CHARGE AND CURRENT.

In elementary physics, matter is made of fundamental building blocks known as atoms and each atom consists of electrons, protons and neutrons. Charge e on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

Points to be noted about electric charge

1. The coulomb is a large unit for charges. In 1C of charge, there are $(1/1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons
2. According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$ C
3. The law of conservation of charge states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the electric charges in a system does not change.

Electric current is the time rate of change of charge, measured in amperes (A)

Mathematically, the relationship between current i, charge q, and time t is $i = \frac{dq}{dt}$ — 1.1 where current is measured in amperes (A), and

1 ampere = 1 coulomb / second.

The charge transferred between time t_0 and t is obtained by integrating both sides of eq 1.1 We obtain

$$q = \int_{t_0}^{t} i\,dt \quad \text{— 1.2}$$

If the current does not change with time, but remains constant, we call it a direct current dc (I)

Direct current dc :- Is a current that remains constant with time

Alternating current (ac) i ⇒ Is a current that varies sinusoidally with time

NB. Current is the movement of charge or the rate of charge flow.

Voltage

To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf). This emf is also known as voltage or potential difference. The voltage V_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b; mathematically,

$$V_{ab} = \frac{dw}{dq} \quad \text{------} \quad 1.3$$

Where w is energy in joules (J) and q is charge in coulombs (C). The voltage V_{ab} or simply v is measured in volts (V).

It is evident that

1 volt = 1 joule / coulomb = 1 newton meter / coulomb

Thus

Voltage (P.d) is the energy required to move a unit charge through an element, measured in volts (V) - Or:

:- Is the energy required to move 1C of charge through an element.

Current and voltage are the two basic variables in electric circuits. Like electric current, a constant voltage is called a dc voltage and is represented by V, whereas a sinusoidally time-varying voltage is called an ac voltage and is represented by v. A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

POWER AND ENERGY

Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves. Thus power and energy calculations are important in circuit analysis.

Power is the rate of expending or absorbing energy, measured in Watts (W)

:- It is the energy supplied or absorbed per unit time. It is also the product of voltage and current

To relate power and energy to voltage and current we write this relationship as

$$p = \frac{dw}{dt} \quad ----\ 1.4$$

(2) Where p is power in watt (W), w is energy in joules (J), and t is time in seconds (S). From equs. (1.1), (1.3) and (1.4), it follows that

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \quad ---\ 1.5$$

$$p = vi \quad --------\ 1.6$$

The power p in eqn 1.6 is a time-varying quantity and is called the instantaneous power. Thus the power absorbed or supplied by an element is the product of the voltage across the element and the current through it.

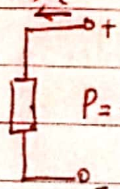
If the power has a + sign — it is being delivered to or absorbed by the element

\- Sign — the power is being supplied by the element

Current direction and voltage polarity play a major role in determining the sign of power.

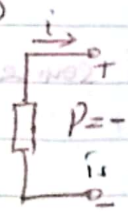
Passive sign convention, current enters through the positive polarity of the voltage. In this case

a) i



$P = +vi$ or $vi > 0$ Implies that the element is absorbing power.

b) i 1.7



$P = -vi$ or $vi < 0$ meaning element is releasing or supplying power

The law of conservation of energy must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero

$$\sum P = 0 \quad \text{---} \quad 1.8$$

This confirms that the total power supplied to the circuit must balance the total power absorbed.

From eqn (1.5), the energy absorbed or supplied by an element from t_0 to time t is

$$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt \quad \text{---} \quad 1.9$$

The electric power utility companies measure energy in watt-hours (Wh), where

$$1\,Wh = 3{,}600\,J$$

Energy is the capacity to do work, measured in joules (J)

CIRCUIT ELEMENTS

An element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are two types of elements found in electric circuits: passive and active elements.

An active element is capable of generating energy while a passive element is not

Active elements :- Generators, batteries and operational amplifiers

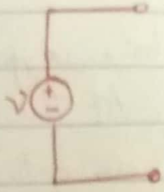
Passive element :- Resistors, Capacitors and inductors

The most important active elements are voltage or current sources that deliver power to the circuit connected to them.

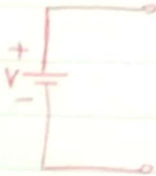
There are two kinds of sources: Independent and dependent sources.

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables.

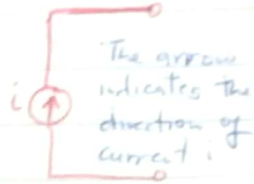
Independent Sources



Time Varying voltage source



dc voltage source

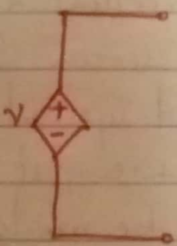


independent current source

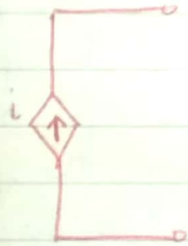
An ideal dependent (or contolled) source is an active element in which the source quantity is controlled by another voltage or curret.

Dependent sources are designated by diamond-shaped symbols,

Dependent sources



dependent Voltage Source



dependent curret source

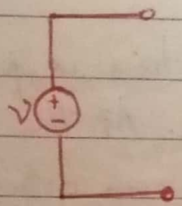
There are four possible types of dependent sources

1. Voltage-controlled voltage source (VCVS)
2. A Current-controlled voltage source (CCVS)
3. Voltage-controlled current source (VCCS)
4. Current-controlled current source (CCCS)

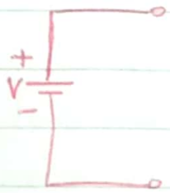
Voltage source comes with polarities (+ -) in it's symbol, while a curret source comes with an arrow, irrespective of what it depends on.

It should be noted that not only do sources supply power to a circuit, they can absorb power from a circuit too.

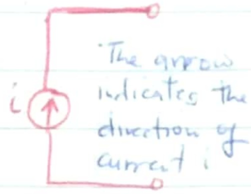
Independent Sources



Time Varying voltage source



dc voltage source



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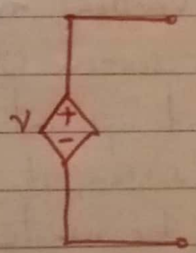
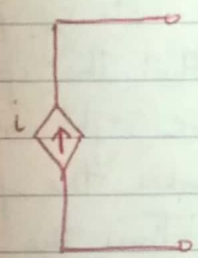
Dependent sources



dependent Voltage Source



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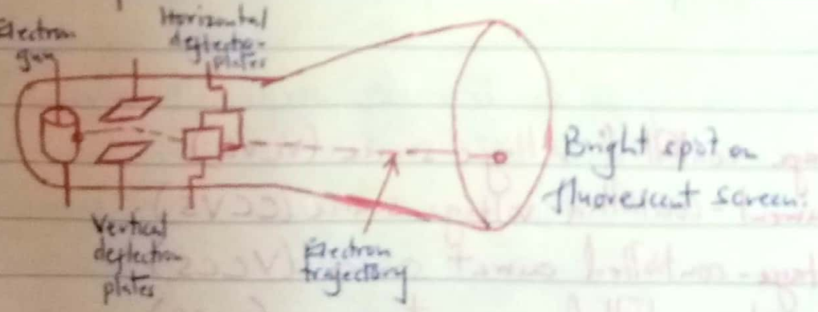
APPLICATIONS

1 TV picture tube

One important application of the motion of electrons is found in both the transmission and reception of TV signals. At the transmission end, a TV camera reduces a scene from an optical image to an electrical signal. Scanning is accomplished with a thin beam of electrons in an iconoscope camera tube.

At the receiving end, the image is reconstructed by using a cathode-ray tube (CRT) located in the TV receiver. The CRT is depicted in figure below. Unlike the iconoscope tube, which produces an electron beam of constant intensity, the CRT beam varies in intensity according to the incoming signal. The electron gun maintained at a high potential, fires the electron beam. The beam passes through two sets of plates for vertical and horizontal deflections so that the spot on the screen where the beam strikes can move right and left and up and down. When the electron beam strikes the fluorescent screen, it gives off light at that spot. Thus the beam can be made to "paint" a picture on the TV screen.



Cathode-ray tube

BASIC LAWS

Ohm's law

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current is known as resistance and is represented by the symbol R.

The resistance of any material with a uniform cross-sectional area A depends of A and it's l, as shown in figure 2.1(a). In mathematical form,

$$R = \rho \frac{l}{A} \quad \text{-------} \quad 2.1$$

Where ρ is the resistivity of the material in ohm-meters. Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities.

Ohm's law states that the voltage V across a resistor is directly proportional to the current i flowing through a resistor.

This relationship is written as

$$v \propto i \quad \text{-------} \quad 2.2$$

Ohm defined the constant of proportionality for a resistor to be the resistance R. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g. if there are changes in the temperature.) Thus,

$$v = iR \quad \text{-------} \quad 2.3$$

which is the mathematical form of Ohm's law. R in eq. 2.3 is measured in the unit of ohms, designated Ω

Short circuit is a circuit element with resistance approaching zero

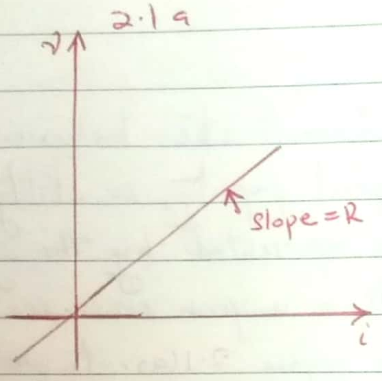
e.g $R \Rightarrow 0 \quad \therefore \quad v = iR = 0$

Open circuit is a circuit element with resistance approaching infinity

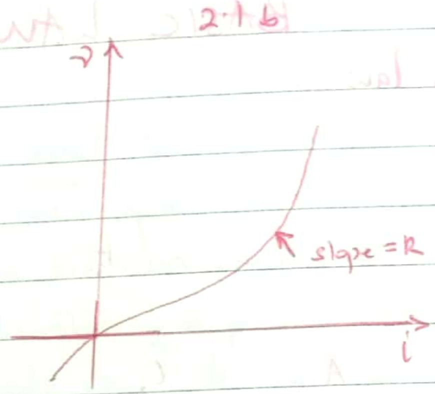
e.g $R \Rightarrow \infty \quad \therefore \quad i = \lim_{R \to \infty} \frac{v}{R} = 0$

NB Not all resistors obey ohm's law. A resistor that obeys ohm's law is known as linear resistor with $i-v$ graph of a straight line passing through the origin figure 2.1 a.

The resistance of nonlinear resistor varies with current figure 2.1b. Example of nonlinear resistance are lightbulb and diode



i-v characteristic of a linear resistor



i-v characteristic of a nonlinear resistor

The reciprocal of resistance R, is the conductance G.

$$G = \frac{1}{R} = \frac{i}{v} \quad \text{------ } 2.4$$

Conductance is a measure of how well an element will conduct electric current. (Is the ability of an element to conduct electric current). It is measured in mhos (℧) or siemens (S)

The power dissipated by a resistor can be expressed in terms of R.

$$p = vi = i^2R = \frac{v^2}{R} \quad \text{------ } 2.5$$

It can also be expressed in terms of G

$$p = vi = v^2G = \frac{i^2}{G} \quad \text{------ } 2.6$$

From eqn 2.5 and 2.6 we note that

1. The power dissipated in a resistor is a nonlinear function of either current or voltage
2. Since R and G are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit. This confirms the idea that a resistor is a passive element, incapable of generating energy.

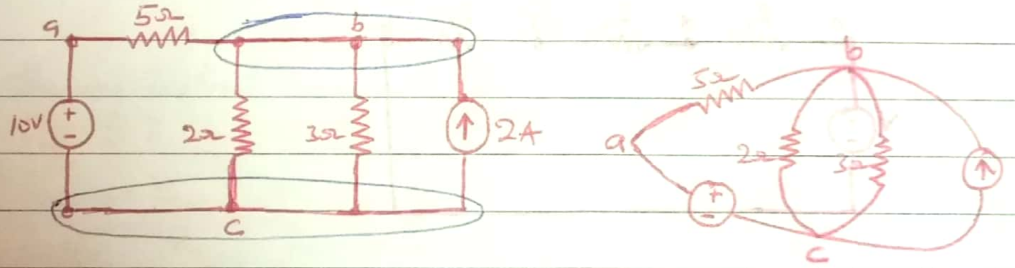
NODES, BRANCHES, AND LOOPS

Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concepts of network topology.

Network is an interconnection of elements or devices, whereas a Circuit is a network providing one or more closed paths.

In network topology, we study the properties relating to the placement of elements in the network and the geometric configuration of the network. Such elements include branches, nodes and loops

A branch represents a single element such as a voltage source or a resistor e.g figure 2.2 has five branches, 10V source, 2A current source and the three resistors



A node is the point of connection between two or more branches. e.g figure 2.2 has three nodes a, b & c.

A loop is any closed path in a circuit. figure 2.2 has six loops with three being independent

A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology

$$b = l + n - 1$$

KIRCHHOFF'S LAWS

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero

$$\sum_{n=1}^{N} i_n = 0$$

Where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node

(The sum of the currents entering a node is equal to the sum of the currents leaving the node)

Kirchhoff's Voltage law (KVL) states that the algebraic sum of all potential differences in a closed loop (or closed circuit) is equal to zero.

