3.5 Smoothing Spatial Filters

3.5.1 Smoothing Linear Filters

Output = average of pixels contained in the neighbourhood of the filter mask

Averaging filters or *lowpass filters*

⇒ Reduce "sharp" transitions in intensities

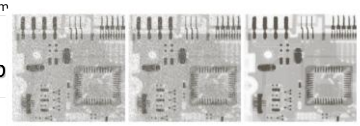
⇒ Application 1: noise reduction

⇒ Application 2: smoothing of false contours

FC caused generally by use of an insufficient number of intensity levels)

⇒ Blurring effect

Edges are characterized by sharp intensity transitions, so averaging filters have this undesirable side effect that they blur images



a b c

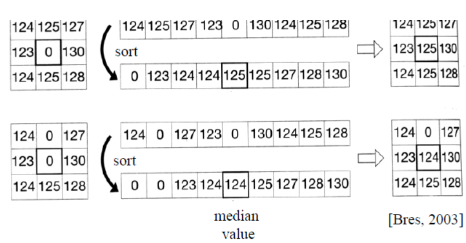
FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

3.5.2 Order-Statistic (Nonlinear) Filters

Ordering (ranking) the pixels contained in the image area encompasses by the filter, then replacing the value of the centre pixel with the value of ranking result

Median filter: replaces the value of a pixel by the median of the intensity values in the neighbourhood. Good noise-reduction capabilities with less smoothing (e.g. *impulse noise*, or *salt-and-pepper noise*)

Median filter :



Max filter, min filter

3

Sharpening Spatial Filters

Previously we have looked at smoothing filters which remove fine detail

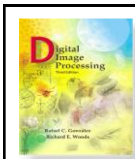
Sharpening spatial filters seek to highlight fine detail

– Remove blurring from images

– Highlight edges

Sharpening filters are based on *spatial differentiation*

The principal objective of sharpening is to highlight transitions in intensity. Uses of image sharpening vary and include applications ranging from electronic printing and medical imaging to industrial inspection and autonomous guidance in military systems. In the last section, we saw that image blurring could be accomplished in the spatial domain by pixel averaging in a neighborhood. Because averaging is analogous to integration, it is logical to conclude that sharpening can be accomplished by spatial differentiation. This, in fact, is the case.



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Chapter 3
Intensity Transformations & Spatial Filtering

3.6 Sharpening Spatial Filters

Objective: highlight transitions in intensity

Spatial (digital) differentiation => enhances edges and other discontinuities

3.6.1 Foundation

Let's study the behavior of first- and second-order derivatives in areas of constant intensities, at the onset and end of discontinuities, and along intensity ramps

Basic definition of first-order derivative of a one-dimensional function f(x):

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

Second derivative: $$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

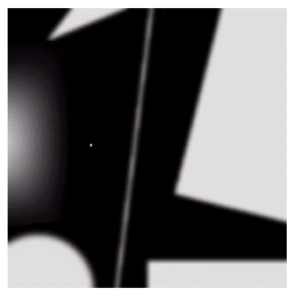
5

Spatial Differentiation

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Differentiation measures the *rate of change* of a function

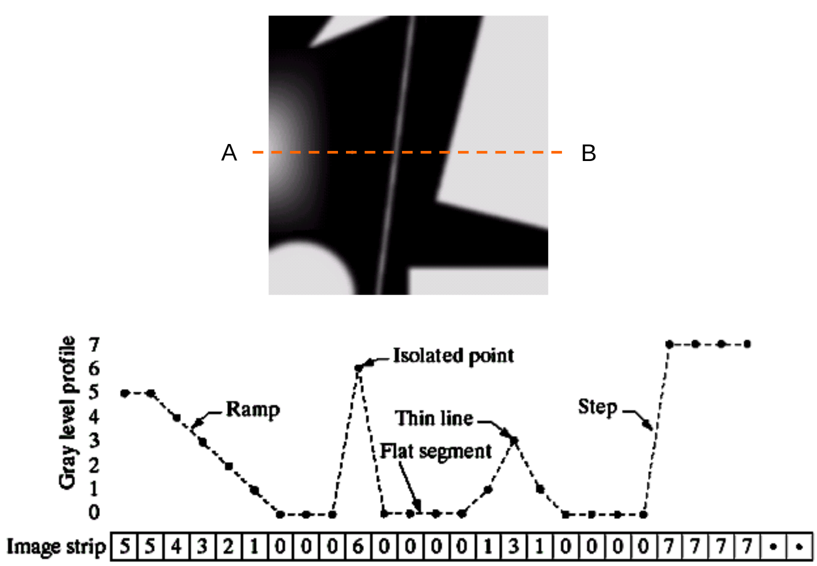
Let's consider a simple 1 dimensional example



