UNIVERSITY OF ELDORET

MATH 212 – Linear Algebra I CAT 1 TIME: 1 Hour 20/10/2022

a) Find the rank of the following matrix:

$$A=\begin{bmatrix}2 & 2 & 2\\ 4 & -2 & 4\\ 6 & 0 & 6\end{bmatrix}$$ [3 marks]

b) Solve the system of linear equations using the Gauss Elimination Method:

$$-x_1+2x_2-3x_3=4$$
$$2x_1-4x_2+6x_3=-8$$ [5 marks]

c) Find the determinant of the following matrix by first suitably partitioning the matrix:

$$A=\begin{bmatrix}6 & 2 & 4\\ -4 & 2 & 0\\ 4 & 8 & 6\end{bmatrix}$$ [7 marks]

Write on both sides of the paper Question..... in either margin

MATH 212 CAT 1 2022 SOLUTIONS

a)

$$\begin{pmatrix} 2 & 2 & 2 \\ 4 & -2 & 4 \\ 6 & 0 & 6 \end{pmatrix} \xrightarrow[R_3-3R_1]{R_2-2R_1} \begin{pmatrix} 2 & 2 & 2 \\ 0 & -6 & 0 \\ 0 & -6 & 0 \end{pmatrix} \xrightarrow[R_3-R_2]{R_2} \begin{pmatrix} 2 & 2 & 2 \\ 0 & -6 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\text{Rank}(A) = 2$

b) In Matrix form:

$$\begin{pmatrix} -1 & 2 & -3 \\ 2 & -4 & 6 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \\ 0 \end{pmatrix}$$

It's Augmented Matrix is given by

$$\left(\begin{array}{ccc|c} -1 & 2 & -3 & 4 \\ 2 & -4 & 6 & -8 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[R_2+2R_1]{R_1} \left(\begin{array}{ccc|c} -1 & 2 & -3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Now we have:

$$\begin{pmatrix} -1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$$

Back substitution

$R_3 \Rightarrow 0x_3 = 0$, Let $x_3 = t$, $t \in \mathbb{R}$

$R_2 \Rightarrow 0x_2 = 0$, Let $x_2 = s$, $s \in \mathbb{R}$

$R_1 \Rightarrow -x_1 + 2x_2 - x_3 = 4$

$x_1 = 2x_2 - x_3 - 4$

$x_1 = 2s - t - 4$

ie $x_1 = 2s - t - 4$

$x_2 = s$

$x_3 = t$, $s, t \in \mathbb{R}$.

Write on both sides of the paper

Question.....

Do not write in either margin

(c)

$$A = \left(\begin{array}{c|cc} 6 & 2 & 4 \\ \hline -4 & 2 & 0 \\ 4 & 8 & 6 \end{array}\right) \Rightarrow$$

$A_{11} = 6$, $A_{12} = [2 \;\; 4]$

$A_{21} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$, $A_{22} = \begin{pmatrix} 2 & 0 \\ 8 & 6 \end{pmatrix}$

$A_{11} = 6, \quad A_{11}^{-1} = \frac{1}{6}$

$$A_{21} A_{11}^{-1} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} 1/6 = \begin{pmatrix} -2/3 \\ 2/3 \end{pmatrix}$$

$$A_{21} A_{11}^{-1} A_{12} = \begin{pmatrix} -2/3 \\ 2/3 \end{pmatrix} [2 \;\; 4] = \begin{pmatrix} -4/3 & -8/3 \\ 4/3 & 8/3 \end{pmatrix}$$

$$A_{22} - A_{21} A_{11}^{-1} A_{12} = \begin{pmatrix} 2 & 0 \\ 8 & 6 \end{pmatrix} - \begin{pmatrix} -4/3 & -8/3 \\ 4/3 & 8/3 \end{pmatrix} = \begin{pmatrix} 10/3 & 8/3 \\ 20/3 & 10/3 \end{pmatrix}$$

$$\left| A_{22} - A_{21} A_{11}^{-1} A_{12} \right| = \left| \begin{pmatrix} 10/3 & 8/3 \\ 20/3 & 10/3 \end{pmatrix} \right| = 100/9 - 160/9 = \frac{-60}{9}$$

$$\det(A) = A_{11} \left| A_{22} - A_{21} A_{11}^{-1} A_{12} \right| = 6 \times \left(-60/9\right)$$

$$\det(A) = -120 \quad -40$$