PES University, Bangalore

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APRIL 2021: IN SEMESTER ASSESSMENT (ISA) B.TECH. IV SEMESTER

UE19MA251- LINEAR ALGEBRA

Mathematics Lab

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Marks : /05

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Signature of the Course Instructor : ______________________________

Topic 1: Gaussian Elimination

1. Solve the following system of equations by Gaussian Elimination. Identify the pivots $2x + 5y + z = 0$, $4x + 8y + z = 2$, $y - z = 3$

Ans.

1.sce

```
clc;
clear;
close;
A=[2,5,1;4,8,1;0,1,-1], b=[0;2;3]
A_aug=[A b]
a=A_aug
n=3

for i=2:n
    for j=2:n+1
        a(i,j)=a(i,j)-a(1,j)*a(i,1)/a(1,1);
end
a(i,1)=0;
end
for i=3:n
    for j=3:n+1
        a(i,j)=a(i,j)-a(2,j)*a(i,2)/a(2,2);
    end
    a(i,2)=0;
end
x(n)=a(n,n+1)/a(n,n);
for i=n-1:-1:1
    sumk=0;
    for k=i+1:n
        sumk=sumk+a(i,k)*x(k);
    end
    x(i)=(a(i,n+1)-sumk)/a(i,i);
end
disp(x(3),x(2),x(1),'The values of x,y,z are ');
disp(a(1,1),a(2,2),a(3,3),'The pivots are');
```

OUTPUT

```
 The values of x,y,z are

   0.5

   0.3333333

  -2.6666667

 The pivots are

  -1.5

  -2.

   2.

-->
```

Topic 2: LU decomposition of a matrix

1. Factorize the following matrices as A = LU

$$A=\begin{pmatrix} 2 & 3 & 1 \\ 4 & 7 & 5 \\ 1 & -2 & 2 \end{pmatrix}$$

Ans.

2a.sce 2b.sce

```
clc;
clear;
close;
A = [2,3,1;4,7,5;1,-2,2];
U=A;
disp(A,'The given matrix is A=');
m=det(U(1,1));
n=det(U(2,1));
a=n/m;
U(2,:)=U(2,:)-U(1,:)/(m/n);
n=det(U(3,1));
b=n/m;
U(3,:)=U(3,:)-U(1,:)/(m/n);
m=det(U(2,2));
n=det(U(3,2));
c=n/m;
U(3,:)=U(3,:)-U(2,:)/(m/n);
disp(U,'The upper triangular matrux is U =');
L=[1,0,0;a,1,0;b,c,1];
disp(L,'The lower triangular matrix is L =');
```

OUTPUT

```
The given matrix is A=

  2.   3.   1.
  4.   7.   5.
  1.  -2.   2.

The upper triangular matrux is U =

  2.   3.   1.
  0.   1.   3.
  0.   0.   12.

The lower triangular matrix is L =

  1.    0.    0.
  2.    1.    0.
  0.5  -3.5   1.

-->
```

2. Solve the system of equations by decomposing A as a product A = LU
   $2x + 3y + z = 8$, $4x + 7y + 5z = 20$, $-2y + 2z = 0$

Ans.

2a.sce 2b.sce

```
clc;
clear;
close;
format('v',5);
A=[2,3,1;4,7,5;0,-2,2];
for l=1:3
    L(l,l)=1;
end
for i=1:3
    for j=1:3
        s=0;
        if j>=i
            for k=1:i-1
                s=s+L(i,k)*U(k,j);
            end
            U(i,j)=A(i,j)-s;
        else
            for k=1:j-1
                s=s+L(i,k)*U(k,j);
            end
            L(i,j)=(A(i,j)-s)/U(j,j);
        end
    end
end
b=[8;20;0];
c=L\b;
x=U\c;
disp(x,'Solution of the given equations is :');
```

OUTPUT

```
 Solution of the given equations is :

   2.
   1.
   1.

-->
```

Topic 3: The Gauss - Jordan method of calculating A^{-1}

1. Find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Ans.

3.sce

```
clc;
clear;
close;
A=[1,0,0;1,1,1;0,0,1];
n=length(A(1,:));
Aug=[A,eye(n,n)];
for j=1:n-1
    for i=j+1:n
        Aug(i,j:2*n)=Aug(i,j:2*n)-Aug(i,j)/Aug(j,j)*Aug(j,j:2*n);
    end
end
for j=n:-1:2
    Aug(1:j-1,:)=Aug(1:j-1,:)-Aug(1:j-1,j)/Aug(j,j)*Aug(j,:);
end
for j=1:n
    Aug(j,:)=Aug(j,:)/Aug(j,j);
end
B=Aug(:,n+1:2*n);
disp(B,'The inverse of A is');

```

<u>OUTPUT</u>

```
 The inverse of A is

   1.    0.    0.
  -1.    1.   -1.
   0.    0.    1.

-->
```

Topic 4: Span of the Column Space of A

1. Identify the columns that span the column space of A

$$A=\begin{pmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{pmatrix}$$

Ans.

