Time inconsistency and non-stationary instantaneous utility: empirical evidence from rural Malawi

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Outline

Empirical evidence of time inconsistency

- Which option do you prefer?
  A) £100 today
  B) £110 next week
- Which option do you prefer?
  C) £100 one year from now
  D) £110 one year and a week from now

Most people choose $A \succ B$ and $D \succ C$, providing evidence of **time inconsistency**: a person's relative preference for well-being at an earlier date over a later date changes according to when she is asked.

This behaviour has been consistently detected in humans, rats and pigeons. (Ainslie, 1974)

- Time inconsistency is a puzzling result as it is in contrast with the predictions of the **discounted utility framework**[1] (DU).
- Most of the DU assumptions are very restricting and have been deeply falsified by empirical evidence.
- Nevertheless, the model is still very popular because of its simplicity, elegance and tractability.

[1] Samuelson, Paul A. "A note on measurement of utility." The review of economic studies 4.2 (1937): 155-161.

DU framework in a nutshell

- Consider an economy that lasts $t = 1, \ldots, T$ periods.
- The decision maker has preferences over consumption profiles $c_t = (c_{t,t}, c_{t+1,t}, \ldots, c_{T,t})$, where $c_{t,s}$ denotes the level of consumption in period t from period s' perspective, with $s, t \in \mathcal{T} = \{1, \ldots, T\}$.
- The agent has an initial endowment s_0.
- Postponing consumption to the next period gives a net return $r > 0$.

DU framework in a nutshell

- Under completeness, transitivity and continuity the preferences over consumption profiles can be represented by an intertemporal utility function:

$$U^t(c_t, \ldots, c_T) = \sum_{k=0}^{T-t} \delta^k u(c_{t+k})$$

- At each period t, the player chooses the optimal consumption profile $c_t^* = (c_{t,t}^*, c_{t+1,t}^*, \ldots, c_{T,t}^*)$ by maximising her intertemporal utility function.
- The initial optimal consumption plan is optimal for all the subsequent periods: $c_{t,s}^* = c_{t,t}^* \forall s, t \in \mathcal{T}$.

Intertemporal choice and DU model

DU model is based on the following set of assumptions:

1. Integration of new alternatives with existing plans
2. Utility independence
3. Consumption independence
4. Stationary instantaneous utility
5. Independence of discounting from consumption
6. Constant discounting and time consistency
7. Diminishing marginal utility and positive time preference

Intertemporal choice and DU model

Almost each assumption is usually violated by the empirical evidence:

1. Limited ability of intertemporal reoptimization
2. Habit formation
3. Preference for spread
4. State-dependent preferences
5. Labeled discount factors
6. Time inconsistency and present bias

- Literature usually explains time inconsistency by relaxing the DU assumption of constant discounting in favour of hyperbolic discounting: a person has a declining rate of time preferences.
- This idea has both sociological and psychological justifications.
- The most famous model of hyperbolic discounting is the (β, δ)[2].
- However, it is unclear why the psychological motives should modify the discount factor rather then the utility function.
- The same phenomenon (and maybe more) can be explained by relaxing the assumption of stationary instantaneous utility.

[2] Laibson, David. "Self-control and saving." Massachusetts Institute of Technology mimeo (1994).

Alternatives to the DU framework

- **Standard DU model**:

$$U^t(c_t, \ldots, c_T) = \sum_{k=0}^{T-t} \delta^k u(c_{t+k})$$

- (β, δ)-**preferences**:

$$U^t(c_t, \ldots, c_T) = \delta u(c_t) + \beta \sum_{k=t+1}^{T} \delta^k u(c_k)$$

- **Dynamic utility**:

$$U^t(c_t, \ldots, c_T) = \sum_{k=0}^{T-t} \delta^k u_t(c_{t+k})$$

Dynamic preferences' update

- My model provides an alternative set-up to deal with time inconsistency by relaxing the assumption of stationary instantaneous utility.
- The novelty consists in the introduction of **law of motion** for the utility function relying on this semi-parametric assumption:

$$U^t(c_t, \dots, c_T) = \sum_{k=0}^{T-t} \delta^k [u(c_{t+k})]^{\beta_t}$$

- The semi-parametric approach allows to understand the main intuition of the phenomenon with a minimal divergence from the DU framework.
- All the other assumptions of the DU framework are retained.

The concavifying parameter

- β_t is an **unexpected** shock to the player's elasticity of intertemporal substitution at time t.
- The agent is **naive**: at any period, $E_t(\beta_t) = 1$.
- We remain agnostic on the reasons why β_t arises.
- β_t has the following law of motion:

$$\beta_t = \begin{cases} 1 & \text{if } t = 1 \\ x_t & \text{if } t > 1 \end{cases}$$

$$x_t = \begin{cases} \beta_t^H \geq 1 & \text{w.p. } \theta \\ 0 < \beta_t^L < 1 & \text{w.p. } (1-\theta) \end{cases}$$

with $\theta \in [0, 1]$. Draws are independent over time.

