UNIVERSITY OF ELDORET

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER EXAMINATION

COURSE CODE: MATH 212

COURSE TITLE: LINEAR ALGEBRA L.

### INSTRUCTION TO CANDIDATES

Answer ALL questions from section A and any THREE from section B

No sharing of scientific calculators.

Do not write on this question paper.

Duration of the examination: 3 hours

### SECTION A (31 MARKS): ANSWER ALL QUESTIONS

#### QUESTION ONE (16 MARKS)

- a) Define a basis S, for a vector space V. (2 Marks)
- b) Let  $(\vec{u}, \vec{v}, \vec{w})$  be an independent set in  $\Re^n$ . Is  $\{\vec{u} + \vec{v}, 2\vec{u} + \vec{w}, \vec{v} 5\vec{w}\}$  linearly independent (4 Marks)
- c) Consider the matrix 1 5 9. What is its rank? 2.

  Marks)
- d) Are all the matrices of the form  $\begin{bmatrix} h & b \\ c & d \end{bmatrix}$  where a + d = 0 a subspace of the vector space of  $2 \times 2$  matrices? (3 Marks)
- e) Find the determinant of the matrix  $\begin{bmatrix} -8 & -4 & 1 \\ 10 & -8 & -4 \end{bmatrix}$  by Laplace expansion method (2 Marks)
- f) Find the inverse of  $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$  by row reduction.

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#### QUESTION TWO (15 MARKS)

- a) Define the following terms as used in transformation.
  - Rank of a linear transformation, 1)
  - Nullity of a linear transformation. (1 Mark)
- b) Show that if  $f: u \to v$  is defined as T(x) = AX, where A is  $n \times m$  and X is  $n \times 1$  vector, then T is a linear transformation (3 Marks)
- Let  $T: \mathbb{R}^2 \to \mathbb{R}^2$  be a linear transformation defined by  $T(x_1, x_2) = (x_1 + 2x_2, 3x_1 x_2)$ . Find the matrix M representing T. (3 marks)
- Determine if T defined as  $T: \mathfrak{R}^1 \to \mathfrak{R}^1$  defined as  $T(x_1, x_2, x_3) = (2x_1 x_2, x_3 x_1 + 1)$  is a linear transformation. (4 Marks).
  - e) Find h such that  $\begin{vmatrix} 2 & h & 1 \\ 3 & 6 & 1 \end{vmatrix}$  is the augmented matrix of an inconsistent system. -16, -XI+1 \$UZV+UZ (3Marks)

## SECTION B - ATTEMPT ANY THREE QUESTIONS IN THIS SECTION

#### QUESTION THREE (13 MARKS)

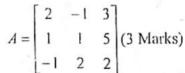
- a) Consider the vectors {(1,4),(2,3),(3,2)}. Are these vectors linearly independent? (3 Marks)
- b) Determine if T defined as  $T: \mathbb{R}^3 \to \mathbb{R}^2$  defined as  $T(x_1, x_2, x_3) = (x_1 + x_3, 2x_2 x_3)$  is a linear transformation. (4 Marks).
- e) Find the basis and dimension of the solution space for the equations

$$x_1 + x_2 - x_3 = 0$$
  
 $-2x_1 - x_2 + 2x_3 = 0$  (6 marks)  
 $-x_1 + x_3 = 0$ 



#### QUESTION FOUR (13 MARKS)

a) Find the basis and dimension of the row space of the matrix





- b) The transformation  $T: \mathbb{R}^3 \to \mathbb{R}^3$  is defined as  $T(x) = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ 
  - Find range Basis for rank of T, (4 Marks)
  - ii) Basis for Kernel of T (4 Marks)
  - iii) Rank of T and Kernel of T (2 Marks)

# QUESTION FIVE (13 MARKS)

Give the complete solution to the system of equations

$$3x - y - 5z = 9$$
$$y - 10z = 0$$

$$-2x+y=-6$$

b) Choose h and k such that the augmented matrix shown has each of the following

$$\begin{bmatrix} 1 & h & 1 & 2 \\ 2 & 4 & 1 & k \end{bmatrix}$$

i) unique solution,

(1 Mark)

infinitely many solutions c)  $S = \{x - x^2 + 2x^3, 1 + 2x + 3x^2, x - x^2 + 2x^3\}$  linearly independent in  $\Re^3$ ? (4 Marks)

# QUESTION SIX (13 MARKS)

a) Find the basis and dimension of the solution space for the equations

$$x_1 - 3x_2 - x_3 = 0$$

$$2x_1 - 6x_2 + 2x_2 = 0$$
 (7 Marks)

$$3x_1 - 9x_2 + 3x_3 = 0$$

b) Is 
$$\{1-3x+2x^2, 1+x+4x^2, 1-7x^2\}$$
 a basis for  $P_2$ ? (6 Marks)

# QUESTION SEVEN (13 MARKS)

a) Determine if (2,1) is the set generated by {(3,1),(2,2)}.

(2 Marks)

$$x + 2y + z = 1$$

b) Consider the system of equations 3x + 2y + z = 2 find;

$$2x - 3y + 2z = 3$$

The adjoint of the matrix of coefficients, i)

(4 Marks)

The determinant of the matrix of coefficients, 11)

(2 Marks)

Inverse of the matrix of the coefficients using (i) and (ii) above. (2 Marks) iii)

Solution of the system of equations using (iii) above. iv)

(3 Marks)

$$\begin{pmatrix} 7 & -7 & 0 \\ -4 & 0 & -7 \\ -13 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} \chi \\ y \\ 2 \end{pmatrix}$$