# Some mathematical logics

**Netural number:** A **natural number** is a positive integer.depending on context **natural number**:

**Without 0**: natural numbers = {1, 2, 3, ...}
**With 0**: natural numbers = {0, 1, 2, 3, ...}

**Even number:** An **even number** is any integer that is exactly divisible by 2. In other words, when you divide an even number by 2, there is no remainder. An even number can be expressed as **2 × n**, where **n** is any integer.

n mod 2 = 0

**Odd number:** An **odd number** is an integer that **cannot be evenly divided by 2**. When divided by 2, it leaves a remainder of 1.

n mod 2 = 1

**Prime number:** A **prime number** is a natural number greater than 1 that has **exactly two distinct positive divisors**. 1 and itself.

**Key Properties:**

- It **cannot** be formed by multiplying two smaller natural numbers (except 1 and itself).
- The number **1 is *not* prime**.
- The **smallest prime number is 2**, which is also the **only even prime**.

**Factor of given number:** A **factor** (or divisor) of a number is an integer that **divides the number exactly** (with no remainder).

**For example:**
**Factors of 12** are:

- 1, 2, 3, 4, 6, 12
  (because all of these divide 12 without leaving a remainder)

**How to find factors of a number:**

1. Start from 1 and go up to the number itself.
2. Check which numbers divide the given number exactly (i.e., number % i == 0).

**Factorial :** A **factorial** (denoted by n!) is the product of all positive integers from 1 to n.

**Definition:**
$n! = n \times (n-1) \times (n-2) \times \ldots \times 1$

**Special case:**
0!=1(by definition)
**Examples:**

- 1!= 1
- 2!=2×1=2
- 3!=3×2×1=6
- 5!=5×4×3×2×1=120

**Armstrongs:** An **Armstrong number** (also known as a **narcissistic number**) is a number that is **equal to the sum of its own digits each raised to the power of the number of digits**.

**General Rule:**
**For an *n*-digit number:**
**Armstrong number⇒abcd…=$a^{number\_of\_digit}$+$b^{number\_of\_digit}$+$c^{number\_of\_digit}$+$d^{number\_of\_digit}$+… Examples:**

- **153**
  It's a 3-digit number:
  13+53+33=1+125+27=1531^3 + 5^3 + 3^3 = 1 + 125 + 27 = 15313+53+33=1+125+27=153
- **9474**
  It's a 4-digit number:
  94+44+74+44=6561+256+2401+256=94749^4 + 4^4 + 7^4 + 4^4 = 6561 + 256 + 2401 + 256 = 947494+44+74+44=6561+256+2401+256=9474
- **370, 371, 407** are also 3-digit Armstrong numbers.

**Palindrome:** A **palindrome** is a number, word, phrase, or sequence that **reads the same forward and backward**.

**For numbers:**

A **palindromic number** stays the same when its digits are reversed.

- **121** → reversed is 121
- **1331** → reversed is 1331
- **123** → reversed is 321 (not a palindrome)

**For words:**

- **"madam"**, **"racecar"**, **"level"** are all palindromes.

For phrases (ignoring spaces and punctuation):

- **"A man, a plan, a canal, Panama"**

**LCM:** **LCM** stands for **Least Common Multiple** — the **smallest multiple** that two or more numbers **share in common**.

**Example:**

Find the LCM of **4** and **6**:

- Multiples of 4: 4, 8, 12, 16, ...
- Multiples of 6: 6, 12, 18, 24, ...
- **LCM = 12**

**How to find LCM:**

1. **Listing multiples** (as above) — good for small numbers.
2. **Prime factorization** — multiply highest powers of all primes involved.
3. **Using formula**:
   $LCM(a,b)=|a\times b| / GCD(a,b)$

**HCF:** **HCF** stands for **Highest Common Factor**, also known as the **Greatest Common Divisor (GCD)**. It is the largest number that divides two or more numbers exactly (without leaving a remainder).

**Example:**

Find the HCF of **12** and **15**:

- Factors of 12: 1, 2, 3, 4, 6, 12
- Factors of 15: 1, 3, 5, 15
- **HCF = 3**

**How to find HCF:**

1. **Listing common factors**: Find the common factors of the numbers and pick the largest.
2. **Prime factorization**: Find the prime factorization of both numbers, then multiply the smallest powers of the common prime factors.
3. **Using the formula**:
   $HCF(a,b)=|a\times b| / LCM(a,b)$

**Fibonacci:--**

The **Fibonacci series** is a sequence of numbers in which each number is the sum of the two preceding ones. It starts like this:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

**Harshad Numbers:**

A **Harshad number** (also known as a **Niven number**) is a number that is **divisible by the sum of its digits**. The term **Harshad** is derived from the Sanskrit words "Har" (joy) and "Shad" (give), meaning "giver of joy.

