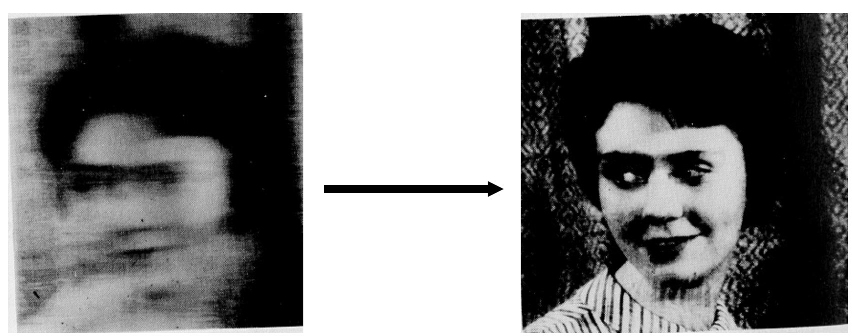
Image Restoration

Credits/References: Gonzalez book on Digital Image Processing, Prof. Bebis CS474 notes

Image Restoration

- Image Enhancement
  - A <u>subjective</u> process using mostly heuristics.
- Image Restoration
  - An <u>objective</u> process assuming a-priori knowledge of the degradation process.

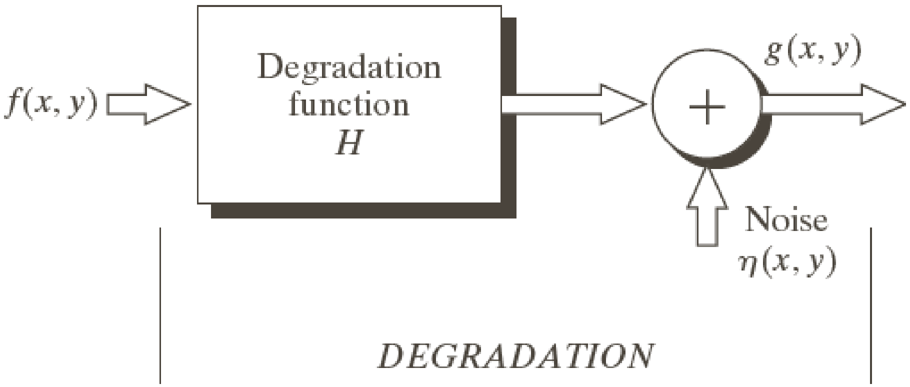
blur due to motion



Modeling Image Degradation

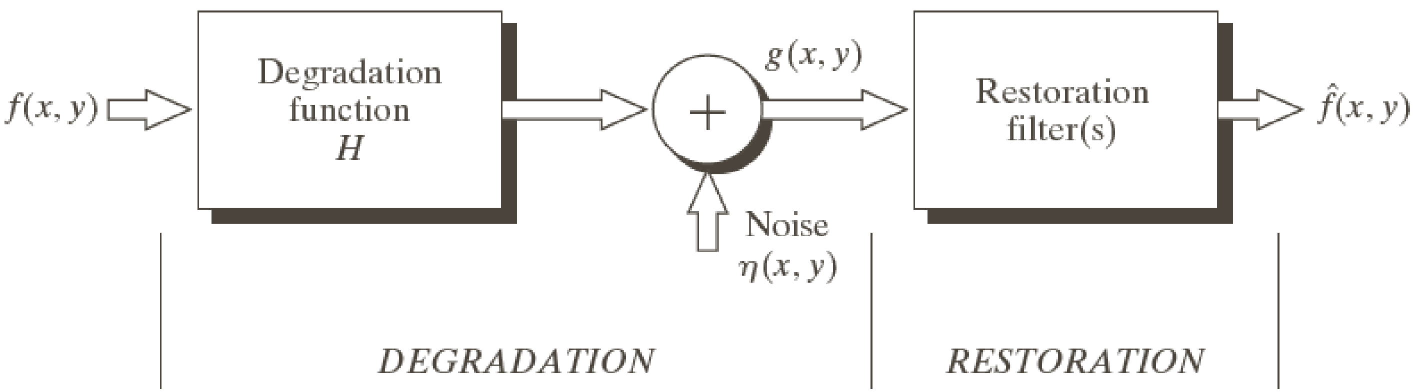
- Degradation process can be modeled through a degradation function **H** and additive noise **η(x,y).**

$$g(x, y) = H[f(x, y)] + n(x, y)$$



Goal of Image Restoration

- Given some knowledge of **H** and noise **η(x,y),** the objective of image restoration is to obtain an estimate of the original image.



Performance Characterization

- How do we characterize the performance of different image restoration algorithms?

Mean Square Error (MSE)

$$\text{MSE}=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f(x,y)-\hat{f}(x,y)\right]^2$$

Warning: do not necessarily imply
"best" in the visual sense.

Signal to Noise Ratio (*SNR*)

$$\text{SNR}=\frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\hat{f}(x,y)^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}[f(x,y)-\hat{f}(x,y)]^2}$$

Assumptions about degradation function H

- H is linear:

$$H[f_1 + f_2] = H[f_1] + H[f_2], \qquad (1)$$

$$H[kf] = kH[f] \qquad (2)$$

- H is shift invariant:

$$\text{If } H[f(x, y)] = g(x, y) \text{ then } H[f(x-a, y-b)] = g(x-a, y-b) \qquad (3)$$

(i.e., shifting the input merely shifts the output by the same amount)

Degradation Model

(assuming linearity and shift invariance)

- Any function f(x,y) can be written as follows:

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)\delta(x-a,y-b)\, da\, db$$

- Then, H[f(x,y)] is:

from (1)

$$H[f(x,y)] = H[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)\delta(x-a,y-b)dadb] =$$

from (2)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H[f(a,b)\delta(x-a,y-b)]dadb = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,b)H[\delta(x-\alpha,y-b)]d\alpha db$$

Degradation Model - Continuous Case (cont'd)

- Suppose

$$H[\delta(x,y)] = h(x,y)$$

impulse response

- Since H(x,y) is shift invariant:

from (3)

$$H[\delta(x-a,y-b)] = h(x-a,y-b)$$

- The

$$H[f(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)h(x-a,y-b)\,da\,db = f(x,y) * h(x,y)$$

Degradation Model - Continuous Case (cont'd)

- Under the assumptions of linearity and shift invariance:

$$g(x, y) = H[f(x, y)] + n(x, y)$$

simplifies to:

Degradation Model: $g(x, y) = f(x, y) * h(x, y) + n(x, y)$

or in freq. domain: $G(u,v) = H(u,v)F(u,v) + N(u,v)$

Noise Properties

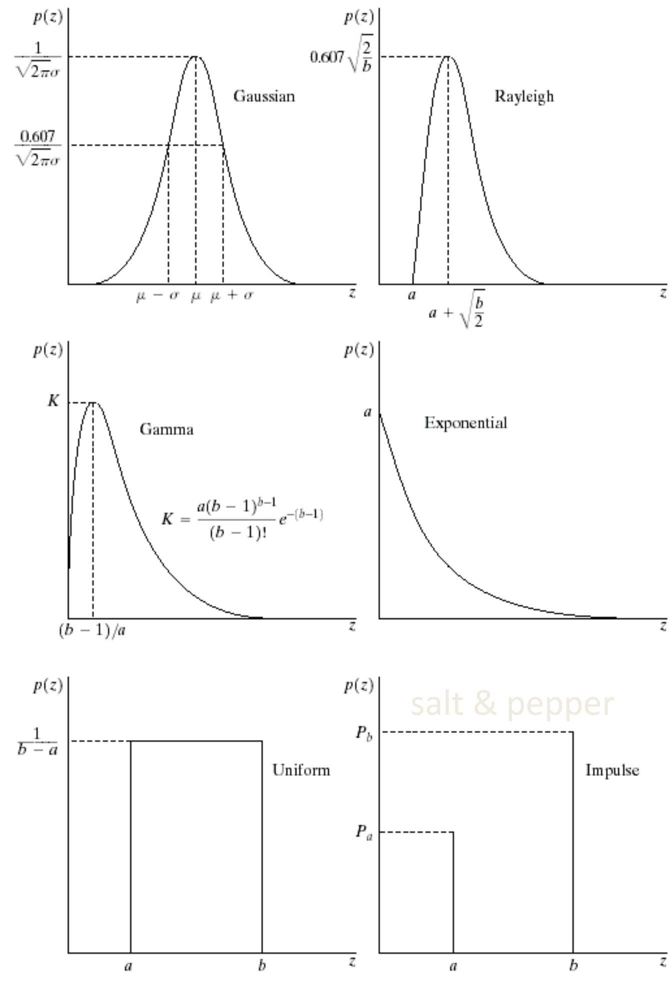
- Noise arises typically during image acquisition and/or transmission.
- For simplicity, we will assume that:

    (1) noise is independent of spatial coordinates

    (2) there is no correlation between pixel values and noise values (not true for periodic noise)

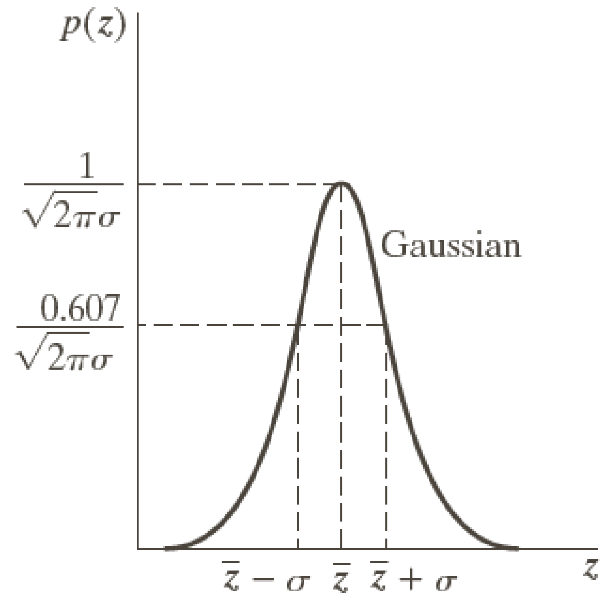
Noise Models

- Many types of noise can be modeled using a probability density function.
- Model is typically chosen based on some understanding of the noise source.



Gaussian noise

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2 / 2\sigma^2}$$



- Gaussian noise arises in an image due to factors such as:

  (1) Electronic circuit noise

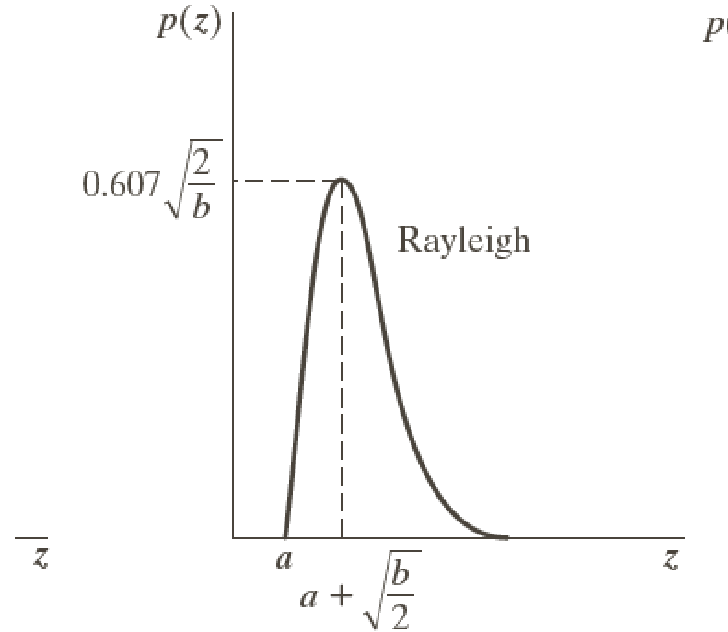
  (2) Sensor noise due to poor illumination and/or high temperature

Rayleigh noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \ge a \\ 0 & z < a \end{cases}$$

$$\bar{z} = a + \sqrt{\pi b/4} \qquad \sigma^2 = \frac{b(4-\pi)}{4}$$

- Typically used to characterize noise in **range** imaging.

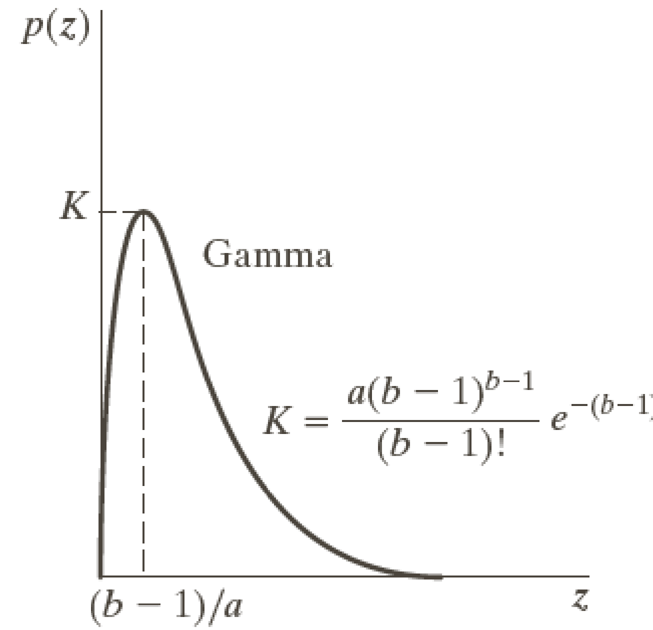


Gamma (Erlang) noise

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$\bar{z} = \frac{b}{a} \qquad \sigma^2 = \frac{b}{a^2}$$

- Typically used to characterize noise in laser imaging.

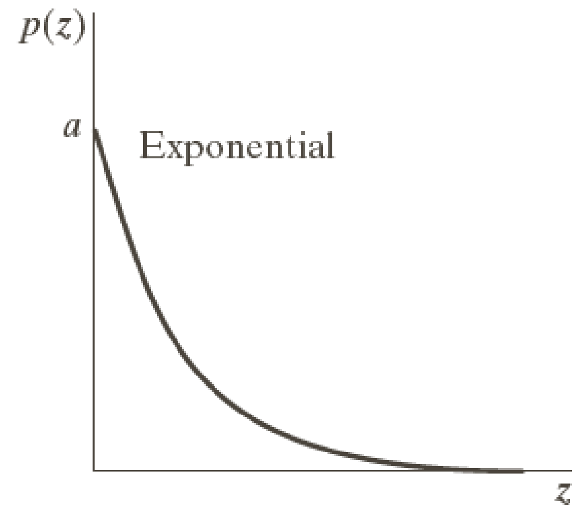


Exponential noise

$$p(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$\bar{z} = \frac{1}{a} \qquad \sigma^2 = \frac{1}{a^2}$$

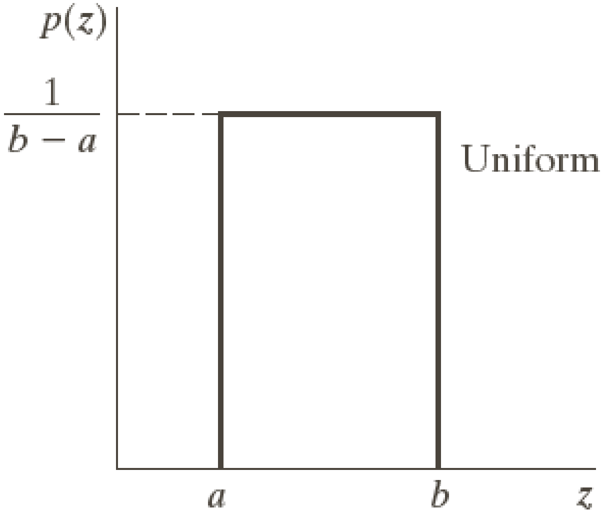
- Typically used to characterize noise in laser imaging.



