### MATH 212 – LINEAR ALGEBRA 1 CAT 2 TIME: 1 HOUR 21/11/2022

- a) State the steps followed in showing that a given set of vectors is a vector space. Hence show that the set of all polynomials of degree  $\leq n$  is a vector space. [5 marks]
- **b)** Given a linear transformation  $T: \mathbb{R}^3 \to \mathbb{R}^3$  defined by  $T(x_1, x_2, x_3) = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ , find the basis for R(T) [5 marks]
- c) Determine if  $S = \{(1,2), (-1,1)\}$  is a basis for  $\mathbb{R}^2$ . [5 marks]

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a) A vector como is a set or moders
a) A vector space is a set of vectors in which addition is defined as
Scalar multiplication such that the set
is closed under vector addition and
Scalar multiplication. Thus given any
Set/collection of vectors say s, to find out
if it's a vector space, check that
i) utves, for all u,ves ii) knes, for all ues, and kelk
a) 114 e 3, 100 an u e 3, and Rein
Now let Pn denote the set of all
Polynomials of degrees &n. Then each of
Such polynomial is a vector of the
form 9,+9,2+9,22+ + 9,27,
Let P(x) and g(x) be such Polynomials
Say Pay = 9. 4 9,2+ 9,22+ + 9,27
260 = bo + b, 21 + b, 22 + + b, 27
Then
P(x) + 2(x) = (9,+6) + (9,+6) x + (92+62) x2 + ···
+ (9n4 bn) xn
which is a Polynomial in Pn
Also K Pa) = K (90+9,x+92x2++9,xn)
= K90+ (K9)2+(K9)22++(K9)21
which is also a polynomial in Pn.
Thus Pr is closed with respect
to both Vector addition and
Scalar multiphredium and is thus
a vector space.

	margin
b) $T: \mathbb{R}^3 \to \mathbb{R}^3$	
$T(x_1, x_2, x_3) = /2 0 - 1 /x_1$	
40-2/22	
000/23/	
He have R(T) = {veV! I ue U for which T(a)=v}	
Let V = (9) Thus if V is in the dange	
C /	
	N
of T, we have $7 \left( \frac{x_1}{x_2} \right) = \left( \frac{9}{5} \right)$	
$\begin{pmatrix} \chi_2 \\ \chi_3 \end{pmatrix} \begin{pmatrix} 5 \\ C \end{pmatrix}$	
1e. 20-1/24 = 9	
40-2 22 6	
600/23/6/	,
6	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
b-2a=0 => b=2a	
If a=t, then b=2t whole stell	
$C(t) \qquad C(1) \qquad C$	
SO K(T) = { 2t , ter } = { t   2   }, te	IR C
The basis for IR (T) 13 thus [1]	
2	
	•

First we check if S is linearly Independent. We have, $K_1Y_1 + K_2 \cdot Y_2 = 0$ ox $K_1(1/2) + K_2(-1,1) = (0,0)$ Thus $K_1 - K_2 = 0$ $2 K_1 + K_2 = 0$ $3 K_2 = 0 = 1 K_2 = 0$ Also $K_1 - K_2 = 0$ ox $K_2 = 0 = 1 K_1 = 0$ Thus $K_1 = K_2 = 0$ and so  S is linearly independent.  Next we check if S spans $\mathbb{R}^2$ Let $(0,b) \in \mathbb{R}^2$ . We have $(1 - 1 + 9) \in \mathbb{R}^2$ . We have $(1 - 1 + 9) \in \mathbb{R}^2$ . We have $(1 - 1 + 9) \in \mathbb{R}^2$ . Or $(1 - 1 + 9) \in \mathbb{R}^2$ Let $(0,b) \in \mathbb{R}^2$ . The have $(0,b) \in \mathbb{R}^2$ . The follows that there is a unique Solution for $(0, C_2 - 9) \in \mathbb{R}^2$ .  It follows that there is a unique Solution for $(0, C_2 - 9) \in \mathbb{R}^2$ .  We therefore analysis for $\mathbb{R}^2$ .  We therefore analysis for $\mathbb{R}^2$ .	
