# Dropping Balls Onto the Floor

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**Abstract**

## 1 Introduction

The aim of this lab is to measure the acceleration of an object in free fall due to the gravitational force of Earth. In order to measure the acceleration due to gravity, two tests were performed: dropping a metal ball from a Drop Box and timing and measuring the distance it fell; and dropping a tennis ball from a balcony and timing the distance it fell.

Kinematics can describe the distance an object travels with constant acceleration as a function of time with $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$, where $x_0$ is the initial position, $v_0$ is the initial velocity, and $a$ is the acceleration of the object. Assuming $x_0$ is the initial height of the bottom of the ball $h$, the end position of the ball is zero, $v_0$ is zero since the ball starts at rest, and $a$ is the negative acceleration of the ball due to gravity since objects fall down, then the kinematics equation can be rearranged to describe the acceleration due to gravity:

1. $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$
2. Assume $x_0 = h$, $x(t) = 0$, $v_0 = 0$, and $a = -g$
3. $0 = h - \frac{1}{2}gt^2$
4. $g = \frac{2h}{t^2}$

Using this formula, a distance $h$ and time $t$ for the object to fall that distance is all that is required to determine the acceleration due to gravity.

## 2 Methods

To minimize error, two experiments were performed: dropping a steel ball from a Drop Box in the lab; and dropping a tennis ball from a balcony.

For the first experiment, the steel ball from a Drop Box in the lab, the height was measured with a two meter meterstick to the bottom of the Drop Box where the ball rests, and the height of the ball was also measured. From this, the distance the ball falls can be calculated as $\Delta x = x_{\text{Height to Drop Box}} - x_{\text{Height of ball}}$.

In order to determine the time the ball is in free fall, each drop was recorded with an iPhone camera. For this first experiment, the camera was filming at 30 frames per second. The time the ball begins falling the frame it initially moves from the Drop Box, and ends the frame it bounces on the ground and reverses direction. iOS allows the exact time of the frame within the video to be calculated, so the total time in free fall is the difference between the end and start time.

The Drop Box holds the ball in its initial position with an electromagnet. When a switch is pressed, the electromagnet turns off, and the only force acting on the ball is gravity, which causes it to fall. See Figure 1 for a visual depiction of the setup.

Since the environment is controlled, only five tests were performed to minimize error across tests. The average height and time were used to calculate $g$, and the standard error across the times was used as part of the error propagation.

For the second experiment, the tennis ball dropped from the balcony, the height was measured with a roll tape measure from the point of impact to the height of the top of the balcony, where the ball was dropped. The ball was initially placed on the balcony before being translated over the balcony, making sure to not move the ball vertically. Then, the ball was released. The point of impact at the bottom was recorded, and the bottom of the roll tape was placed at the point of impact. Then, the other end of the roll tape was placed at the point of drop, and the measurement obtained is the true height.

Similarly, for the second experiment, an iPhone camera filmed the whole drop, and the time was obtained in the same fashion as the first experiment. Unlike the first experiment, this iPhone filmed at 60 frames per second. See Figure 2 for a visual depiction of the setup.

Since the outdoors are less controlled, ten measurements were taken instead of five. This allows the inherit randomness of drop to drop variations, such as a gust of wind pushing the ball, be minimized as much as possible in the average over the ten tests. The drop to drop variations are accounted for in the error propagation.

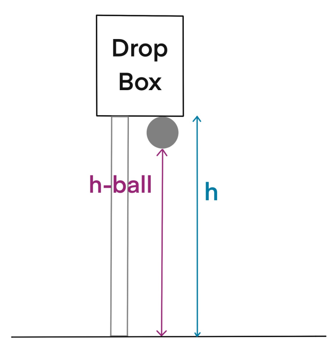


Figure 1: Drop Box setup

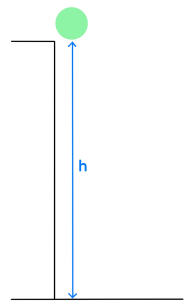


Figure 2: Balcony drop setup

Note: the figures are *not* to scale.

# 3 Results

## 3.1 Drop Box

These are the results from the Drop Box experiment.

Table 1: Drop Box results

| Run | Height (cm) | Time (s) |
|---|---|---|
| 1 | 105.9 | 0.51 |
| 2 | 105.9 | 0.51 |
| 3 | 105.9 | 0.51 |
| 4 | 105.9 | 0.53 |
| 5 | 105.9 | 0.47 |
| Average | 105.90 | 0.505 |

The calculated result of $g$ using the averages is $8.3 \pm 2.3 \frac{\mathrm{m}}{\mathrm{s}^2}$.

## 3.2 Balcony

These are the results from the balcony experiment.

Table 2: Balcony drop results

| Run | Height (cm) | Time (s) |
|---|---|---|
| 1 | 560.0 | 1.01 |
| 2 | 560.0 | 1.05 |
| 3 | 560.0 | 1.01 |
| 4 | 560.0 | 1.07 |
| 5 | 560.0 | 1.00 |
| 6 | 559.6 | 1.05 |
| 7 | 560.0 | 1.06 |
| 8 | 559.6 | 1.06 |
| 9 | 559.8 | 1.03 |
| 10 | 560.2 | 1.05 |
| Average | 559.92 | 1.039 |

The calculated result of $g$ using the averages is $10.4 \pm 0.728 \frac{\mathrm{m}}{\mathrm{s}^2}$.

