

Skeleton Extraction of 2D Objects Using Shock Wavefront Detection

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Abstract. This paper proposes a method for computing the medial axis transform (MAT) or the skeleton of a general 2D shape using a technique with a high performance, based on a distance transform computation from the shape's boundaries. The distance transform is computed propagating a wavefront from the boundary, and the skeleton is obtained detecting the points where the wavefronts collide themselves, and applying connectivity rules during the process. This method has two main advantages: the efficiency and the preservation of the skeleton properties.

1 Introduction

The medial axis transform (MAT) or skeleton of a geometric figure is a set of curves that approximate the local symmetry axis of this figure. The concept of medial axis consists in the reduction of geometric shape information to the minimum. The first definition was given by Blum [1], who stated the analogy with a prairie fire: the MAT is the set of points where the propagation wavefront initiated from the shape boundary “intersects itself”. Despite the extreme simplicity of this definition, the implementation in digital images without loss of important properties is surprisingly difficult. For this reason there exists a large number of algorithms and methods in the literature to address this problem. The relevance of MATs was introduced initially by Blum mainly for visual perception analysis and it is used especially for pattern recognition.

Formally there exist four definitions for medial axis transform, all of them equivalent, described by Montanari in [2]. In the first of them the medial axis transform is the prairie fire model proposed by Blum commented above where the fire lines represent the propagation fronts. The second definition states that the MAT is the set of ridges of the distance map constructed from the shape boundary. A distance map from a set of objects in an image, is the image where the value of each pixel is the distance to the nearest object [3],[4]. In the third definition, one of the more common, the MAT is the set of centers of the maximal discs of the figure, where a maximal disc is a circumference contained in the shape for which there exists no other circumference inside the shape that contains it. The last model define the MAT as the set of points that do not belong to any straight line segment connecting other points to their respective closest boundary points.

The principal MAT properties are the connectivity, the preservation of the main topological features of the shape, i.e. to not lose any main branch in the skeleton and to not add irrelevant branches to it, and the Euclidean equidistance of every point of the skeleton to two or more points on the boundary. Regarding the discrete MAT, it is desirable to obtain one pixel thickness results to accomplish the narrowness property. Obviously in a discrete domain the equidistance property is not possible to obtain, so it is necessary to relax this property to get the most approximated result.

Methods to construct skeletons are well described in the literature, and can be divided into three categories. First, the topological thinning methods, Ammann [5], Zhang [6], that work eroding iteratively the shape until the skeleton is obtained. The criterion used to delete a point is local, so it is necessary to take care in some cases because the skeleton is a global property of the shape. These methods are computationally heavy, because a great number of iterations are needed. Second, the distance maps based methods, Arcelli [7], compute the MAT as the ridges of the DT computed from the boundary, as stated in the second Montanari's definition, and detecting the ridges as local maxima points in the distance maps. The main problem with these methods is the detection of saddle points, where two or more ridges intersect, so they are local maxima in some directions, but local minima in the directions tangent to the ridges that intersect, so these points are likely to be undetected, and the skeleton could be disconnected. Finally the Voronoi diagrams based methods that became very popular at the beginning of the 90's, see Ogniewicz [8], Brandt [9], Sugihara [10] and Kimmel [11]. It is well known that the skeleton of a polygonal figure can be obtained from the Voronoi diagram of the polygon edges. In the case of an arbitrary shape, it is necessary to determine which segments of the shape should be separated to generate the Voronoi diagram. This is achieved in [8] by means of a residual function that avoids the branch generation in the skeleton from points that are close in the boundary. In [11] the Voronoi diagram is generated from boundary segments separated by curvature local maxima.

2 Method

In this paper we propose a method to extract the skeleton from a 2D shape, where the distance map from the shape boundaries is computed using a propagation scheme and then, the skeleton is constructed detecting the points where the collision from the wavefronts occurs. The distance map used in this paper is taken from a previous work [12], where a wavefront from each object is started and propagated until the domain is completely filled. This technique is based on ordered propagation [13] for the sake of efficiency, and instead of using the classical approach of bucket sorting to implement the propagation fronts, we use a double list structure to handle the wavefronts. This is more efficient than raster scan, and the distances are a good approximation to the Euclidean, and they are equivalent to those obtained by Danielsson's method [4].

The way of detecting the points where the wavefront intersection takes place is carried out by a non propagation criterion. The discrete propagation fronts

initiated from the shape boundary, are implemented with a list of pixels at Euclidean distance d from it, where d is an integer number, and these pixels are the ones which have real distance values closer to d inside the shape. The wavefront at distance d from the boundary will generate the wavefront at distance $d + 1$, as long as there exist more inner non visited pixels. When a pixel can not be propagated deeper into the shape, it is automatically labeled as member of the skeleton. With this criterion the skeleton obtained is not connected and has not one pixel thickness.

The connectivity is not assured because two effects. The first one appears because points which should be labeled as belonging to the MAT, are not detected in cases where the propagation fronts coming from opposite directions do not intersect at the same pixel but adjacent ones, as in figure 1(a). The second effect happens when the shape becomes wider as illustrated in figure 1(b). The first effect is corrected a posteriori using the distance map computed, connecting the pixels of the computed MAT through the points with maxima values in the distance map. The second effect is solved in the propagation process, by connecting the extremal points of the skeleton computed in d (if any) with its nearest point in the front at distance d .

