Information Theory in Economics and Investment

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***Abstract*—**

I. Introduction

Big results in information theory applied to economics and investing. Hopefully a look at open problems.

II. Information Theory in Economics

A. Rational Inattention - Modeling behavior in the absence of perfect information

Discuss rational inattention (especially in the context of typical models of behavior in economics).

Also discuss results and newer papers.

B. Robustness

Similar motivation to rational inattention.

Again discuss results and newer papers.

C. Credit Risk Modeling

Info theory used to develop AIC - used in all sorts of model validation.

One paper also uses AIC to look at the predictability of the stock market historically.

III. Information Theory in Investment

Some of the results below deal with the value or influence of information on invenstment in a somewhat less information-theoretic centered approach (though in the abstract they still present a quantification of information, and so they are worth considering/contrasting with the other results).

A. Value of Information

B. Value of Information in Biology

The results above are extended and used to model a population as a financial portfolio in [10]. The growth rate of a population is, using virtually the same setup, bounded by the mutual information between a set of variables representing the environment, and some signal in the environment, $I(X_t; Y_t)$. Notably, this bound is shown to not hold when considering that in contrast to financial models, biological populations must process information at an individual level. This leads to the result that

$$I(X_t; Y_t) \leq I_p^{q_{env}, q_{in}} \leq I(X_t; X_t')$$

i.e. in general the information gathered by any member of a population is less than the collective information gathered by the entire population.

C. Universal Portfolios

Consider a stock market vector given by

$$\boldsymbol{x} = (x_1, x_2, ..., x_m)^t$$

where x_i is the price relative of a given stock - the ratio of its closing to opening price - for a given day.

A portfolio is defined as

$$\boldsymbol{b} = (b_1, b_2, ..., b_m)^t, b_i \geq 0, \Sigma b_i = 1.$$

where each b_i represents the proportion of current wealth invested in stock i.

Finally, the factor by which wealth increases in a given investment period, S, can then be defined as $S = \Sigma b_i x_i$ A straightforward comparison can be made between any two investment strategies by comparing S for various scenarios.

In [3], a strategy is shown that achieves S_n (S over a sequence of n stock vectors) equal to,in the first order of the exponent, the best constant rebalanced portfolio S_n^*.

D. Influence of Side Information in Investment

additional result from optimal portfolios above

E. Cost of Achieving the Best Portfolio in Hindsight

running refs list: [12], [2], [7], [1],
[8], [6], [11], [13], [4], [5],
[9]

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