Study of Atlantic herring and mackerel migration due to temperature changes in the North Sea

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Abstract

We devise a model to study the migration of Atlantic herring and mackerel. By ignoring the fact that fish have brains, we use an advection-diffusion equation to model the migration of said fish across the North Sea in search of their ideal temperatures. It is found that both species of fish tend to diffuse out into the North Sea, and move outside of the general fishable area. Instead of ending up in territorial waters, many fish instead end up in what is likely the open ocean. We recommend that fisheries either invest in long-range vessels for a short-term solution, increasing mariculture to allow for mackerel and herring to grow inland—if that is at all possible—or relocating to Norway.

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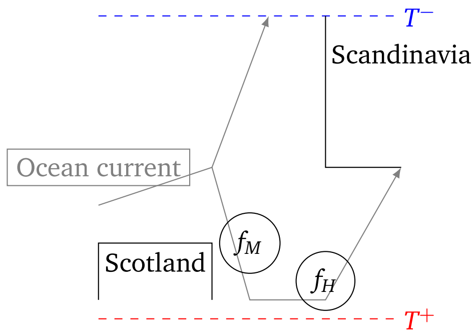


Figure 1: Generic, rough representation of the model; f_M is the mass-distribution of mackerel, and f_H is the mass-distribution of herring. Current derived from [5].

1. Modelling using the advection-diffusion equation

Atlantic herring and mackerel are two distinct, commercially important types of fish. They are a major source of food for a variety of people in Northern Europe, and are, as previously discussed, extremely important for the regions economy. Climate change, like all things, is very much capable of driving them out from their current habitat into different waters, far away from where most Scottish fisheries can effectively catch them.

Considering that both herring and mackerel tend to swim in schools—ignoring their sentience—we can treat both species of fish as a classical, ideal gas. Given this, and how they tend to move *away* from sources of higher heat (T^+) to those of lower heat (T^-), we can assume that they can both diffuse and spread out away from each other, and move with the current via advection. We therefore use the aptly named advection-diffusion equation [1],

$$\nabla \cdot (D\nabla f - \mathbf{v}f) - \frac{\partial f}{\partial t} + S = 0\,, \tag{1}$$

where f is the mass-distribution, $\mathbf{v}$ is the velocity vector of f, D is the diffusion coefficient, and S models the temperature sources within the North Sea.

Taking a page out of semiconductor physics [2], we use the Einstein relation

$$D = kT\mu\,, \tag{2}$$

where k is a statistical constant we will touch on later, T is the absolute temperature of the system, and $\mu = v/F$ is the mobility of the particles within the system. Given that each fish is moving through seawater, we can approximate μ using the drag equation [3]

$$F = \frac{1}{2}\rho C_D A v^2 \,.$$

We can approximate fish as a cylinder of length L, and diameter of$L/3$. Given that the ends of cylinders have a surface area $A = \pi d^2/2$, and $d = L/3$—making $A = L^3/18$—we find that

$$F \approx \frac{1}{36}\pi\rho C_D L^2 v^2 \,,$$

where C_D is the drag coefficient of a cylinder, v is the average speed for a fish, and ρ is the density of seawater. Therefore,

$$\mu \approx \frac{1}{36\pi\rho C_D L^2 v} \,,$$

making

$$D \approx \frac{kT}{36\pi\rho C_D L^2 v} \,. \tag{3}$$

Under normal circumstances, k would be k_B, the Boltzmann constant—however, given that it is incredibly small, we can instead use the average translational kinetic energy to determine that [4]

$$\frac{3}{2}kT = \frac{1}{2}mv^2 \,,$$

where m is the fish's mass; kT is therefore

$$kT = \frac{1}{3}mv^2 \,.$$

Finally, we can approximate D further using our new value of kT,

$$D \sim \frac{mv}{108\pi\rho C_D L^2} \,,$$

turning eq. 1 into

$$\nabla \cdot \left(\frac{mv}{108\pi\rho C_D L^2} \nabla f - \mathbf{v} f \right) - \frac{\partial f}{\partial t} + S = 0 \,. \tag{4}$$

Since oceans have currents, and fish can swim, it is important to make a model of $\mathbf{v}$ that incorporates both of those factors. We start by using the average speed v used prior and assume that it is a constant swimming speed in zero-current conditions. Ocean currents are factored in using a rather basic model derived from [5], as can be seen with fig. 1.