6

Spatial Differentiation

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



7

1^{st} Derivative

The formula for the 1^{st} derivative of a function is as follows:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

It's just the difference between subsequent values and measures the rate of change of the function

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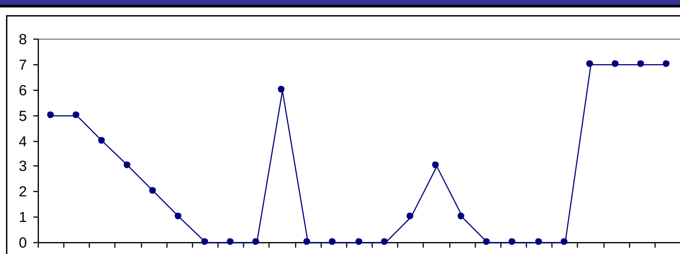
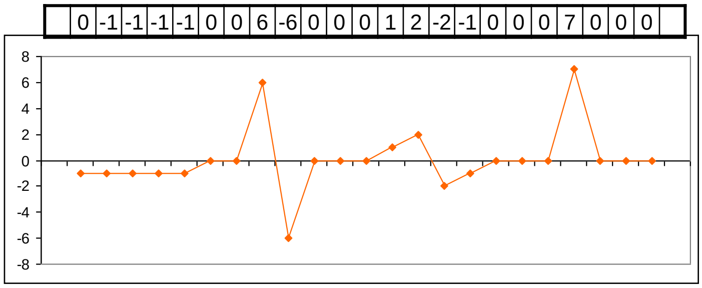
1^{st} Derivative (cont…)



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
	0	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0	



9

2nd Derivative

The formula for the 2nd derivative of a function is as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

Simply takes into account the values both before and after the current value

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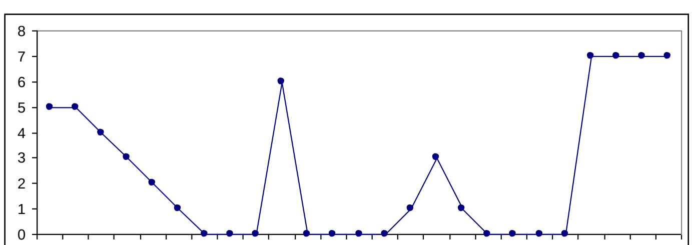
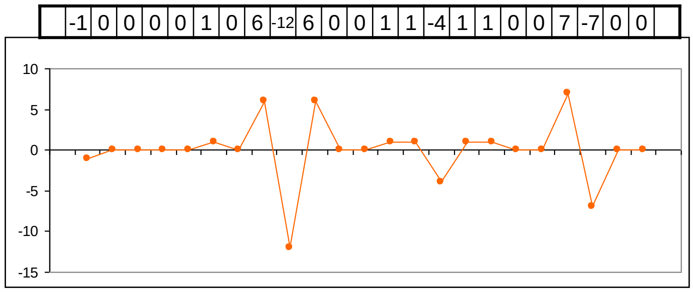
2nd Derivative (cont…)



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
	-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	



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Using Second Derivatives For Image Enhancement

The 2nd derivative is more useful for image enhancement than the 1st derivative

- Stronger response to fine detail
- Simpler implementation
- We will come back to the 1st order derivative later on

The first sharpening filter we will look at is the *Laplacian*

- Isotropic
- One of the simplest sharpening filters
- We will look at a digital implementation

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The Laplacian

The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

where the partial 1st order derivative in the x direction is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and in the y direction as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

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The Laplacian (cont…)

So, the Laplacian can be given as follows:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