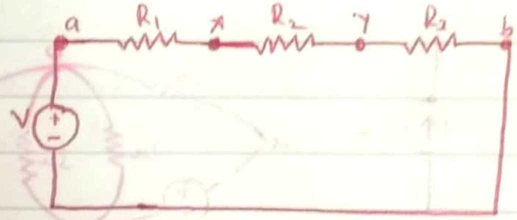
$$\sum_{m=1}^{M} v_m = 0$$

Where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.

B SERIES AND PARALLEL RESISTORS

Resistors in Series

Consider the circuit below



If the current I is flowing through the circuit between points a and b then applying Ohm's law to each of the resistors, gives

$V_{ax} = IR_1$, $V_{xy} = IR_2$, $V_{yb} = IR_3$

The total voltage between points a and b is

$$V_{ab} = V_{ax} + V_{xy} + V_{yb}$$
$$= IR_1 + IR_2 + IR_3$$
$$= I(R_1 + R_2 + R_3)$$

Meaning that the three resistors can be replaced by an equivalent resistor R_{eq}

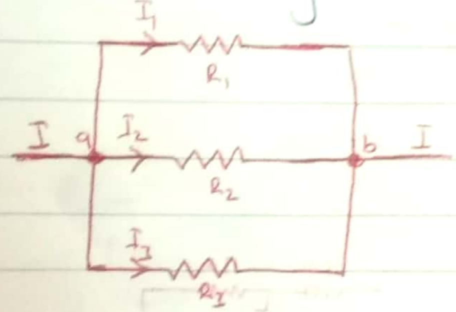
$\therefore V_{ab} = I R_{eq}$

but $R_{eq} = R_1 + R_2 + R_3$

Thus the equivalent resistance of any number of resistors in series equals the sum of the individual resistances

Resistors in parallel

Consider the diagram below



Let the currents flowing in the resistors R_1, R_2 and R_3 be I_1, I_2 and I_3. Then

$$I_1 = \frac{V_{ab}}{R_1}, \quad I_2 = \frac{V_{ab}}{R_2}, \quad I_3 = \frac{V_{ab}}{R_3}$$

Let the equivalent current I_{eq} be

$$I_{eq} = I_1 + I_2 + I_3$$

$$= \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

$$= V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Let the equivalent resistor R_{eq} be

$$I = \frac{V_{ab}}{R_{eq}} = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

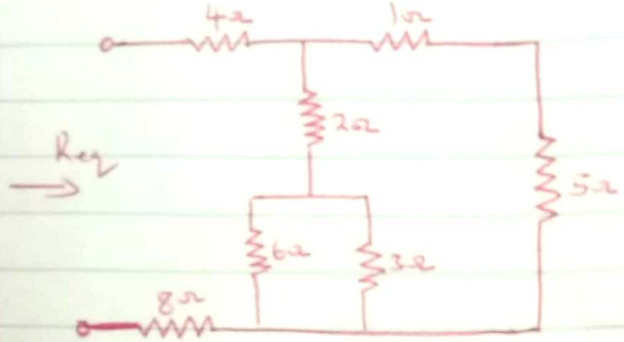
$$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq} = \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3}$$

In general, for any number of resistors in parallel, the reciprocal of the equivalent resistance equals the sum of the reciprocals of their individual resistances

Example

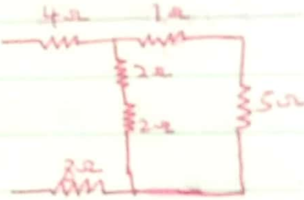
Find the equivalent resistance R_{eq} of the circuit below



Soln.

1. 6Ω and 3Ω are parallel

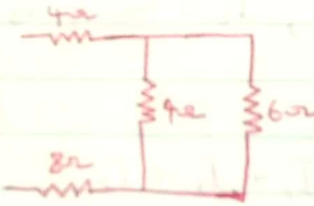
$$6\Omega // 3\Omega = \frac{6 \times 3}{6+3} = 2\Omega$$



2. 1Ω and 5Ω are in series

$$1\Omega + 5\Omega = 6\Omega$$

and

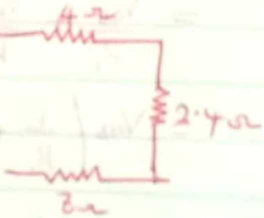


3. 2Ω and 2Ω are in series

$$2\Omega + 2\Omega = 4\Omega$$

4. 4Ω and 6Ω are in parallel

$$4\Omega // 6\Omega = \frac{4 \times 6}{4+6} = 2.4\Omega$$



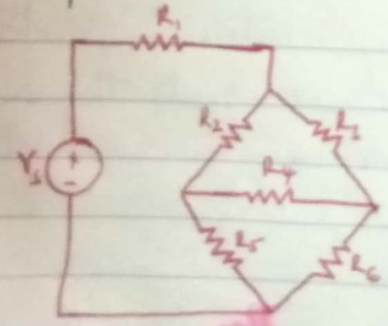
5. lastly 4Ω, 2.4Ω and 8Ω are in series

$$\therefore 4\Omega + 2.4\Omega + 8\Omega = 14.4\Omega$$

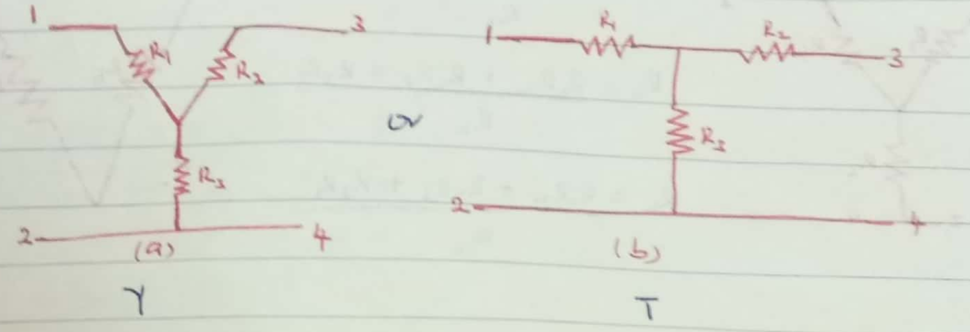
$$R_{eq} = 14.4\Omega$$

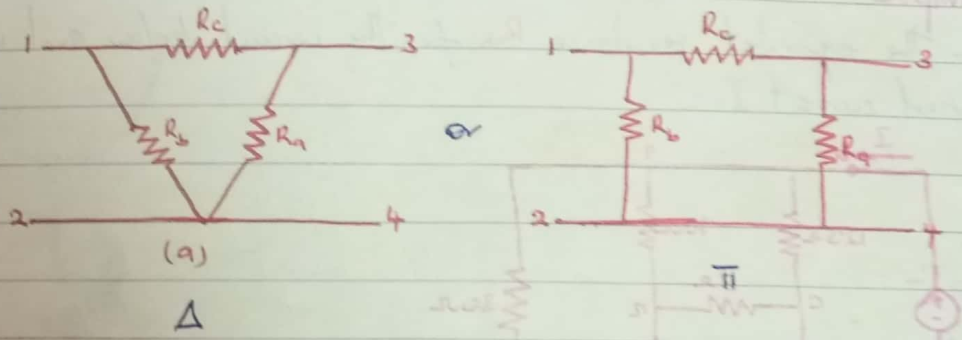
WYE - DELTA TRANSFORMATIONS

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. like in the circuit below

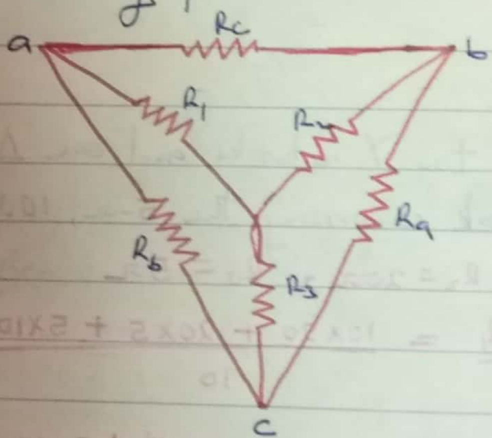


Many circuits like the above can be simplified by using three-terminal equivalent networks. These are the wye (Y) or tee (T) network and the delta (Δ) or pi (π) network.

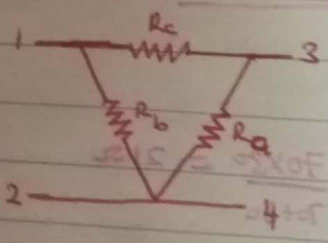




When converting from Δ or π to Y or T and vice-versa



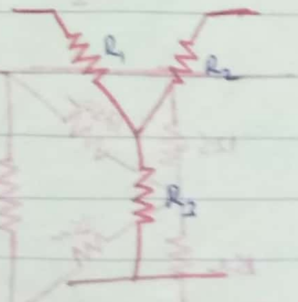
Delta to Y conversion



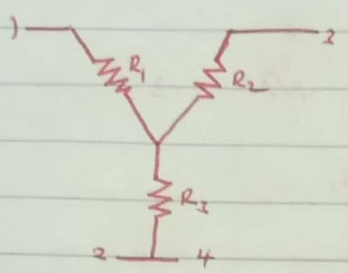
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



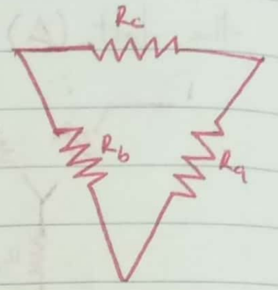
Wye to Delta conversion



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

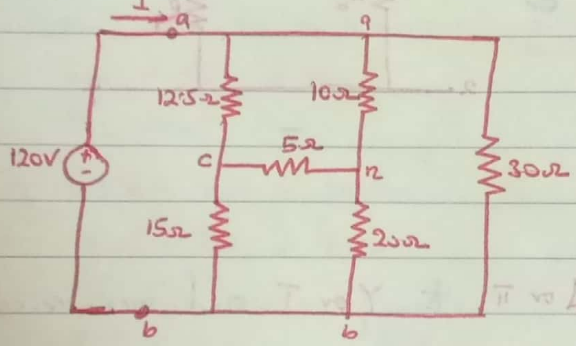
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$



Example

Obtain the equivalent resistance R_{ab} for the circuit below and use it to find current I



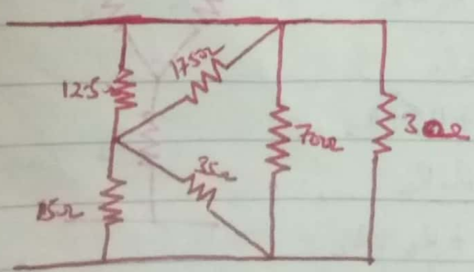
Soln.

In this circuit, there are two Y networks and one Δ network. If we convert the Y network comprising the 5-Ω, 10Ω and 20Ω resistors. taking $R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 5\Omega$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10\times20 + 20\times5 + 5\times10}{10} = 35\Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{10\times20 + 20\times5 + 5\times10}{20} = 17.5\Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{10\times20 + 20\times5 + 5\times10}{5} = 70\Omega$$



$$70 // 30 = \frac{70\times30}{70+30} = 21\Omega$$

$$12.5 // 17.5 = \frac{12.5\times17.5}{12.5+17.5} = 7.2917\Omega$$

$$15 // 35 = \frac{15\times35}{15+35} = 10.5\Omega$$

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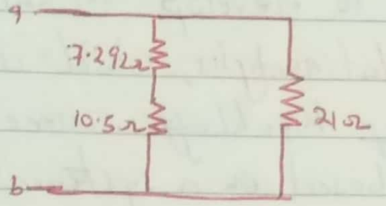
Equivalent circuit is as shown



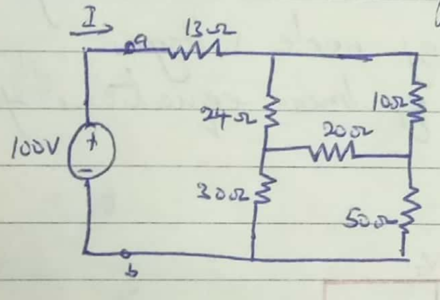
$$R_{ab} = (7.292 + 10.5) // 21$$

$$= \frac{17.792 \times 21}{17.792 + 21} = 9.632\,\Omega$$

Therefore $I = \frac{V_s}{R_{ab}} = \frac{120\,V}{9.632\,\Omega} = 12.458\,A.$

~~Applications~~ Assignment

For the network below find R_{ab} and I Ans 40Ω, $2.5A$



Application

Resistors are often used to model devices that convert electrical energy into heat or other forms of energy. Such devices include conducting wire, lightbulbs, electric heaters, stoves, ovens and loudspeakers.

CIRCUIT ANALYSIS

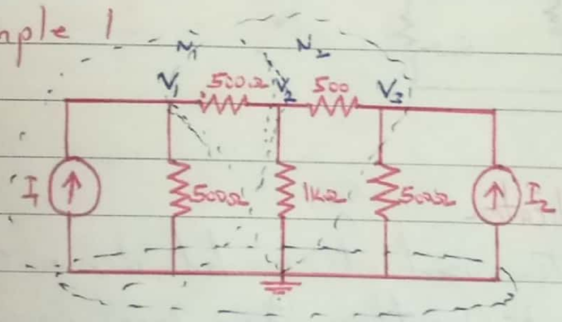
The fundamental laws of circuit theory (Ohm's law and Kirchhoff's laws) helps us to develop techniques for circuit analysis namely Modal analysis, which is based on a systematic application of Kirchhoff's current law (KCL) and Mesh analysis, which is based on a systematic applicat[ion] of Kirchhoff's voltage law (KVL).