Uniform noise

$$p(z) = \begin{cases} \dfrac{1}{b-a} & a \le z \le b \\ 0 & otherwise \end{cases}$$

$$\bar{z} = \frac{a+b}{2} \qquad \sigma^2 = \frac{(b-a)^2}{12}$$

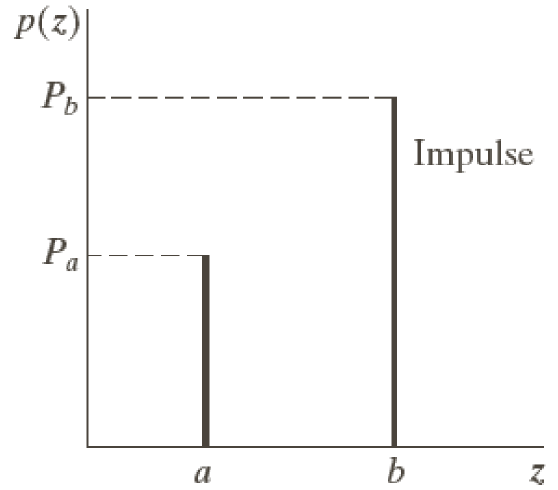


- Least used in practice.

- Useful as the basis for random number generators.

Impulse noise

$$p(z)=\begin{cases} P_a & z=a \\ P_b & z=b \\ 0 & otherwise \end{cases}$$



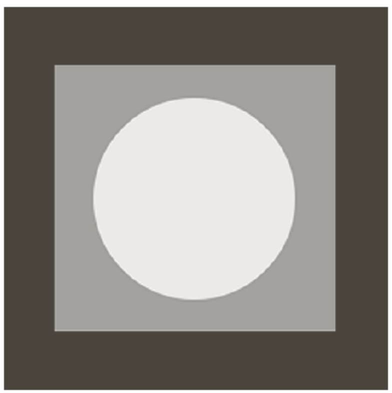
• If P_a=P_b, a=0, and b=255, then this is salt and pepper noise.

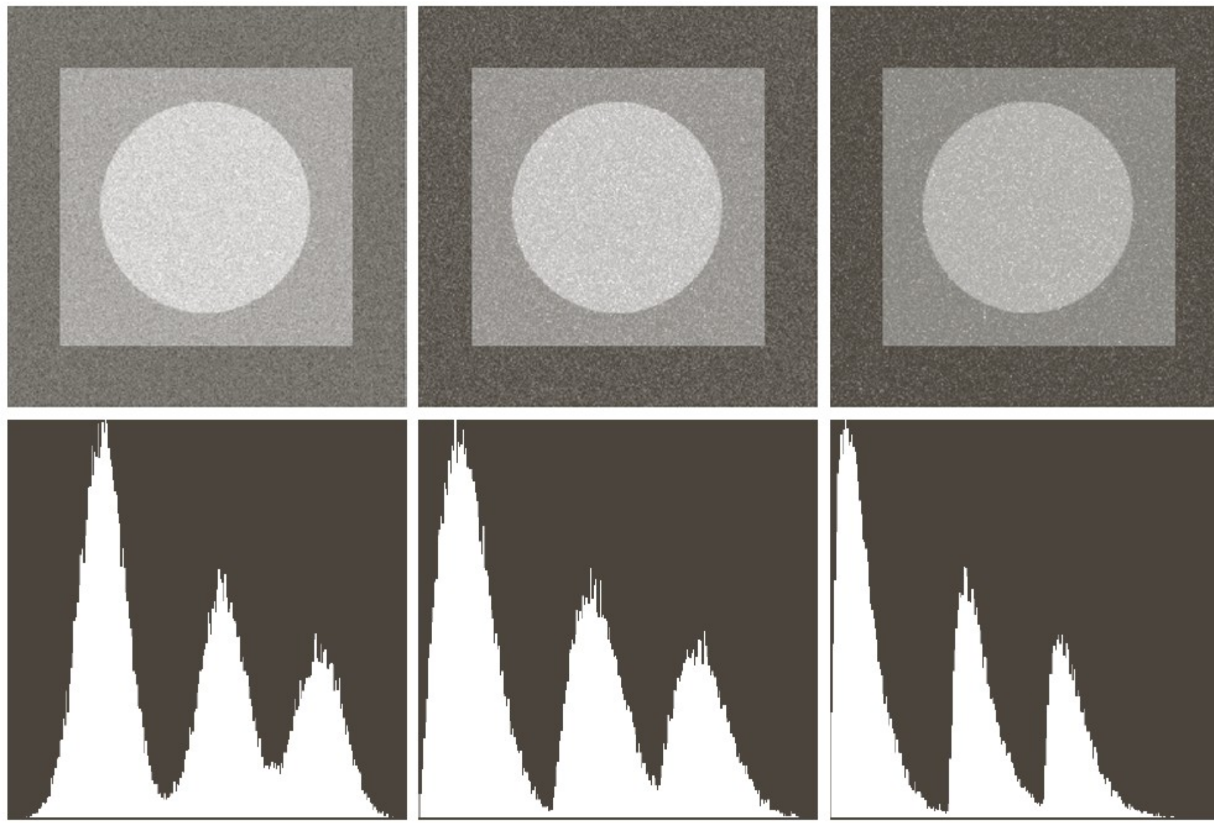
•Common in situations where quick transients (e.g., faulty switching), takes place during imaging.

Examples

noise corrupted images and their histograms

test pattern



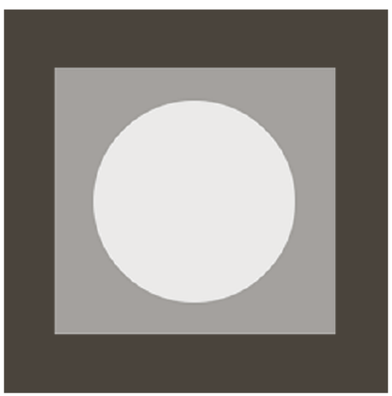


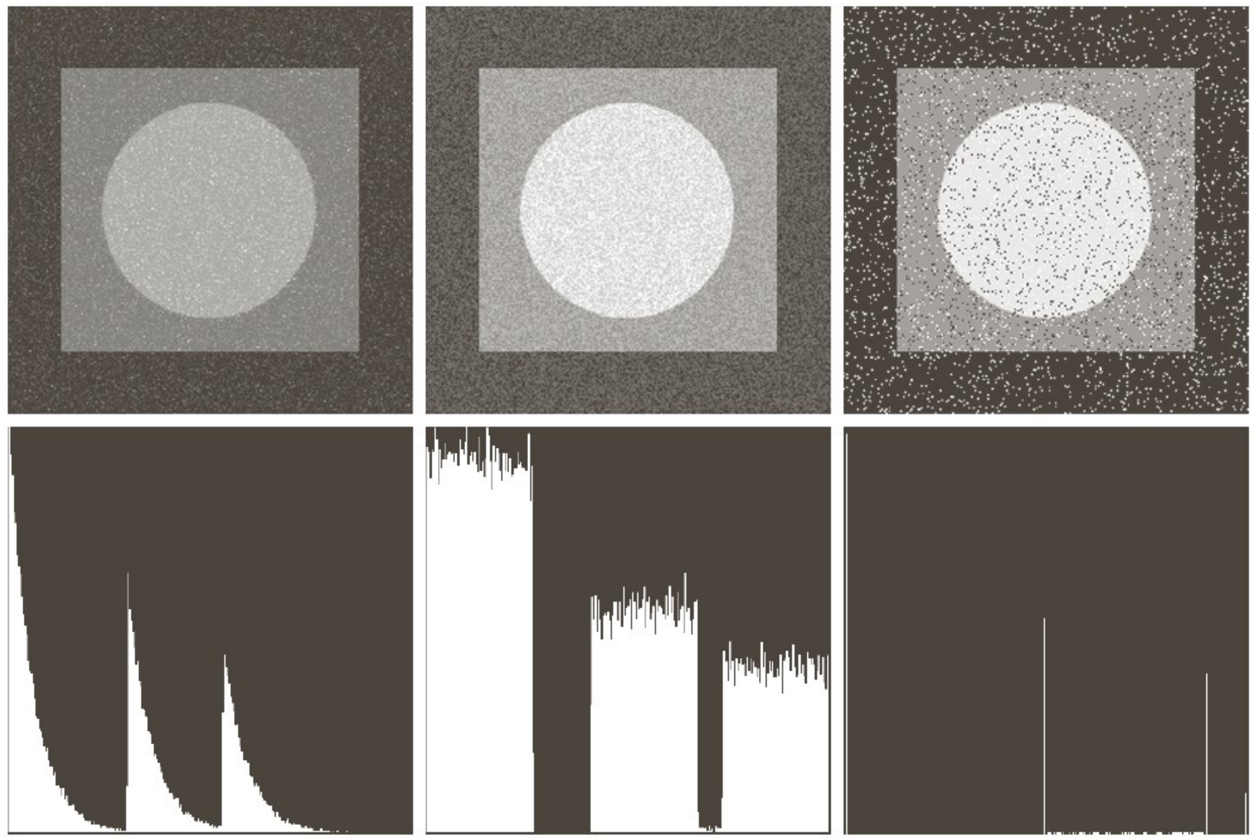
Gaussian Rayleigh Gamma

Examples (cont'd)

