecast the following exchange rates: Euro against the US Dollar (EURUSD) and the Japanese Yen against the US Dollar (JPYUSD).

Section 2 presents the forecast models considered in our study; Section 3 describes the data used in our study; Section 4 discusses model selection using the Granger Causality test and Predictive Least Squares; Section 5 considers forecast combination; Section 6 presents the forecasts produced by the models uses; Section 7 evaluates the forecasts based on mean squared error and the Diebold-Mariano test; Finally, Section 8 will discuss the limitations of our study.

2 Forecast Models

In our study, we use three different models to generate forecasts for JPYUSD and EURUSD. The models were estimated using ordinary least squares (OLS) using rolling estimation windows to generate forecasts for horizons of one, six and twelve months. These forecast horizons correspond to available forward rate data.

We compare two structural models against the random walk, and provide rationale for introducing our own structural model incorporating VIX to forecast exchange rates.

2.1 Random Walk

Meese and Rogoff (1983) established the difficulty of beating naive random walk model in forecasting exchange rates. Their research showed that structural models based on economic theories do not provide better forecasts than the random walk in terms of smaller forecast errors and higher accuracy, even where the forecasts are based on actual realized values of the variables.

In their paper, Meese and Rogoff studied different models by comparing forecasts at one, three, six and twelve-month horizons for major exchange rates. These forecasts were made using rolling regressions that re-estimated the coefficients (using OLS) for every forecast period using the most up-to-date information available at the time of a given forecast.

2.2 Taylor Rule Differentials

The Taylor Rule Fundamentals model is used by central banks in setting short-term nominal interest rates to achieve target levels of inflation and output. According to this model, the central bank will raise the short-term nominal interest rate if inflation and/or output is higher than target levels. Conversely, the central bank will lower the short-term nominal interest rate when enacting expansionary monetary policy to stimulate demand when inflation and/or output falls below the target levels. Following Taylor (1993), the model is specified as follows:

$$\bar{i}_t = \pi_t + \phi(\pi_t - \bar{\pi}) + \gamma y_t^g + \bar{r} \tag{1}$$

$$= (\bar{r} - \phi\bar{\pi}) + (1 - \phi)\pi_t) + \gamma y_t^g \tag{2}$$

$$= \mu + \lambda\pi_t + \gamma y_t^g \tag{3}$$

Where,
$\bar{i}_t$ = Target short-term nominal interest rate
π_t = Actual inflation rate
$\bar{\pi}$ = Target inflation rate
y_t^g = Output gap = Cyclical component of real GDP estimated as deviation from HP Trend of real GDP
$\bar{r}$ = Equilibrium real interest rate

The Taylor Rule Differentials model is an alternative to the Fundamentals model. The original proposal of this model in *Engel et al.* posited the coefficients for the Taylor rule as such:

$$i_t - i_t^* = \alpha + 1.5(\pi_t - \pi_t^*) + 0.5(y_t^g - y_t^g*) \tag{4}$$

Uncovered Interest rate Parity (UIP) is a no-arbitrage condition which states that expected returns on deposits in home and foreign countries must be equal when compared in home currency. UIP is expressed as $(1 + i_t) = (1 + i_t^*)\frac{e_{t+1}}{e_t}$. This means that if the interest rate in foreign country is higher, the exchange rate would adjust to make the two returns equal in terms of home currency. Thus, the foreign currency would be expected to depreciate relative to the home currency if the foreign country interest rate is higher.

Thus, from the Differentials model, the Taylor Rule exchange rate forecasting equation is written as:

$$\delta e_{t+1} = e_{t+1} - e_t = \omega + \omega_1(1.5(\pi_t - \pi_t^*) + 0.5(y_t^g - y_t^{g*})) + \varepsilon_t \tag{5}$$

Where e_{t+1} is the log of the U.S. dollar nominal exchange rate determined as the U.S. price of foreign currency.

For our study, we aim to estimate rather than posit the coefficients on inflation and output gap of each country. We use the following equation to estimate the Taylor Rule exchange rate model in this study:

$$\delta e_{t+1} = e_{t+1} - e_t = \beta_0 + \beta_1\pi_t + \beta_2\pi_t^* + \beta_3 y_t^g + \beta_4 y_t^{g*} + \varepsilon_t \tag{6}$$

2.3 Taylor's Rule + VIX

The second structural model uses Taylors Rule Model with the Volatility Index (VIX) as an additional explanatory variable in the forecast model:

$$e_{t+1} = \beta_0 + \beta_1\pi_t + \beta_2\pi_t^* + \beta_3 y_t^g + \beta_4 y_t^{g*} + VIX_t + \varepsilon_t \tag{7}$$

The VIX, which is constructed using the implied volatility of S&P 500 index options derived from price inputs of these options, represents the markets expectations of 30-day future volatility. Although the VIX is measured using the price of the put options on the S&P 500, it is widely recognized as a measure of global volatility. This is based on the fact that the volatility implied by options on various stock indices around the world appears to be highly correlated (Londono & Wilson. (2018)). As shown below, volatility indices similar to the VIX, but based on stock indices of other major developed countries such as Germany (VDAX), Japan (VXJ), the U.K (VFTSE) and Switzerland (VSMI). Historical VIX data was extracted from Bloomberg.

	Global volatility	U.S. (VIX)	Germany (VDAX)	Japan (VXJ)	U.K. (VFTSE)	Switzerland (VSMI)
Global volatility	1					
U.S.	0.98	1				
Germany	0.85	0.86	1			
Japan	0.8	0.82	0.73	1		
U.K.	0.93	0.95	0.94	0.79	1	
Switzerland	0.87	0.88	0.95	0.8	0.95	1

Figure 1: Correlation between Global Volatility Indices

In order to explore the relationship between such volatility indices and currency movements, we first consider the relationship between the VIX and the US Dollar. Lequeux and Menon (2010) showed that there exists a positive relationship between the VIX level and the volatility of the US dollar against G10 currencies.