MODAL ANALYSIS

Steps of Nodal Analysis

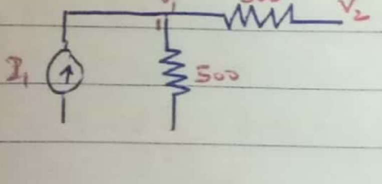
1. Choose a reference (ground) node
2. Assign node voltages to the other nodes
3. Apply KCL to each node other than the reference node; express currents in terms of node voltages
4. Solve the resulting system of linear equations for the node voltages.

Example 1



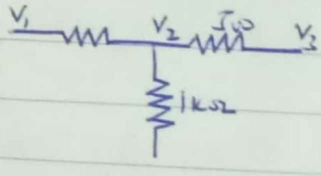
KCL at Node 1

$$I_1 = \frac{V_1 - V_2}{500} + \frac{V_1}{500}$$



at Node 2

$$\frac{V_1 - V_2}{500} + \frac{V_2}{1000} + \frac{V_2 - V_3}{500} =$$



at Node 3

$$I_2 = \frac{V_3 - V_2}{500} + \frac{V_3}{500}$$

Solve the equations to find the nodal voltages V_1, V_2 and V_3

at Node 1

$$I_1 = \frac{V_1 - V_2}{500} + \frac{V_1}{500}$$

$$500I_1 = V_1 - V_2 + V_1$$

$$500I_1 = 2V_1 - V_2$$

$$2V_1 = 500I_1 + V_2$$

$$V_1 = 250I_1 + \frac{V_2}{2} \quad \text{----(i)}$$

at Node 3

$$I_2 = \frac{V_3 - V_2}{500} + \frac{V_3}{500}$$

$$500I_2 = V_3 - V_2 + V_3$$

$$500I_2 = 2V_3 - V_2$$

$$2V_3 = 500I_2 + V_2$$

$$V_3 = 250I_2 + \frac{V_2}{2} \quad \text{----(iii)}$$

At node 2

$$\frac{V_2 - V_1}{500} + \frac{V_2}{1000} + \frac{V_2 - V_3}{500} = 0$$

$$2V_2 - 2V_1 + V_2 + 2V_2 - 2V_3 = 0$$

$$5V_2 - 2V_1 - 2V_3 = 0 \quad \text{----(ii)}$$

Using eqns (i) & (iii) to substitute in eqn (ii)

$$5V_2 - 2\left(250I_1 + \frac{V_2}{2}\right) - 2\left(250I_2 + \frac{V_2}{2}\right) = 0$$

$$5V_2 - 500I_1 - V_2 - 500I_2 - V_2 = 0$$

$$3V_2 - 500I_1 - 500I_2 = 0$$

$$3V_2 = 500I_1 + 500I_2$$

$$V_2 = \frac{500}{3}I_1 + \frac{500}{3}I_2$$

$$V_2 = 166.7I_1 + 166.7I_2 \quad \text{----(iv)}$$

finding V_1 and V_2

$$V_1 = 250I_1 + \frac{V_2}{2}$$

$$V_1 = 250I_1 + \frac{1}{2}\left(166.7I_1 + 166.7I_2\right)$$

$$V_1 = 250I_1 + 83.4I_1 + 83.4I_2$$

$$V_1 = 333.4I_1 + 83.4I_2 \quad \text{---(v)}$$

$$V_3 = 250I_2 + \frac{V_2}{2}$$

$$V_3 = 250I_2 + \frac{1}{2}\left(166.7I_1 + 166.7I_2\right)$$

$$V_3 = 250I_2 + 83.4I_1 + 83.4I_2$$

$$V_3 = 83.4I_1 + 333.4I_2 \quad \text{---(vi)}$$

Therefore

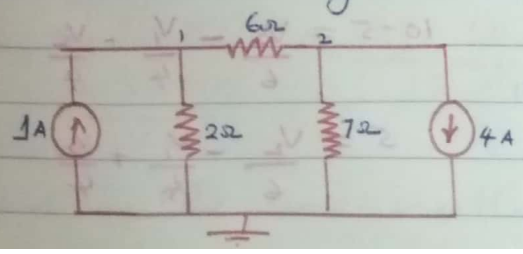
$$V_1 = 333.4I_1 + 83.4I_2$$

$$V_2 = 166.7I_1 + 166.7I_2$$

$$V_3 = 83.4I_1 + 333.4I_2$$

Example 2

Obtain the node voltages in the circuit below



Soln.

Node 1

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = V_1 - V_2 + 3V_1$$

$$6 = 4V_1 - V_2 \quad \text{------ (i)}$$

Node 2.

$$-4 = \frac{V_2 - V_1}{6} + \frac{V_2}{7}$$

$$-168 = 7V_2 - 7V_1 + 6V_2$$

$$-168 = 13V_2 - 7V_1 \quad \text{------ (ii)}$$

$$6 = 4V_1 - V_2$$

$$-168 = -7V_1 + 13V_2$$

$$\Rightarrow \quad 78 = 52V_1 - 13V_2$$

$$-168 = -7V_1 + 13V_2$$

$$-90 = 45V_1$$

$$\therefore V_1 = -2$$

Equating V_1 in eqn (i)

$$6 = 4V_1 - V_2$$

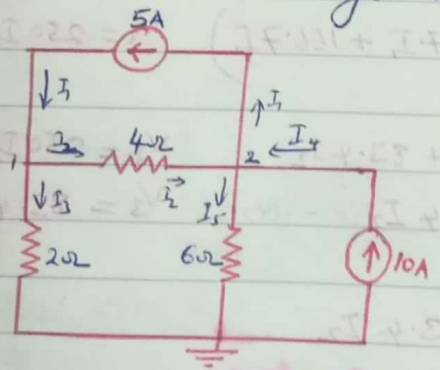
$$6 = -8 - V_2$$

$$V_2 = -8 - 6$$

$$V_2 = -14$$

Example 3

Calculate the node voltages in the circuit shown below



At node 1

$$I_1 = I_2 + I_3$$

$$5 = \frac{V_1 - V_2}{4} + \frac{V_1}{2}$$

$$20 = V_1 - V_2 + 2V_1$$

$$\cancel{20 = 5V_1 - 5V_2 + 10V_1}$$

$$20 = 3V_1 - V_2 \quad \text{---- (1)}$$

At node 2

$$I_2 + I_4 = I_1 + I_5$$

$$\frac{V_1 - V_2}{4} + 10 = 5 + \frac{V_2}{6}$$

$$10 - 5 = \frac{V_2}{6} - \frac{V_1}{4} + \frac{V_2}{4}$$

$$5 = \frac{V_2}{6} - \frac{V_1}{4} + \frac{V_2}{4}$$

Multiplying by 24

$120 = 4V_2 - 6V_1 + 6V_2$

$120 = 10V_2 - 6V_1$

$60 = 5V_2 - 3V_1$ - - - - - (ii)

Using elimination technique

$$\begin{aligned} &20 = 3V_1 - V_2 \\ + \;&\underline{60 = -3V_1 + 5V_2} \\ &80 = 4V_2 \\ &\underline{V_2 = 20V} \end{aligned}$$

Substitute V_2 in eqn 1

$20 = 3V_1 - V_2$

$20 = 3V_1 - 20$

$3V_1 = 20+20$

$V_1 = \frac{40}{3} \simeq 13.33V$

$\therefore V_2 = 20V$ & $V_1 = 13.33V$.

Using Cramers rule

Take eqn (1) and (ii) in a matrix form

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

The determinant of the matrix is

$$\Delta = \begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} = 15 - 3 = 12$$

Obtain V_1 and V_2 as

$$V_1 = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{12} = \frac{100+60}{12} = 13.33V$$

$$V_2 = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{12} = \frac{180+60}{12} = 20.$$

Which is the same result.

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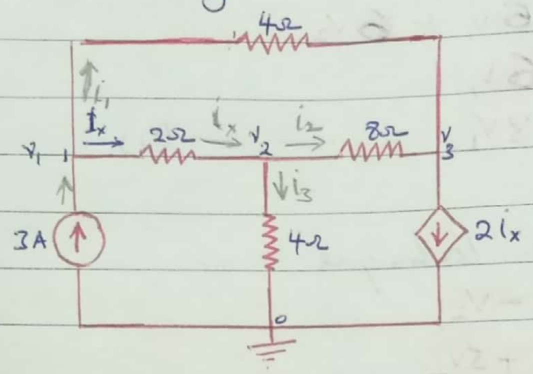
Example 4.

Determine the voltages at the nodes of the figure below



At node 1,

$3 = i_1 + i_x \quad \Rightarrow \quad 3 = \dfrac{v_1 - v_3}{4} + \dfrac{v_1 - v_2}{2}$

Multiplying by 4 and rearranging terms, we get

$3v_1 - 2v_2 - v_3 = 12$ ---- (i)

At node 2,

$i_x = i_2 + i_3 \quad \Rightarrow \quad \dfrac{v_1 - v_2}{2} = \dfrac{v_2 - v_3}{8} + \dfrac{v_2 - 0}{4}$

Multiplying by 8 and rearranging terms, we get

$-4v_1 + 7v_2 - v_3 = 0$ ---- (ii)

At node 3,

$i_1 + i_2 = 2i_x \quad \Rightarrow \quad \dfrac{v_1 - v_3}{4} + \dfrac{v_2 - v_3}{8} = \dfrac{2(v_1 - v_2)}{2}$

Multiplying by 8, rearranging terms, and dividing by 3, we get

$2v_1 - 3v_2 + v_3 = 0$ ---- (iii)

We have three simultaneous equations to solve to get the node voltages v_1, v_2 and v_3. We shall solve the equations in two ways

Method 1

Using the elimination technique. Add eqns (i) & (iii)

$$\begin{aligned} 3v_1 - 2v_2 - v_3 &= 12 \\ 2v_1 - 3v_2 + v_3 &= 0 \\ \hline 5v_1 - 5v_2 \quad\quad &= 12 \end{aligned}$$ ---- (iv)

$$v_1 - v_2 = \frac{12}{5} = 2.4$$

Adding eqn (ii) & (iii)

$$-4v_1 + 7v_2 - v_3 = 0$$
$$2v_1 - 3v_2 + v_3 = 0$$
$$-2v_1 + 4v_2 = 0$$
$$2v_1 = 4v_2$$
$$v_1 = 2v_2 \quad \text{--- (v)}$$

Substituting (v) into (iv)

$$5(2v_2) - 5v_2 = 12$$
$$10v_2 - 5v_2 = 12$$
$$5v_2 = 12$$
$$v_2 = \frac{12}{5} = 2.4$$

$$v_1 = 2v_2 \Rightarrow v_1 = 2(2.4) \Rightarrow v_1 = 4.8$$

Taking eqn (iii)

$$2v_1 - 3v_2 + v_3 = 0$$
$$2(4.8) - 3(2.4) + v_3 = 0$$
$$9.6 - 7.2 + v_3 = 0$$
$$2.4 + v_3 = 0$$
$$v_3 = -2.4$$

$$v_1 = 4.8, \quad v_2 = 2.4, \quad v_3 = -2.4.$$

Method 2. Cramers rule.

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

From this we obtain

$$v_1 = \frac{\Delta_1}{\Delta}, \quad v_2 = \frac{\Delta_2}{\Delta}, \quad v_3 = \frac{\Delta_3}{\Delta}$$

Where Δ, Δ_1, Δ_2 and Δ_3 are the determinants to be calculated as follows. To calculate the determinant of a 3X3 matrix, we repeat the first two rows and cross multiply

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \\ -3 & -2 & -1 \\ -4 & 7 & -1 \end{vmatrix}$$

$$= 21 - 12 + 4 + 14 - 9 - 8 = 10$$

Similarly, we obtain

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \\ 12 & -2 & -1 \\ 0 & 7 & -1 \end{vmatrix} = 84 + 0 + 0 - 0 - 36 - 0 = 48$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & 1 \\ 2 & 0 & 1 \\ 3 & 12 & -1 \\ -4 & 0 & 1 \end{vmatrix} = 0 + 0 - 24 - 0 - 0 + 48 = 24$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \\ 3 & -2 & 12 \\ -4 & 7 & 0 \end{vmatrix} = 0 + 144 + 0 - 168 - 0 - 0 = -24$$

Thus, we find

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8\,V \qquad v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4\,V$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4\,V$$

Nodal analysis with Voltage sources

1. If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source
2. If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode; we apply both KCL and KVL to determine the node voltages

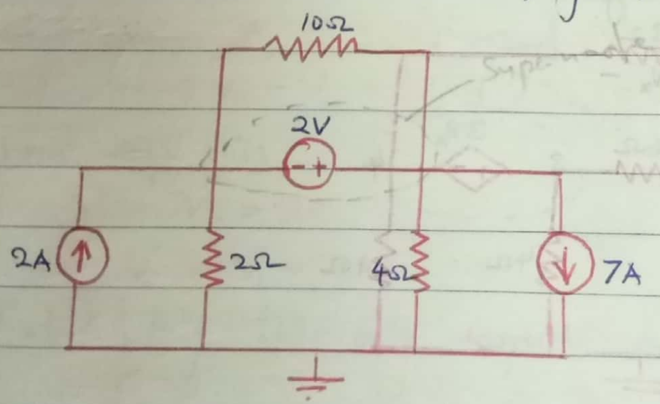
A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

