UNIVERSITY OF ELDORET

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER EXAMINATION

COURSE CODE: MATH 212

COURSE TITLE: LINEAR ALGEBRA I

INSTRUCTION TO CANDIDATES

Answer **ALL** questions from section A and any **THREE** from section B

No sharing of scientific calculators.

Do not write on this question paper.

Duration of the examination: 3 hours

SECTION A (31 MARKS): ANSWER ALL QUESTIONS

QUESTION ONE (16 MARKS)

a) Define a basis S, for a vector space V. (2 Marks)

b) Let $(\vec{u}, \vec{v}, \vec{w})$ be an independent set in $\Re^n$. Is $\{\vec{u}+\vec{v}, 2\vec{u}+\vec{w}, \vec{v}-5\vec{w}\}$ linearly independent? (4 Marks)

c) Consider the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 9 \\ 2 & 4 & 6 \end{bmatrix}$. What is its rank? 2. (2 Marks)

Square matrix
number of rows = number of columns

d) Are all the matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a+d=0$ a subspace of the vector space of 2×2 matrices? (3 Marks)

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e) Find the determinant of the matrix $\begin{bmatrix} 0 & -1 & -1 \\ -8 & -4 & 1 \\ 10 & -8 & -4 \end{bmatrix}$ by Laplace expansion method. (2 Marks)

f) Find the inverse of $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$ by row reduction. (3 Marks)

10

$6-\frac{3}{2}h=0$ $\quad 3 \quad \frac{3}{2}\cdot 2 \quad 7-\frac{3}{6}\cdot 4$

$\frac{2}{3}\times\frac{3}{2}h=6\times\frac{2}{3}$ $\quad 3-\frac{3}{2}\cdot 7$

$6-\frac{3}{2}\cdot h$

QUESTION TWO (15 MARKS)

a) Define the following terms as used in transformation

i) Rank of a linear transformation, (1 Mark)

ii) Nullity of a linear transformation. (1 Mark)

b) Show that if $T: u \to v$ is defined as $T(x) = AX$, where A is $n\times m$ and X is $n\times 1$ vector, then T is a linear transformation (3 Marks)

c) Let $T: \Re^2 \to \Re^2$ be a linear transformation defined by $T(x_1, x_2) = (x_1 + 2x_2, 3x_1 - x_2)$. Find the matrix M representing T. (3 marks)

4 d) Determine if T defined as $T: \Re^3 \to \Re^2$ defined as $T(x_1, x_2, x_3) = (2x_1 - x_2, x_3 - x_1 + 1)$ is a linear transformation. (4 Marks).

e) Find h such that $\begin{bmatrix} 2 & h & \vdots & 4 \\ 3 & 6 & \vdots & 7 \end{bmatrix}$ is the augmented matrix of an *inconsistent* system. (3Marks)

$-16.$ $\quad -x_1+1 \quad u_2 + v_3$

SECTION B – ATTEMPT ANY THREE QUESTIONS IN THIS SECTION

QUESTION THREE (13 MARKS)

35

a) Consider the vectors $\{(1,4),(2,3),(3,2)\}$. Are these vectors linearly independent? (3 Marks)

b) Determine if T defined as $T: \Re^3 \to \Re^2$ defined as $T(x_1, x_2, x_3) = (x_1 + x_3, 2x_2 - x_3)$ is a linear transformation. (4 Marks).

c) Find the basis and dimension of the solution space for the equations

$x_1 + x_2 - x_3 = 0$

$-2x_1 - x_2 + 2x_3 = 0$ (6 marks)

$-x_1 + x_3 = 0$

0

QUESTION FOUR (13 MARKS)

a) Find the basis and dimension of the row space of the matrix

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 5 \\ -1 & 2 & 2 \end{bmatrix}$$ (3 Marks)

0

b) The transformation $T: \Re^3 \to \Re^3$ is defined as $T(x) = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Find

i) Basis for ~~rank~~ range of T, (4 Marks)

ii) Basis for Kernel of T (4 Marks)

iii) Rank of T and Kernel of T (2 Marks)

QUESTION FIVE (13 MARKS)

a) Give the complete solution to the system of equations

$3x - y - 5z = 9$

$y - 10z = 0$ (5 Marks)

$-2x + y = -6$

b) Choose h and k such that the augmented matrix shown has each of the following

$\left[\begin{array}{cc|c} 1 & h & 2 \\ 2 & 4 & k \end{array}\right]$ (2 Marks)

i) unique solution. (1 Mark)

ii) no solution,

iii) infinitely many solutions (1 Mark)

c) $S = \{x - x^2 + 2x^3, 1 + 2x + 3x^2, x - x^2 + 2x^3\}$ linearly independent in $\Re^3$? (4 Marks)

QUESTION SIX (13 MARKS)

a) Find the basis and dimension of the solution space for the equations

$x_1 - 3x_2 - x_3 = 0$

$2x_1 - 6x_2 + 2x_2 = 0$ (7 Marks)

$3x_1 - 9x_2 + 3x_3 = 0$

-6+3

2 -(-2)

b) Is $\{1 - 3x + 2x^2, \; 1 + x + 4x^2, \; 1 - 7x^2\}$ a basis for P_2? (6 Marks)

QUESTION SEVEN (13 MARKS)

a) Determine if (2,1) is the set generated by $\{(3,1),(2,2)\}$. (2 Marks)

b) Consider the system of equations
$$x + 2y + z = 1$$
$$3x + 2y + z = 2$$
$$2x - 3y + 2z = 3$$
find;

i) The adjoint of the matrix of coefficients, (4 Marks)

ii) The determinant of the matrix of coefficients, (2 Marks)

iii) Inverse of the matrix of the coefficients using (i) and (ii) above. (2 Marks)

iv) Solution of the system of equations using (iii) above. (3 Marks)

$$\begin{pmatrix} 7 & -7 & 0 \\ -4 & 0 & 7 \\ -13 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$y = -4 + 0 + 21$

$x = 7 + -14 + 0$

$= \frac{17}{-14}$

$x = -7 \quad x = 1/$

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