We can easily build a filter based on this

0	1	0
1	-4	1
0	1	0

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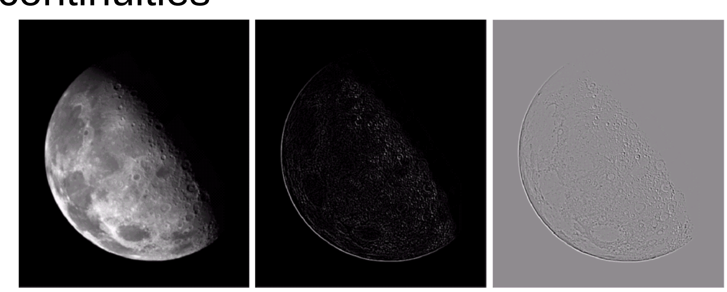
The Laplacian (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities



Original Image | Laplacian Filtered Image | Laplacian Filtered Image Scaled for Display

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But That Is Not Very Enhanced!

The result of a Laplacian filtering is not an enhanced image

We have to do more work in order to get our final image

Subtract the Laplacian result from the original image to generate our final sharpened enhanced image

$$g(x, y) = f(x, y) - \nabla^2 f$$

Laplacian Filtered Image Scaled for Display

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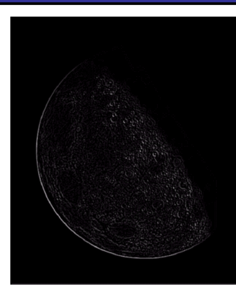
Laplacian Image Enhancement

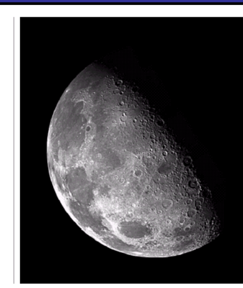
Original Image

-



Laplacian Filtered Image

=



Sharpened Image

In the final sharpened image edges and fine detail are much more obvious

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Laplacian Image Enhancement

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

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Simplified Image Enhancement

The entire enhancement can be combined into a single filtering operation

$$
\begin{aligned}
g(x,y) &= f(x,y) - \nabla^2 f \\
&= f(x,y) - [f(x+1,y) + f(x-1,y) \\
&\qquad + f(x,y+1) + f(x,y-1) \\
&\qquad - 4f(x,y)] \\
&= 5f(x,y) - f(x+1,y) - f(x-1,y) \\
&\qquad - f(x,y+1) - f(x,y-1)
\end{aligned}
$$

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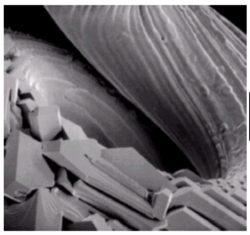
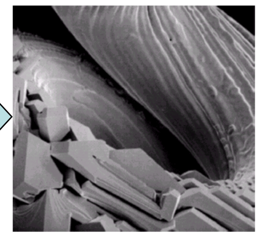
Simplified Image Enhancement (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

This gives us a new filter which does the whole job for us in one step



0	-1	0
-1	5	-1
0	-1	0



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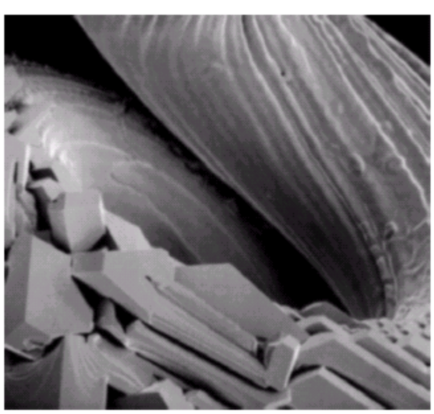
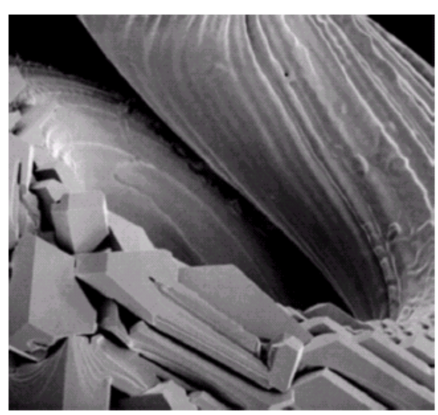
Simplified Image Enhancement (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)





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Variants On The Simple Laplacian