Note the following properties of a supernode

1. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages
2. A supernode has no voltage of it's own
3. A supernode requires the application of both KCL and KVL

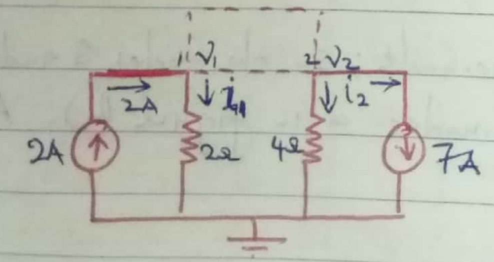
Example 5

For the circuit shown below, find the node voltages



The supernode contains 2V source, nodes 1 and 2, and the 10Ω resistor. Applying KCL to the supernode as shown in b gives

b)



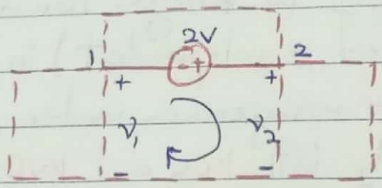
$2 = i_1 + i_2 + 7$

Expressing i_1 and i_2 in terms of node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7$$

$$\Rightarrow 8 = 2v_1 + v_2 + 28 \quad \text{or} \quad v_2 = -20 - 2v_1 \quad \text{---(1)}$$

To get the relationship between v_1 and v_2, we apply KVL to the circuit in (c). Going round the loop, we obtain

c) 

$$-v_1 - 2 + v_2 = 0$$

$$v_2 = v_1 + 2 \quad \text{---(ii)}$$

From eqn (i) and (ii) we write

$$-20 - 2v_1 = v_1 + 2$$

$$3v_1 = -22$$

$$v_1 = -7.333V.$$

but $v_2 = v_1 + 2 \Rightarrow v_2 = -7.333 + 2$

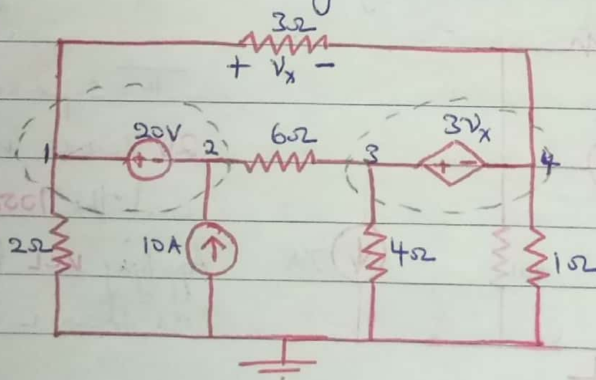
$$= -5.333V$$

NB.

The 10Ω resistor does not make any difference because it is connected across the super-node.

Example 6.

Find the node voltages in the circuit below.

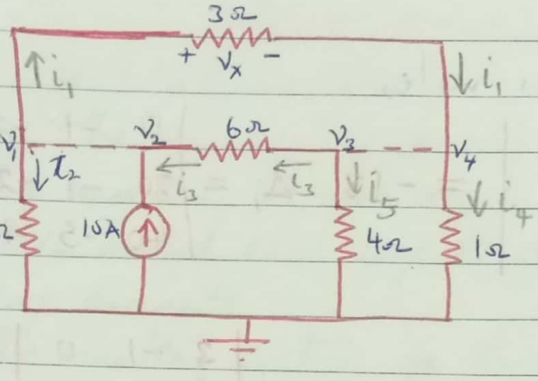


Sol.

Nodes 1 and 2 form a supernode; so do nodes 3 and 4. We apply KCL to the two supernodes as in figure (b). At supernode 1-2,

$$i_3 + 10 = i_1 + i_2$$

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

b) 

$$5v_1 + v_2 - v_3 - 2v_4 = 60 \quad \text{---(i)}$$

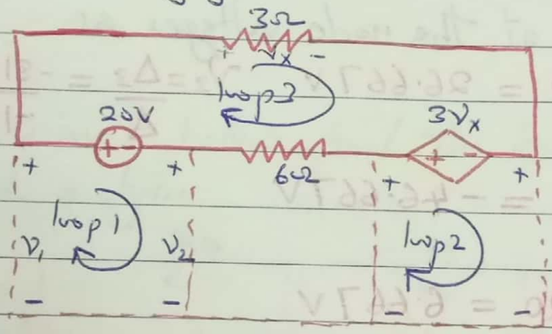
At supernode 3-4,

$$i_1 = i_3 + i_4 + i_5$$

$$\frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4} \quad \text{---(ii)}$$

$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$

We now apply KVL to the branches involving the voltage sources as shown in figure (c). For loop 1



$$-v_1 + 20 + v_2 = 0$$

$$v_1 - v_2 = 20 \quad \text{---(iii)}$$

For loop 2

$$-v_3 + 3v_x + v_4 = 0$$

But $v_x = v_1 - v_4$

so that

$$3v_1 - v_3 - 2v_4 = 0 \quad \text{-----(iv)}$$

For loop 3

$$v_x - 3v_x + 6i_3 - 20 = 0$$

But $6i_3 = v_3 - v_2$ and $v_x = v_1 - v_4$. Hence

$$-2v_1 - v_2 + v_3 + 2v_4 = 20 \quad \text{-----(v)}$$

From eqn (iii)

$$v_1 - v_2 = 20$$

$$v_2 = v_1 - 20$$

Substituting this into eqn (i) and (ii) respectively.

$$6v_1 - v_3 - 2v_4 = 80 \quad \text{-----(vi)}$$

and $$6v_1 - 5v_3 - 16v_4 = 40 \quad \text{-----(vii)}$$

Eqns (iv) (vi) and (vii) can be cast into a matrix form as

$$\begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} \begin{vmatrix} v_1 \\ v_2 \\ v_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 80 \\ 40 \end{vmatrix}$$

Using Cramer's rule,

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18 \qquad \Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{vmatrix} = -480$$

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{vmatrix} = -3120 \qquad \Delta_4 = \begin{vmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{vmatrix} = 840$$

Thus, we arrive at the node voltages as

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.667\,V \qquad V_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.333\,V$$

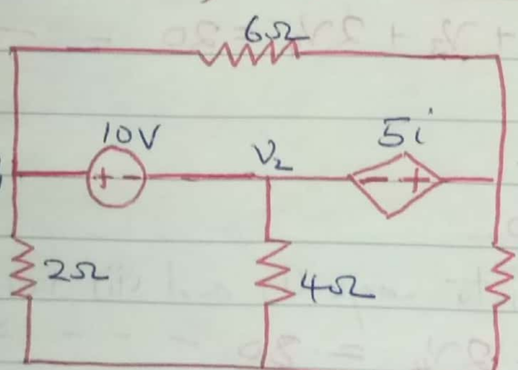
$$V_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.667\,V$$

and $V_2 = V_1 - 20 = 6.667\,V$

Assignment

Find V_1, V_2 and V_3 in the circuit below using nodal analysis

Ans $V_1 = 3.043\,V$, $V_2 = -6.956\,V$, $V_3 = 0.6522\,V$



Mesh Analysis

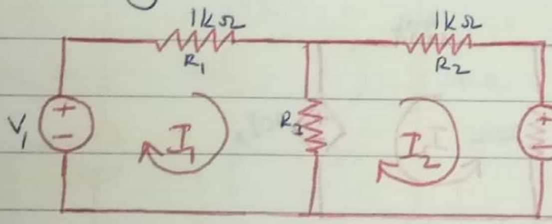
Steps of Mesh analysis

1. Identify the number of basic meshes
2. Assign a current to each mesh
3. Apply KVL around each loop to get an equation in terms of the loop currents
4. Solve the resulting system of linear equations

A mesh is a loop which does not contain any other loops within it.

Example. 1

For the circuit below find the branch currents I_1 and I_2 using mesh analysis



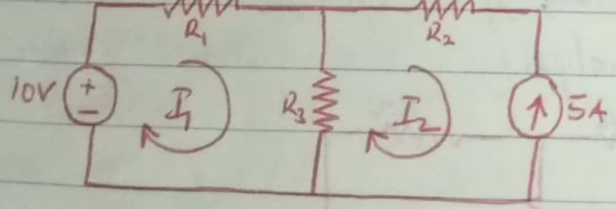
Mesh equations

$$-V_1 + I_1R_1 + (I_1 - I_2)R_3 = 0$$

$$I_2R_2 + V_2 + (I_2 - I_1)R_3 = 0$$

Example 2 — Case 1

When a current source exists in one mesh.



Loop 1

$$-10 + 4I_1 + 6(I_1 - I_2) = 0$$

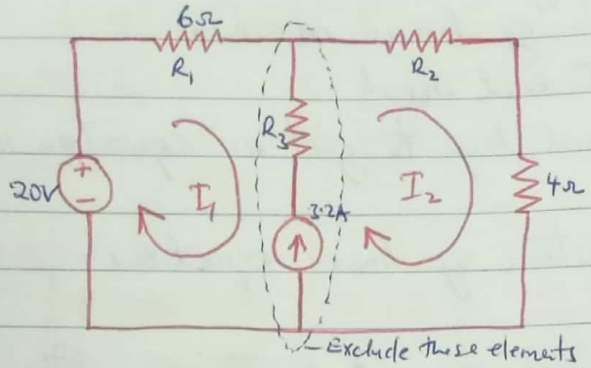
Loop

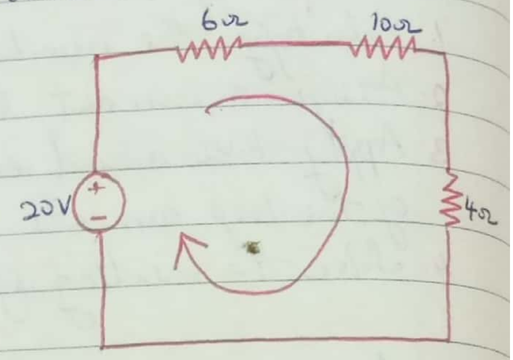
$$I_2 = -5A$$

No need to write a loop equation

Case 2.

When a current source exists between two meshes.





$-20 + 6I_1 + 10I_2 + 4I_2 = 0$

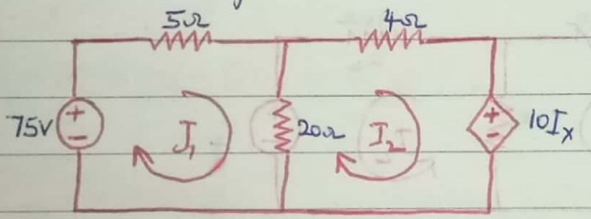
$I_2 = I_1 + 6$

$I_1 = -3.2\text{ A}$

$I_2 = 2.8\text{A}$

Case 3

Mesh with dependent sources



$-75 + 5I_1 + 20(I_1 - I_2) = 0$

$10I_x + 20(I_2 - I_1) + 4I_2 = 0$

$I_x = I_1 - I_2$

$-75 + 5I_1 + 20(I_1 - I_2) = 0$

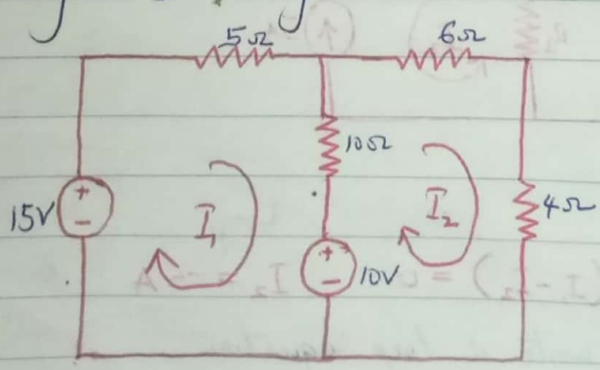
$10(I_1 - I_2) + 20(I_2 - I_1) + 4I_2$

$I_2 = 5\text{A}$

$I_1 = 7\text{A}$

Example 3

For the circuit below, find the branch currents I_1, I and I_2 using mesh analysis



use Elimination or Cramers rule

Soln.

Mesh I

$-15 + 5I_1 + 10(I_1 - I_2) + 10 = 0$

$-5 + 15I_1 - 10I_2 = 0$

$15I_1 - 10I_2 = 5$

$3I_1 - 2I_2 = 1$

Mesh 2

$6I_2 + 4I_2 + 10(I_2 - I_1) - 10 = 0$

$20I_2 - 10I_1 = 10$

$2I_2 - I_1 = 1$

Elimination method

$3I_1 - 2I_2 = 1$

$-I_1 + 2I_2 = 1$

$2I_1 = 2$

$I_1 = 1A$

$3I_1 - 2I_2 = 1$

$3 - 2I_2 = 1$

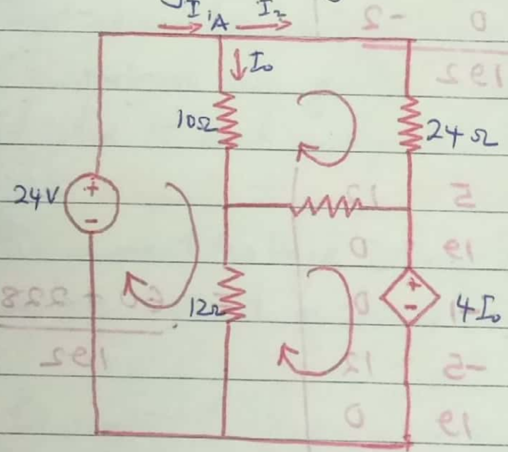
$-2I_2 = -2$

$I_2 = 1$

$\therefore I_1 = 1A, \; I_2 = 1A$

Example 4

Use mesh analysis to find the current I_0 in the circuit below



Soln :

Mesh 1

$-24 + 10(I_1 - I_2) + 12(I_1 - I_3) = 0$

$-24 + 10I_1 - 10I_2 + 12I_1 - 12I_3 = 0$

$22I_1 - 10I_2 - 12I_3 = 24$

$11I_1 - 5I_2 - 6I_3 = 12.$

Mesh 2.