2. Solving the model numerically

Given how the eq. 4 is, except for certain circumstances, impossible to solve analytically, we *must* solve it numerically. Luckily, Andrea La Spina [6] has written an excellent solver for the advection-diffusion equation in MATLAB.

The initial conditions were randomized for the various fish—as it is impossible to know where all fish are—and were solved using the forward Crank-Nicolson method. The only parameters the simulation depended on are v and D, which makes simulations exceedingly easy.

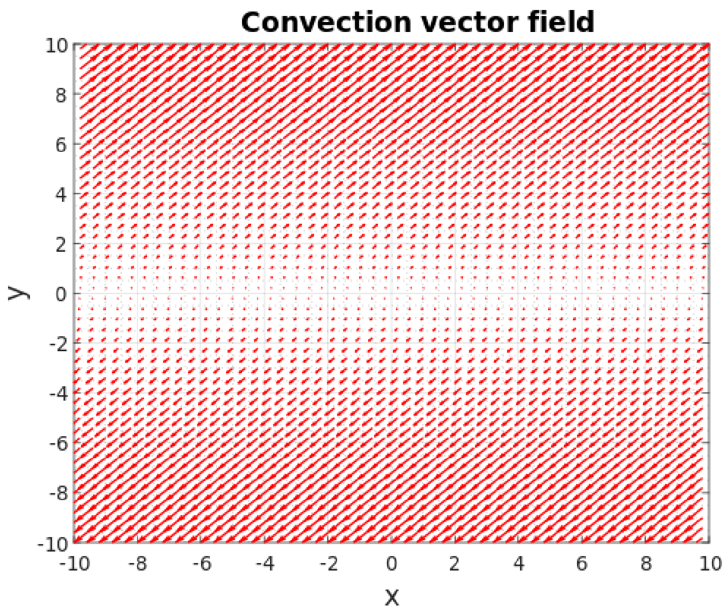
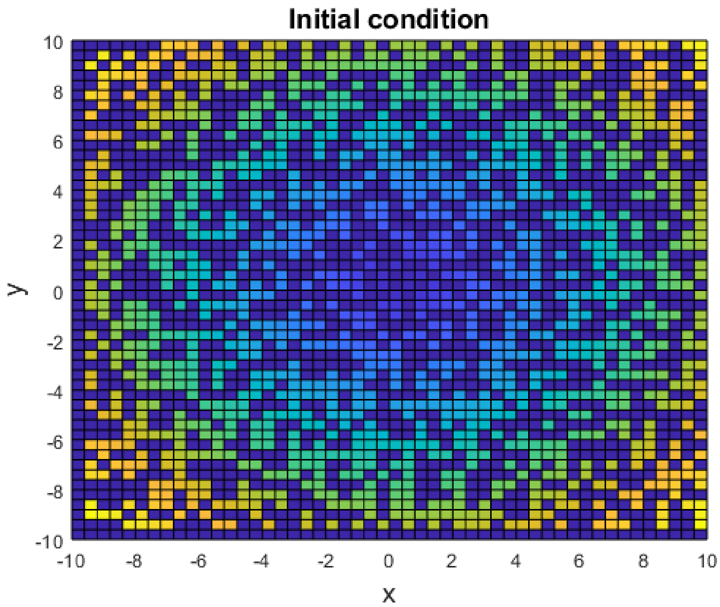
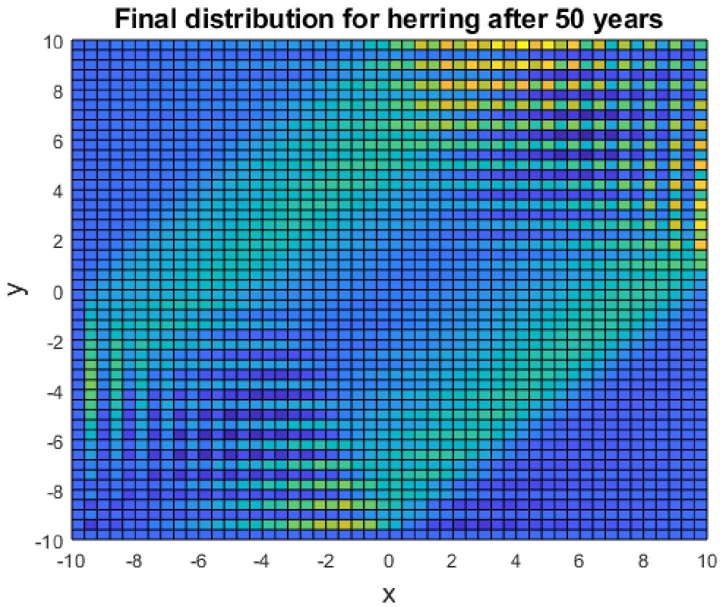