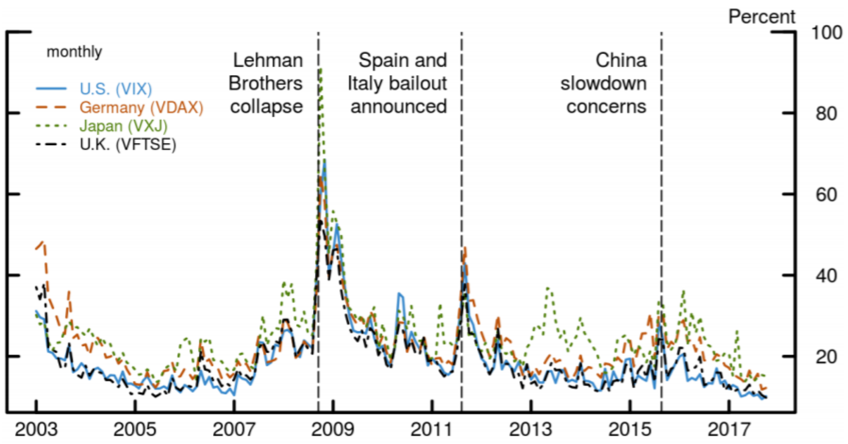


Figure 2: Correlation between VIX and Exchange Rate Movements US dollar against G10 currencies

Hence, we can reasonably conclude that movements of the VIX are likely to correspond to exchange rate fluctuations. In other words, expectations of high volatility (as indicated by the VIX) would lead to capital flight from emerging markets to developed markets, as risk off sentiment weighs on global financial markets.

Our intuition behind selecting the VIX to include as an additional explanatory variable is that we believe the Taylor's Rule does not fully capture the **short run** fluctuations in exchange rate. The idea of VIX tracking exchange rate movements was first suggested by De et al., who write about "risk-off episodes", which refer to time periods where there is an increase in global uncertainty. Increasing uncertainty creates an increase in demand and thus flow towards "safe-haven" assets, of which include currency pairs such as USDJPY.

The 2015 paper empirically determines the VIX index as a trigger point in defining "risk-off episodes", and tracks immediate currency fluctuations up to 12 weeks after it. Our study differs from De et al.'s study in that we are incorporating the VIX index as a structural measure combined with Taylor's rule to capture the immediate movements in exchange rates which Taylor's rule alone does not.

3 Data

This project employed various macroeconomic variables in the Taylor Rule model. The highest frequency available for the data is monthly. We used point sample data rather than averaged data in order to avoid introducing correlations not present in the original series. Following Working (1960), even if daily exchange rates are serially uncorrelated and followed a random walk, a series of monthly averages of daily rates will exhibit positive serial correlation.

We also used seasonally unadjusted data to ensure that the coefficients of variables are estimated on a consistent basis. This is because the use of data that has been seasonally adjusted using different methods is likely to distort the estimates.

Historical output data for the U.S, Japan and Germany was extracted from the IMF's International Financial Statistics (IFS) Database. In particular, for US output data, we used the IFS database rather than FRED for consistency (i.e. all output data was sourced from the same database). Monthly time series data for the VIX was extracted from Yahoo.com, while data for variables such as the exchange rates, interest rates, and inflation was extracted from the FRED database. The relevant series extracted from FRED are shown in the table below.

Time Series Variables from FRED				
No.	FRED Code	Name	Frequency	Seasonal Adjustment
1	CPIAUCNS	Consumer Price Index for All Urban Consumers: All Items in US City Average	Monthly	Not Seasonally Adjusted
2	FEDFUNDS	Effective Federal Funds Rate	Monthly	Not Seasonally Adjusted
3	EXJPUS	Japan / U.S Foreign Exchange Rate	Monthly	Not Seasonally Adjusted
4	JPNCPIALLMINMEI	Consumer Price Index of All Items in Japan	Monthly	Not Seasonally Adjusted
5	IRSTCI01JPM156N	Interbank Rate for Japan	Monthly	Not Seasonally Adjusted
6	EXUSEU	U.S / Euro Foreign Exchange Rate	Monthly	Not Seasonally Adjusted
7	DEUCPIALLMINMEI	Consumer Price Index of All Items in Germany	Monthly	Not Seasonally Adjusted
8	IRSTCI01DEM156N	Interbank Rate for Germany	Monthly	Not Seasonally Adjusted
9	CP0000EZ19M086NEST	Harmonized Index of Consumer Prices: All Items for Euro Area	Monthly	Not Seasonally Adjusted
10	IRSTCI01EZM156N	Interbank Rate for Euro Area	Monthly	Not Seasonally Adjusted

3.1 Constructing the Datasets and Models

The output gap can be defined as the deviation of actual real GDP from an estimate of its potential level. As there is no presumed definition of potential output used by central banks in their interest rate reaction functions, we estimated potential output of a country using its industrial production index measure as a proxy, as in Molodtsova & Papell (2009). Following Molodtsova & Papell, we applied the Hodrick-Prescott (HP) Filter to the time series data for industrial production for Germany (which was used a proxy to represent economies in the European Union that use the Euro), Japan, and the U.S.