$10(I_2 - I_1) + 24I_2 + 4(I_2 - I_3) = 0$

$10I_2 - 10I_1 + 24I_2 + 4I_2 - 4I_3 = 0$

$-10I_1 + 38I_2 - 4I_3 = 0$

$-5I_1 + 19I_2 - 2I_3 = 0$

Mesh 3

$+4I_0 + 12(I_3 - I_2) + 4(I_3 - I_2) =$

But at node A $I_0 = I_1 - I_2$

$+4(I_1 - I_2) + 12(I_3 - I_2) + 4(I_3 - I_2) = 0$

$4I_1 - 20I_2 + 16I_3 = 0$

$I_1 - 5I_2 + 4I_3 = 0$

Using Cramer's rule

$$\begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ 1 & -5 & 4 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

Obtaining the determinant

$$\begin{vmatrix} 11 & -5 & -6 \\ -5 & -19 & -2 \\ 1 & -5 & 4 \\ 11 & -5 & -6 \\ -5 & -19 & -2 \end{vmatrix}$$

$\Delta \Rightarrow 418 - 30 - 10 - 114 - 22 - 50 = 192$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 12 & 5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{vmatrix}}{192} = \frac{456-24}{192} = \frac{432}{192}$$

$I_1 = 2.25\,A.$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{vmatrix}}{192} = \frac{24+120}{192} = \frac{144}{192}$$

$I_2 = 0.75\,A.$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 11 & 5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{vmatrix}}{192} = \frac{60+228}{192} = \frac{288}{192}$$

$I_3 = 1.5\,A$

Thus $I_1 = 2.25\,A$, $I_2 = 0.75\,A$ $I_3 = 1.5\,A$

and $I_0 = I_1 - I_2 = 2.25 - 0.75$

$= 1.5\,A.$

CIRCUIT THEOREMS

A major advantage of analyzing circuits using Kirchhoff's laws is that circuits can be analyzed without tampering with it's original configuration. A major disadvantage of this approach is that, for a large, complex circuit, tedious computation is involved. To handle the complexity some theorem's have been developed over years to simplify circuit analysis.

Such theorems include Thevenin's and Norton's theorems. In addition to circuit theorem, we look at the concept of linearity, superposition, source transformation and maximum power transfer.

SUPERPOSITION THEOREM

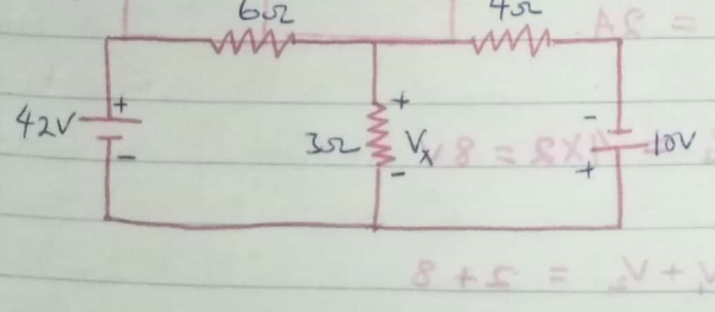
States that

The voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to each independent source acting alone.

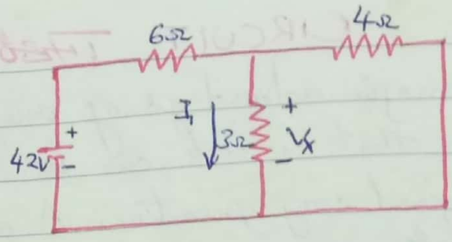
Steps to superposition theorem

1. To find the current/voltage in any branch, take one source at a time and replace the rest of the sources by their internal resistances (if given)
2. Calculate the current/voltage by any method (mesh/nodal/KVL/KCL)
3. Now calculate the current/voltage in the same branch by taking the other source in the circuit and replacing the rest of the sources by their internal resistances.
4. Repeat steps 1 and 2 until all the sources have been considered
5. Total current/voltage in the given branch = algebraic sum of all the currents/voltages in the branch due to all the current sources.

Example 1

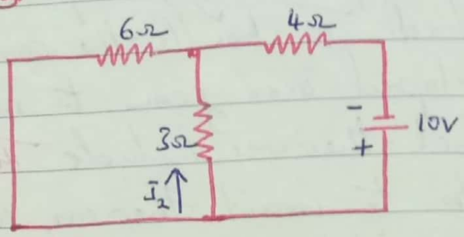


Considering 42V source only
(10V source SC)



$$V_x = \frac{\frac{3\times4}{3+4}}{6+\frac{3\times4}{3+4}} \times 42 = 9.33$$

Only 10V source connected
(42V source SC)



$$V_x = \frac{\frac{6\times3}{6+3}}{4+\frac{6\times3}{6+3}} \times 10 = 3.33$$

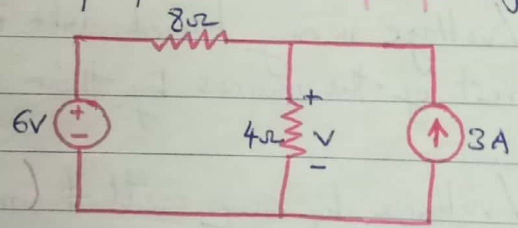
Total voltage $V_x = V_{x(42)} + V_{x(10)}$

$= 9.33 - 3.33$

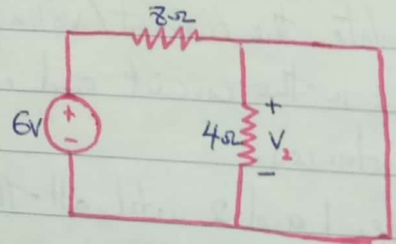
$= 6V$

Example 2

Use the superposition theorem to find V in the circuit below

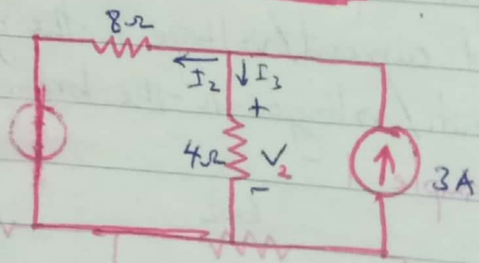


Considering 6V source only
(3A source SC)



$$V_1 = \frac{4}{4+8} \times 6 = 2V$$

Taking 3A source only
(6V source SC)



$$I_3 = \frac{8}{4+8} \times 3 = 2A.$$

Hence $V_2 = 4I_3 = 4\times2 = 8V$

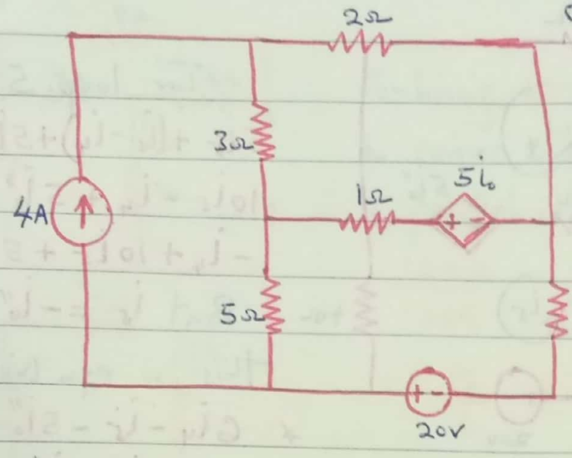
And we find

$V = V_1 + V_2 = 2+8$

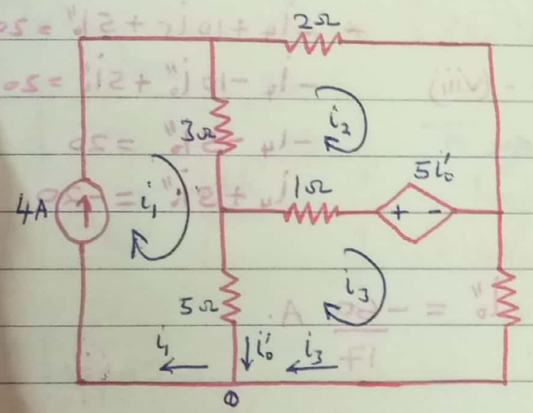
$= 10V.$

Example 3

Find i_o in the circuit below using superposition



The dependent source must be left intact. Taking the 4A source and SC (20V)



We apply mesh analysis in order to obtain i_o'

For loop 1

$i_1 = 4A$ ---(i)

For loop 2.

$3(i_2 - i_1) + 2(i_2) - 5i_o' + 1(i_2 - i_3) = 0$

$3i_2 - 3i_1 + 2i_2 - 5i_o' + i_2 - i_3 = 0$

$6i_2 - 3i_1 - 5i_o' - i_3 = 0$

$-3i_1 + 6i_2 - i_3 - 5i_o' = 0$ ------(ii)

For loop 3.

$5(i_3 - i_1) + (i_3 - i_2) + 5i_o' + 4i_3 = 0$

$5i_3 - 5i_1 + i_3 - i_2 + 5i_o' + 4i_3 = 0$

$10i_3 - 5i_1 - i_2 + 5i_o = 0$

$-5i_1 - i_2 + 10i_3 + 5i_o' = 0$ --(iii)

But at node 0

$i_3 = i_1 - i_o' = 4 - i_o'$ ------(iv)

Substituting eqn (i) and (iv) into eqns (ii) and (iii)

$-3i_1 + 6i_2 - i_3 - 5i_o = 0$

$-12 + 6i_2 - (4 - i_o') - 5i_o = 0$

$-12 + 6i_2 - 4 + i_o' - 5i_o = 0$

$6i_2 - 4i_o' = 16$

$3i_2 - 2i_o' = 8$ ---(v)

$-5i_1 - i_2 + 10i_3 + 5i_o' = 0$

$-20 - i_2 + 10(4 - i_o') + 5i_o' = 0$

$-20 - i_2 + 40 - 10i_o' + 5i_o' = 0$

$20 - i_2 - 5i_o' = 0$

$i_2 + 5i_o' = 20$ ---(vi)

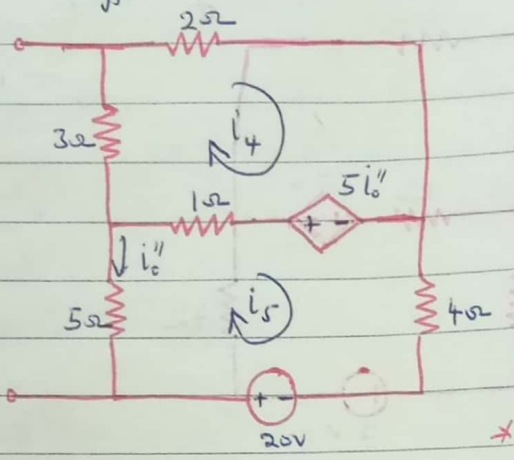
Two simultaneous equations

$3i_2 - 2i_o' = 8$

$i_2 + 5i_o' = 20$ (vii)

Solving the eqns we get $i_o' = \frac{52}{17}A$

We turn off 4A current source



For loop 5

$5i_5 + (i_5 - i_4) + 5i_o'' + 4i_5 - 20 = 0$

$10i_5 - i_4 + 5i_o'' = 20$

$-i_4 + 10i_5 + 5i_o'' = 20$ --- (ix)

But $i_5 = -i_o''$ substituting this in eqn (viii) and (ix)

$* \ 6i_4 - i_5 - 5i_o'' = 0$

$6i_4 + i_o'' - 5i_o'' = 0$

$6i_4 - 4i_o'' = 0$ --- (x)

$* \ -i_4 + 10i_5 + 5i_o'' = 20$

$-i_4 - 10i_o'' + 5i_o'' = 20$

$-i_4 - 5i_o'' = 20$

$i_4 + 5i_o'' = -20$

For loop 4

$3i_4 + 2i_4 - 5i_o'' + (i_4 - i_5) = 0$

$3i_4 + 2i_4 - 5i_o'' + i_4 - i_5 = 0$

$6i_4 - i_5 - 5i_o'' = 0$ ----- (viii)

The two simultenous eqns are

$6i_4 - 4i_o'' = 0$

$i_4 + 5i_o'' = -20$

We solve them to get $i_o'' = -\frac{60}{17}$ A.

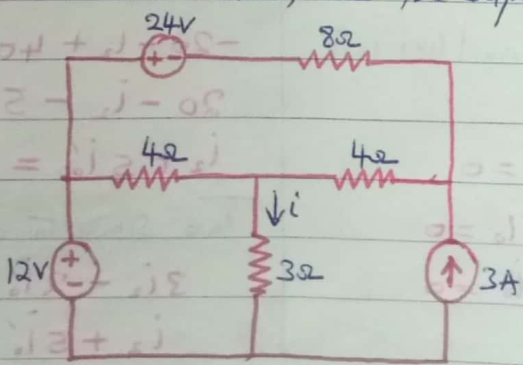
But $i_o = i_o' + i_o''$

$$= \frac{52}{17} + \left(-\frac{60}{17}\right)$$

$$= -\frac{8}{17} = -0.4706 \text{ A}.$$

Example 4

For the circuit below, use the superposition theorem to find i.