noise corrupted images and their histograms

test pattern





Exponential Uniform Salt & Pepper

Estimation of noise parameters

- Estimate the noise model parameters from a small patch of reasonably constant background intensity.
- For impulse noise, estimate probability of black/white pixels (i.e., choose a mid-gray region).

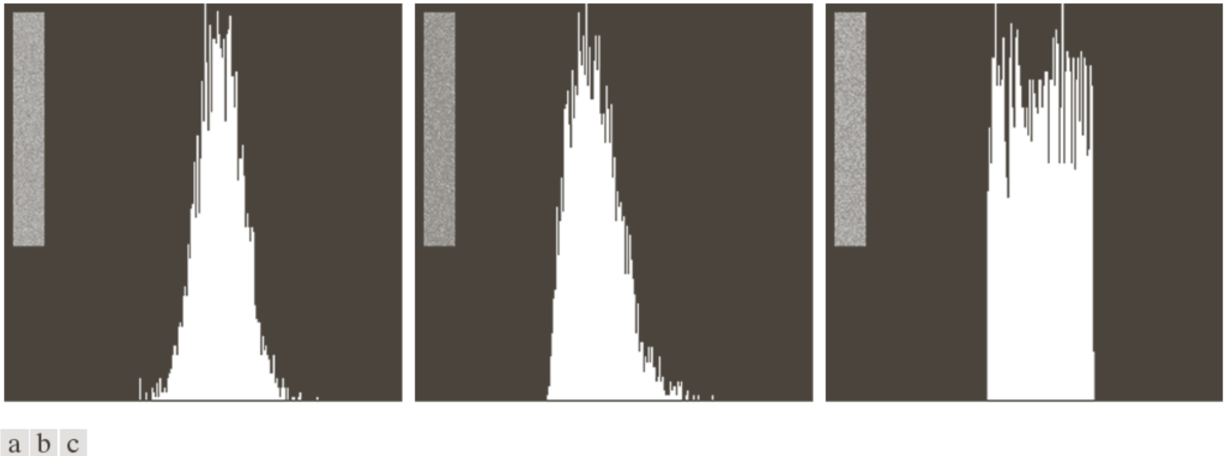


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

(1) Estimate mean and variance

(2) Compute a and b (i.e., parameters of noise distribution)

Restoration in the presence of noise only

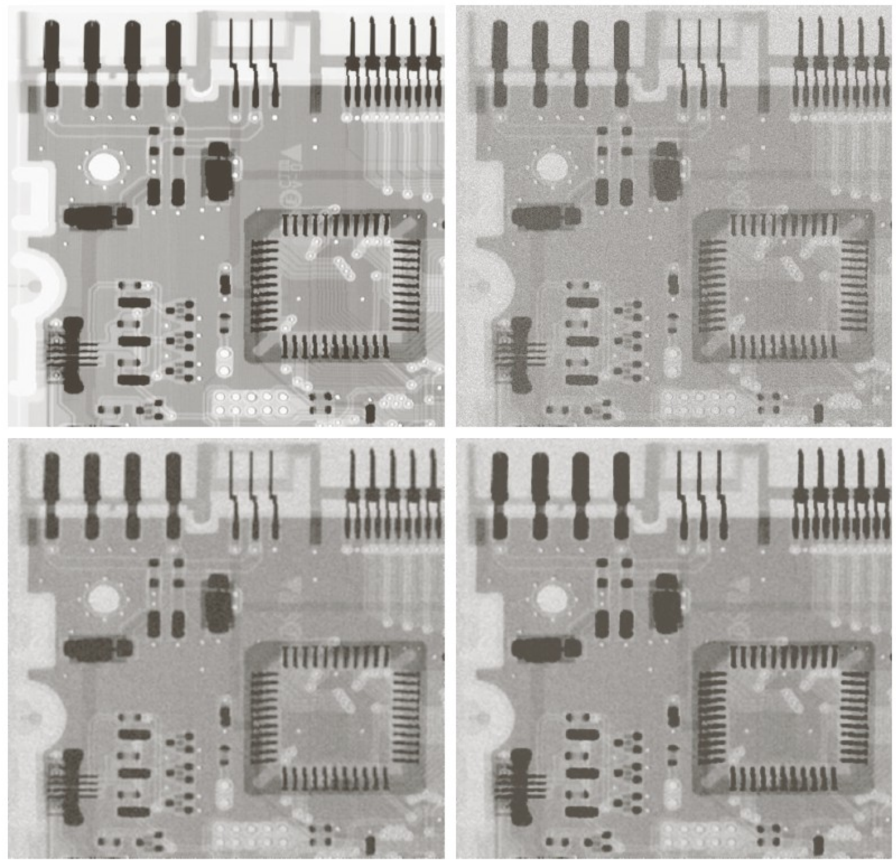
- If noise can be estimated, then subtract it from the input image:

$$\text{estimate } n(x,y)$$

$$g(x,y) = f(x,y) + n(x,y) \rightarrow \hat{f}(x,y) = g(x,y) - \hat{n}(x,y)$$

  – Frequency-domain filters (band-reject, band-pass, etc.)

Arithmetic/Geometric mean filters



a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Geometric mean filters tend to preserve more details (i.e., less blurring)

m x n mask

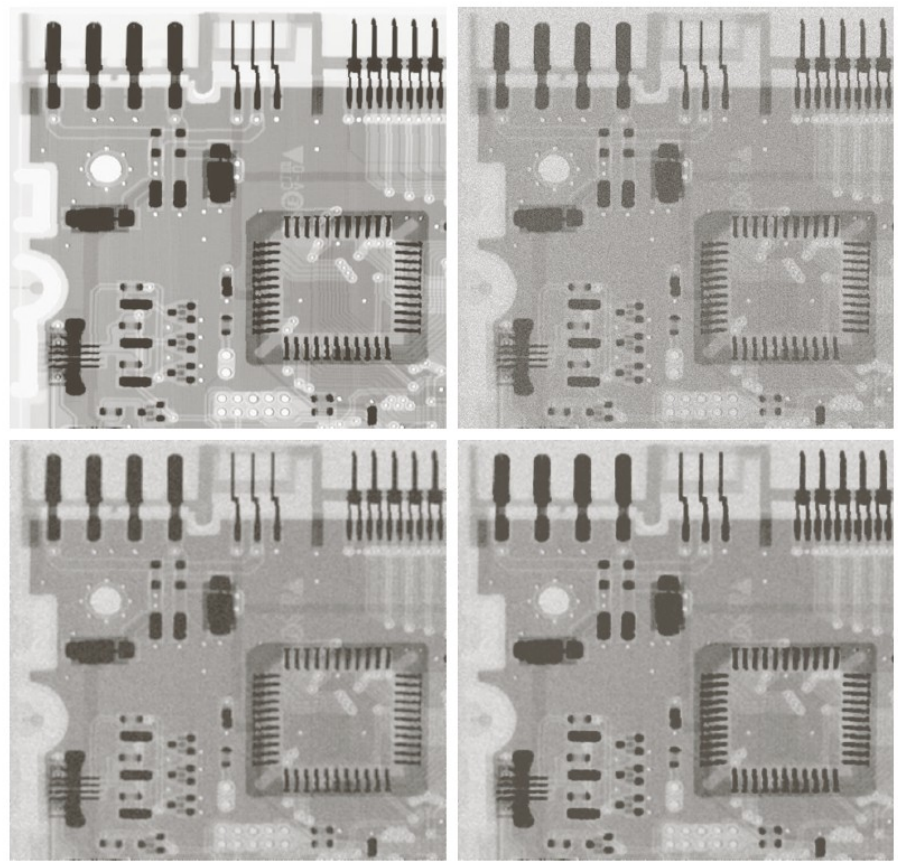
$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{x,y}} g(s,t)$$