First we check if S is linearly modependent. We have, $K_1V_1 + K_2V_2 = 0$ or $K_1(1/2) + K_2(-1,1) = (0,0)$ Thus $K_1 - K_2 = 0$ $2K_1 + K_2 = 0$ $2K_1 + K_2 = 0$ $3K_2 = 0 = )K_2 = 0$ $3K_2 = 0 = )K_2 = 0$ Also $K_1 - K_2 = 0$ or $K_2 = 0 = )K_1 = 0$ Thus $K_1 = K_2 = 0$ and so  S is linearly independent.  Next we check if S spans $\mathbb{R}^2$ Let $(0,b) \in \mathbb{R}^2$ . We have $(2V_1 + C_2V_2 = (0,b))$ $(2V_1 + C_2V$	$(c)$ $S = \{(1,2), (-1,1)\}$
Independent. We have, $K_1V_1 + K_2V_2 = 0$ ox $K_1(1/2) + K_2(-1,1) = (0,0)$ Thus $K_1 - K_2 = 0$ $2K_1 + K_2 = 0$ $2K_1 + K_2 = 0$ $2K_1 + K_2 = 0$ $3k_2 = 0 = 0$ $2k_2 - 2R_1 = 0$ $3k_2 = 0 = 0$ $3k_2 = 0$ $3$	First we check if S is linearly
$K_{1}V_{1} + K_{2}V_{2} = 0$ or $K_{1}(1/2) + K_{2}(-1,1) = (0,0)$ Thus $K_{1} - K_{2} = 0$ $2 K_{1} + K_{2} = 0$ $2 K_{1} + K_{2} = 0$ $2 K_{1} + K_{2} = 0$ $3 k_{2} = 0 = 0$ $3 k_{2} - 2R_{1}(0) = 0$ $3 k_{2} - 2R_{1}(0) = 0$ $3 k_{3} - 2R_{1}(0) = 0$ $3 k_{4} - 2R_{1}(0) = 0$ $3 k_{5} - 2R_{1}(0) = 0$	
Thus $K_1 - K_2 = 0$ $2 K_1 + K_2 = 0$ $2 K_1 + K_2 = 0$ $3 k_2 = 0 = 0$ $3 k_2 = 0 = 0$ $3 k_2 = 0 = 0$ $4 k_2 - 2R_1 = 0$ $4 k_2 - 2R_2 $	
$2 k_1 + k_2 = 0$ $(1 - 1 : 0)$ $(2 - 1 : 0)$ $(2 - 1 : 0)$ $(3 : 2 - 2R_1)$ $(0 : 3 : 0)$ $(3 : 2 = 0)$ $(4 : 0)$ $(5 : 2 = 0)$ $(5 : 2 = 0)$ $(5 : 2 = 0)$ $(5 : 3 : 2 = 0)$ $($	
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	(1 -1:0)
also $K_1-K_2=0$ or $K_2=0$ => $K_1=0$ Thus $K_1=K_2=0$ and so  S is lineary independent.  Next we check if s spans $\mathbb{R}^2$ Let $(a,b)\in\mathbb{R}^2$ , we have $(a,b)\in\mathbb{R}^2$ $(a,b)\in\mathbb{R}^2$ It follows that those is a unique solution for $(a,b)\in\mathbb{R}^2$ Solution for $(a,b)\in\mathbb{R}^2$ We those for $(a,b)\in\mathbb{R}^2$	$(2   1   0   R_2 - 2R_1   0   3   0  $
Thus $K_1 = K_2 = 0$ and so  S is linearly independent.  Next We Check if S spans $\mathbb{R}^2$ Let $(a,b) \in \mathbb{R}^2$ . We have $(a,b) \in \mathbb{R}^2$ . On a solution for $(a,b) \in \mathbb{R}^2$ . Solution for $(a,b) \in \mathbb{R}^2$ . We thorefore and we have $(a,b) \in \mathbb{R}^2$ .	$3k_2 = 0 = ) k_2 = 0$
Next We Check & S Spans $\mathbb{R}^2$ Let $(a,b) \in \mathbb{R}^2$ . We have	
Next we check if S spans $\mathbb{R}^2$ Let $(a,b) \in \mathbb{R}^2$ . We have $(a,b) \in \mathbb{R}^2$ . Unique $(a,b) \in \mathbb{R}^2$ . U	
Let $(a,b) \in \mathbb{R}^2$ . We have $C_1V_1 + C_2V_2 = (a,b)$ $1e. C_1-C_2 = 9$ $2c_1+c_2=b$ $\begin{pmatrix} 1 & -1 & 9 & R_1 \\ 2 & 1 & b \end{pmatrix} \xrightarrow{R^2 \to 1} \begin{pmatrix} 1 & -1 & 9 \\ 2 & 1 & b \end{pmatrix}$ It follows that there is a unique  Solution for $C_1$ , $C_2$ and $C_3$ $C_4$ $C$	Sis lineary Independent.