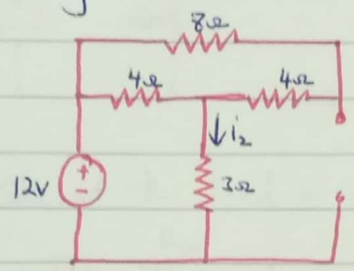


We have three sources. Let

$i = i_1 + i_2 + i_3$ –

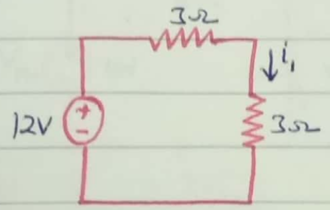
Where i_1, i_2 and i_3 are due to the 12V, 24V and 3A sources respectively

Using 12V source, (SC 24V and OC 3A voltage and current sources)



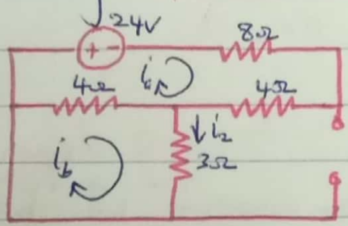
Combining 4Ω (on the right hand side) in series with 8Ω gives 12Ω. The 12Ω in parallel with 4Ω gives

$$\frac{12 \times 4}{12+4} = 3\Omega$$



$$i_1 = \frac{12}{6} = 2A.$$

Taking 24V source (SC 12V and OC 3A voltage and current sources)



To get i_2, apply mesh analysis

loop 1

$$4(i_a - i_b) + 24 + 8i_a + 4i_a = 0$$

$$16i_a - 4i_b = -24$$

$$4i_a - i_b = -6. \quad \text{---(i)}$$

loop 2.

$$4(i_b - i_a) + 3i_b = 0$$

$$7i_b - 4i_a = 0 \quad \text{---(ii)}$$

$$i_a = \frac{7}{4} i_b \quad \text{---(iii)}$$

Using eqn (iii) in (i)

$$4\left(\frac{7}{4} i_b\right) - i_b = -6$$

$$7i_b - i_b = -6$$

$$6i_b = -6$$

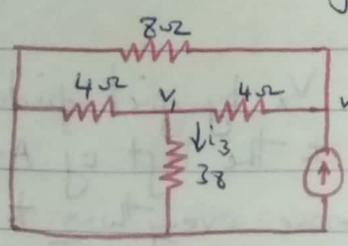
$$i_b = -1 \quad \text{---(V)}$$

Therefore

$$i_2 = i_b = -1 \quad \text{---(iv)}$$

loop 3

We use nodal analysis



(Vi)

$$3 = \frac{V_2}{8} + \frac{V_2 - V_1}{4} \Rightarrow 24 = 3V_2 - 2V_1$$

$$\frac{V_2 - V_1}{4} = \frac{V_1}{4} + \frac{V_1}{3}$$

$$V_2 = \frac{10}{3} V_1 \quad \text{---(Vii)}$$

Substituting eqn (vii) into (vi)

$3V_2 - 2V_1 = 24$

$3\left(\frac{10}{3}V_1\right) - 2V_1 = 24$

$10V_1 - 2V_1 = 24$

$8V_1 = 24$

$\underline{\underline{V_1 = 3V}}$

Thus $i_3 = \frac{V_1}{3\Omega} = \frac{3}{3} = 1A$

Thus $i = i_1 + i_2 + i_3$

$= 2 - 1 + 1$

$= \underline{\underline{2A}}$

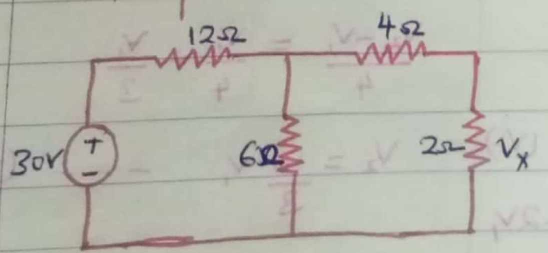
THEVENIN'S THEOREM

Steps to follow

1. Identify the load which may be a resistor or a part of the circuit
2. Replace the load with an open circuit
3. Calculate V_{oc}, this V_{TH}
4. Turn off all independent voltage and currents sources in the linear 2-terminal circuit
5. Calculate the equivalent resistance of the circuit. This is R_{TH}

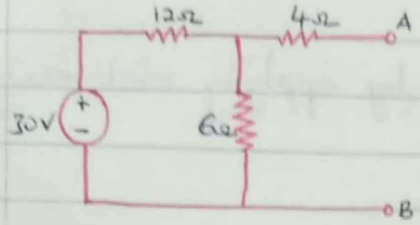
The voltage through and voltage across the load in series with V_{TH} and R_{TH} is the loads actual current and voltage in the original circuit.

Example 1



Find V_x by first finding V_{TH} and R_{TH} to the left of A-B

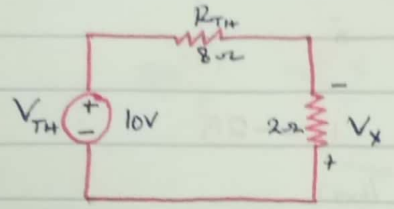
Remove everything to the right of A-B



$$V_{AB} = \frac{30 \times 6}{6+12} = 10V$$

$$R_{TH} = \frac{12 \times 6}{12+6} + 4 = 8\Omega$$

Connect the load in order to find V_x



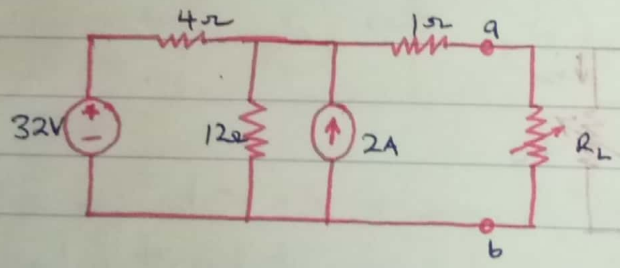
$$V_x = \frac{10 \times 2}{2+8} = 2V \approx \frac{10}{2+8} \times 2 = 2V$$

NB

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} \qquad V_L = R_L I_L = \frac{R_L}{R_{TH} + R_L} V_{TH}.$$

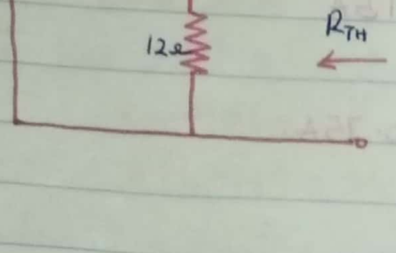
Example 2.

Find the Thevenin equivalent circuit of the figure below, to the left of the terminals a-b. Then find the current through $R_L = 6, 16,$ and 36Ω



Soln

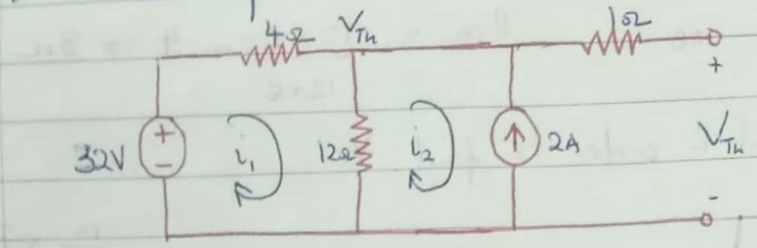
We find R_{TH} by turning off the 32V voltage source (replacing it with a short circuit) and the 2A current source (replacing it with an open circuit). The circuit becomes



$$R_{TH} = (4 \,//\, 12) + 1 = 4\Omega$$

$$= \frac{4 \times 12}{4+12} + 1 = 4\Omega$$

To find V_{TH}, Consider the circuit by applying mesh analysis to the two loops



$$-32 + 4i_1 + 12(i_1 - i_2) = 0 \qquad i_2 = -2A$$

Solving for i_1, we get $i_1 = 0.5A$. Thus,

$$V_{TH} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30V$$

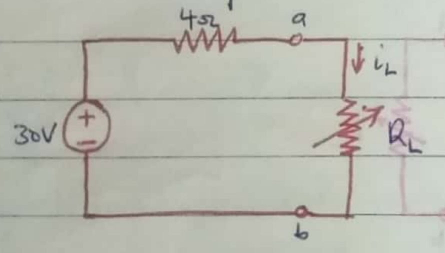
Alternatively, it is even easier to use nodal analysis. We ignore the 1Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

$$96 - 3V_{Th} + 24 = V_{Th}$$

$$V_{Th} = 30V$$

The Thevenin equivalent circuit is as shown below.



The current through R_L is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$, $I_L = \frac{30}{10} = 3A$

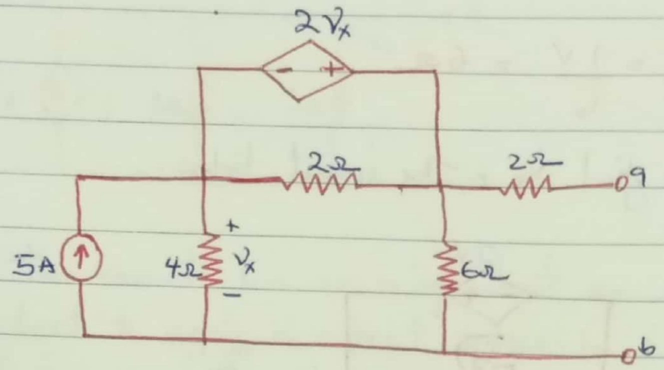
When $R_L = 16$ $I_L = \frac{30}{20} = 1.5A$

When $R_L = 36$ $I_L = \frac{30}{40} = 0.75A$.

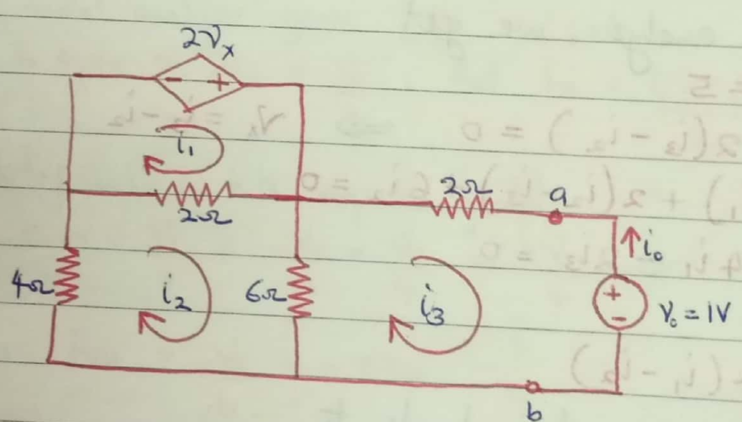
Scanned with CamScanner

Example 3.

Find the Thevenin equivalent of the circuit below.



This circuit contains a dependent source. To find R_{Th}, we set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, we excite the network with a voltage source v_o connected to the terminals as shown below.



We may set $v_o = 1V$ to ease calculation, since the circuit is linear. Our goal is to find the current i_o through the terminals, and then obtain $R_{Th} = 1/i_o$ (Alternatively, we may insert a 1A current source, find the corresponding voltage v_o, and obtain $R_{Th} = v_o/1$.)

Applying the mesh analysis to loop 1 in the circuit above

$$-2v_x + 2(i_1 - i_2) = 0 \quad \text{or} \quad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence, $i_1 = -3i_2$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

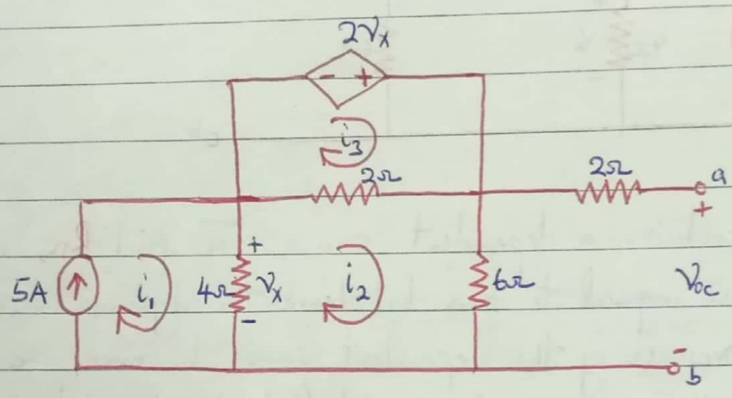
$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solving these equations gives $i_3 = -\frac{1}{6}$ A.

But $i_o = -i_3 = \frac{1}{6}$ A.

Hence, $R_{Th} = \frac{1\,V}{i_o} = 6\Omega$

To get V_{Th}, we find v_{oc} in the circuit below.



Apply mesh analysis, we get

$i_1 = 5$

$-2v_x + 2(i_3 - i_2) = 0 \implies v_x = i_3 - i_2$

$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$

or $12i_2 - 4i_1 - 2i_3 = 0$

But

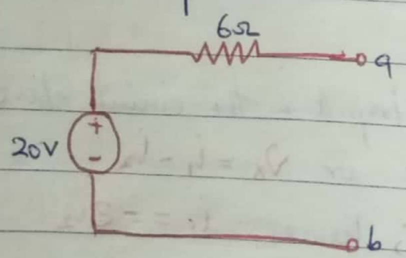
$v_x = 4(i_1 - i_2)$

Solving these equations leads to

$i_2 = \frac{10}{3}$.

