

$$\Delta f = \nabla \cdot \nabla f = \text{tr} \left(\begin{bmatrix} \frac{\partial^2 f}{\partial u^2} & \frac{\partial^2 f}{\partial u \partial v} \\ \frac{\partial^2 f}{\partial v \partial u} & \frac{\partial^2 f}{\partial v^2} \end{bmatrix} \right) = \frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}$$