4 Methodology

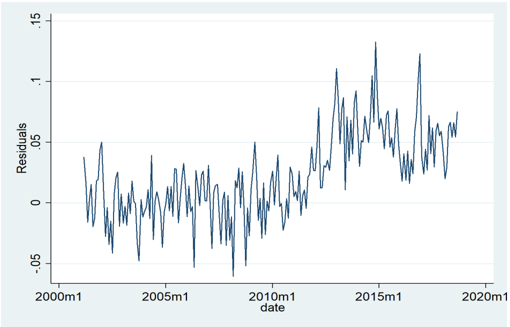
4.1 Errors: Checking for Serial Autocorrelation

Before moving forward in our analysis, we checked the errors of all potential models for comparison: Taylor's Rule, Taylor's Rule & increasing number of lags of VIX (up to 3 lags). We visually examined the time series for each model's residuals and the autocorrelation function as well as conducted the Ljung-Box Q test. These measures will determine whether each of the potential models have errors that are serially correlated or not.

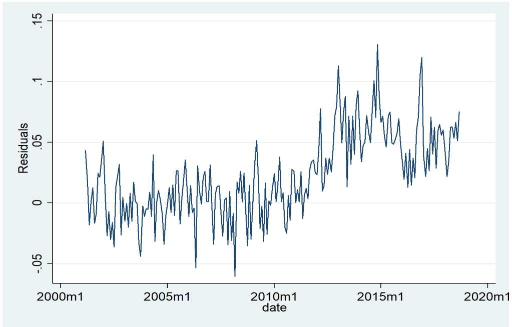
Henceforth, reference to JPYUSD and EURUSD within the potential models refer to δe_t, i.e. the difference between the log of the currency from one period to the next.

4.1.1 JPYUSD

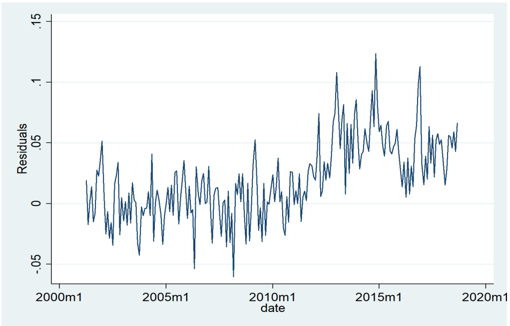
For JPYUSD, the residual plots and the residual autocorrelation functions for all potential models are as follows, where TR refers to Taylor's rule:



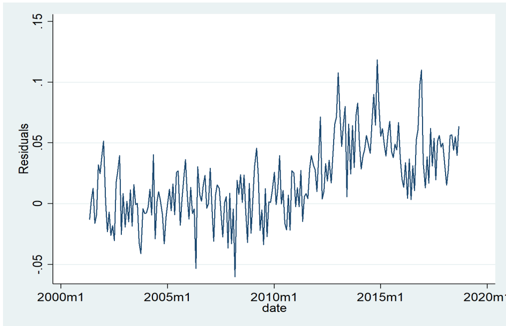
(a) TR



(b) TR + 1 lag of VIX



(c) TR + 2 lags of VIX



(d) TR + 3 lags of VIX

Figure 3: Residual plots for all potential models for JPYUSD

From Figure 3, the residual plots shows some persistence in all potential models. The Auto Correlation Function (ACF) plots (not shown) also show that the autocorrelation of errors are significantly different from zero with a slow decline. Both of these indicate that all the models have errors that are serially correlated. To confirm this, we used the Ljung-Box Q test and examine the Q-statistic at the 13th lag for all the models. The results are as follows:

Model	Q-statistic on errors	White noise - Y/N?
Taylor's rule	0.000	N
Taylor's rule + 1 lag of VIX	0.000	N
Taylor's rule + 2 lags of VIX	0.000	N
Taylor's rule + 3 lags of VIX	0.000	N

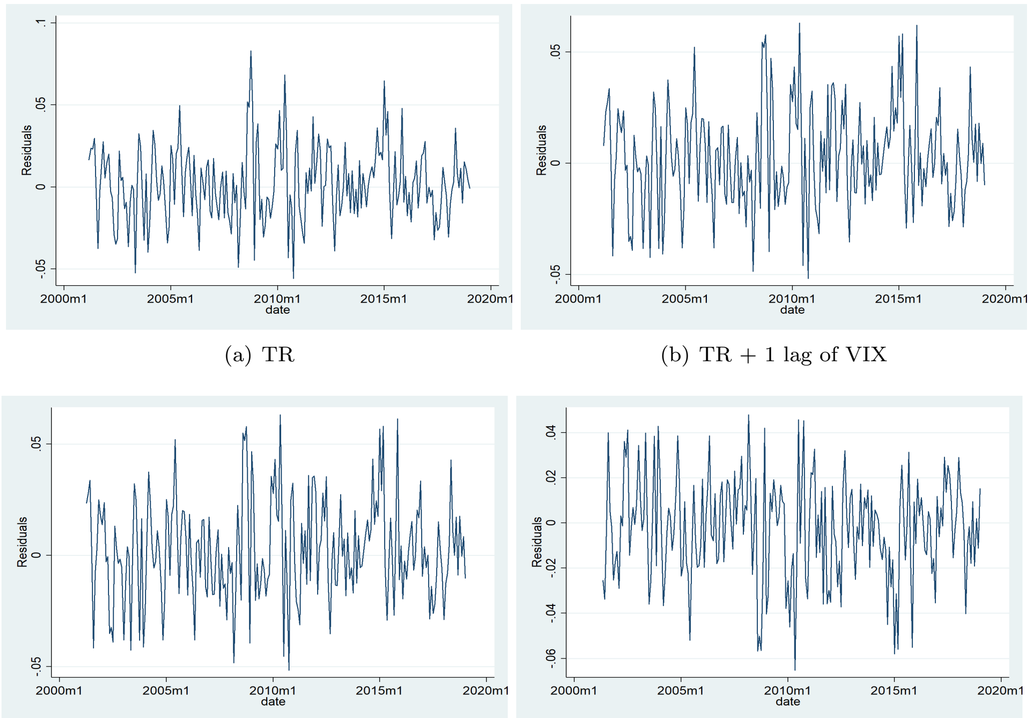
Table 1: Ljung-Box Q test results for JPYUSD

As results from the Ljung-Box test show that Q-stat are 0 up to 3 decimal places for relevant lags of errors in all models, we must reject the null hypothesis that the residuals are serially uncorrelated white noise. Thus, all testing on the JPYUSD models takes into account serially correlated errors and uses HAC errors.