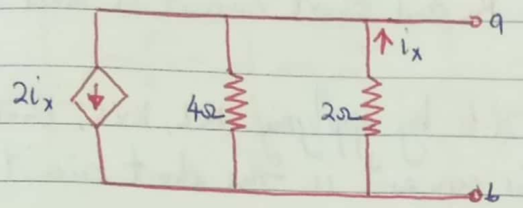
Hence $V_{Th} = v_{oc} = 6i_2 = 20V$

The Thevenin equivalent is as shown below.

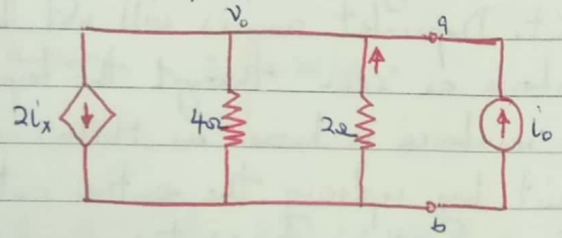


Example 4.

Determine the Thevenin equivalent of the circuit below.



Since this circuit has no independent sources, $V_{Th} = 0V$. To find R_{Th}, it is best to apply a current source i_o at the terminals as shown below



Applying nodal analysis gives

$$i_o + i_x = 2i_x + \frac{v_o}{4} \quad \text{(i)} \qquad \text{But } i_x = \frac{0 - v_o}{2} = \frac{-v_o}{2} \quad \text{(ii)}$$

Substituting eqn (i) into eqn (ii) yields

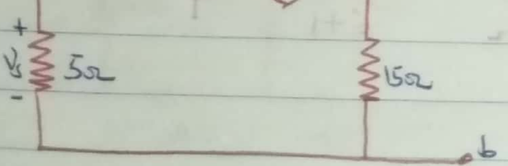
$$i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = \frac{-v_o}{4} \quad \text{or} \quad v_o = -4i_o$$

Thus $R_{TH} = \frac{v_o}{i_o} = -4\Omega$

The negative value of the resistance shows that the circuit is supplying power according to the passive sign convention. Of course resistors cannot supply power (they absorb); it is the dependent sources that supplies the power.

Assignment

Obtain the thevenin equivalent of the circuit below.



Answer

$V_{Th} = 0V$

$R_{Th} = -7.5\Omega$

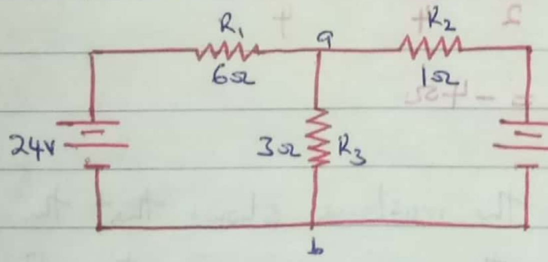
NORTON'S THEOREM

Steps to Norton's theorem

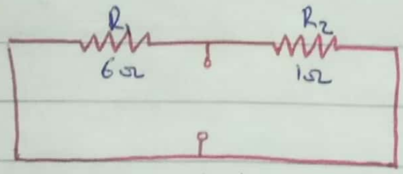
1. Remove the load resistor R_L and short circuit it and name the terminal a-b
2. Find the current through a-b by applying KCL, KVL, ohm's law or superposition principle. This current is the short circuit current and is known as Norton's equivalent current I_N
3. Calculate Norton's resistance by:
   a) Setting all independent voltage sources as short circuit and current sources open circuit. Dependent sources will not be changed.
   b) Calculate the resistance as "seen" through the terminals a-b into the network. This resistance is known as Norton's equivalent R_N
4. Draw equivalent circuit by replacing the entire network by Norton's equivalent current I_N in parallel with Norton's equivalent R_N and connect the load resistance R_L

Example 1

Find the current through 3Ω resistor by Norton's theorem for the network shown below



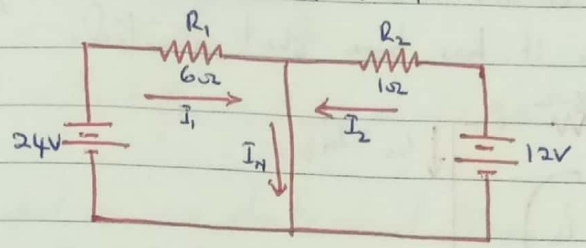
1. Remove 3Ω resistor and re-draw the circuit



R_1 & R_2 are in parallel

$$R_N = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 1}{6+1} = \frac{6}{7} = 0.857\Omega$$

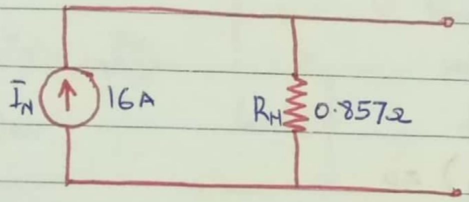
2. Short circuit terminal a-b



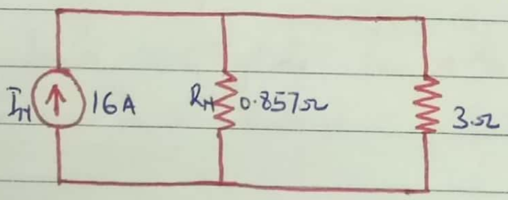
$$I_N = I_1 + I_2 = \frac{24}{6} + \frac{12}{1}$$

$$= 16\ A.$$

3. Draw Norton's equivalent circuit



4. Connect R_3 and calculate through it. Apply current divide rule

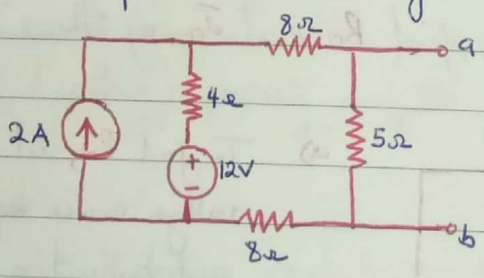


$$I_L = I_N \times \frac{R_N}{R_N + R_L} = 16 \times \frac{0.857}{0.857 + 3}$$

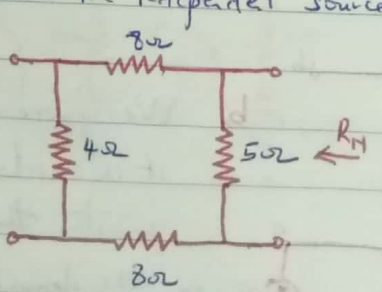
$$= 3.55\ A.$$

Example 2

Find Norton equivalent circuit of the figure shown below.



1. We find R_N in the same way we find R_{TH} is the Thevenin equivalent circuit. Set the independent sources equal to zero.

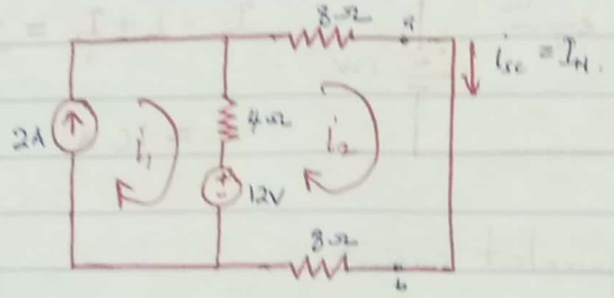


$$R_N = 5 // (8+4+8)$$

$$= \frac{5 \times 20}{5+20} = \frac{100}{25}$$

$$= 4\ \Omega$$

To find I_N we short circuit terminals a and b and ignore the 5Ω resistor because it has been short circuited.



Applying mesh analysis

$i_1 = 2A$

$8i_2 + 8i_2 - 12 + 4(i_2 - i_1) = 0$

$16i_2 - 12 + 4i_2 - 4i_1 = 0$

$20i_2 - 12 - 4i_1 = 0$

but since $i_1 = 2$

$20i_2 - 12 - 8 = 0$

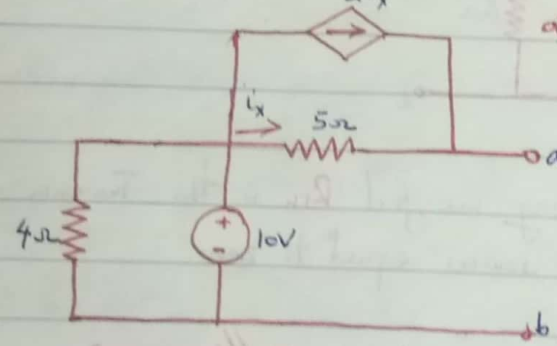
$20i_2 - 20 = 0$

$20i_2 = 20$

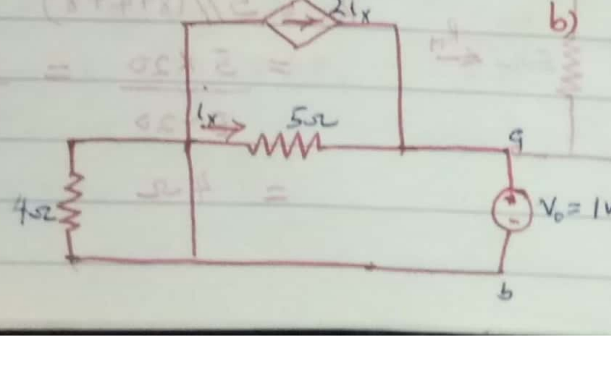
$i_2 = 1A.$

Example 3

Using Norton's theorem, find R_N and I_N of the circuit below at terminals a-b



a) To find R_N, set the independent voltage source equal to zero and connect a voltage source of $V_o = 1V$ to the terminals

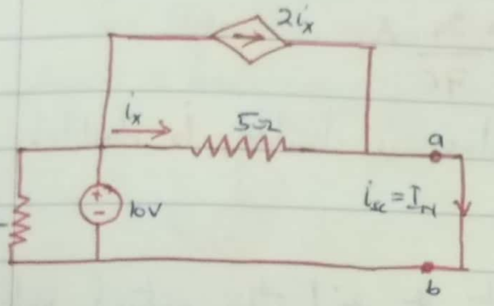


b) We ignore 4Ω resistor because it is short-circuited. The 5Ω resistor, the voltage source, and the dependent current source are all in parallel.

Hence, $i_x = \frac{V_o}{5} = \frac{1}{5} = 0.2$ At node a, $-i_o = i_x + 2i_x = 3i_x = 0.6$

and $R_{Th} = \frac{V_o}{i_o} = \frac{1}{-0.6} = -1.67\Omega$

To find I_N, we short-circuit terminals a and b and find the current i_{sc} as shown below.



c) From this figure 4Ω resistor, the 10V voltage source, the 5Ω resistor, and the dependent current source are all in parallel.

Hence $i_x = \frac{10-0}{5} = 2A$

At node a, KCL gives $i_{sc} = i_x + 2i_x = 2+4 = 6A$.

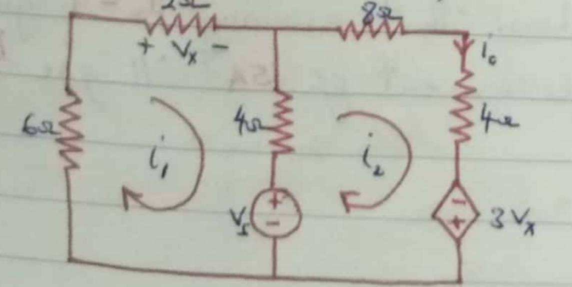
Thus $I_N = 6A$.

LINEARITY PROPERTY

Linearity is the property of an element describing a linear relationship between cause and effect. The property is a combination of both the homogeneity (scaling) property and the additivity property.

Example 1

For the circuit below, find i_o when $V_s = 12V$ and $V_s = 24V$



Soln.

Applying KVL to the two loops, we obtain

$12i_1 - 4i_2 + V_s = 0$ --- (1)

$-4i_1 + 16i_2 - 3V_x - V_s = 0$ --- (2)

But $V_x = 2i_1$ eqn (2) becomes

$-10i_1 + 16i_2 - V_s = 0$ --- (3)

Adding eqn 1 and 3 yields

$2i_1 + 12i_2 = 0 \Rightarrow i_1 = -6i_2$

Substituting this in eqn 1 we get

$-76i_2 + V_s = 0 \Rightarrow i_2 = \frac{V_s}{76}$

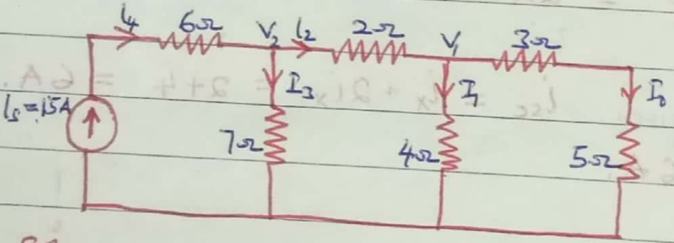
When $V_s = 12V$, $i_o = I_2 = \frac{12}{76}$ A

When $V_s = 24V$, $I_o = I_2 = \frac{24}{76}$ A.

Showing that when the source value is double, i_o doubles.

