

Advanced Heuristics

Questions on “Can’t get no satisfaction” by Brian Hayes

1. Can get satisfaction

1.1 Give a model of satisfaction for each of the following boolean formulae.

a) $(p \rightarrow \neg q)$

b) $(\neg p \vee q) \wedge \neg r$

c) $(\neg(p \vee q) \wedge r) \wedge \neg(\neg r \vee \neg q)$

d) $(\neg p \vee \neg q) \wedge (r \rightarrow \neg q)$

e) $r \wedge \neg(p \vee r)$

f) $\neg q \vee (\neg r \wedge \neg p)$

g) $(\neg p \vee \neg q) \wedge (r \vee \neg q)$

h) $q \vee (r \vee p)$

1.2 Which of the formulae in 1.1 are in Disjunctive Normal Form? Which are in Conjunctive Normal Form?

1.3 Put all the formulae which are not in any of the two forms in CNF.

1.4 For all formulae of 1.1, how many clauses and variables do they have (after applying 1.3) ?

1.5 Write a boolean formula, not necessarily in any form,

a) With 3 variables and 2 clauses

b) With 3 variables and 3 clauses

c) With 3 variables and 4 clauses

1.6 Write a boolean formula in Conjunctive Normal Form (CNF). Make sure you add some negations (\neg) too.

- a) With 7 variables and 2 clauses
- b) With 4 variables and 3 clauses
- c) With 6 variables and 4 clauses
- d) With 2 variables and 7 clauses
- e) With 4 variables and 3 clauses
- f) With 5 variables and 2 clauses.

1.8 The following boolean formulae are randomly generated 3-SAT. Can you tell which ones are (un)satisfiable?

- a) With 2 variables and 5 clauses
- b) With 4 variables and 3 clauses
- c) With 5 variables and 2 clauses.

1.9 There is a mistake in paragraph 3 of the section 'Phase Transitions'. The answer can be found in Gent & Walsh 1994, in the first paragraph of section 2.

- a) Can you find it?
- b) Even with this mistake removed, there is still a reproducibility problem – in both papers - when generating Random 3SAT-formulas.

2. Algorithms

2.1. The following algorithms operate on an array of n integers. If n increases, how does the run-time of these algorithms increase? Write $O(f(n))$.

- a) Finding the smallest integer in the array
- b) Sorting the numbers in the array with bubblesort
- c) Calculating the average value of the array
- d) Putting the integers into two groups of equal value (if you add up the integers)
- e) Sorting the numbers in the array with mergesort

f) Printing all subsets of size $\frac{1}{2}n$

g) Finding a set of integers in the array that sums up to 131

2.2 Which of the above algorithms would you classify as P, which as NP?

2.3. What does it mean if a problem is NP-complete?

2.4 Why is SAT important for NP-completeness?

3. Phase Transitions

3.1 Look up and explain what kind of phase transitions can be found in

a) the state of water, in respect to temperature.

b) the ferromagnetism in nickel, in respect to temperature (hint: "Curie-point").

c) random graphs, in respect to connectivity (hint: Erdős-Rényi)

d) logistic map for biological populations, in respect to the reproductive rate.

e) the solvability in a Random 3-sat formula, in respect to the ratio clauses-to-variables.