Thus, all testing on the JPYUSD models will be taking into account serially correlated errors.

4.1.2 EURUSD

For EURUSD, the residual plots and the residual autocorrelation functions for all potential models are as follows:



(a) TR

(b) TR + 1 lag of VIX

(c) TR + 2 lags of VIX

(d) TR + 3 lags of VIX

Figure 4: Residual plots for all potential models for EURUSD

Visually, the plot of the residuals from Figure 4 seems to follow white noise distribution, thus indicating white noise errors. A further examination of the ACF plots of these models (not shown) also shows that the autocorrelation of errors are mostly not significantly different from zero as most estimations fall within the Bartlett Bands, with only a few lags of errors showing autocorrelation that is significantly different from zero. However, contrary to these, the results of the Ljung-Box Q test indicates that the residuals of the 4 models in question are serially correlated. The Q test results are shown in Table 2.

As results from the Ljung-Box test show that Q-stat < 0.05 for relevant lags of errors in all models, we must reject the null hypothesis that the residuals are serially uncorrelated white noise. Thus, all testing on the EURUSD models must take into account serially correlated errors and use HAC errors.

Model	Q-statistic on errors	White noise - Y/N?
Taylor's rule	0.001	N
Taylor's rule + 1 lag of VIX	0.017	N
Taylor's rule + 2 lags of VIX	0.018	N
Taylor's rule + 3 lags of VIX	0.033	N

Table 2: Ljung-Box Q test results for EURUSD

4.2 Selection Based on Granger Causality

We tested the joint significance of variables in each model using F test to establish Granger Causality of the variables on exchange rate. To test the null hypothesis that the variables do not Granger-cause exchange rate, we first regressed exchange rate on all lags of variables used in each model, using HAC Newey-West errors to account for the serial correlation between residuals. If the F-statistic for all lags of X jointly exceeds the critical value, this means that the lags of the variable are jointly significant, and so we can reject the null hypothesis of no Granger Causality between the variable and exchange rate.

It is also worthwhile to mention that while each component of the Taylor's rule Fundamentals model can be individually tested for Granger Causality, it is more intuitive to measure the 4 variables (inflation of both domestic and foreign countries, and the output gap of both the domestic and foreign countries) jointly using the F-test to establish Granger Causality on JPYUSD and EURUSD.

4.2.1 Granger Causality tests on JPYUSD models

The results of the Granger Causality tests are as follows:

Model	F-statistic	Granger Causes - Y/N?
Taylor's rule	3.150	Y**
1 lag of VIX	5.090	Y**
2 lags of VIX	2.810	Y*
3 lags of VIX	1.910	N
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 3: Granger Causality test results for JPYUSD models

Based on the results from Table 3, the variables that Granger-cause movement in JPYUSD are: joint components of Taylor's rule, 1 lag of VIX and 2 lags of VIX. Also, we adopted a lower confidence level for the Granger Causality test, as we note that 2 lags of VIX jointly will Granger cause movements in JPYUSD only at the 10% significance level.

4.2.2 Granger Causality tests on EURUSD models

The results of the Granger Causality tests are as follows:

Model	F-statistic	Granger Causes - Y/N?
Taylor's rule	0.320	N
1 lag of VIX	3.730	Y*
2 lags of VIX	2.880	Y*
3 lags of VIX	1.920	N
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 4: Granger Causality test results for EURUSD models

Based on the results from Table 4, the variables that Granger cause movement in EURUSD are: 1 lag of VIX and 2 lags of VIX. Following the approach adopted for the Granger Causality tests on potential JPYUSD models, we decided on a lower confidence level for the Granger Causality test. We note that both 1 lag and 2 lags of VIX will Granger cause movements in JPYUSD only at the 10% significance level.

4.3 Selection Based on Predictive Least Squares

We selected the optimal number of lags in each model based on pseudo out-of-sample (POOS) forecast errors using a Predictive Least Squares (PLS) approach. The PLS approach allows us to estimate out-of-sample forecast accuracy directly using POOS forecast errors, as opposed to estimating out-of-sample mean squared forecast errors by adjusting the in-sample mean squared error with a degrees of freedom penalty as is done in calculating AIC and BIC. As such, PLS can be compared across models with different dependent variables (e.g. Y or log(Y)), unlike AIC and BIC which can only be compared across models with the same dependent variable.

Our dataset for JPYUSD [1] starts from 2001m1 and has a total of 212 observations. We used data start-

[1] We used a slightly different time period for EURUSD considering the data available to us, the procedure followed is identical

ing from 2001m3 as we needed 2 lags of VIX and ensured that all 3 ADL models were estimated using the same number of observations. For the first rolling window we used data from 2001m3 to 2009m10 (104 observations) to estimate the parameters of the models and make the first direct POOS forecasts for 2009m11. Then we discarded the first observation (2001m3) and added in the last observed value (2009m11) to make the next direct forecast for 2009m12. Using this rolling window procedure until 2014m4 (end of our holdout sample), with 104 observations each time, we produced a series of 53 direct forecasts between 2009m11 and 2014m4. Comparing the forecasted values with their actual values, we calculated the pseudo out-of-sample forecast errors and the PLS criterion values for each model.