Characterisation of the β-transformation

Let us consider the following items:

- a consumption set $C \subseteq \mathbb{R}_+$
- a C^2 utility function $u : C \to \mathbb{R}_+$, with $u'(c) \geq 0$ and $u''(c) \leq 0$
- a real number $\beta > 0$
- a function $v : C \times \mathbb{R} \to \mathbb{R} : v(c, \beta) = u(c)^\beta$.

Some restrictions on β must be imposed so that $v(c)$ still represents convex preferences. In particular, the following is true:

Proposition 1

The maximum shock each agent can tolerate while retaining convex preferences is measured by the **resilience parameter** $\hat{\beta} = 1 - \frac{u''(c) \cdot u(c)}{u'(c)^2}$.

Characterisation of the β-transformation

Proof.

Consider $v(c) = u(c)^{\beta}$. This function represents convex preferences iff $v''(c) \leq 0$:

$$v''(c) = \beta u(c)^{(\beta-1)} \cdot \left[u''(c) + (\beta - 1)\frac{u'(c)^2}{u(c)} \right] \leq 0$$

$$\Rightarrow \beta \in (0, \hat{\beta})$$

where $\hat{\beta} = 1 - \frac{u''(c) \cdot u(c)}{u'(c)^2}$. $\square$

The resilience parameter

- $\hat{\beta}$ depends on both the value of c and the shape of u. People can have different values of $\hat{\beta}$ because either their preferences are represented by different utility functions or because they have the same utility function but made different choices.
- the larger the interval $(1, \hat{\beta})$, the more likely the individual is to retain convex preferences after a big shock. When making a choice of c under u, the decision maker implicitly determines the maximum shock she can tolerate while keeping standard behaviour.
- Since $\hat{\beta}$ depends on c, $\hat{\beta}$ will generally be time-dependent in the intertemporal choice framework.

The resilience parameter

Focus on the second term of $\hat{\beta}$:

$$\frac{u''(c) \cdot u(c)}{u'(c)^2} = \frac{u''(c)}{u'(c)} \cdot \frac{u(c)}{u'(c)}$$

- $\frac{u''(c)}{u'(c)} = \frac{d \log(u'(c))}{dc}$: percentage change in marginal utility
- $\frac{u'(c)}{u(c)} = \frac{d \log(u(c))}{dc}$: percentage change in level utility

The term measures the **elasticity of the marginal utility with respect to the level utility**. So, it evaluates how much the power transformation can bend the utility function before making it linear.

Evolution of the EIS

- We expect the β-transformation to modify the elasticity of intertemporal substitution (EIS), which we denote as $\gamma(c_t; \beta_t)$.
- $\gamma(c_1; 1) = \gamma_1$ denotes the EIS at $t = 1$, *before* any β-transformation.
- $\gamma(c_t; \beta_t) = \gamma_t$ with $t \neq 1$ denotes the EIS of the instantaneous utility at time t *after* the transformation.
- Our goal is to study the behaviour of γ_t assuming $\beta_t \in (0, \hat{\beta_t})$.

Evolution of the EIS

- The abstract definition of elasticity of intertemporal substitution[3] is:

$$\gamma_v = -\frac{v'(c)}{c \cdot v''(c)} \tag{1}$$

where $v'(c)$ and $v''(c)$ denote the first and the second order derivative of v evaluated at point c, respectively.

- Since $v(c, \beta) = u(c)^{\beta}$, (1) is equivalent to:

$$\gamma_v = -\frac{u'(c)}{c\left[u''(c) + (\beta - 1) \cdot \frac{u'(c)^2}{u(c)}\right]} \tag{2}$$

- Notice that $\beta < \hat{\beta} \Rightarrow \gamma_v \geq 0$ and $\lim_{\beta \to \hat{\beta}} \gamma_v = \infty$.

[3] Hall, Robert E. "Intertemporal Substitution in Consumption." Journal of Political Economy 96, no. 2 (1988): 339–57.

Evolution of the EIS

Proposition 2

Assume $v(c) = u(c)^\beta$ *and* $\beta \in (0, \hat{\beta})$. *Then* $\gamma(c, \beta)$ *is increasing in* β.

Proof.

We want to show that for any pair (β_1, β_2) and for any $c \in C$, $\beta_1 < \beta_2 \Rightarrow \gamma(c, \beta_1) \leq \gamma(c, \beta_2)$. Let us assume by contradiction this is not the case: $\exists c \in C$ and a pair (β_1, β_2) with $\beta_1 < \beta_2$ such that $\gamma(c, \beta_1) > \gamma(c, \beta_2)$. From Equation (2), this means:

$$-\frac{u'(c)}{c\left[u''(c) + (\beta_1 - 1) \cdot \frac{u'(c)^2}{u(c)}\right]} > -\frac{u'(c)}{c\left[u''(c) + (\beta_2 - 1) \cdot \frac{u'(c)^2}{u(c)}\right]} \tag{3}$$

Using the fact that $\beta_1 \in (0, \hat{\beta})$ and $\beta_2 \in (0, \hat{\beta})$, Equation (3) simplifies to $\beta_1 > \beta_2$, contradicting our initial statement. □

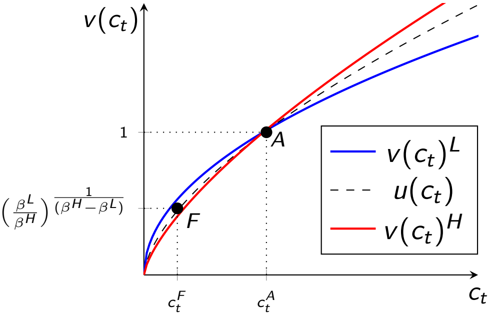
Application to CRRA

Assume $u(c) = \frac{c^{(1-\sigma)}}{1-\sigma}$, with $\sigma \in (0, 1)$.