Let $(a,b) \in \mathbb{R}^2$ . We have $C_1V_1 + C_2V_2 = (a,b)$ $1e. C_1-C_2 = 9$ $2c_1+c_2=b$ $\begin{pmatrix} 1 & -1 & 9 & R_1 \\ 2 & 1 & b \end{pmatrix} \xrightarrow{R^2 \to 1} \begin{pmatrix} 1 & -1 & 9 \\ 2 & 1 & b \end{pmatrix}$ It follows that there is a unique  Solution for $C_1$ , $C_2$ and $C_3$ $C_4$ $C$	
$\begin{array}{c} G_1V_1+G_2V_2=(Q,b)\\ \\ 1e. G_1(1,2)+G_2(-1,1)=(Q,b)\\ \\ 1e. G_1-G_2=Q\\ \\ 2G_1+G_2=b\\ \\ \\ 1-1!Q_1R_1 \rightarrow (1-1!Q_1)\\ \\ 21!b_1R_2R_1 \rightarrow (0.3!b-2Q_1)\\ \\ If follows that there is a unique \\ Solution for G_1G_2 and SO\\ \\ S_1G_2G_1 = 1R^2\\ \\ We therefore Gardude that S_1S_1\\ \end{array}$	
1e. $q(1,2) + C_2(-1,1) = (q,b)$ .  1e. $C_1 - C_2 = q$ $2 c_1 + C_2 = b$ $\begin{pmatrix} 1 & -1 &   & q \\ 2 & 1 &   & b \end{pmatrix} \xrightarrow{R_2 - 2q} \begin{pmatrix} 0 & 3 &   & b - 2q \end{pmatrix}$ It follows that there is a unique Solution for $C_1$ , $C_2$ and $S_0$ S $Spans R^2$ We therefore Conclude that $S$ is	
IR. $C_1 - C_2 = 9$ $2C_1 + C_2 = b$ $1 - 1 \cdot 9 \cdot R_1 \rightarrow (1 - 1 \cdot 9)$ $2 \cdot 1 \cdot b \cdot R_2 \cdot 2R_1 \rightarrow (0 \cdot 3 \cdot b - 29)$ If follows that there is a unique Solution for $C_1$ , $C_2$ and $S_0$ $S$ Spans $IR^2$ We therefore Conclude that $S$ is	$G_1V_1+G_2V_2=(G,b)$
2 C <sub>1</sub> + C <sub>2</sub> = b  (1 -1   9   R <sub>1</sub> ) (1 -1   9  (2   1   b)   R <sub>2</sub> = 2R <sub>1</sub> (0 3   b - 29)  If follows that there is a unique  Solution for C <sub>1</sub> , C <sub>2</sub> and so  S spans IR <sup>2</sup> We therefore Conclude that S is	1e. G(1,2) + Cz(-1,1) = (a,b)
[1 -1; 9 R1 (1-1; 9)  [2 1; 5 R2-2R] (0 3; 5-29)  It follows that there is a unique  Solution for C, C2 and so  S spans IR <sup>2</sup> We therefore Conclude that S is	$R$ , $C_1 - C_2 = 9$ matte of knowledge and innevation)
It follows that there is a unique Solution for C, Cz and so  S spans IR2  We therefore Conclude that S is	
It follows that there is a unique solution for G, Cz and so  S spans IR <sup>2</sup> We therefore Conclude that S is	
Solution for C, Cz and so  S spans 182  We therefore Conclude that S is	2 1 5 22 0 3 5-29
Solution for C, Cz and so  S spans 182  We therefore Conclude that S is	
S span: 1R2 We therefore Conclude that S is	It Jollows That There is a Unique
we therefore Conclude that S is	Golution for C, Cz and So
a basis for ik-	
	4 basis for 1k