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**Impact of Randomization on Social Welfare and Electoral System Outcomes**

Introduction:

During the course of this semester, we have studied many different electoral systems, both in theoretical numbers and in case studies of real-life outcomes. One critical feature that sets these two groups apart is the presence of personal biases evident in the preferences of real people: there is always some level of polarization, and different groups can sometimes vote strategically to increase the representation of their favored party/candidate or decrease the representation of their least desired party/candidate.

With this in mind, I wonder what outcomes might look like if there were no parties, no polarization in views, and no personal preference in voting. In a society where everyone was the same and shared the same values, there would be no incentive to vote strategically, and since the candidates running for election would also be the same as the people voting, I would anticipate that the preferences would be somewhat randomly calculated.

In this case, I hypothesize that outcomes from a computerized randomly generated set of preferences will closely resemble the preferences from real people that are similar in as many ways as possible. To compare outcomes, I will calculate the winners for three social welfare functions and three electoral systems for a computerized randomly generated group and again for a set of real people, and check for the variation in winners within each group as well as between both sets of data.

I anticipate a high degree of variation in winners within each group – if scores are randomized, no one candidate will stand out significantly more than the others win all of the systems or functions – and a low degree of variation between each group – I anticipate that the case study results will match the computerized results after controlling for personal bias. In addition to analyzing the results, I will also try to determine the best social welfare function or electoral system based on the results, if there is one.

Methods:

*Part 1: Computerized Preferences*

Dataset 1: In my theoretical society, there will be five candidates in the race for one elected position and fifteen voters. First, I will create a matrix where each voter randomly assigns each candidate a score between 0-100, thus using a score voting system. Using these scores, I will calculate social welfare outcomes through the Benthamite, Rawlsian, and Egalitarian winners. The Benthamite/Utilitarian winner will be calculated by finding the highest average score for each candidate. The Rawlsian winner will be calculated by first by finding the lowest score that each candidate received and then picking the candidate with the highest minimum score. This function picks the winner around the philosophy of picking a candidate that maximizes the utility of the least off member. The Egalitarian winner will be calculated first by finding the maximum score that each candidate received and subtracting the lowest score the candidate received from it – the candidate with the shortest distance between the maximum and minimum value will be the winner. This function picks the winner around the philosophy of picking the candidate that has the least polarized scores.

Dataset 2: Using the same numbers from Dataset 1, I will adjust the scores for intensity of preference by dividing each score by the total number of points allotted by each voter and multiplying by 100. The purpose of this dataset is to determine whether the vote proportionality could affect the social welfare outcomes. Essentially, the matrix will now demonstrate preferences such that each voter has 100 total points that must be distributed between the five candidates. I will recalculate winners for each of the social welfare functions. Next, I will calculate the electoral system winners based on Single Member Plurality, Borda Count, and Instant Runoff.

*Part 2: Case Study*

Dataset 3: In the second part of my code, I will create fake profiles for each of the five candidates and analyze preferences from real people after getting their scores through a google form. The characteristics of interest will include gender, age, race, children, birth month, favorite color, home state, and pets. The reason I chose these characteristics is because they are neural, uncontroversial features that should not provoke polarizing sentiments. The reason I did not add more than eight characteristics was because I wanted to make sure that the people in my sample set took the time to read through the candidate profiles and did not want to overload information onto them.

I will gather preferences from six women that are undergraduate students at Emory University (age 20-21), in Delta Phi Epsilon sorority, and were raised in the state of Georgia. Since I will only be getting preference scores from six people, it’s a small sample size for which the results will not be able to be generalized to a larger group; it will not have a significant p-value. However, by controlling for as many confounding variables as possible, the scores can be attributed more confidently to randomization rather than personal differences. In this case, I will essentially be controlling for age, location, occupation, and personality to some degree since they are all members of the same sorority, and therefore attempting to eliminate variability in personal biases in scoring. After getting information on their preferences through a google form, I will recalculate social welfare functions (Benthamite, Rawlsian, Egalitarian) and electoral systems (SMP, Borda Count, Instant Runoff) based on their preferences.

Results:

*Part 1: Computerized Scores*

In Dataset 1, there was a high degree of variability in social welfare outcomes. Candidate E was the winner for highest total sum and Utilitarian/Benthamite functions, but Candidate B was the winner for the Rawlsian function, and Candidate C was the winner for the Egalitarian function. Since each of the functions has an entirely different method of calculating a winner, the variability is not surprising.

