University of Eldoret

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UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

REGULAR EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE

COURSE CODE: MATH 212

COURSE TITLE: LINEAR ALGEBRA I

DATE: 2ND DECEMBER, 2021 **TIME:** 12.00 NOON – 3.00 PM

INSTRUCTIONS TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF (4) PRINTED PAGES. PLEASE TURN OVER

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2021/2022 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER EXAMINATION

COURSE CODE: **MATH 212**

COURSE TITLE: **LINEAR ALGEBRA I.**

INSTRUCTION TO CANDIDATES

Answer **ALL** questions from section A and any **THREE** from section B.

No sharing of scientific calculators.

Do not write on this question paper.

Duration of the examination: 3 hours

SECTION A (31 MARKS): ANSWER ALL QUESTIONS

QUESTION ONE (16 MARKS)

a) Obtain the inverse of the matrix below using the reduced-row echelon approach.

$$A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(5 Marks)

b) Determine the value of k so that the system

$$x_1 - 3x_3 = -3$$
$$2x_1 + kx_2 - x_3 = -2 \qquad \text{has;}$$
$$x_1 + 2x_2 + kx_3 = 1$$

i) No solution.
ii) Many solutions.
iii) Unique solution.

(6 Marks)

c) If $f(x) = \sin x$ and $g(x) = \sin 2x$, find the Wronskian of $f(x)$ and $g(x)$ at $x_0 = \frac{\pi}{4}$

(5 Marks)

QUESTION TWO (15 MARKS)

a) Let $V = P_3$, the space of all polynomials of degree ≤ 3. Let W be the set with all such polynomials but with a constant zero term. Determine if W is a vector subspace of P_3.

(4 Marks)

b) Determine if $T: \Re^2 \to \Re^2$ defined by $T(x_1, x_2) = (x_1 + 1, x_2)$ is a linear transformation.

(5 Marks)

c) Determine if $S = \{2 + x + x^2, x - 2x^2, 2 + 3x - x^2\}$ is linearly independent in P_2 .(6 Marks)

SECTION B – ATTEMPT ANY THREE QUESTIONS IN THIS SECTION

QUESTION THREE (13 MARKS)

Let $T: \Re^3 \to \Re^3$ be defined by

$$T(X) = \begin{bmatrix} 2 & 0 & 1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ Find,}$$

a) Basis for the range of T. (6 Marks)

b) Basis for the kernel of T. (5 Marks)

c) Rank of T. (1 Mark)

d) Nullity of T. (1 Mark)

QUESTION FOUR (13 MARKS)

Find the basis and the dimension of the solution space for the equations,

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$

$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$

$$x_1 + x_2 - 2x_3 - x_5 = 0$$

$$x_3 + x_4 + x_5 = 0$$

(13 Marks)

QUESTION FIVE (13 MARKS)

a) Determine if the set $S = \{(1,2,1), (2,9,0), (3,3,4)\}$ is a basis for $\Re^3$. (7 Marks)

b) Consider the vectors $v_1 = (1,1,-1)$, $v_2 = (4,0,1)$ $v_3 = (3,-1,2)$,

a basis of the subspace

i) Find the subspace spanned by the above vectors. (5 Marks)

ii) Write the remaining vectors as a linear combination of the vectors in the basis. (1 Mark)

QUESTION SIX (13 MARKS)

a) Find the determinant of the following matrix using Laplace expansion;

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 2 & 1 & 0 & 2 \\ 2 & 2 & -1 & 0 \end{bmatrix}$$

(5 Marks)

b) Find the inverse of the matrix below by first getting its adjoint;

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 2 \\ 1 & 0 & 3 \end{bmatrix}$$

(8 Marks)

QUESTION SEVEN (13 MARKS)

a) Find the rank of the following matrix

$$A = \begin{bmatrix} 3 & 4 & 2 & 1 \\ 7 & 3 & 1 & 0 \\ 4 & 4 & -2 & 1 \\ 2 & 1 & 3 & 0 \end{bmatrix}$$

(5 Marks)

b) Solve the following system of equations by first getting the inverse;

$$3x_1 + 2x_2 + 3x_3 = 1$$
$$2x_1 - 2x_2 + 4x_3 = 3$$
$$4x_1 + 5x_2 - x_3 = -2$$

(8 Marks)