

## Lab 2

EE141, Digital Signal Processing, Fall 2025

### Problem 1: Reviewing and verifying properties of the DTFT

In this problem, we consider the discrete time Fourier transform given by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}.$$

We also consider that if  $x[n]$  is a causal sequence, this sum reduces to

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} x[n]e^{-j\omega n}, \quad (1)$$

and we can further truncate the above sum to

$$X(e^{j\omega}) \approx \tilde{X}(e^{j\omega}) \triangleq \sum_{n=0}^{\infty} x[n]e^{-j\omega n} \quad (2)$$

if  $x[n]$  decays to zero as  $n \rightarrow \infty$ . For this problem, we let  $x[n] = 0.8^n u[n]$  and  $y[n] = 0.7^n \cos(\frac{\pi n}{6})u[n]$ .

**a)** Solving analytically for  $x[n]$ , we observe

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} 0.8^n e^{-j\omega n}$$

as  $x[n]$  is a causal signal, shown in (1). We can further simplify to

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} (0.8e^{-j\omega})^n,$$

where we observe this as a converging sum, as  $a = 0.8 < \infty$ . Continuing with our idea of infinite geometric series, we can simplify this even further to

$$X(e^{j\omega}) = \frac{1}{1 - 0.8e^{-j\omega}}.$$

We can then use this in MATLAB to solve for  $X(e^{j\omega})$  in an analytical manner, using our `w = linspace(-pi, pi, K)` where  $K = 500$ .

**b)** Picking a large enough  $N$  s.t.  $0.9^N \approx 0$ , we can numerically compute  $X(e^{j\omega_k})$  using (2).