Currency Pair	random walk	Taylor's Rule	1 lag of VIX	2 lags of VIX
JPYUSD	1.948	2.103	2.081	2.042
EURUSD	0.01727	0.01792	0.01766	0.01772

Table 5: PLS Criterion Values for JPYUSD and EURUSD

For JPYUSD, the PLS values choose random walk as the best model, followed closely by Taylor's Rule with 2 lags of VIX. We will use Taylor's Rule with 2 lags of VIX for the subsequent forecasts for JPYUSD. For EURUSD, the criterion picks random walk as the best model as well, and Taylor's Rule with 1 lag of Vix as a close second. We will use Taylor's Rule with 1 lag of VIX as the model for the OOS forecasts for EURUSD.

5 Forecast Combination

Using the POOS forecasts produced in the previous section, we used the Granger-Ramanathan Regression method to determine the weights to be assigned to forecasts from each model in the combined model. We used this method of forecast combination as the regression takes into account correlations among forecasts, which are likely to be present given that the two structural models used both include variables from Taylor's Rule. Secondly, introducing a constant into the regression also takes into account that the forecasts generated by the structural models may not be free of bias. Lastly, this regression method allows us to combine models with different dependent variable. We accounted for the difference in the forecasted variable (log differential vs exchange rate) by transforming the differentials back to actual values.

First, we regressed the actual value of exchange rate on the forecasts from each model using a constrained regression. The coefficients of the forecasts were obtained using OLS, and any forecast with negative coefficients were dropped from the model. The regression is repeated until the remaining forecasts all have positive coefficients, and these coefficients represent the weight to be assigned to each remaining model when they are combined.

5.1 Granger-Ramanathan Forecast Combination Results for JPYUSD

The initial results from the constrained regression on the JPYUSD forecasts are shown below.

Constrained linear regression   Number of obs = 53
Root MSE = 1.9076

(1) f_rw_jpy + trhat_exrate + vix1hat_exrate + vix2hat_exrate = 1

jpy_exrate~a	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
f_rw_jpy	.6683863	.2320901	2.88	0.006	.2019838	1.134789
trhat_exrate	-.5854611	.6788259	-0.86	0.393	-1.949613	.7786905
vix1hat_ex~e	-2.149703	1.639661	-1.31	0.196	-5.444725	1.145319
vix2hat_ex~e	3.066778	1.666938	1.84	0.072	-.2830586	6.416615
_cons	.1677078	.2650085	0.63	0.530	-.3648468	.7002623

Figure 5: Granger-Ramanathan Forecast Regression Results for JPYUSD

Based on the results we dropped the Taylor's Rule and Taylor's Rule with 1 lag of VIX forecasts.

Constrained linear regression   Number of obs = 53
Root MSE = 1.9252

(1) f_rw_jpy + vix2hat_exrate = 1

jpy_exrate~a	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
f_rw_jpy	.6219672	.226671	2.74	0.008	.1669062	1.077028
vix2hat_ex~e	.3780328	.226671	1.67	0.101	-.0770281	.8330938
_cons	.1847676	.2672215	0.69	0.492	-.3517019	.7212372

Figure 6: Granger Ramanathan Forecast Regression Results for JPYUSD

Finally we obtained the ideal weights on the random walk and Taylor's Rule with 2 lags of VIX forecasts as 0.62 and 0.38 respectively. This aligns with our analysis using the PLS criterion which selected random walk as the best model, followed by Taylor's Rule with 2 lags of VIX as the second best.

5.2 Granger-Ramanathan Forecast Combination Results for EURUSD

The initial results from the constrained regression on the EURUSD forecasts are shown below.

Since the coefficients on Taylor's Rule and Taylor's Rule with 2 lags of VIX were negative, we dropped these models and regressed again.

Based on the results, the ideal forecast combination for EURUSD includes a weighted sum of random walk and Taylor's Rule with 1 lag of VIX.

```
Constrained linear regression                   Number of obs     =         53
                                                Root MSE          =     0.0169

 ( 1)  f_rw_eur + trhat_exrate + vix1hat_exrate + vix2hat_exrate = 1
```

eur_exrate~a	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
f_rw_eur	3.039454	1.148039	2.65	0.011	.7323824	5.346526
trhat_exrate	-2.80979	1.263891	-2.22	0.031	-5.349673	-.2699068
vix1hat_ex~e	2.772753	3.150787	0.88	0.383	-3.558991	9.104497
vix2hat_ex~e	-2.002417	3.014037	-0.66	0.510	-8.059351	4.054517
_cons	.0037515	.0028185	1.33	0.189	-.0019126	.0094156

Figure 7: Granger-Ramanathan Forecast Regression Results for EURUSD

```
Constrained linear regression                   Number of obs     =         53
                                                Root MSE          =     0.0174

 ( 1)  f_rw_eur + vix1hat_exrate = 1
```

eur_exrate~a	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
f_rw_eur	.7350253	.4881524	1.51	0.138	-.2449815	1.715032
vix1hat_ex~e	.2649747	.4881524	0.54	0.590	-.7150321	1.244981
_cons	.000373	.002451	0.15	0.880	-.0045476	.0052936

Figure 8: Granger-Ramanathan Forecast Regression Results for EURUSD

6 Out of Sample Forecast Performance

As mentioned above, we used data from 2001m1 to 2014m4 as our in-sample observations, and data from 2009m11 to 2014m4 was used in POOS forecasting. After choosing our forecasting model using the above-mentioned model selection methods and the selected sample of observations, we generated out-of-sample (OOS) forecasts from 2014m5 to 2018m8 to evaluate the performance of our chosen forecasting model.