Then, the following results:

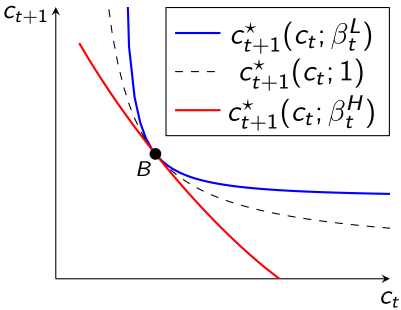
1. $\hat{\beta}_t = \hat{\beta} = \frac{1}{1-\sigma}$
2. $\gamma_t = (1 + \sigma\beta_t - \beta_t)$
3. $u(c_t)_t^{\beta} = \left[\frac{c^{(1-\sigma)}}{1-\sigma}\right]^{\beta_t} = \frac{\beta_t}{(1-\sigma)^{(\beta_t-1)}} \frac{c_t^{(1-\gamma_t)}}{1-\gamma_t}$

Comparative statics



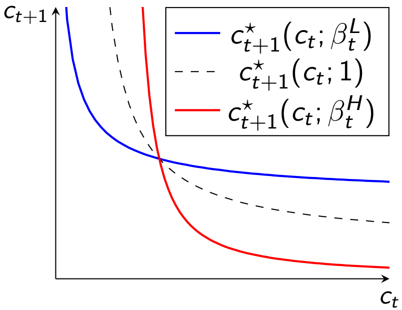
- The value functions are both increasing, and they cross at the point c_A s.t. $u(c_t^A) = 1$
- v^H is steeper than v^L as long as $u(c_t) \geq \left(\frac{\beta^L}{\beta^H}\right)^{\frac{1}{(\beta^H-\beta^L)}}$
- v^L is unambiguously more concave than v^H, which becomes convex if $\beta^H \geq \hat{\beta}$

The bending effect



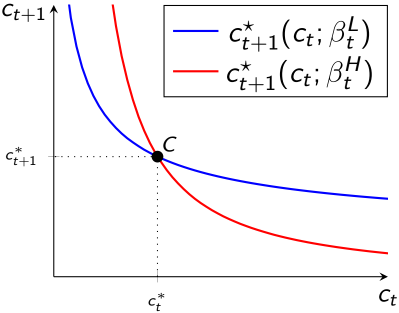
- If $\beta_t < 1$ ($\beta_t > 1$), the indifference curve becomes unambiguously **more** (less) **eccentric**.
- As the EIS increases (decreases), the agent requires weakly less (more) future consumption — given the same level of current consumption — in order to stick on the same utility level.
- If $\beta_t \geq \hat{\beta}_t$, the indifference curve becomes concave.

The tilting effect



- If $\beta_t < 1$ ($\beta_t > 1$), the indifference curve is **tilted to the left** (right).
- The net effect of the change in EIS depends on the level of current consumption.
- If $\beta_t < 1$ ($\beta_t > 1$), the change in future consumption required to stay on the same utility level is lower (higher) the higher is the amount of current consumption.

Single-crossing condition



Single-crossing condition for time-dependent indifference curves:

$$\left| \frac{\partial c_{t+1}}{\partial c_t}\Big|_{\beta_t \geq 1} \right| > \left| \frac{\partial c_{t+1}}{\partial c_t}\Big|_{\beta_t < 1} \right|$$

with $\lim_{c_t \to 0} \left[c^\star_{t+1}(c_t;\beta^H_t) - c^\star_{t+1}(c_t;\beta^L_t) \right] \geq 0$

Dynamic optimality conditions

At each period t, optimality requires:

$$\left[\frac{u(c_{t+k})}{u(c_t)}\right]^{(\beta_t - 1)} \cdot \frac{u'(c_{t+k})}{u'(c_t)} = [\delta(1+r)]^k \tag{4}$$

- The marginal rate of substitution at time t of consumption in any two periods depends on the realisation of β_t.
- It might be optimal to revise the consumption plan at each period.
- Crucially, this could lead to present-biased behaviour as well as feature-biased.

Summary

1. Non-stationary instantaneous utility leads to different optimal solutions with respect to the theoretical benchmark.
2. Optimality usually requires a repeated revision of the consumption plan.
3. At each period t, the decision maker implicitely defines her resilience parameter, which is only partially determined by her intrinsic baseline preferences.
4. The net effect of a shock to the EIS can be split in a bending effect and a tilting effect.
5. According to the realisation of the shock, the revised optimal plan could either increase or decrease current consumption, explaining both present-biased and future-biased behavior.