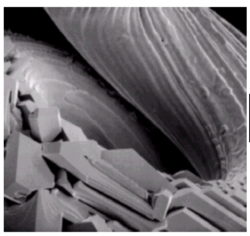
There are lots of slightly different versions of the Laplacian that can be used:

0	1	0
1	-4	1
0	1	0

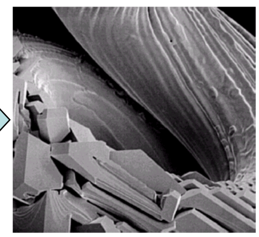
Simple Laplacian

1	1	1
1	-8	1
1	1	1

Variant of Laplacian



-1	-1	-1
-1	9	-1
-1	-1	-1



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

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1^{st} Derivative Filtering

Implementing 1^{st} derivative filters is difficult in practice

For a function $f(x, y)$ the gradient of f at coordinates (x, y) is given as the column vector:

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

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1st Derivative Filtering (cont…)

The magnitude of this vector is given by:

$$\nabla f = mag(\nabla \mathrm{f})$$
$$= \left[G_x^2 + G_y^2\right]^{1/2}$$
$$= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$

For practical reasons this can be simplified as:

$$\nabla f \approx \left|G_x\right| + \left|G_y\right|$$

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1st Derivative Filtering (cont…)

There is some debate as to how best to calculate these gradients but we will use:

$$\nabla f \approx \left|(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)\right|$$
$$+ \left|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)\right|$$

which is based on these coordinates

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

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Sobel Operators

Based on the previous equations we can derive the *Sobel Operators*

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

To filter an image it is filtered using both operators the results of which are added together

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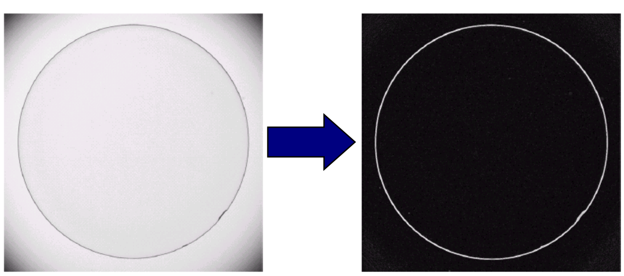
Sobel Example

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



An image of a contact lens which is enhanced in order to make defects (at four and five o'clock in the image) more obvious

Sobel filters are typically used for edge detection

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Unsharp Masking and Highboost Filtering

- Blur the original image *f(x,y)*:

$$f(x,y) \Rightarrow \bar{f}(x,y)$$

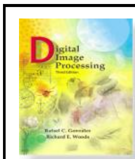
- Subtract the blurred image from the original = Mask!

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

- Add the mask to the original

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

k = 1: Unsharp Masking
k > 1: Highboost Filtering



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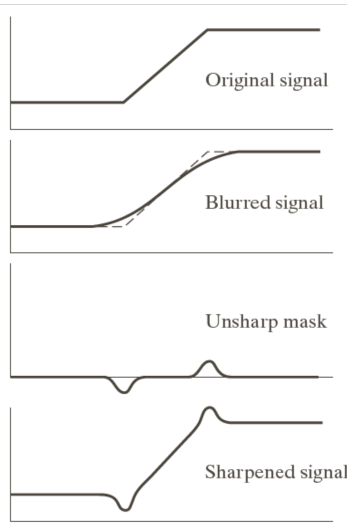

a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

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a
b
c
d
e

FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

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Gradient

- First derivatives in image processing are implemented using the magnitude of the gradient
- For a function *f(x,y)*, the gradient of *f* at coordinates *(x,y)* is defined as 2D column *vector*

Points in the direction of the greatest rate of change!

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- Magnitude (length) of this vector is given by:

$$M(x,y) = \text{mag}(\nabla \mathbf{f}) = \left[G_x^2 + G_y^2\right]^{\frac{1}{2}} = \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{\frac{1}{2}}$$

Gradient Image

$$M(x,y) \approx |G_x| + |G_y|$$

Approximation for practical reasons!