For JPYUSD we chose the Taylor's Rule combined with 2 lags of VIX as it was only marginally beaten by random walk. For EURUSD, we chose Taylor's Rule with 1 lag of VIX. For both we used a rolling estimation window to generate 1-month ahead, 6-month ahead and 12-month ahead forecasts of the log differential of the exchange rate. We chose the rolling estimation window to ensure that parameters were estimated on most recent and relevant data. This first rolling window consisted of all available data prior to 2014m4 to make a direct forecasts for 2014m5 (1-month ahead), 2014m10 (6-month ahead) and 2015m4 (12-month ahead). We then proceeded in a similar manner of deleting and adding observations to make the other direct forecasts until 2018m8. We stopped at 2018m8 due to unavailability of credible output gap data for Japan beyond this date.

6.1 Direct OOS forecast results for JPYUSD log differential

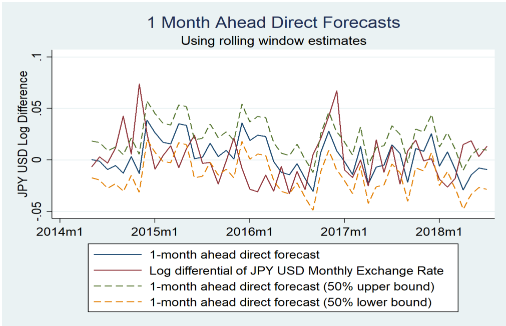
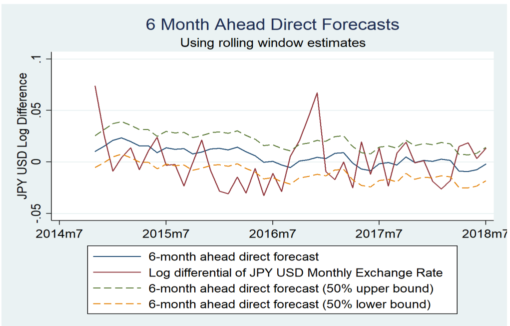
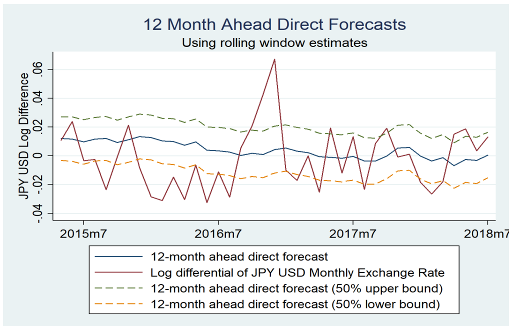


Figure 9: 1-month ahead direct forecasts of JPYUSD log differential



(a) 6-month ahead



(b) 12-month ahead

Figure 10: 6 and 12-month ahead direct forecasts of JPYUSD log differential

6.2 Direct OOS forecast results for EURUSD log differential

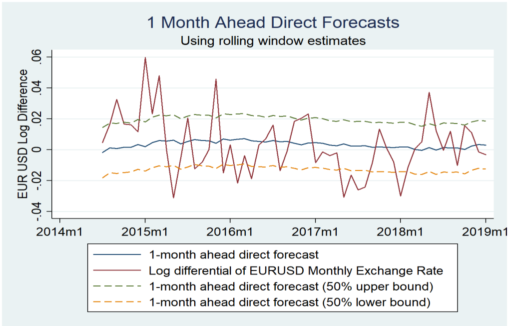
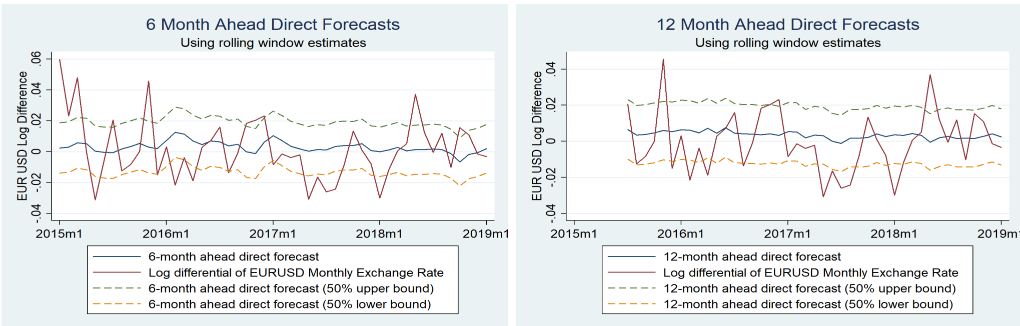


Figure 11: 1-month ahead direct forecasts of EURUSD log differential



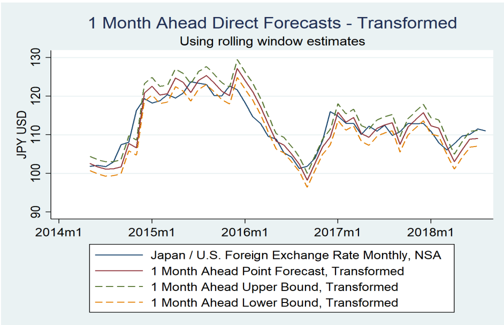
(a) 6-month ahead (b) 12-month ahead

Figure 12: 6 and 12-month ahead direct forecasts of EURUSD log differential

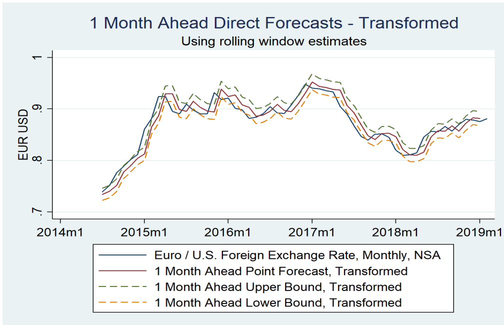
6.3 Transformation of log differential forecasts to exchange rate values

Since our model predicted the one period change in the log of the exchange rate, we transformed the dependent variable back to actual exchange rate values by using the observed exchanged rate in the previous period. This makes sense for the 1 month ahead direct forecast as the last period value would be available at the time of forecasting and will allow for such a transformation. The results of the transformation of the 1-month ahead direct forecasts of JPYUSD and EURUSD are shown below in Figure 13.