The skeletonization simplified pseudo code is as follows.

Input: Q : The set of n objects belonging to the boundary shape
Output: Sk : The set of points corresponding to the skeleton

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for  $i = 1$  to  $n$  (Initialization)
     $F_i^0 = \mathbf{q}_i$ 
end for
 $d=0$ 
while (There exist non visited elements in  $\overline{Q}$ ) do
    propagate front  $F^d$ :  $F^d \rightarrow F^{d+1}$ 
    for  $i = 1$  to number of elements in  $F^d$ 
        if ( $\mathbf{p}_i$  agrees shock criterion)
            add  $\mathbf{p}_i$  to  $Sk$ 
        end if
    end for
    if ( $Sk \neq \emptyset$ )
        For each extremal point in  $Sk$ :  $\mathbf{e}$ , add the nearest
        point to  $\mathbf{e}$  that belongs to  $F^d$ , to  $Sk$ 
    end if
     $d=d+1$ 
end while
Thinning( $Sk$ ) (one pass)
Reconnection( $Sk$ )

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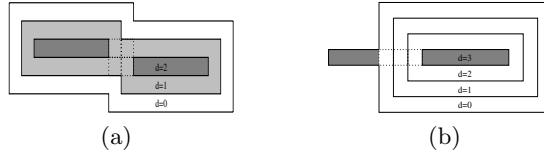


Fig. 1. Disconnection effects: wavefronts coming from opposite directions do not intersect at the same pixel but adjacent ones (a), and aperture zones (b)

F^d represents the propagation front at distance d and \overline{Q} represents the interior pixels of the shape.

The narrowness property is easily achieved by a regular thinning which is highly efficient because only a small percentage of the image has to be scanned, and only one thinning iteration is needed.

3 Results and Conclusions

Figure 2 shows the distance map obtained from the boundaries of a 2D object, and the skeleton extracted with our method. In figure 3, it is shown other results, where the skeleton from different 2D shapes are successfully obtained using shock wavefront detection. This algorithm presents several advantages. First, the MAT is computed very efficiently because it uses an ordered propagation scheme [13] which is a very fast operation. The post-processing steps, connection and thinning, are computed also very fast because a very small number of points are involved and only one thinning iteration is needed. Second, the topological properties of the shape are always preserved as shown in our results. Third, irrelevant branches in the MAT do not appear, i.e. it is not very sensitive to boundary noise, and no pruning of the resulting skeleton is needed. And finally this method can be also extended straightforwardly to 3D shapes, obtaining the medial surface transform.

On the other hand, the skeleton branches obtained with our method do not start in the boundary, in general, but they could be extended if necessary. Skele-

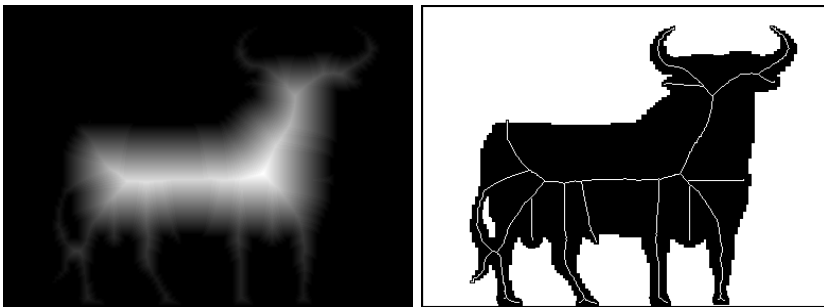


Fig. 2. Distance Transform from the boundary of a shape and skeleton

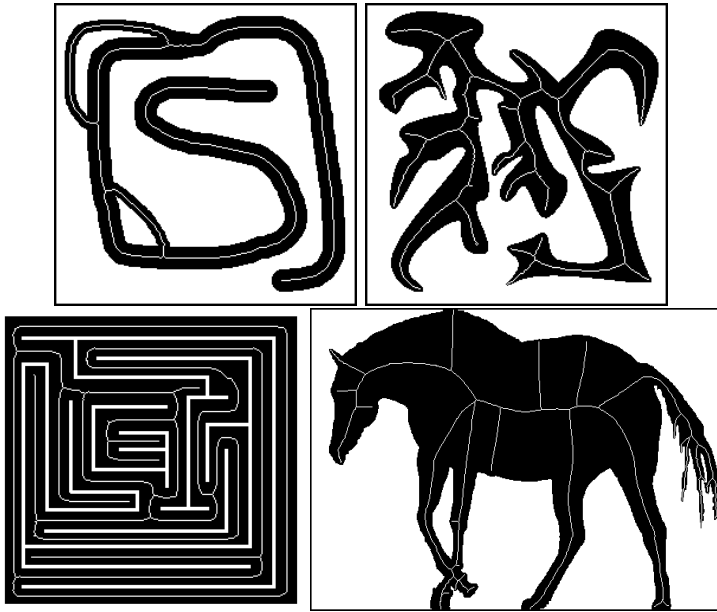


Fig. 3. Medial axis transform in several shapes

tons are used in multiple applications, from motion path planning, to pattern recognition and shape analysis, and we strongly believe that the method presented here is suitable for many of those applications, especially when the efficiency is one of the key factors.

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