Landing a Ball in a Cup Using Projectile Motion

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In this lab, we used a ramp and a track on a table to predict the point of impact of a ball that rolled down said ramp, off said track, and onto the ground. To predict the point of impact, we performed ten trial runs where the ball rolled through a set of photogates located at the end of the track to obtain an initial velocity in the x direction, but without hitting the ground. We then used that velocity to predict how far the ball would travel off the track before being one cup height above the ground, and placed a cup at that point. When we performed the experiment again in the same manner as in the trial run, this time letting the ball run off the track and off the table, passing through the center of the mouth of the cup, thus landing in the cup.

1 Introduction

The aim of this lab was to have a ball roll off a track and into a cup on the ground. To get the ball rolling, it started from the top of a ramp, rolled down the ramp when released to gather velocity in the x direction, ran along a horizontal track with a pair of photogates to measure the speed, along with ensuring the y component of the velocity was zero, and then rolled off the end of the track to land in a cup on the ground. To know where to place the cup, we calculated the speed of the ball by repeatedly releasing it from the ramp and having it run through a pair of photogates to measure the time. Knowing the time it took for the ball to travel between the photogates t_1, and by measuring the distance between the photogates d, we calculated the average velocity in the x direction using $\bar{v}_{x0} = \frac{d}{\bar{t}_1}$ where $\bar{t}_1$ is the average time measurement of ten trial runs.

In order to know where to place the cup, we needed to obtain the time in free fall. To get the time of the ball in free fall, we used the kinematic equation for constant acceleration in the y direction and solved for the free fall time t_2:

$$y_f = y_0 + v_{0y}t_2 + \frac{1}{2}a_y t_2^2 \tag{1}$$

$$\text{Assume } y_f = 0,\ y_0 = h,\ v_{0y} = 0, \text{ and } a_y = -g \tag{2}$$

$$0 = h - \frac{1}{2}gt_2^2 \tag{3}$$

This ultimately yields an equation for solving for t_2 using only known quantities:

$$t_2 = \sqrt{\frac{2h}{g}} \tag{4}$$

Using a known value for t_2, we can substitute into the distance equation to solve for x_f assuming the ball starts falling at $x_0 = 0$ with no x acceleration:

$$x_f = \frac{d}{t_1}\sqrt{\frac{2h}{g}} \tag{5}$$

Then, we placed a cup x_f meters away from the table, released the ball from the top of the ramp in the same manner as in the trial runs (the runs to obtain the x component of the ball's velocity), and witnessed the ball land perfectly in the cup.

2 Methods

The main source of uncertainty in this lab was the initial release of the ball onto the ramp to gain acceleration. To minimize the error, we pushed the ball up the back face of the ramp until it started falling on the main side of the ramp. We did this as to impart minimal initial velocity on the ball, so the ball would accelerate for the same amount of time and reach the same end speed.

We performed ten trial runs to obtain an average velocity in the x direction. We used ten runs since there was a slight run to run variation, and obtaining the average over ten runs allowed for an accurate prediction of v_{x0} for the actual run. For the trial runs, we stopped the ball from falling onto the ground by placing a member at the end of the track to collect the ball after it passed through the photogates, allowing us to get a measurement of t_1, but without allowing the ball to fall to the ground.

To measure the distance at which to place the cup on the ground, we positioned the end of the track to be at the edge of the table. We used one meterstick to measure the height from the ground to the top of the track to obtain the height of the track. Additionally, we measured the height of the cup, since we want the ball to fall into the mouth of the cup rather than the base of the cup (if we did not account for the height of the cup, the ball would hit the side of the cup instead of going into the cup). While we had the meterstick lined up with the edge of the table, we used a second meterstick positioned on the ground to measure the x distance from the start of the track, placing the cup at the calculated distance and in line with the line the ball would travel through.

With the cup positioned at the predicted point of impact, we released the ball in the same manner as the trial runs. With the measurements done properly, we released the ball, it rolled down the track, off the edge, and passed through the center of the cup, landing in the cup.

Here is the experiment setup:

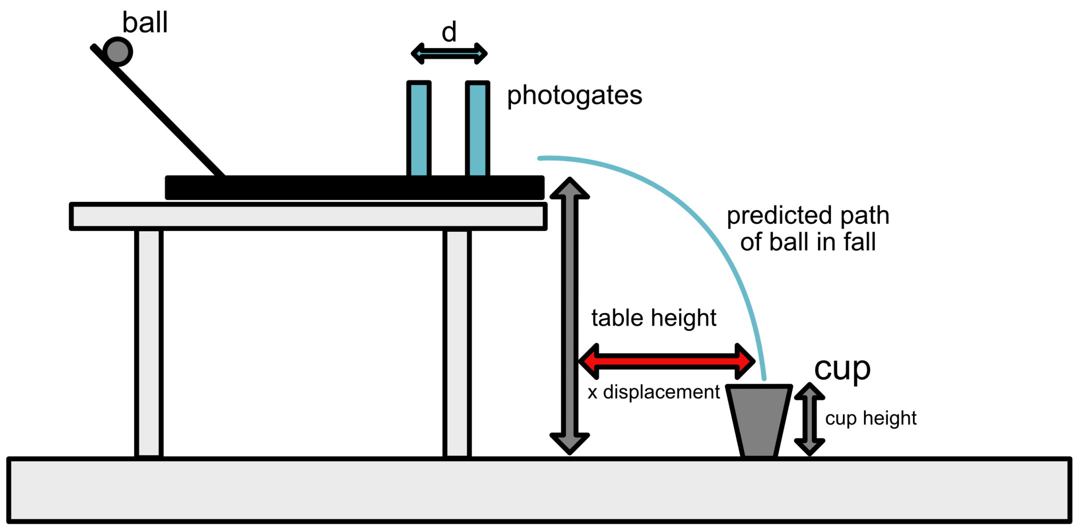

Figure 1: Track setup

3 Results

For all calculations, we assume g to be $9.81\frac{\mathrm{m}}{\mathrm{s}^2}$.