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Gradient

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

A 3x3 region of image intensity values

Roberts Cross gradient operators

$$M(x,y) \approx |z_9 - z_5| + |z_8 - z_6|$$

Sobel Operators

$$M(x,y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

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Gradient

- All mask coefficients sum to zero, as expected of a derivative operator (areas of constant intensity)
- Masks of even sizes are awkward to implement
  - Lack of center of symmetry
- To filter an image, it is filtered using both vertical (*y*) and horizontal (*x*) operators and the results are added together to obtain the magnitude of the gradient
- Computation of partial derivatives are linear operations
  - Implemented as sum of products
- Computation of the Gradient is non-linear
  - *M(x,y)* involves squaring/square roots or absolute values

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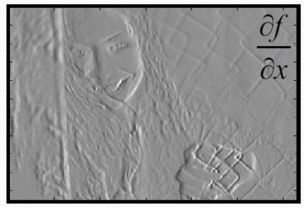
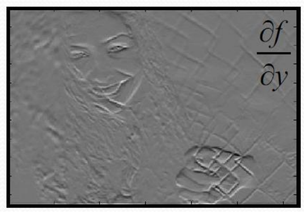
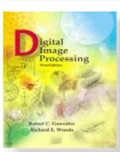
Gradient – Example1





$$\sqrt{\frac{\partial f}{\partial x}^2 + \frac{\partial f}{\partial y}^2}$$



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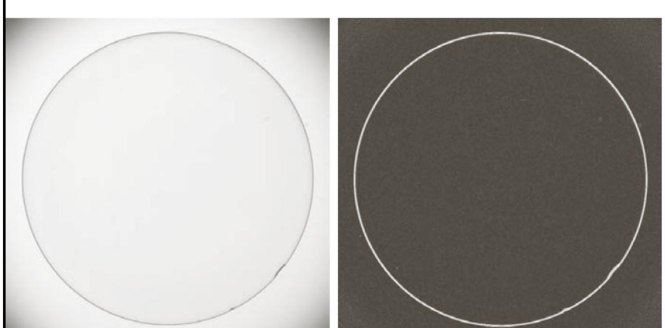

a b

FIGURE 3.42 (a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)

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1^{st} & 2^{nd} Derivatives

Comparing the 1^{st} and 2^{nd} derivatives we can conclude the following:

- 1^{st} order derivatives generally produce thicker edges
- 2^{nd} order derivatives have a stronger response to fine detail e.g. thin lines
- 1^{st} order derivatives have stronger response to grey level step
- 2^{nd} order derivatives produce a double response at step changes in grey level

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Combining Enhancement Methods

- Use Laplacian to highlight fine detail
  - Also produce noisier results than the gradient
- Use gradient to enhance prominent edges
  - Gradient has a stronger response in ramps and steps areas than does the Laplacian
  - Response of the gradient to noise is lower than Laplacian
  - Response to noise can be lowered by smoothing the gradient with an averaging filter
- Combining Laplacian and gradient operators
  - Smooth the gradient and multiply it by the Laplacian image (preserve details in the strong areas while reducing noise in the flat areas)
  - The above result is added to the original image

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Combining Spatial Enhancement Methods

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Successful image enhancement is typically not achieved using a single operation

Rather we combine a range of techniques in order to achieve a final result

This example will focus on enhancing the bone scan to the right

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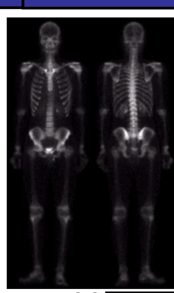
Combining Spatial Enhancement Methods (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

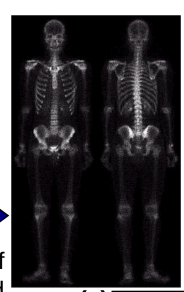


(a)

Laplacian filter of bone scan (a)

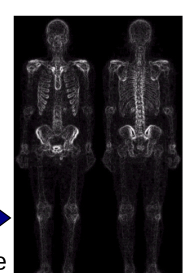
(b)

Sharpened version of bone scan achieved by subtracting (a) and (b)



(c)

Sobel filter of bone scan (a)



(d)

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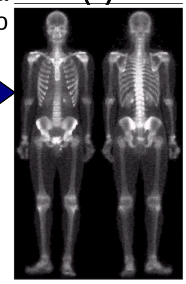
Combining Spatial Enhancement Methods (cont…)