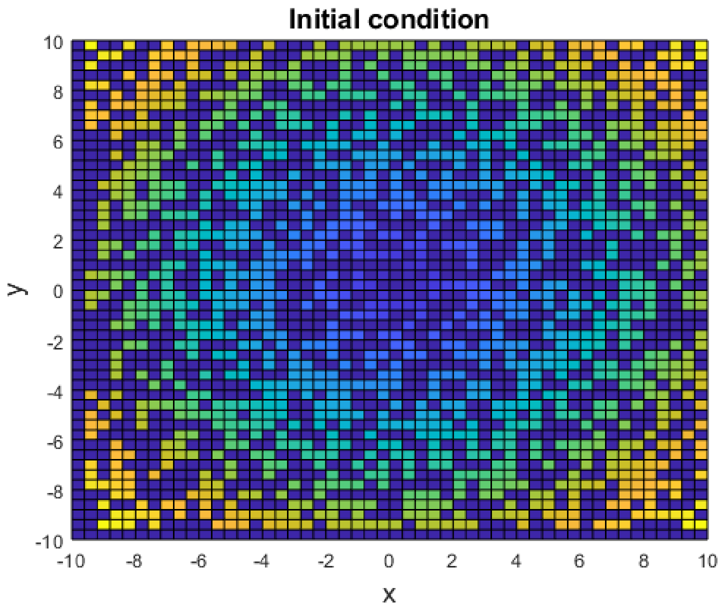
Figure 2: Convection field used in the simulations.

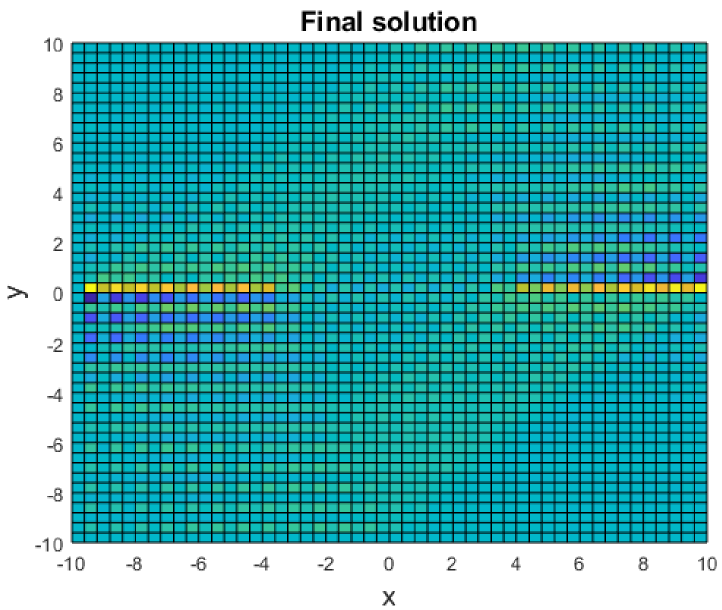
2.1. Solutions for herring





2.2. Solutions for mackerel





3. Table of parameters

Table 1: L and v taken from [7], C_D from [8], and ρ from [9]. Due to the lack of sufficient publications listing the average weight of the fish in question, we decided to make m the maximum listed weight on Wikipedia article for each respected species subtracted by a small, arbitrary amount.

Fish	C_D	ρ (kg/m^3)	L (m)	m (kg)	v (km h^{-1})	D (kg m s^{-1})
Herring	0.5	1.03	0.3	1.0	6.0	0.3948
Mackerel	0.5	1.03	0.5	2.5	11.0	0.1563

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