We also performed this transformation on the 6-month and 12-month ahead forecasts by assuming we are evaluating the performance of our forecasts (generated 6 months or 12 months back) in the relevant period, with available observed values of the exchange rates in the previous month. These results are seen in Figure 14.

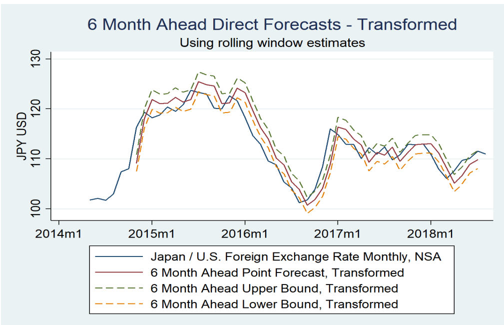


(a) JPYUSD

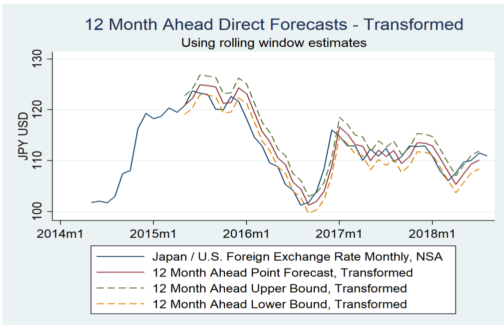


(b) EURUSD

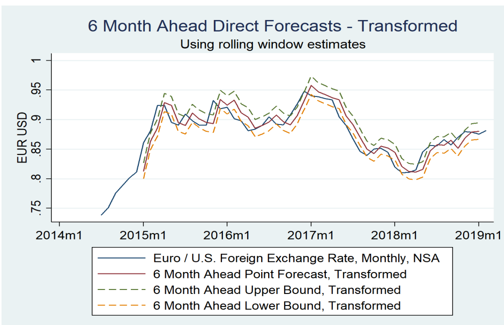
Figure 13: 1-month ahead direct forecasts of JPYUSD and EURUSD transformed



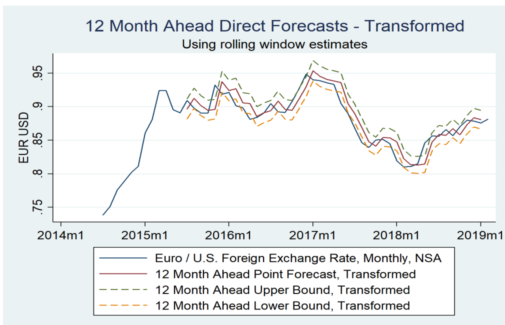
(a) 6-month ahead



(b) 12-month ahead



(c) 6-month ahead



(d) 12-month ahead

Figure 14: 6 and 12-month ahead direct forecasts of JPYUSD and EURUSD transformed

7 Forecast Evaluation

In this section, we evaluate our chosen model for each currency against the random walk model for exchange rates. We compare the two forecasts based on (i) mean squared error, for forecast precision, and (ii) the Diebold-Mariano test, for forecast accuracy.

7.1 Mean Squared Error

We evaluated forecast precision using the mean square error (MSE), implicitly assuming that our forecasts follow a squared loss function. Although mean error (ME) and mean absolute error (MAE) are also measures of forecast accuracy, they are incomplete measures because they do not provide any indication as to the error variance. Using these measures to evaluate forecasts could lead us to pick a model with a small mean error but a large error variance. This can be circumvented by using MSE, which measures overall forecast accuracy by incorporating both mean error and the error variance. We transformed the MSE of structural from log to linear errors in order to compare the MSE of different models in the same units.

As discussed in Section 6.3, we decided to use the forecasts generated from our models, i.e. $\Delta\hat{e}_t$, to derive a forecast for the exchange rate, $\hat{s}_{t+1}$. This is to ensure a common basis of comparison between the random walk model and the model we have chosen. As such, we calculate squared loss, and thus the MSE, also based on these derived forecasts of exchange rates, $\hat{s}_{t+1}$. The forecast horizon for comparison of the models was chosen to be 1-month ahead direct forecasts, as we felt that the 6-month ahead and 12-month ahead direct forecasts were bounded by more assumptions than that of 1-month ahead forecasts. More specifically, beyond the 1-month direct forecasts, increasing uncertainty on the components of the chosen model for both JPYUSD and EURUSD would mean the values determined from these forecasts are less reliable than that of the 1-month ahead direct forecasts.

7.1.1 MSE of JPYUSD models

The model chosen based on PLS was the random walk model, followed closely by the model incorporating Taylor's rule and 2 lags of VIX. Thus, MSE will be compared against the two models on the 1-month ahead forecasts. The MSE of the random walk model was found to be 6.566 and that of the chosen model at 10.300. Since the random walk model has a lower MSE, it would mean the random walk is a more precise forecast for JPYUSD as compared to the model with Taylor's rule and 2 lags of VIX.

7.1.2 MSE of EURUSD models

For EURUSD, the same calculations were done based on the random walk model as well as the Taylor's rule and 1 lag of VIX model. These two models were that which minimized PLS. The MSE of the random walk model is .0002903 while that of the chosen model is .0003060. It seems that the random walk is marginally more accurate than the chosen model to predict EURUSD, i.e. the model with Taylor's rule and 1 lag of VIX.

