

Lab 2

EE141, Digital Signal Processing, Fall 2025

Problem 1: Reviewing and verifying properties of the DTFT

In this problem, we consider the discrete time Fourier transform given by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}.$$

We also consider that if $x[n]$ is a causal sequence, this sum reduces to

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} x[n]e^{-j\omega n}, \quad (1)$$

and we can further truncate the above sum to

$$X(e^{j\omega}) \approx \tilde{X}(e^{j\omega}) \triangleq \sum_{n=0}^{\infty} x[n]e^{-j\omega n} \quad (2)$$

if $x[n]$ decays to zero as $n \rightarrow \infty$. For this problem, we let $x[n] = 0.8^n u[n]$ and $y[n] = 0.7^n \cos(\frac{\pi n}{6})u[n]$.

a) Solving analytically for $x[n]$, we observe

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} 0.8^n e^{-j\omega n}$$

as $x[n]$ is a causal signal, shown in (1). We can further simplify to

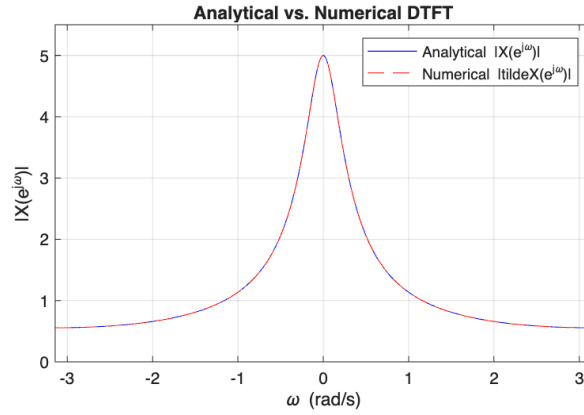
$$X(e^{j\omega}) = \sum_{n=0}^{\infty} (0.8e^{-j\omega})^n,$$

where we observe this as a converging sum, as $a = 0.8 < \infty$. Continuing with our idea of infinite geometric series, we can simplify this even further to

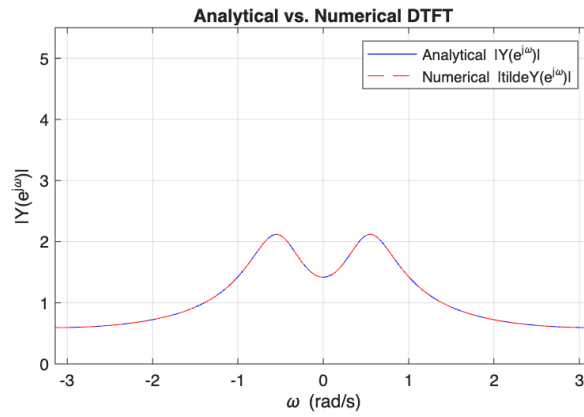
$$X(e^{j\omega}) = \frac{1}{1 - 0.8e^{-j\omega}}.$$

We can then use this in MATLAB to solve for $X(e^{j\omega})$ in an analytical manner, using our `w = linspace(-pi, pi, K)` where $K = 500$.

- b)** Picking a large enough N s.t. $0.9^N \approx 0$ (used $N = 50$), we use MATLAB to numerically compute $X(e^{j\omega_k})$ using (2).
- c)** Plotting the analytical and numerical solutions, we observe the following:



d) Doing the same for $y[n]$, we observe the following plot:



e) We now let $z[n] = 2x[n] + 3y[n]$. Analytically computing $Z(e^{j\omega_k})$ and numerically computing $\tilde{Z}(e^{j\omega_k})$ using the same ω_k as in the previous parts, we observe the following magnitude and sum plots:

