TIME COMPLEXITY (MASTER'S THEOREM)

The Master Theorem applies to recurrences of the following form:

$$T(n) = aT(n/b) + f(n)$$

where $a \geq 1$ and $b > 1$ are constants and $f(n)$ is an asymptotically positive function.

There are 3 cases:

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a} \log^k n)$ with[1] $k \geq 0$, then $T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$.
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ with $\epsilon > 0$, and $f(n)$ satisfies the regularity condition, then $T(n) = \Theta(f(n))$.
   Regularity condition: $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n.

a) T (n) = 3T (n/2) + n

Answer: T (n) = Θ (n^lg 3) (Case 1)

b) T (n) = 64T (n/8) − n ^2 (log n)

Answer: T (n) = Does not apply (f(n) is not positive)

c) T (n) = 2nT (n/2) + n^n

Answer: T (n) = Does not apply (a is not constant)

d) T (n) = 3T (n/3) + n/2

Answer: T (n) = Θ (n log n) (Case 2)

e) T (n) = 7T (n/3) + n^2

Answer: T (n) = Θ (n^2) (Case 3)