7.2 Diebold-Mariano Test

The Diebold-Mariano (DM) test is used to test if the differences in MSE of the models are significant enough to conclude that one model is superior. After regressing the differential on a constant, the T test is used to test the hypothesis that the expected loss differential is zero. Using the Ljung-Box Q Test to check if the loss differentials are serially correlated, HAC errors are used if the differentials are serially correlated and robust errors are used if the differentials are serially uncorrelated.

Our setup for the DM test is as follows: we used the random walk model as the first model and the chosen model as the second model. The differential is calculated as the difference between the squared loss of the random walk model and that of the chosen model. This means that after regressing the expected loss differential on a constant, a negative differential coefficient concludes the random walk model as the more accurate model. The result is subject to a t-test rejecting the null hypothesis that the differential is zero, as mentioned above.

7.2.1 DM Test on JPYUSD models

First, we determined whether the loss differentials of the 1-month ahead forecast are serially correlated, using the Ljung-Box Q test at the 7th lag. The Q-statistic was found to be 6.708 with a probability of 45.99%, which means we cannot reject the null that the autocorrelation of loss differentials are equal to zero. We move on to conduct the DM test with robust errors, which yielded a differential coefficient of -3.736 and p-value of 1.2%. The low p-value indicates we can reject the null hypothesis that the differential is zero, and can conclusively say that the random walk forecast is the better forecast compared to our chosen model for predicting JPYUSD.

Since the POOS window size is small (less than 100), we also made the small sample correction following the formula proposed by Harvey, Leybourne and Newbold (1998). The t-statistic of the differential increased significantly to -2.584, but the conclusion remains the same as the p-value remained low at 1.27%. After correcting for the small POOS window size, we can still conclusively determine that the random walk model is the better model at predicting movements of JPYUSD.

7.2.2 DM Test on EURUSD models

Similar to the section above, we conduct the Ljung-Box Q test on the differentials to determine whether there is serial autocorrelation. The Q-statistic was 7.0484 with a probability of 42.39%, thus we cannot reject the null that the autocorrelation of loss differentials are equal to zero.. We again conduct the DM test with robust errors, where the differential coefficient was found to be -0.0000157, i.e. ≈ 0. This differential was found to be not significantly different from zero with a p-value of 36.3%.

With a similar POOS window size as JPYUSD, the correction was also made for the DM test for the EURUSD model comparison. The correction caused the t-statistic of the differential to be more negative, at -0.912, however the conclusion remained the same - we still do not reject the null hypothesis that the differential is zero due to a high p-value of 58.1%. This means that the random walk model and our chosen model which incorporates Taylor's rule with 1 lag of VIX are not significantly different from one another at predicting movements of EURUSD.

8 Limitations

8.1 Limitations regarding Dataset

In our study, we used data from Germany to estimate the Taylor Rule exchange rate model. As the largest economy in Europe, the movement of variables in Germany has the largest effect on the Euro, and so Germany is the best available proxy in studying the movements of EURUSD. However, as the official currency of 19 member states in the European Union, the Euro is not only affected by the variables in Germany. Ideally, we should use data aggregated from all 19 states that share the Euro, weighted either by the relative GDP or relative foreign transaction volume of each state. However, even if we were to do so, the method of aggregation could also lead to inconsistencies in the data as there is no formal aggregation readily available to us from a credible source.

Additionally, the forecasting ability of the VIX could be studied more extensively by analyzing the relationship between the VIX and exchange rate movements of more currency pairs. It may be possible that certain currency pairs have stronger relationship to the VIX than others.

Lastly, as the currencies studied in this paper float freely against one another, the exchange rates are in constant fluctuation. Ideally, we should use data of higher frequency so as to study more of the movement in exchange rates. However, our study was limited by the availability of data as the highest frequency available for macroeconomic indicators is monthly.

8.2 Limitations regarding Methodology

Limitations in using the Granger Causality test: The Granger Causality test is performed on in-sample observations. Thus, even if Granger Causality is established for in-sample observations, this causality may not hold in OOS observations.

Limitations of PLS: The disadvantages of using PLS in model selection is that it tends to overestimate the true mean square forecast error (MSFE) and tends to be over-parsimonious. Moreover, the result of PLS is highly sensitive to the number of observations used in estimating the model and number of observations used to test POOS forecasts.

9 Conclusion

As discussed in the paper, this report investigates the idea of using VIX as an additional component of exchange rate analysis. Using JPYUSD and EURUSD as our case studies, we determined that incorporating VIX combined with Taylor's rule within an exchange rate forecasting model does beat the standard fundamentals-based forecasting model on Taylor's rule alone, based on MSE of forecasts. However, when compared to the workhorse benchmark of exchange rate forecasting, the random walk model, JPYUSD is conclusively better forecasted with the random walk model as compared to our chosen model to forecast exchange rates.

While we acknowledge that the random walk is the benchmark model when it comes to forecasting exchange rates, it is worthwhile to note our results in this report. More specifically, for EURUSD, when comparing the random walk model with our chosen model (Taylor's rule with 1 lag of VIX), the DM test could not conclusively determine one model to be significantly better than the other. This highlights a potential area for further study - incorporating VIX into a forecasting model could beat the random walk model with other "safe-haven" currency pairs such as USDCHF and AUDJPY. Given more time and resources, we would have explored incorporating VIX into current exchange rate forecasting models on more currency pairs. Additionally, we would more comprehensively compare our models with the different measures of Taylor's rule to generate a model that would consistently beat the random walk benchmark model for exchange rate forecasting.

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