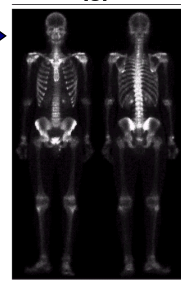
Result of applying a power-law trans. to (g)

(h)



Sharpened image which is sum of (a) and (f)

(g)



The product of (c) and (e) which will be used as a mask

(f)

(e)

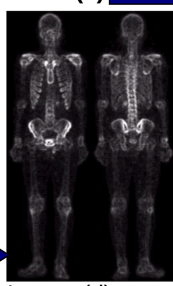


Image (d) smoothed with a 5*5 averaging filter

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Combining Spatial Enhancement Methods (cont…)

Compare the original and final images

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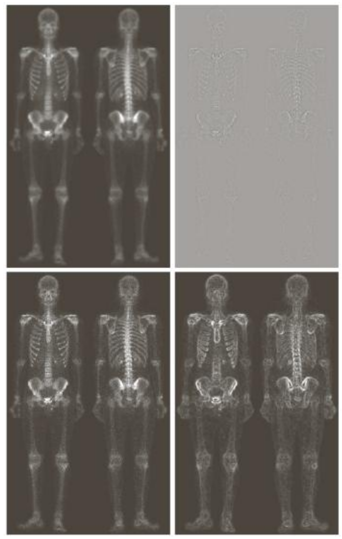

a b
c d

FIGURE 3.43
(a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).

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e f
g h

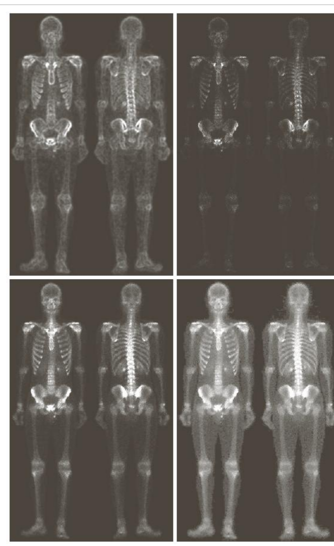

FIGURE 3.43
(Continued)
(e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

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Joseph Fourier

Joseph's father was a tailor in Auxerre
Joseph was the ninth of twelve children
His mother died when he was nine and his father died the following year

Fourier demonstrated talent on math at the age of 14.
In 1787 Fourier decided to train for the priesthood - a religious life or a mathematical life?
In 1793, Fourier joined the local Revolutionary Committee

Born: 21 March 1768 in Auxerre, Bourgogne, France
Died: 16 May 1830 in Paris, France

Credits: Xin Li Professor, Lane Dept of CSEE, West Virginia University

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Fourier's "Controversy" Work

Fourier did his important mathematical work on the theory of heat (highly regarded memoir *On the Propagation of Heat in Solid Bodies*) from 1804 to 1807

This memoir received objection from Fourier's mentors (Laplace and Lagrange) and not able to be published until 1815

Napoleon awarded him a pension of 6000 francs, payable from 1 July, 1815. However Napoleon was defeated on 1 July and Fourier did not receive any money

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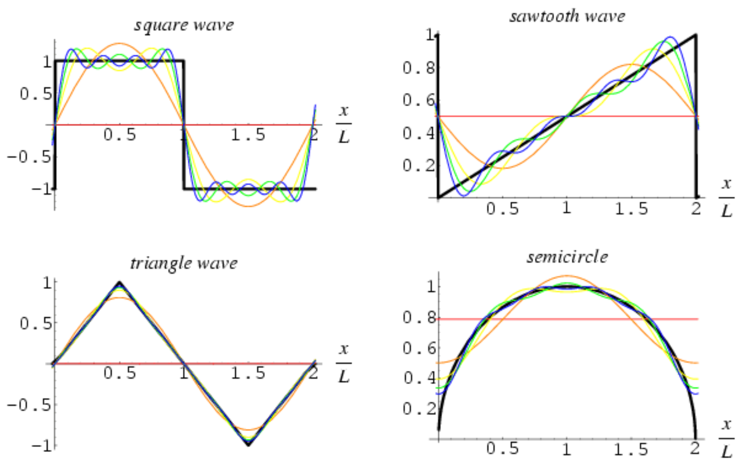
Examples



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