4.sce

```
clc;
clear;
close;
disp('The given matrix is ');
a=[1,3,3,2;2,6,9,7;-1,-3,3,4];
disp(a);
a(2,:)=a(2,:)-(a(2,1)/a(1,1))*a(1,:);
a(3,:)=a(3,:)-(a(3,1)/a(1,1))*a(1,:);
disp(a);
a(1,:)=a(3,:)-(a(3,2)/a(2,3))*a(2,:);
disp(a);
for i=1:3
    for j=i:4
        if(a(i,j)<>0)
            disp('is a pivot column',j,'column')
            break
        end
    end
end
```

<u>OUTPUT</u>

```
The given matrix is

   1.    3.    3.    2.
   2.    6.    9.    7.
  -1.   -3.    3.    4.

   1.    3.    3.    2.
   0.    0.    3.    3.
   0.    0.    6.    6.

   0.    0.    6.    6.
   0.    0.    3.    3.
   0.    0.    6.    6.

column

   3.

is a pivot column

column

   3.

is a pivot column

column

   3.

is a pivot column

-->
```

Topic 5: The Four Fundamental Subspaces

1. Find the four fundamental subspaces of

$$A=\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

Ans.

5.sce

```
clc;
clear;
close;
A=[1,2,0,1;0,1,1,0;1,2,0,1];
disp(A,'A=');
[m,n]=size(A);
disp(m,'m=');
disp(n,'n=');
[v,pivot]=rref(A);
disp(rref(A));
disp(v);
r=length(pivot);
disp(r,'rank=');
cs=A(:,pivot);
disp(cs,'Column Space=');
ns=kernel(A);
disp(ns,'Null Space=');
rs=v(1:r,:)';
disp(rs,'Row Space=');
lns=kernel(A');
disp(lns,'Left Null Space=');

```

<u>OUTPUT</u>

```
A=

  1.   2.   0.   1.
  0.   1.   1.   0.
  1.   2.   0.   1.

m=

  3.

n=

  4.

  1.   0.  -2.   1.
  0.   1.   1.   0.
  0.   0.   0.   0.

  1.   0.  -2.   1.
  0.   1.   1.   0.
  0.   0.   0.   0.

rank=

  2.

Column Space=

  1.   2.
  0.   1.
  1.   2.

Null Space=

  0.          -0.8660254
 -0.4082483    0.2886751
  0.4082483   -0.2886751
  0.8164966    0.2886751

Row Space=

  1.   0.
  0.   1.
 -2.   1.
  1.   0.

Left Null Space=

 -0.7071068
  1.106D-16
  0.7071068

-->
```

Topic 6: Projections by Least Squares

1. Find the solution x = (C , D) of the system Ax = b and the line of best fit C + Dt = b given

$$A=\begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \quad b=(1,\,1,\,3)$$

Ans.

6.sce

```
clc;
clear;
close;
A=[1 -1;1 1;1 2];
disp(A,'A=');
b=[1;1;3];
disp(b,'b=');
x=(A'*A)\(A'*b);
disp(x,'x=');
C=x(1,1);
D=x(2,1);
disp(C,'C=');
disp(D,'D=');
disp('The line of best fit is b=C+Dt');
```

OUTPUT

```
A=

  1.  -1.
  1.   1.
  1.   2.

b=

  1.
  1.
  3.

x=

  1.2857143
  0.5714286

C=

  1.2857143

D=

  0.5714286

The line of best fit is b=C+Dt

-->
```

Topic 7: The Gram - Schmidt Orthogonalization

1. Apply the Gram – Schmidt process to the following set of vectors and find the orthogonal matrix:

(1, 1, 0) , (1, 0, 1) , (0, 1, 1)

Ans.

7.sce

```
clc;
clear;
close;
A=[1,1,0;1,0,1;0,1,1];
disp(A,'A=');
[m,n]=size(A);
for k=1:n
    V(:,k)=A(:,k);
    for j=1:k-1
        R(j,k)=V(:,j)'*A(:,k);
        V(:,k)=V(:,k)-R(j,k)*V(:,j);
    end
    R(k,k)=norm(V(:,k));
    V(:,k)=V(:,k)/R(k,k);
end
disp(V,'Q=');
```

<u>OUTPUT</u>

```
 A=

   1.   1.   0.
   1.   0.   1.
   0.   1.   1.

 Q=

   0.7071068   0.4082483  -0.5773503
   0.7071068  -0.4082483   0.5773503
   0.          0.8164966   0.5773503

-->
```

Topic 8: Eigen values and Eigen vectors of a given square matrix

1. Find the Eigen values and the corresponding Eigen vectors of the following matrix

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

Ans.

8.sce

```
clc;
clear;
close;
A=[8,-6,2;-6,7,-4;2,-4,3];
lam=poly(0,'lam');
lam=lam;
charMat=A-lam*eye(3,3);
disp(charMat,'The characteristic Matrix is');
charPoly=poly(A,'lam');
disp(charPoly,'The characteristic Ploynomial is');
lam=spec(A);
disp(lam,'the eigen values of A are');
function[x,lam]=eigenvectors(A)
    [n,m]=size(A);
    lam=spec(A)';
    x=[];
    for k=1:3
        B=A-lam(k)*eye(3,3);
        C=B(1:n-1,1:n-1);
        b=-B(1:n-1,n);
        y=C\b;
        y=[y;1];
        y=y/norm(y);
        x=[x,y];
    end
endfunction
[x,lam]=eigenvectors(A);
disp(x,'The eigen vectors of A are');
```

OUTPUT

```
The characteristic Matrix is


  8 -lam  -6        2


 -6        7 -lam  -4


  2       -4        3 -lam


The characteristic Ploynomial is

                         2     3
 -7.128D-14 +45lam -18lam  +lam

the eigen values of A are

  1.584D-15
  3.
  15.

The eigen vectors of A are

  0.3333333  -0.6666667   0.6666667
  0.6666667  -0.3333333  -0.6666667
  0.6666667   0.6666667   0.3333333

-->
```