1(a) $[\ ] \vdash A \rightarrow (B \rightarrow (B \land A))$

$$\dfrac{\dfrac{B \in [A, B]}{[A, B] \vdash B} \text{Ass} \quad \dfrac{A \in [A, B]}{[A, B] \vdash A} \text{Ass.}}{\dfrac{[A, B] \vdash B \land A}{\dfrac{[A] \vdash B \rightarrow (B \land A)}{[\ ] \vdash A \rightarrow (B \rightarrow (B \land A))} \rightarrow\text{-Intro}} \rightarrow\text{-Intro}} \land\text{-Intro}$$

1(b) $[\,] \vdash (A \vee A) \rightarrow A$

$$\dfrac{\dfrac{A \in [A \vee A]}{[A \vee A] \vdash A \vee A} \text{ Ass.} \quad \dfrac{A \in [A \vee A, A]}{[A \vee A, A] \vdash A} \text{ Ass.} \quad \dfrac{A \in [A \vee A, A]}{[A \vee A, A] \vdash A} \text{ Ass.}}{\dfrac{\dfrac{\qquad\qquad}{[A \vee A] \vdash A} \text{ } \vee\text{-Elim}}{[\,] \vdash (A \vee A) \rightarrow A} \rightarrow \text{Intro}}$$

1(c) $[A] \vdash (A \rightarrow B) \rightarrow B$

$$\dfrac{\dfrac{\dfrac{A \in [A, A \rightarrow B]}{[A, A \rightarrow B] \vdash A} \text{ Ass.} \qquad \dfrac{A \rightarrow B \in [A, A \rightarrow B]}{[A, A \rightarrow B] \vdash A \rightarrow B} \text{ Ass.}}{[A, A \rightarrow B] \vdash B} \rightarrow \text{Elim}}{[A] \vdash (A \rightarrow B) \rightarrow B} \rightarrow \text{Intro}$$

② (a) $[ ] \vdash \backslash x.\ \backslash y.\ (y, x) : A \rightarrow (B \rightarrow (B, A))$

$$\frac{[x:A, y:B](y) = B}{[x:A, y:B] \vdash y : B}\ var \qquad \frac{\dfrac{[x:A, y:B](x) = A}{[x:A, y:B] \vdash x : A}\ var}{}$$

$$\frac{[x:A, y:B] \vdash (y, x) : (B, A)}{[x:A] \vdash \lambda y.\ (y, x) : B \rightarrow (B, A)}\ lambda$$

$$\frac{[x:A] \vdash \lambda y.\ (y, x) : B \rightarrow (B, A)}{[ ] \vdash \lambda x \lambda y\ (y, x) : A \rightarrow (B \rightarrow (B, A))}\ lambda$$

2(b)

$$\dfrac{[X:(A|A)](x)=(A|A)}{[x:(A|A)]\vdash x:(A|A)}\text{ var}\qquad\dfrac{[x:(A|A),y:A](y)=A}{[x:(A|A),y:A]\vdash y:A}\text{ var}\qquad\dfrac{[x:(A|A),z:A](z)=A}{[x:(A|A),z:A]\vdash z:A}\text{ var}$$

$$\dfrac{\rule{0pt}{1em}}{[X:(A|A)]\vdash \text{case } x \text{ of }\{\text{left }y\to y;\ \text{right }z\to z\}:A}\text{ case}$$

$$\dfrac{\rule{0pt}{1em}}{[\ ]\vdash \lambda x.\ \text{case } x \text{ of }\{\text{left }y\to y;\ \text{right }z\to z\}:(A|A)\to A}\text{ lambda}$$

2(c) $[x:A] \vdash \lambda f\, f\, x : (A \to B) \to B$

$$\cfrac{\cfrac{[x:A,\, f:(A\to B)](f) = (A\to B)}{[x:A,\, f:(A\to B)] \vdash f:(A\to B)}\ var \qquad \cfrac{[x:A,\, f:(A\to B)](x) = A}{[x:A,\, f:(A\to B)] \vdash x:(A\to B)}\ var}{\cfrac{[x:A,\, f:(A\to B)] \vdash f\, x : B}{[x:A] \vdash \lambda f\, f\, x : (A\to B) \to B}\ lambda}\ App.$$