Lab 3

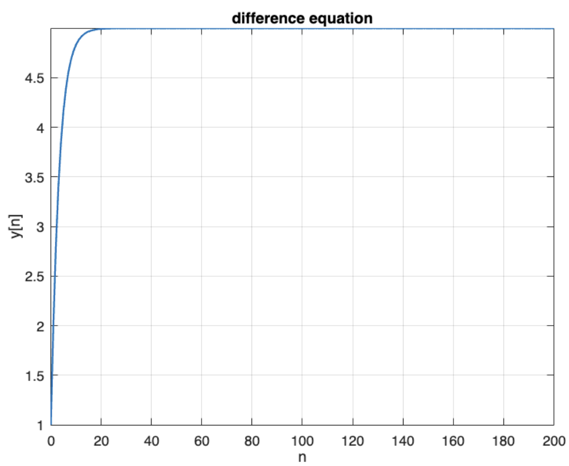
Problem 1: Evaluating stability of second order systems

In this problem, we will evaluate the stability of LTI systems described by a second order difference equation.

a) From the provided block diagram, we can infer that the difference equation must be

$$x[n] = y[n] - y[n-1] + c_0 y[n-2]. \tag{1}$$

b) If we allow $x[n] = u[n-10]$ and $c_0 = 0.2$, we can evaluate the output $y[n]$ recursively. The output, $y[n]$, is plotted below for $n = 0, 1, 2, ..., 200$:



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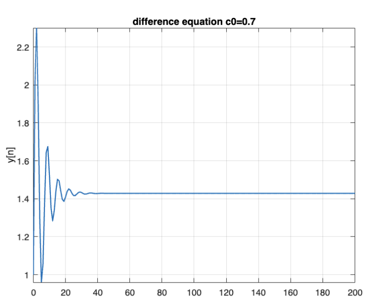
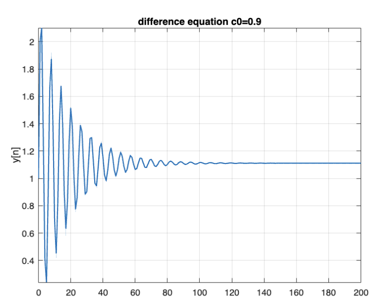
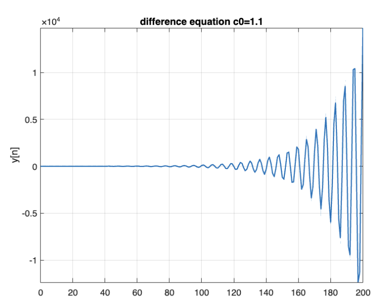
c) Repeating part (a) with $c_0 = 0.7$, $c_0 = 0.9$, and $c_0 = 1.1$, we observe the following plots:



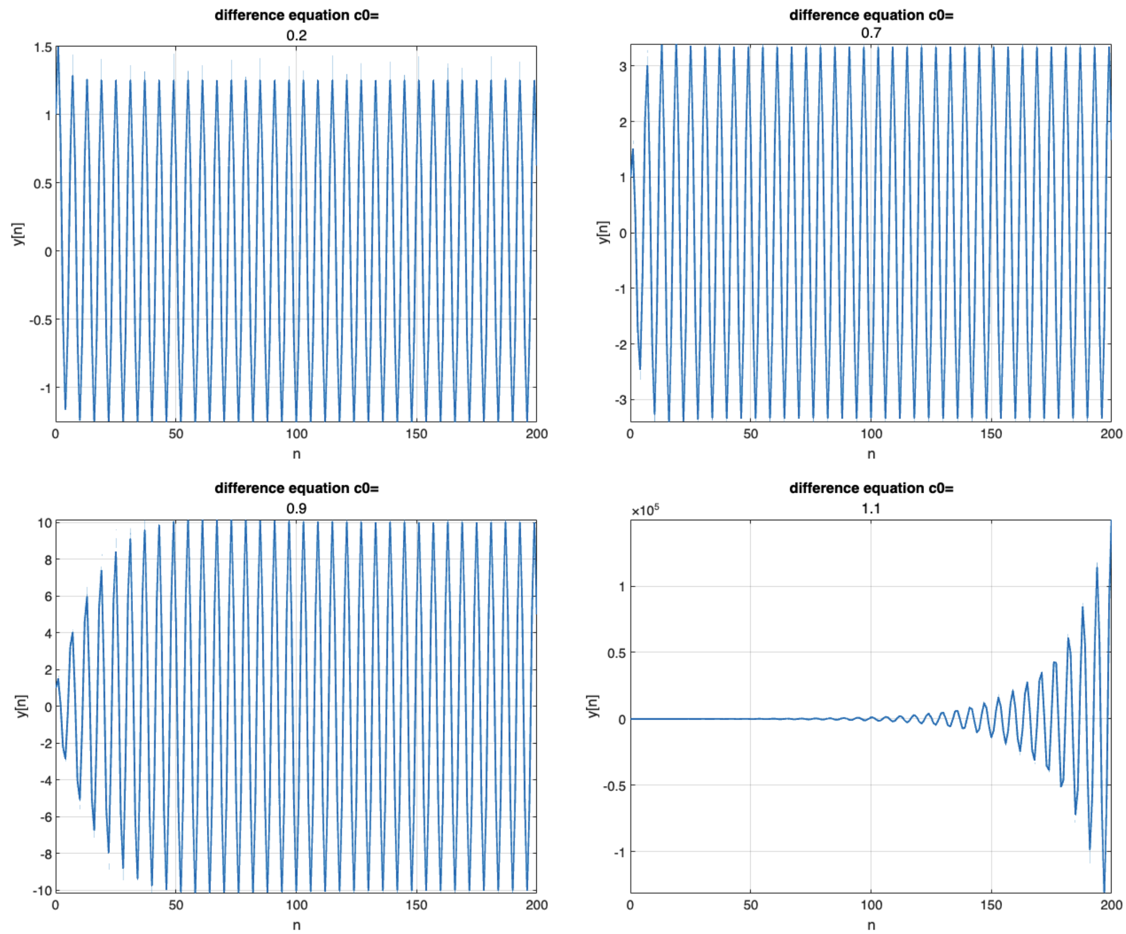




We can directly observe that when $c_0 < 1$, the plots of $y[n]$ oscillate before damping to convergence as $n \to \infty$, more accurately described by $\lim_{n\to\infty} y[n] =$

0. On the other hand, the plot of $y[n]$ with $c_0 = 1.1$ begins at stability before oscillating to divergence as n increases, analogous to $\lim_{n\to\infty} y[n] = \infty$. We are also able to directly observe that as we increase the value of c_0, the maximum value within the plotted range decreases.

d) Repeating this process for $c_0 = 0.2, 0.7, 0.9, 1.1$ and with $x[n] = \cos[\frac{\pi n}{3}]$, we observe the following plots:



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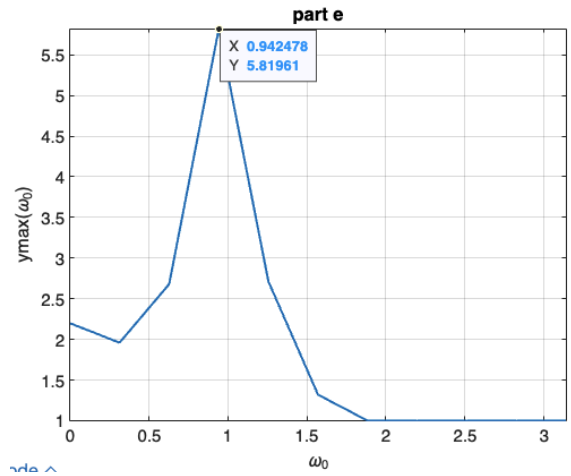
We see here that the amplitude of $y[n]$ increases with an increased c_0. Furthermore, we observe that when $c_0 = 1.1$, $y[n]$ becomes unbounded as $n \to \infty$, with its amplitude also approaching ∞.

e) In this part, we will create an array for w_0 and repeat the procedure with $x[n] = cos(\omega_0 n)$, with $c_0 = 0.8$. The plot of $y_{\max}(\omega_0)$ versus ω_0 is shown below:



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Here, we can observe that the peak of $y_{\max}(\omega_0)$ is at $\omega_0 \approx 0.942$.