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{x,y}} g(s,t) \right]^{\frac{1}{mn}}$$

Arithmetic/Geometric mean filters

a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



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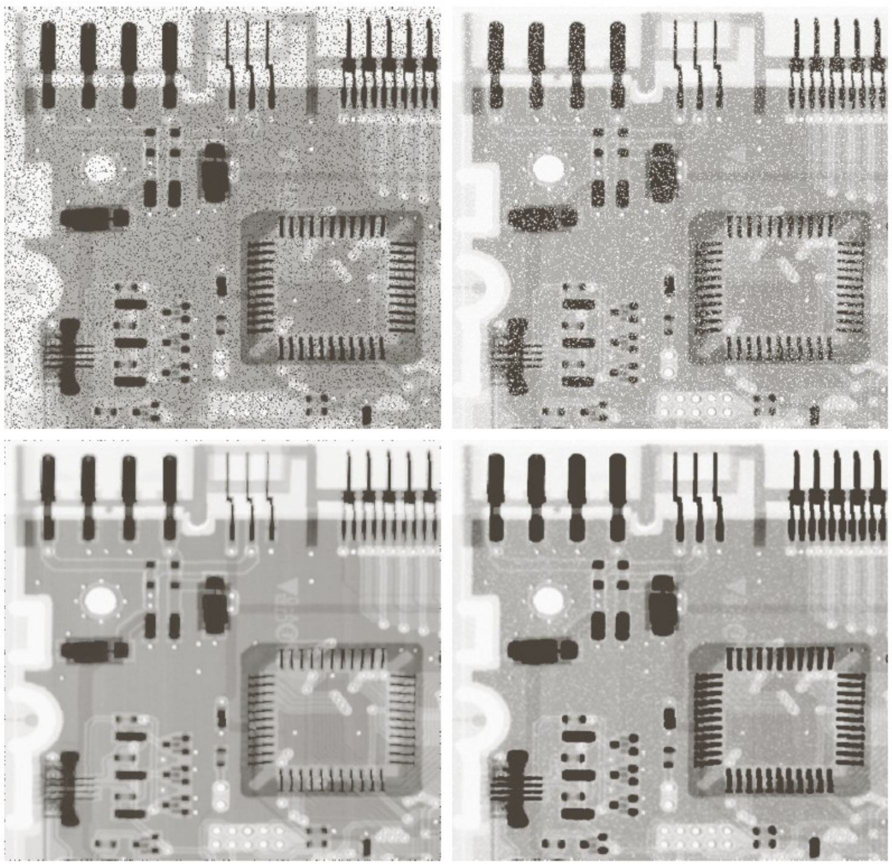
$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{x,y}} g(s,t) \right]^{\frac{1}{mn}}$$

Contra-Harmonic filters



Q=1.5

Q= -1.5

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Q: order of filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{x,y}} [g(s,t)]^{Q+1}}{\sum_{(s,t)\in S_{x,y}} [g(s,t)]^{Q}}$$

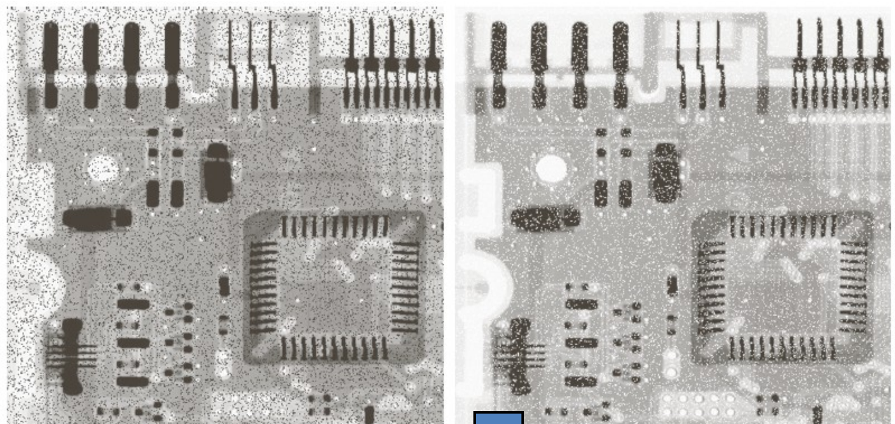
Good for removing salt or pepper noise (but not both simultaneously)

Q>0 → pepper noise
Q<0 → salt noise

If Q=0, same as arithmetic filter!

Contra-Harmonic filters (cont'd)

Example of selecting the wrong sign :

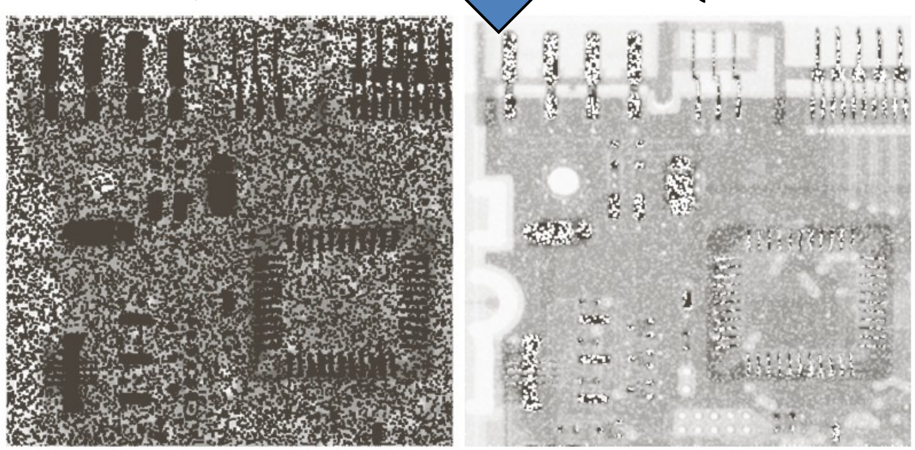


a b
c d

FIGURE 5.8
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability.

Q= -1.5

Q= 1.5



a b

FIGURE 5.9
Results of selecting the wrong sign in contraharmonic filtering.
(a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$.
(b) Result of filtering 5.8(b) with $Q = 1.5$.