Here is the results of our trial runs:

Table 1: Trial run results

Run	Time (s)
1	0.0366
2	0.0359
3	0.0375
4	0.0392
5	0.0367
6	0.0378
7	0.0391
8	0.0369
9	0.0387
10	0.0362
Average ($\bar{t}_1$)	0.0375
Standard error	0.000380

Here is the prediction of x_f using the other measurements:

Table 2: Predicted values

Quantity	Amount
d	0.075m
$\bar{t}_1$	0.0375s
Cup height	0.124m
Table height	0.936m
h	0.812m
x_f	0.814m
Error	0.0263m

When we released the ball, it rolled down the ramp, off the track, and landed in the cup.

4 Discussion

Our ball landed in the cup. The small amount of error reflects that fact.

The amount of error for the time measurement is small because we used a photogate. The error for time is as follows:

Table 3: Photogate error

Error source	Value
$\sigma_{t,stat}$	0.000380s
$\sigma_{t,res}$	0.00005s
$\sigma_{t,sys}$	0.001s
σ_t	0.00107s

We used the standard error of the trial runs for the time measurement for the statistical error for time, since there was a small amount of variability between trial runs. The resolution error of the photogate is half of the resolution of the photogate (the resolution was 0.0001s). The systematic error is half of the resolution of the measuring tape for each gate's placement. We account for this error with σ_d.

We used the same measuring device for both d and h, used in the same manner. Thus, they have the same error values, i.e. $\sigma_h = \sigma_d$. The error for the meterstick is as follows:

Table 4: Meterstick error

Error source	Value
$\sigma_{h,stat}$	0m
$\sigma_{h,res}$	0.0005m
$\sigma_{h,sys}$	0.001
σ_h	0.0012m

We assumed that the statistical error for the meterstick is zero, since repeated measurements yield the same result. Each group member independently verified each measurement to minimize statistical error. The resolution error of the meterstick is half the resolution of the meterstick (the resolution was 0.0005m). We determined the systematic error of the meterstick to be 0.001m, since there were two measurements in the photogate, each one being within half of the resolution of the meterstick, and measuring the proper height of the track was hard because we held the meterstick at a close but not perfect 90° since we did not have a speed square to ensure a 90° angle between the meterstick and ground.

One major source of error not accounted for properly in the error calculation is the error of placing the cup in the correct spot. We calculated the error for the experiment using the following equation:

$$\sigma_x^2 = \left(\frac{1}{t_1}\sqrt{\frac{2h}{g}}\right)^2 \sigma_d^2 + \left(-\frac{d}{t_1^2}\sqrt{\frac{2h}{g}}\right)^2 \sigma_t^2 + \left(\frac{d}{2t_1}\sqrt{\frac{2}{gh}}\right)^2 \sigma_h^2 \tag{6}$$

That source of error is not accounted for in any of the aforementioned measurements: σ_h is the error in the height measurement, and σ_d is the error in the photogate distance measurement. In other words, the error of placing the cup at the right place is not represented in Equation 6. We took steps in the procedure to minimize this error, like using a second meterstick to align x_0, but we did not assign a numerical value to the error.

However, in the end, we managed to put a ball in a cup.

5 Sample Calculations

We performed all calculations in a spreadsheet. We recorded the ten trial runs from `A4:A13`.

To calculate the average of the ten runs, we used `AVERAGE(A4:A13)`. To calculate the height the ball would fall, we subtracted the table height from the cup height. To calculate the predicted x_f, we did `A2/A15*SQRT(2*D2/9.81)`.

To calculate $\sigma_{t,stat}$, we used `STDEV(A4:A13)/sqrt(10)`. To calculate σ_t and σ_h, we took the square root of the sum of squares of the related error values. For example, we calculated σ_t with `=SQRT(B17^2+B18^2+B19^2)`. Finally, we used the spreadsheet version of Equation 6 to calculate the error, which is as follows:

```
SQRT((1/A15*SQRT(2*D2/9.81))^2*H20^2+
     (-A2/A15^2*SQRT(2*D2/9.81))^2*B20^2+
     (A2/2*A15*SQRT(2/9.81*D2))^2*E20^2)
```

All of the values are available in Figure 2.

	A	B	C	D	E	F	G	H
1	photogate dist.	cup height	table height	height diff	predicted x_f			
2	0.075	0.124	0.936	0.812	0.8146142429			
3	time_1							
4	0.0366							
5	0.0359							
6	0.0375							
7	0.0392							
8	0.0367							
9	0.0378							
10	0.0391							
11	0.0369							
12	0.0387							
13	0.0362							
14	average	velocity						
15	0.03746	0.4994666667						
16								
17	\sigma_t,stat	0.000380409136		\sigma_h,stat	0			
18	\sigma_t,res	0.00005		\sigma_h,res	0.0005			
19	\sigma_t,sys	0.001		\sigma_h,sys	0.001		same as \sigma_h	
20	\sigma_t	0.001071079414		\sigma_h	0.001118033989		\sigma_d	0.001118033989
21								
22	error prediction	0.02626748783						

Figure 2: Spreadsheet