In Dataset 2, after adjusting for intensity, all of the winners remained consistent with Dataset 1 except with the Egalitarian function, where Candidate B won instead of Candidate C. This result is also not surprising because adjusting for intensity significantly changes the value of the minimum and maximum scores, which are the only two numbers that the Egalitarian function takes into account. For example, when looking at minimum scores, voter 3 gave Candidate C a score of 3, which translated into a 1.08 after adjusting for intensity, but voter 14 gave Candidate A a score of 2, which translated to a 1.41 after adjusting for intensity. Although voter 14’s raw score had a lower value than voter 4’s, the value after adjusting for intensity was higher. On the other hand, when looking at maximum values, voter 6 gave Candidate B a score of 100, which translated into a 27.17 after adjusting for intensity, and voter 15 gave Candidate D a score of 94, which translated into a 41.05 after adjusting for intensity. Although voter 6’s raw score was 7 points higher in value than voter 15’s, the value after adjusting for intensity was 14 points lower than voter 15’s.

For the electoral system outcomes, Candidate E won in all three of the systems – SMP, Borda Count, and Instant Runoff. However, it is important to note that there was one instance where a voter gave two candidates the same score – voter 14 gave Candidate C and Candidate E a score of 42. Since 42 was also the highest score that the voter gave, I randomly picked Candidate C to receive the top vote for SMP and Borda count purposes. With this outcome, Candidate C was ranked first 1/15 times in the SMP results, and it’s Borda count score was a 4 while Candidate E’s score was 3. However, regardless of which candidate was picked to receive the top preferences, the results would not have changed. Even after being at a disadvantage due to losing the tie, Candidate E still won SMP with 5/15 of the vote (the runner up was Candidate A with 4/15 of the vote) and Borda count with a total score of 39 (the runner up was Candidate A with a total of 32). If Candidate E had received the extra point, it would only have further helped it win the election.

For Instant Runoff, since no candidate had the SMP majority, I went through two rounds of elimination: in the first round, Candidate C was eliminated since it only had 1/15 of the SMP vote, and it’s vote went to Candidate E. In the next round, Candidate B was eliminated since it only had 2/15 of the SMP majority. After this elimination, before Candidate E won by attaining the majority with 8/15 of the vote.

This process would have looked a little different if Candidate E had won the tie for top preference with voter 14 because Candidate C would have a SMP score of 0/15 and there would only be one round of elimination instead of two. Regardless of the tie outcomes, Candidate E wins.

*Part 2: Case Study*

In Dataset 3, there was very little variability with Candidate D being the winner for the highest total sum, Utilitarian/Benthamite, and Rawlsian function as well as all of the electoral systems. The only function that Candidate D did not win was the Egalitarian social welfare function, where Candidate A won. There was no need for an instant runoff in this case because Candidate D already had the SMP majority with 4/6 of the vote.

However, there is some variability in my electoral values due to ties. Voter 2 gave two candidates (A and C) a score of 80 and three candidates (B, D, and E) a score of 90; voter 4 gave two candidates (A and C) a score of 80. The only tie for top preference was with voter 2, and it was a three-way with a score of 90. In this case, I randomly picked Candidate B to receive the top preference, thus making its SMP score a 2/6. However, this choice had no implication on the results – even with the tie disadvantage, Candidate D still won with 4/6 of the vote. If Candidate E had received the top preference, it would have an SMP score of 1/16 and still lose to Candidate D. If Candidate E had received the top preference, it would have further help it win with 5/6 of the vote.

Since this was already a rather small sample set, I was hesitant to randomly assign a Borda count winner for three different ties like I did in Part 1 of my data since it had a greater potential to skew the winner than SMP. Instead, I used a different variation of Borda count where the maximum value that I assigned was the number of unique scores assigned by each voter minus 1. For example, since voter 2 only had two scores that were unique (80 and 90), I assigned a score of 1 to the candidates that received a score of 90 and a score of 0 to the candidates that received a score of 80. Essentially, people that received the same score from the same voter received the same Borda count score.

Analysis:

*Variability:*

If my hypothesis was correct, there would be a high degree of variation within each group (computerized and case study) and a low degree of variation between the groups.

For social welfare functions: in Dataset 1, there is a different winner for each social welfare function; in Dataset 2, two out of three functions have the same winner; in Dataset 3, two out of three functions have the same winner. Since no candidate is declared the winner in all of the social welfare functions in any of the datasets, I would say that my hypothesis was true.

For electoral system outcomes: in the computerized scores, Candidate E wins in all three systems; in the case study, Candidate D wins in all three systems. Since one candidate is declared the winner in both sets of electoral systems, there is no variability and I would say that my hypothesis was not satisfied.

Between the two groups, there is a similar pattern of winners. Dataset 2 and 3 both have one candidate win 2/3 social welfare functions, and both groups have one candidate that wins all of the electoral systems. The only area of difference is between Dataset 1 and 3 in social welfare outcomes, where Dataset 1 has a different winner for all functions and Dataset 3 has a different winner for one out of three functions. Because there is only one area of variation, I would say that my hypothesis is true in this case. Additionally, since there was never a case where one candidate won all of the social welfare functions, there is a higher degree of variation there compared to electoral systems.