All of the calculated results contain the true value of $g = 9.81 \frac{\mathrm{m}}{\mathrm{s}^2}$ within the calculated error propagation.

# 4 Discussion

Ultimately, the purpose of this lab was to calculate the value of $g$. The value of $g$ was obtained in both experiments; $g = 9.81 \frac{\mathrm{m}}{\mathrm{s}^2} \in 8.3 \pm 2.3 \frac{\mathrm{m}}{\mathrm{s}^2}$ and $g = 9.81 \frac{\mathrm{m}}{\mathrm{s}^2} \in 10.4 \pm 0.728 \frac{\mathrm{m}}{\mathrm{s}^2}$. However, there is a good amount of error.

## 4.1 Calculating error

Error was calculated using the standard formula for error propagation $\sigma$: $\sigma = \sqrt{\sum_L \frac{\partial g}{\partial L}^2 \sigma_L^2}$ for all measurements $L$, and $\sigma_L = \sqrt{\sigma_{sys,L}^2 + \sigma_{res,L}^2 + \sigma_{stat,L}^2}$. Since there are two measurements, height and time, the formula for error propagation is $\sigma = \sqrt{\left(\frac{2}{t^2}\right)^2 \sigma_h^2 + \left(\frac{-4h}{t^3}\right)^2 \sigma_t^2}$.

The error within a measurement has three components: resolution, systematic, and statistical.

Resolution error is simply half of the resolution of the measuring device. The roll tape has a resolution of 0.2cm, so the resolution error is 0.1cm. For a 30 frames per second video, the resolution is half of the length of a frame, or $\frac{1}{60}$s.

Systematic error is the amount of variability between measurements. For the distance measurements, our group used the resolution error, since we were confident in our measurements of distance. Additional repeated measurements of the same known objects yielded values within the resolution error. For the video measurements, one frame of error was given for the drop, since noticing the beginning of the ball's movement is tricky, and one frame of error was given for the time it took to hit the ground, since the ball was headed down in one frame and up in the other: the actual bounce was never recorded. This gives two frames of error. For a 30 frames per second video, the systematic error is 2 frames, or $\frac{2}{30}$s.

Statistical error is the amount of variability by external factors, such as wind. To account for this variability, we use the estimate for standard error $\sigma_{sys} = \frac{\sigma}{\sqrt{n}}$, where $\sigma$ is the standard deviation of the measurements and $n$ is the number of measurements.

For the Drop Box experiment, only five drops were used to minimize the run to run variability since the lab is a controlled environment. However, for the balcony experiment, ten drops were used since there were more sources of systematic error, such as wind, not holding the ball at the perfect height (i.e. moving the ball slightly up or down while bringing it into the dropping position from the balcony ledge).

## 4.2 Drop Box error

This result is surprising. Our group expected this to have the smallest error of the two experiments since it was the most controlled of the two experiments, but instead it had the largest error of the two experiments. Here is the calculated error propagation:

Table 3: Error propagation for the Drop Box experiment (units omitted for intermediate numbers)

| | |
|---|---|
| $\sigma_h$ | 0.0707 |
| $\sigma_{h,sys}$ | 0.05 |
| $\sigma_{h,stat}$ | 0 |
| $\sigma_{h,res}$ | 0.05 |
| $\sigma_t$ | 0.0694 |
| $\sigma_{t,sys}$ | 0.0667 |
| $\sigma_{t,stat}$ | 0.00980 |
| $\sigma_{t,res}$ | 0.0167 |
| $\frac{\partial g}{\partial h}$ | 7.842 |
| $\frac{\partial g}{\partial t}$ | -32.89 |
| $\left(\frac{\partial g}{\partial h}\right)^2 \sigma_h^2$ | 0.3075 |
| $\left(\frac{\partial g}{\partial t}\right)^2 \sigma_t^2$ | 5.213 |
| Error prediction | $\pm 2.3 \frac{\text{m}}{\text{s}^2}$ |

The cause for the large error is the large magnitude of $\left(\frac{\partial g}{\partial t}\right)^2 \sigma_t^2$, suggesting a majority of the error arose in the measurement of time. For future labs, using a higher frame rate in video recordings can reduce the amount of error in video recordings, since it becomes easier with more frames to identify the frame the ball begins to fall and the frame the ball impacts the ground, leading to an overall smaller $\sigma_{t,sys}$. With a 60 frames per second camera, the error propagation drops to $\pm 1.3 \frac{\text{m}}{\text{s}^2}$.

Originally, this experiment was supposed to use a photogate to get an exact time for the ball to drop, but the photogate was out of commission for this lab. Had that been used instead of the iPhone, the error propagation would be approximately $\pm 0.93 \frac{\text{m}}{\text{s}^2}$.

## 4.3 Balcony error

The amount of error in this lab is about what our group expected. However, there are other factors that did not factor into our calculation of error prediction in this lab.

The big element of error in this lab that is unaccounted for is air resistance. As the tennis ball falls faster, the amount of drag on the tennis ball increases exponentially. To minimize air resistance, the lowest balcony available was used to allow the ball to fall from a consistent height, but not high enough to the point where air resistance becomes a significant factor on the amount of time it takes for the tennis ball to hit the ground.

For the tennis ball, this is a significant concern, since the height of the drop is quite large and the tennis ball is relatively light compared to its size, especially when compared to the metal ball dropped from the Drop Box. Since the tennis ball is relatively light compared to its size, the overall force of gravity acting on the tennis ball is small, so even a small amount of drag starts to have an effect on the acceleration and fall duration.