Real Numbers

Ex 1.1, 1

Express each number as a product of its prime factors:

(i) 140

2	**140**
2	70
5	35
7	7
	1

Hence,

$$140 = 2 \times 2 \times 5 \times 7$$

$$= 2^2 \times 5 \times 7$$

Express each number as a product of its prime factors:

(ii) 156

2	**156**
2	78
3	39
13	13
	1

Hence,

$\mathbf{156} = 2 \times 2 \times 3 \times 13$

$\mathbf{= 2^2 \times 3 \times 13}$

Express each number as a product of its prime factors:

(iii) 3825

3	**3825**
3	1275
5	425
5	85
17	17
	1

Hence,

$$\mathbf{3825} = 3 \times 3 \times 5 \times 5 \times 17$$

$$\mathbf{= 3^2 \times 5^2 \times 17}$$

Express each number as a product of its prime factors:

(iv) 5005

5	**5005**
7	1001
11	143
13	13
	1

Hence,

$$\mathbf{5005 = 5 \times 7 \times 11 \times 13}$$

Express each number as a product of its prime factors:

(v) 7429

$$
\begin{array}{r|l}
17 & \mathbf{7429} \\ \hline
19 & 437 \\ \hline
23 & 23 \\ \hline
 & 1
\end{array}
$$

Hence,

$$\mathbf{7429 = 17 \times 19 \times 23}$$

Ex 1.1, 2

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91

Finding HCF

$$\begin{array}{r|l} 2 & \mathbf{26} \\ \hline 13 & 13 \\ \hline & 1 \end{array} \qquad\qquad \begin{array}{r|l} 7 & \mathbf{91} \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$26 = 2 \times \boxed{13}$$

$$91 = 7 \times \boxed{13}$$

∴ H.C.F = 13

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(ii) 510 and 92

Finding HCF

$$\begin{array}{r|l} 2 & \mathbf{510} \\ \hline 3 & 255 \\ \hline 5 & 85 \\ \hline 17 & 17 \\ \hline & 1 \end{array} \qquad\qquad \begin{array}{r|l} 2 & \mathbf{92} \\ \hline 2 & 46 \\ \hline 23 & 23 \\ \hline & 1 \end{array}$$

$$510 = \boxed{2} \times 3 \times 5 \times 17$$

$$92 = \boxed{2} \times 2 \times 23$$

∴ **H.C.F = 2**

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(iii) 336 and 54

Finding HCF

$$\begin{array}{c|l} 2 & \mathbf{336} \\ \hline 2 & 168 \\ \hline 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{c|l} 2 & \mathbf{54} \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$336 = \boxed{2} \times 2 \times 2 \times 2 \times \boxed{3} \times 7$$

$$54 = \boxed{2} \times 3 \times 3 \times \boxed{3}$$

∴ **H.C.F** $= 2 \times 3$

= 6

Ex 1.1, 3

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Finding HCF

2	**12**
2	6
3	3
	1

3	**15**
5	5
	1

3	**21**
7	7
	1

$12 = 2 \times 2 \times \boxed{3}$

$15 = \boxed{3} \times 5$

$21 = \boxed{3} \times 7$

∴ H.C.F = 3

Ex 1.2, 1

Prove that $\sqrt{5}$ is irrational.

We have to prove $\sqrt{5}$ is irrational

Let us **assume the opposite**,

i.e., $\sqrt{5}$ is rational

Hence, $\sqrt{5}$ can be written in the form $\frac{a}{b}$

where a and b (b≠ 0) are ***co-prime*** (no common factor other than 1)

Hence,

$$\sqrt{5} = \frac{a}{b}$$

$$\sqrt{5}\,b = a$$

Squaring both sides

$$(\sqrt{5}b)^2 = a^2$$

Prove that $3 + 2\sqrt{5}$ is irrational.

We have to prove $3 + 2\sqrt{5}$ is irrational

Let us assume the **opposite**,

i.e., **$3 + 2\sqrt{5}$ is rational**

Hence, $3 + 2\sqrt{5}$ can be written in the form $\frac{a}{b}$

where a and b ($b \neq 0$) are ***co-prime*** (no common factor other than 1)

Hence,

$$\mathbf{3 + 2\sqrt{5} = \frac{a}{b}}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$2\sqrt{5} = \frac{a - 3b}{b}$$

Prove that the following are irrationals :

(i) $\frac{1}{\sqrt{2}}$

We have to prove $\frac{1}{\sqrt{2}}$ is irrational

Let us assume the **opposite**,

i.e., $\frac{1}{\sqrt{2}}$ **is rational**

Hence, $\frac{1}{\sqrt{2}}$ can be written in the form $\frac{a}{b}$

where a and b (b≠ 0) are ***co-prime*** (no common factor other than 1)

Hence,

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\frac{b}{a} = \sqrt{2}$$

(ii) $7\sqrt{5}$

We have to prove $7\sqrt{5}$ is irrational

Let us assume the **opposite,**

i.e., **$7\sqrt{5}$ is rational**

Hence, $7\sqrt{5}$ can be written in the form $\frac{a}{b}$

where a and b (b≠ 0) are ***co-prime*** (no common factor other than 1)

Hence,

$$\mathbf{7\sqrt{5} = \frac{a}{b}}$$

$$\sqrt{5} = \frac{1}{7} \times \frac{a}{b}$$

(iii) $6 + \sqrt{2}$

We have to prove $6 + \sqrt{2}$ is irrational

Let us assume the **opposite,**

i.e., **$6 + \sqrt{2}$ is rational**

Hence, $6 + \sqrt{2}$ can be written in the form $\frac{a}{b}$

where a and b (b≠ 0) are ***co-prime*** (no common factor other than 1)

Hence,

$$\boldsymbol{6 + \sqrt{2} = \frac{a}{b}}$$

$$\sqrt{2} = \frac{a}{b} - 6$$