*Computerized vs. Case Study*

In the case study, one thing that I noticed was that the scores were skewered within the higher range. The lowest score that any voter gave a candidate was a 30, and the average assigned score was a 73.16. On the other hand, the average assigned score for the computerized dataset was a 50.78 and it followed a normal distribution. I think this positive skew is a result of people being more forgiving when it comes to scoring people and a general sense of believing people are good. It could also be a result of a confounding characteristic that they share due to same values.

There was also a higher propensity for voters in the case study to create ties between candidates – in the case study with 6 voters, there was one three-way tie and two two-way ties, and with the computerized dataset with 15 voters, there was only one two-way tie. However, as explained in the results, none of these ties had the potential to alter the winner.

*Is there a best function or system?*

Based on these results, it is difficult to declare one system or function as the best. In my hypothetical society where every voter is the same and has the same values, the Utilitarian/Benthamite function might be the best function just because there is no polarization and having the highest average score means that the average utility that the population would gain is the highest.

However, in a deeply polarized society, this might not be the best function because it could be the case that half the population has an extremely high utility and half the population has an extremely low utility, and the average score can be misleading. In this situation, I think the Egalitarian function would be the best function because it would try to maximize utility by finding the least polarizing candidate. However, one downfall of the calculation method that I used is that if all of the voters gave one Candidate a 0, the distance between the maximum and minimum value would be 0 and that Candidate would win the election. Therefore, a candidate having consistently low scores could win using this function because the difference between the maximum and minimum values would be the smallest.

It is comforting that both groups produced the same winner for all of the electoral systems being tested because it means that winners are selected based on consistently being the best instead of by chance of which system was being used. However, since all of them produced the same winner, it is difficult to say which system worked the best because they all resulted in the same outcome.

While there may not be one function or system that works the best under all circumstances, one function that I think is the least helpful is the Rawlsian function. While it is meant to maximize the utility of the least well-off voter, there can always be a better function to select a winner, whether the population is polarized, seeks representation from smaller parties, or has some other agenda in mind. All that it takes for a candidate to win in this situation is having the highest minimum score, which means that if a candidate gets a score of 3 out of 100 from every voter but the minimum value for every other candidate is a 2, that candidate would win the election even though its average score is a 3 out of 100 and every other candidate is higher.

*Axiomatic analysis:*

Based on the axioms detailed under Arrow’s Theorem, the score voting system violates universal/unrestricted domain because voters sometimes cannot give two or more candidates the same score at all since it would pose a problem during Borda count, they cannot give two or more candidates the same highest score because it would pose a problem during SMP, and they cannot give two or more candidates the same lowest score because it may pose a problem when calculating the Rawlsian winner. Dataset 1 and 3 do not violate IIA because an increase in one candidate’s score would not affect any other candidate, but Dataset 2 does violate IIA because an increase in one candidate’s score leads to a decrease in another candidate’s score since values are proportional. The Egalitarian social welfare function violates Pareto because every single voter could give one candidate a score of 0 and another candidate their highest score, but the candidate receiving a 0 would win because the distance between the maximum and the minimum would be zero.

Extension:

One way in which I could make a stronger argument in this project is by extending my datasets. In Dataset 1 and 2, I could create a loop, randomly generate scores several times, and then use the average scores as my final dataset. With a larger sample size, the p-value would be smaller and more generalizable. Similarly, I could try to expand my sample size in Dataset 3 for the same purposes.

Alternatively, I could conduct an experiment in my case study where I have a control group of people selected without controlling for personal biases and a treatment group that controls for as many demographic and personal features as possible. I could give them both the candidate profiles, ask them to score the candidates, and then compare the results between both groups to determine the effect of personal biases in randomized candidate features. Taking this further, I could also create a candidate profile that included politically controversial stances and measure the effect in scores between the two groups.

Conclusion:

To conclude, the degree of variation in winners is higher for social welfare functions compared to electoral systems, and the variation in winners between computerized random scores and a sample of people with similar personal backgrounds is small. Different functions and systems work better with different populations, which makes it difficult to determine the best method of picking a winner.

Appendix A: Dataset 1 A screenshot of a cell phone

Description automatically generated

Appendix B: Dataset 2

A close up of text on a white background

Description automatically generated

Appendix C: Part 1 Borda Count Ranked Preferences

A screenshot of a cell phone

Description automatically generated

Appendix D: Fake candidate profiles

A screenshot of a cell phone

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Appendix E: Dataset 3

A screenshot of a cell phone

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Appendix F: Part 2 Borda Count Ranked Preferences

A screenshot of a cell phone

Description automatically generated