- **18** → Sum of digits = **1 + 8 = 9** → **18 is divisible by 9**
- **21** → Sum of digits = **2 + 1 = 3** → **21 is divisible by 3**
- **19** → Sum of digits = **1 + 9 = 10** → **19 is NOT divisible by 10**

**Anagrams Number:**

An **anagram** is a word or phrase formed by rearranging the letters of another word or phrase, using all the original letters exactly once

- **"listen"** → **"silent"**
- **"race"** → **"care"**
- **"evil"** → **"vile"**
- **"dormitory"** → **"dirty room"** (ignoring spaces)

**Neon Number:**

A **Neon Number** is a number where the sum of the digits of its square is equal to the original number.

- **9** → Square = **9 × 9 = 81**
  - Sum of digits of 81 → **8 + 1 = 9**
- **12** → Square = **12 × 12 = 144**
  - Sum of digits of 144 → **1 + 4 + 4 = 9**

**Peterson Numbers:**

A **Peterson number** is a number where the sum of the factorials of its digits equals the number itself.

1. **145**
   - Digits: **1, 4, 5**
   - Factorial Sum: **1! + 4! + 5! = 1 + 24 + 120 = 145**
2. **Other Peterson Numbers**: **1, 2, 145** (There are very few!)

**Spy Numbers**

A **Spy Number** is a number where the **sum of its digits** is equal to the **product of its digits**

1. **112**
   - Digits: **1, 1, 2**
   - Sum = **1 + 1 + 2 = 4**
   - Product = **1 × 1 × 2 = 4**
2. **123**
   - Digits: **1, 2, 3**

- Sum = **1 + 2 + 3 = 6**
- Product = **1 × 2 × 3 = 6**

**<u>Sunny number</u>**

A *sunny number* is a number that is one less than a perfect square. In other words, a number N is sunny if there exists an integer n such that:

$N+1=n^2$

For example:

- 3 is a sunny number because 3+1=4, and 4 is a perfect square (since 2^2=4).
- 8 is another sunny number because 8+1=9, and 9 is a perfect square (since 3^2 = 9)

**<u>Leap year:</u>** A **leap year** is a year that has **366 days** instead of the usual 365. The extra day is added to **February**, making it **29 days long** instead of 28.

**Rules to determine a leap year:**

A year is a leap year if:

- It is **divisible by 4**,
  **but not divisible by 100**,
  **unless it is also divisible by 400**.

**In short:**

- **Leap year:** 2000, 2016, 2020, 2024
- **Not a leap year:** 1900, 2100 (divisible by 100 but not by 400)

**Examples:**

- **2024** is a leap year → divisible by 4, not by 100
- **1900** is **not** a leap year → divisible by 100, but not by 400
- **2000** is a leap year → divisible by 400

| **Leep year-Concept** | | | | |
|---|---|---|---|---|
| **Year** | **Day** | **hour** | **minutes** | **seconds** |
| 1-Year | 365 | 5 | 48 | 47.5 |
| 2-Year | 365 | 5 | 48 | 47.5 |
| 3-Year | 365 | 5 | 48 | 47.5 |
| 4-Year | 365 | 5 | 48 | 47.5 |
| | **Remain times** | 20 | 192 | 190 |
| | | | 195/60=3h,15m | 190/6= 3min,10sec |
| | Approx 1-day which is added in | **23** | **15** | **10** |

| | | | | |
|---|---|---|---|---|
| | every 4years | Add some time | | |
| leep year | 366 | | | |
| | | | | |
| | negative time in leep year(-) | extra added time | 44 | 50 |
| | | | | |
| 100 year | | | 44*25=1100 | 50*25=1250 |
| | | | | 1250/60 |
| | | | 1120/60 | 20 minutes, 50 sec |
| | | | 18hours,40 minutes | |
| | | 18 | 40 | 50 |
| 100-years | 1-day remove in feb month | Not a Leep year | | |
| | | 5 | 19 | 10 |
| | | | | |
| | **( approx 6hour behind in 100th year)** | **5** | **19** | **10** |
| | **( approx 6hour behind in 100th year)** | **5** | **19** | **10** |
| | **( approx 6hour behind in 100th year)** | **5** | **19** | **10** |
| | | **20** | **76** | **40** |
| **400 years** | **Now add 1-day in 400th year** | **21** | **16** | **40** |
| | | **That's why it is leep year** | | |
| | | **2** | **44** | **20** |