Order statistics filters

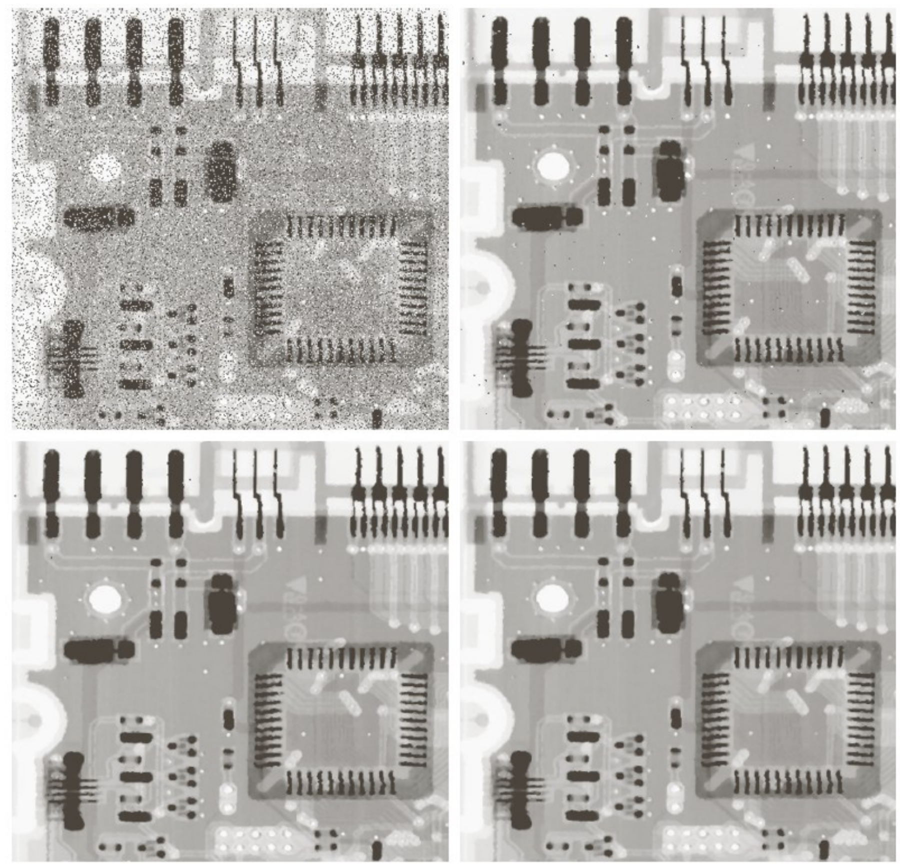
- Their response is based on the ordering (ranking) of the pixels contained in an area covered by the filter.
- Order statistics filters are nonlinear spatial filters.

Median filter

Multiple passes can improve results

$$\hat{f}(x,y) = \underset{(s,t)\in S_{x,y}}{median}\left\{g(s,t)\right\}$$

Less blurring compared to arithmetic filters



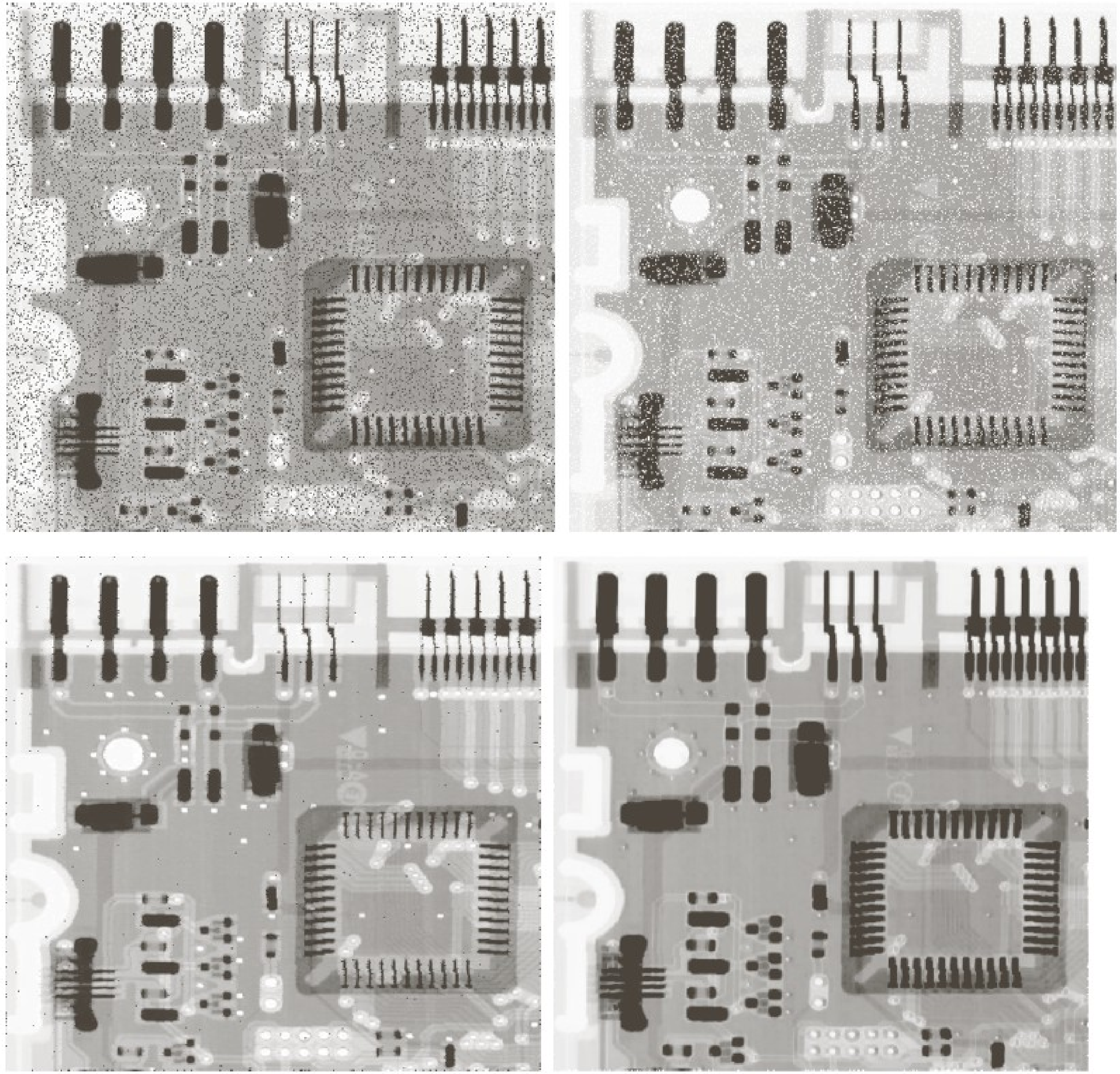
a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Max/Min filters

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$



Max filter is good for pepper noise

Min filter is good for salt noise

Alpha-trimmed mean filter

Good for multiple types of noise (e.g., Uniform and salt-and-pepper noise)

Assume an *m x n* neighborhood:

0<=d<=mn-1

(1) Disregard d/2 lowest and d/2 highest values
(2) Average the remaining values

Special cases:
d=0 → arithmetic mean
d=mn-1 → median

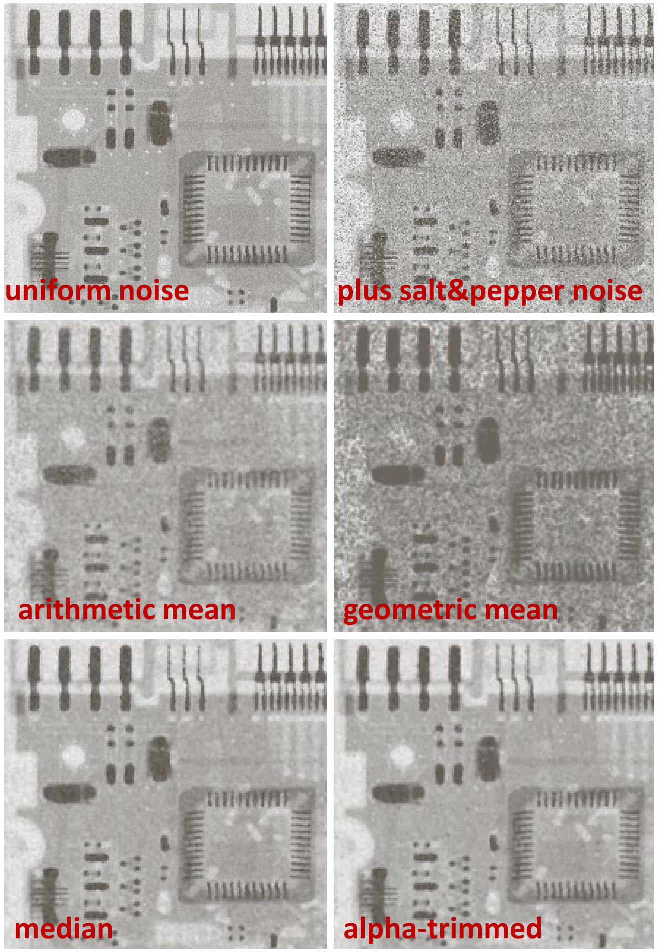
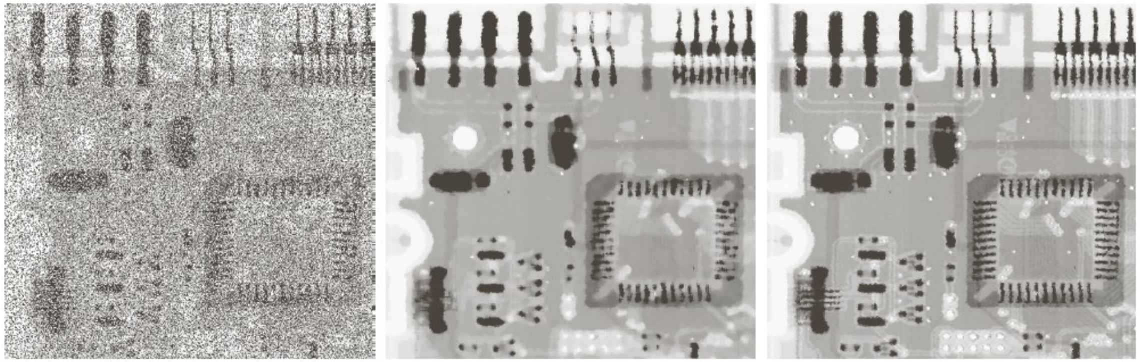


FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

Adaptive Filters

- Adaptive filters have superior performance compared to non-adaptive filters.
  - Non-fixed (i.e., adaptive) parameters
  - Have higher complexity.



a b c

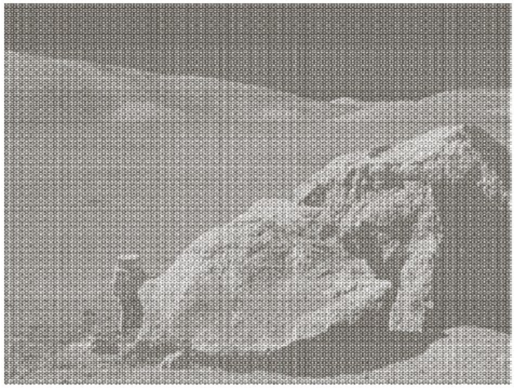
FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Adaptive median filtering:

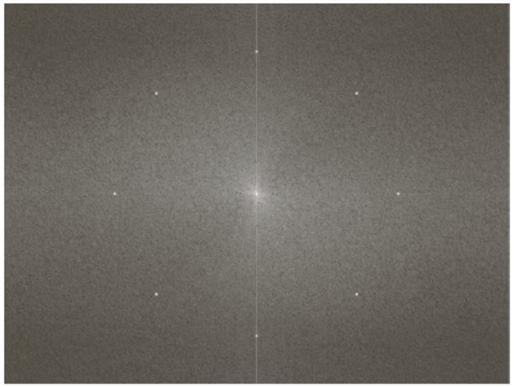
Periodic noise

- Arises from interferences (e.g., electrical or electromechanical) during image acquisition.
- Can be analyzed and filtered quite effectively in the frequency domain using a band-reject filter.

image corrupted by sinusoidal noise



spectrum of noisy image



Band-reject filters

Ideal Butterworth Gaussian

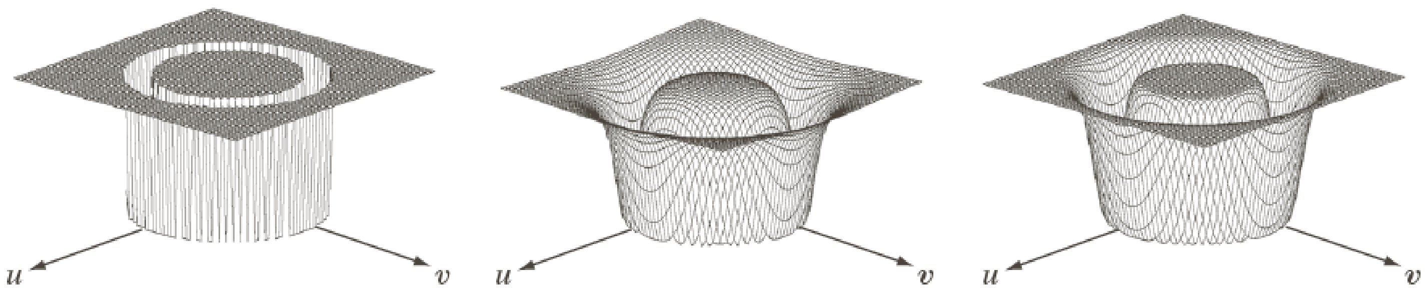
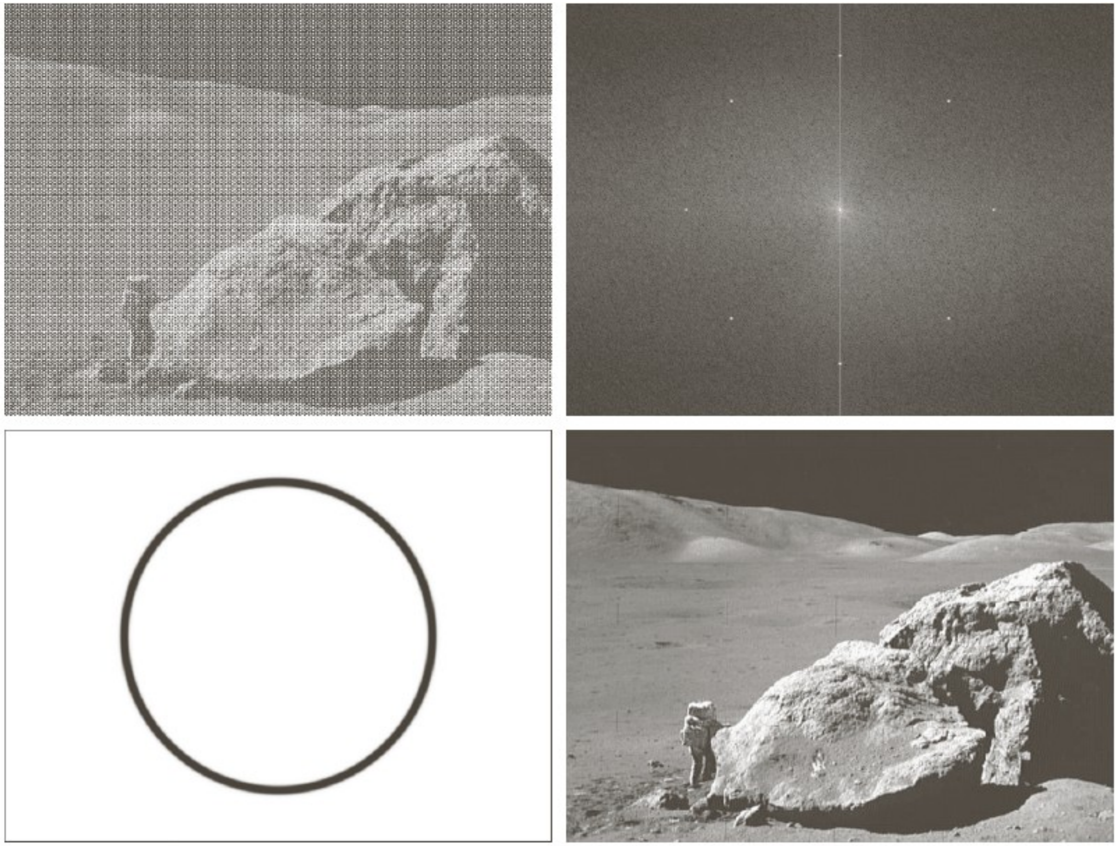


TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

Band-reject filters (cont'd)



a b
c d

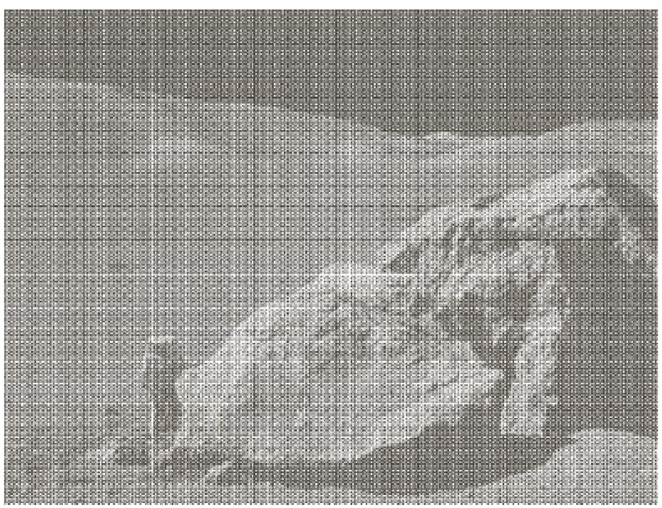
FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Bandpass filters

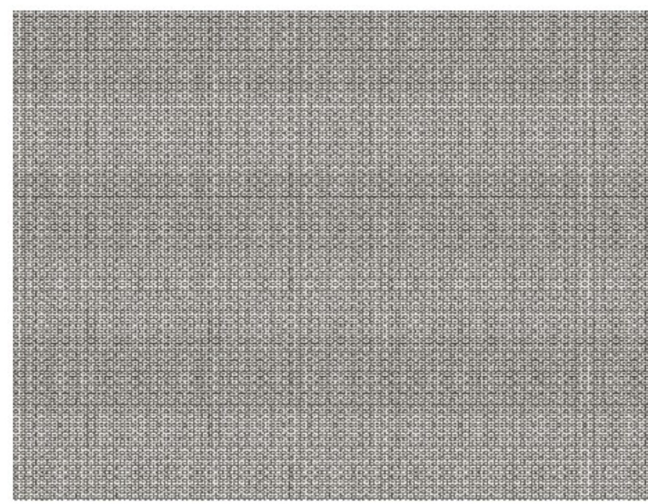
- Performs the opposite operation of a band-reject filter.

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

- Useful in isolating the effect of specific frequencies in an image.



noise pattern



Estimating degradation H

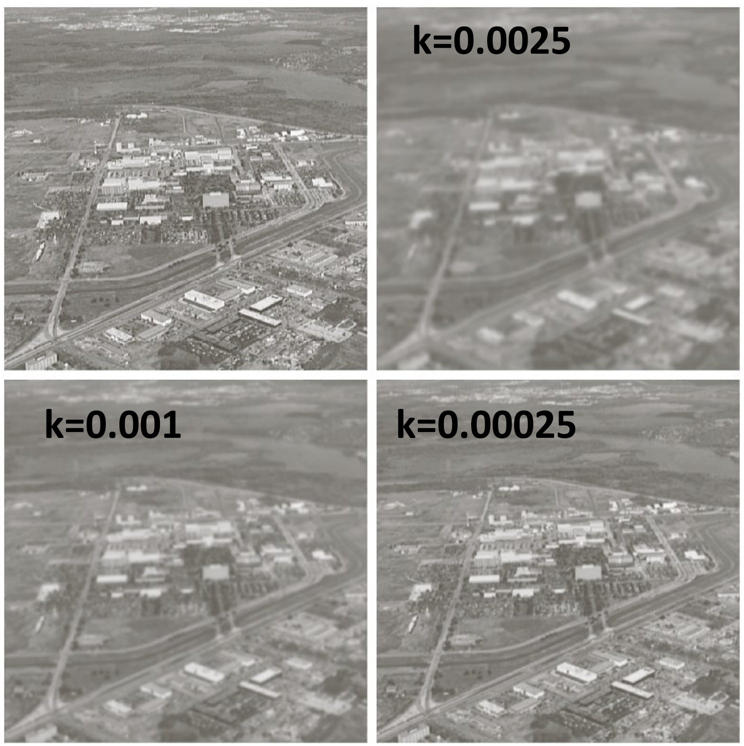
$$\mathbf{G(u,v)=H(u,v)F(u,v) + N(u,v)}$$

- Typically, H is modeled mathematically; let’s look at

  two examples:
  - Atmospheric turbulence
  - Motion blurring

Degradation due to environmental conditions

- Atmospheric turbulence (Hufnagel and Stanley [1964])



a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

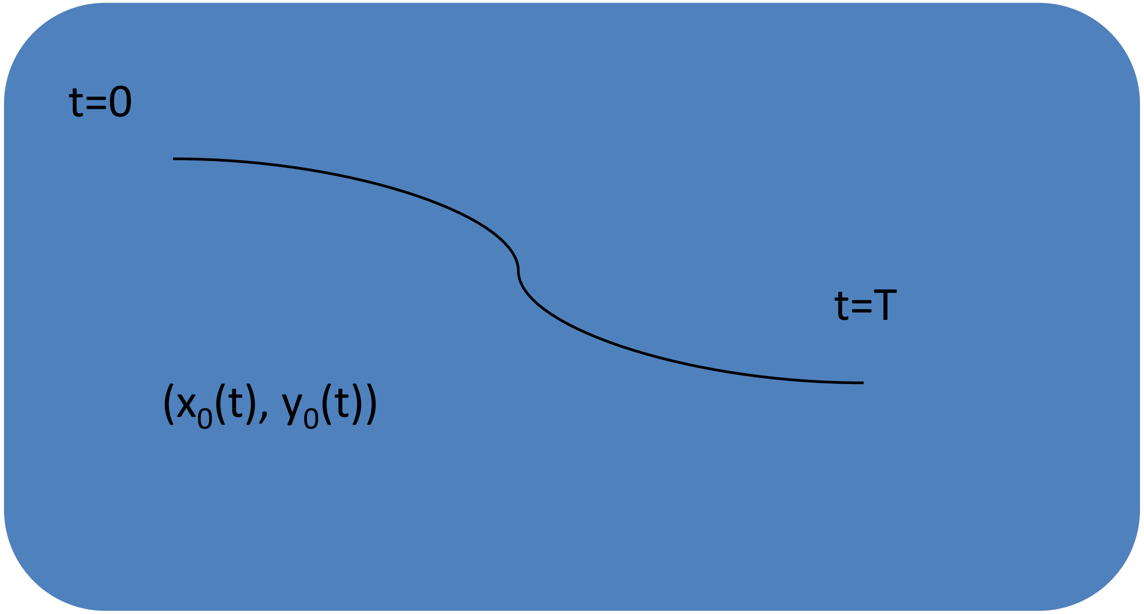
G(u,v)=H(u,v)F(u,v) + N(u,v)

where:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

Degradation due to uniform linear motion

- Consider the case of camera/object planar motion (2D) where $x_0(t)$ and $y_0(t)$ is the motion trajectory.



e.g., camera motion

Degradation due to uniform linear motion

- If T is the exposure time (i.e., time interval during which the camera shutter is open), then the output image g(x,y) is:

$$g(x,y) = \int_0^T f(x - x_0(t), y - y_0(t))dt$$

- Taking the FT: G(u,v)=H(u,v)F(u,v)

where $$H(u,v) = \int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}dt$$

Degradation due to uniform linear motion (cont'd)

- If $x_0(t)=\alpha t/T$ and $y_0(t)=0$, then:

$$H(u,v)=\int_0^T e^{-j2\pi(ux_0(t))}dt=\int_0^T e^{-j2\pi u\alpha t/T}dt=\frac{T}{\pi u\alpha}\sin(\pi u\alpha)e^{-j\pi u\alpha}$$

- If $x_0(t)=\alpha t/T$ and $y_0(t)=bt/T$, then:

$$H(u,v)=\frac{T}{\pi(u\alpha+vb)}\sin(\pi(ua+vb))e^{-j\pi(ua+vb)}$$

Degradation due to uniform linear motion (cont'd)

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

G(u,v)=H(u,v)F(u,v) where:

$$H(u,v) = \frac{T}{\pi(u\alpha + vb)} \sin(\pi(ua + vb))e^{-j\pi(ua+vb)}$$