Example 2

Assume $I_o = 1A$ and use linearity to find the actual value I_o in the circuit below.



Soln.

If $I_o = 1A$, then $V_1 = (3+5)I_o = 8V$ and $I_1 = \frac{V_1}{4} = 2A$

applying KCL at node 1 gives $I_2 = I_1 + I_o = 3A$

$V_2 = V_1 + 2I_2 = 8 + 6 = 14V$ ($I_3 = \frac{V_2}{7} = 2A$

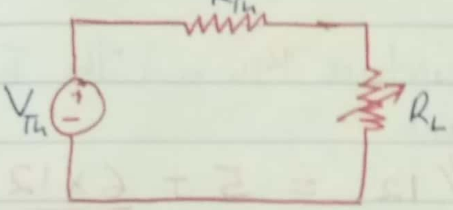
Applying KCL at node 2 gives $I_4 = I_3 + I_2 = 5A$

Therefore, $I_s = 5A$, This shows that assuming $I_o = 1$ gives $I_s = 5A$; the actual source current of 15A will give $I_o = 3A$ as the actual value.

MAXIMUM POWER TRANSFER.

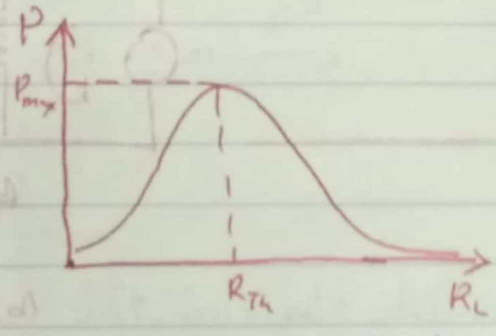
In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load.

The thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. Consider the circuit below.



$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched below



We notice that the power is small for small or large values of R_L but maximum for some value R_L between 0 and ∞

From the figure we notice that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}.

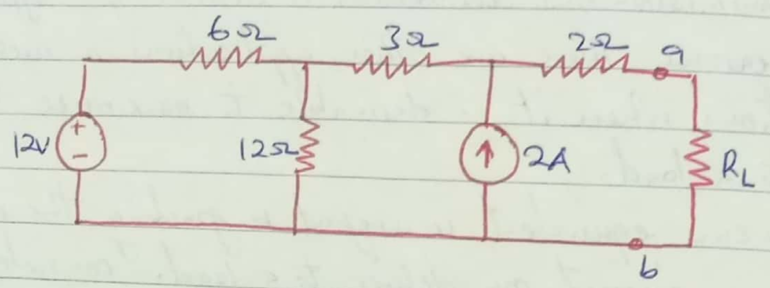
Maximum power is transfered to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$)

$$\therefore \quad P_{max} = \frac{V_{Th}^2}{4R_{Th}} \quad \text{only when } R_L = R_{Th}$$

When $R_L \neq R_{Th}$ $\quad P = i^2 R = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$

Example 1.

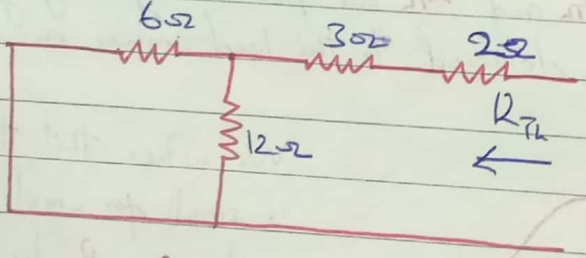
Find the value of R_L for maximum power transfer circuit below. Find the maximum power.



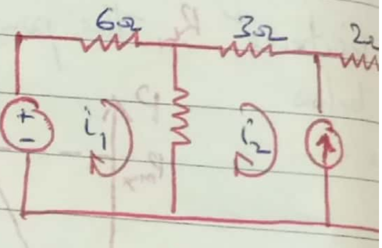
Soln.

We find the Thevenin resistance R_{Th} and the Thevenin V V_{Th} across the terminals a-b.

$$R_{Th} = 2+3+6 \,//\, 12 = 5 + \frac{6\times 12}{18} = 9\Omega$$



(a)



(b)

To get V_{Th}, we consider the circuit in (b). Applying mesh analysis

$$-12 + 18i_1 - 12i_2 = 0 \qquad i_2 = -2A$$

Solving for i_1, we get $i_1 = -\frac{2}{3}$.

Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$$

$$V_{Th} = 22V$$

For maximum power transfer

$$R_L = R_{Th} = 9\Omega$$

and the maximum power is

$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4\times 9} = 13.44W.$$

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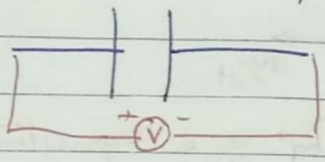
CAPACITORS AND INDUCTORS

Capacitors.

A capacitor is a passive element designed to store energy (electric charge) in it's electric field. Capacitors are used extensively in electronics, communications, computers and power systems e.g they are used in tuning circuits for radio receivers and as dynamic memory elements in computer systems.

A capacitor consists of two plates separated by an insulator (dielectric) i.e air, ceramic, paper or mica.



When a voltage source V is connected to the capacitor, the capacitor is said to store electric charge. The amount of charge stored Q is directly proportional to the applied voltage V so that

$$Q = CV \quad \text{- - - - - (1)}$$

C is the constant of proportionality known as capacitance of the capacitor. Unit of capacitance is Farad (F).

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F)

$$C = \frac{\varepsilon A}{d} \quad \text{- apply only to parallel plate capacitors} \quad \text{- - - (2)}$$

A – Surface area of each plate, d – distance between the plates
ε – The permittivity of the dielectric material between the plates

To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides.

$$i = \frac{dq}{dt} \quad \text{- - - - - (3)}$$

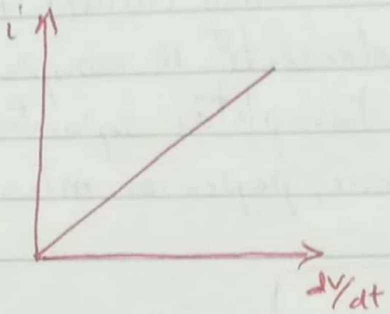
differentiating both sides of eqn (1)

$$\frac{dq}{dt} = C\frac{dv}{dt}$$

$$i = C\frac{dv}{dt} \quad - - - - - (4)$$

Capacitors that satisfy eqn (4) are said to be linear

current volta



current-voltage relationship of a capacitor.

The voltage-current relation can be obtained by integrating
sides of eqn. (4)

$$i = C\frac{dv}{dt} \quad \Rightarrow \quad dv = \frac{1}{C}\, i\, dt$$

$$\int dv = \frac{1}{C}\int i\, dt$$

$$v = \frac{1}{C}\int_{-\infty}^{t} i\, dt \quad - - - 5$$

or

$$v = \frac{1}{C}\int_{t_0}^{t} i\, dt + v(t_0) \quad - - - 6$$

Where $v(t_0) = \frac{q(t_0)}{C}$ is the voltage across the capacitor at time

The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt} \quad - - - - (7)$$

The energy stored in a capacitor is therefore

$$w = \int_{-\infty}^{t} p\, dt = C\int_{-\infty}^{t} v\frac{dv}{dt}\, dt$$

$$= C\int_{-\infty}^{t} v\, dv = \frac{1}{2}Cv^2\Big|_{t=-\infty}^{t} \quad - - - (8)$$

Note that $v(-\infty) = 0$ because the capacitor was uncharged at $t = -\infty$, Thus

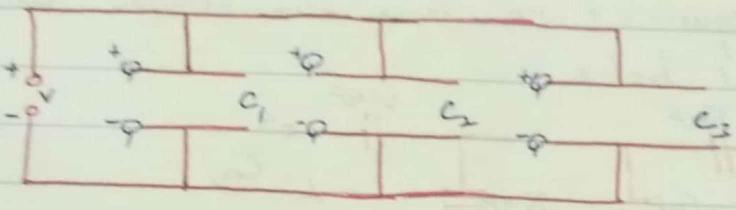
$$w = \frac{1}{2} C v^2 \quad \text{---} \quad (9)$$

Using eqn (1) we can write eqn (9) as

$$w = \frac{q^2}{2C}$$

DC THROUGH A CAPACITOR.

Capacitors in Parallel



Consider capacitors C_1, C_2 and C_3 arranged in parallel. The applied pd V is the same across each plate but the charges are different.

$$Q_1 = C_1 V, \quad Q_2 = C_2 V, \quad Q_3 = C_3 V$$

The total charge $Q = Q_1 + Q_2 + Q_3$

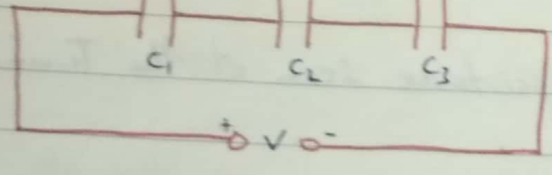
$$= C_1 V + C_2 V + C_3 V$$

$$= V(C_1 + C_2 + C_3)$$

Therefore the equivalent capacitance C is

$$C = C_1 + C_2 + C_3$$

Capacitors in Series



For the capacitors arranged in series, the charge is the same across each plate but the voltage is different.

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3}$$

$$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

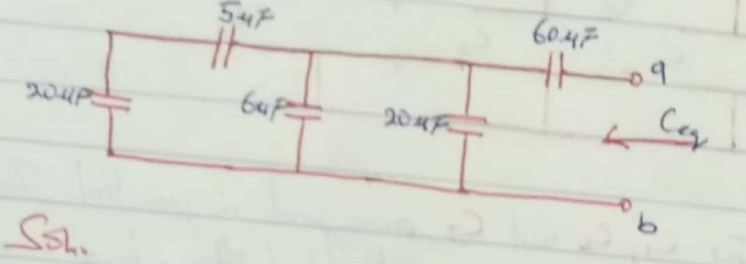
$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

Therefore the equivalent capacitance C is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Example 1

Find the equivalent capacitance seen between terminals a and b of the circuit below



Soln.

20uf and 5uf capacitors are in series

$$C = \frac{20 \times 5}{20+5} = 4\,uf.$$

4uf are in parallel with 6uf and 20uf capacitance

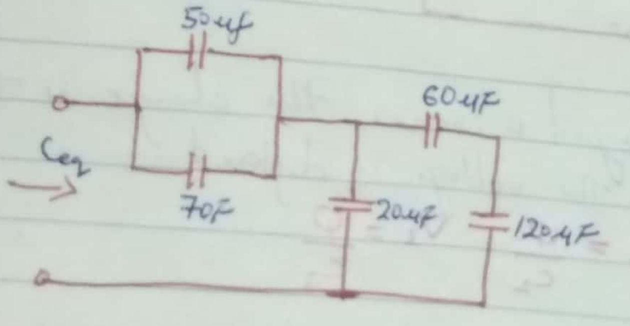
$$C = 4 + 6 + 20 = 30\,uf$$

30uf is in series with 60uf capacitor

$$C_{eq} = \frac{30 \times 60}{30+60} = 20\,uf$$

Assignment

Find the equivalent capacitance seen at the terminals of the circuit below



Answer

40uF

AC THROUGH A CAPACITOR.

Consider capacitor plates being continually charged, discharged and charged the other way round by the alternating voltage of the mains. Let a p.d V be applied across a capacitor of capacitance C and let it's value at time t be given by

$$V = V_m \sin 2\pi f t,$$

V_m is the peak voltage and f is the frequency of the supply. The charge Q on the capacitor at time t is

$$Q = CV$$

The current I flowing in the capacitor is then given by

$$I = \text{rate of change of charge}$$

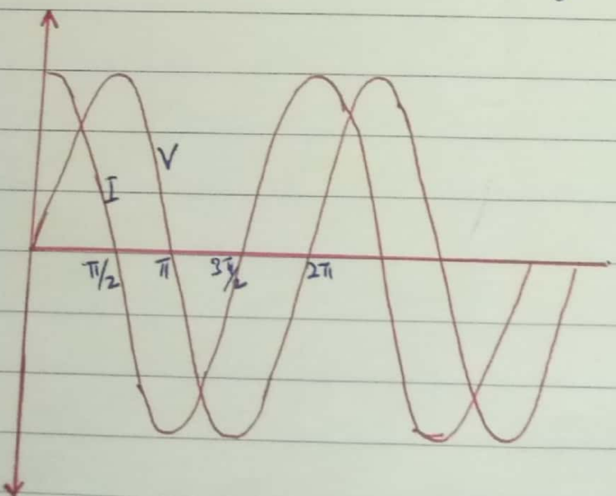
$$= \frac{dQ}{dt} = \frac{d(CV)}{dt}$$

$$= \frac{C\,dV}{dt} = C\frac{d}{dt}\{V_m \sin(2\pi f t)\}$$

$$= 2\pi f C V_m \cos(2\pi f t)$$

Where $2\pi f C V_m = I_m$ the peak value

The current flowing "through" the capacitor (cosine function) leads the applied p.d (a sine function) by one quarter of a cycle.



Consider the ratio $\frac{V_m}{I_m} = \frac{V_m}{2\pi f C V_m} = \frac{1}{2\pi f c} = \frac{1}{wC}$

This expression resembles $\frac{V}{I} = R$ which defines resistance with $\frac{1}{2\pi f C}$ replacing R. This quantity is taken as a measure of the

opposition of a capacitor to a.c and is called capacitive react
X_c. That is

$$X_c = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$