Advanced Heuristics

Questions on "On the threshold" by Brian Hayes

1. Algorithms

1.1. The following algorithms operate on a graph* of *v* vertices and *e* edges. The graph is connected, there are no loose vertices. If *v* increases, how does the run-time of these algorithms increase?

a) finding the most connected vertex in the graph.

b) finding a path that visits every vertex exactly once.

c) finding the minimal number of edges you need to remove to break the graph in two pieces.

d) calculating the average degree of the graph's vertices.

e) determining whether the graph is the same as some given graph with the same numbers of vertices and edges.

f) finding a path that visits every edge exactly once.

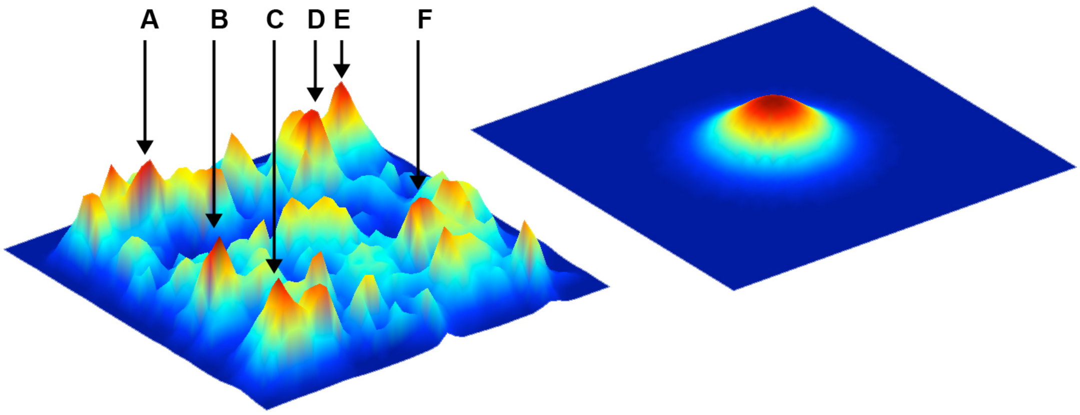
g) calculating the average path length (edges to be traversed) between any two vertices in the graph.

* *The term 'graph' refers to the network structure, not to the diagram.*

1.2 Which of the above algorithms would you classify as P, which as NP?

2. Graph* Colouring & Fitness Landscapes

Graph1 has ten vertices and is connected. By adding edges, graph2 is created. An exhaustive algorithm has coloured the vertices in all possible combinations. In many coloured graphs, conflicts exist. The number of conflicts of a graph is the number of connected vertex pairs that have the same colour. A fitness landscape was drawn, in which the peaks represent the colourings with no conflicts.



a) Which of the two fitness landscapes belongs to which graph?

b) How many edges does Graph 1 have?

c) How many edges does Graph 2 have?

d) The graph corresponding to the left fitness landscape turned out to be 3-colourable (R,G,B). The six perfect colourings given below correspond to a letter in the left landscape. Which to which?

1) R,R,B,B,G,B,G,B,G,B
2) R,R,G,B,G,B,R,B,R,B
3) R,R,B,G,R,B,G,B,G,B
4) B,G,R,G,B,R,G,B,G,G
5) G,G,R,G,B,R,G,B,G,R
6) R,B,R,G,R,R,G,B,G,G

e) Motivate why you chose the pairings as you did.

f) Between the zero-conflict configurations of B and C lay a number of colourings with terrible fitness scores. Do you expect these colourings to look anything like B or C? Motivate your answer.

g) Which algorithm would you use to colour Graph1? Which not?

h) Which algorithm would you use to colour Graph2? Which not?

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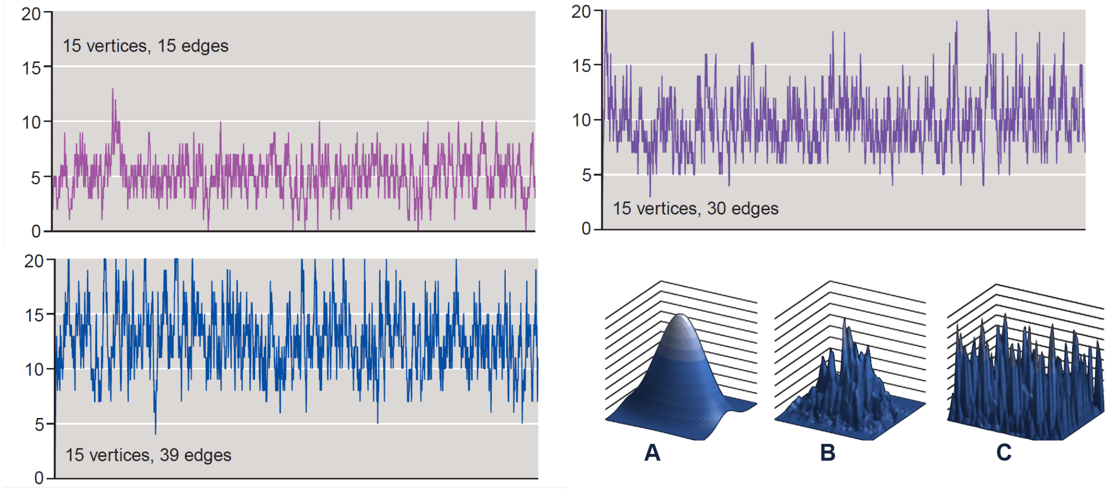
3. Graph* Colouring & Fitness Landscapes

Below are the three random walks from Brian Hayes' article. Bottom right are some fitness landscapes of graphs with 15 vertices.



a) What is a random walk?

b) In what respect is a random walk similar/different from random sampling?

c) Which random walk touches the most solutions? How come?

d) Which random walk touches the most conflictuous colourings? How come?

* *The term 'graph' is intentionally avoided because of its ambiguity in english.*

4. Magnetic spins

a) Explain how interaction between spins can be viewed as message passing.

b) (P.16) "The trouble is ... into a stable state". Explain why this is 'trouble'.

c) What other trouble should this remind you of? I mean algorithmic optimizational trouble, and it again has a lot to do with the physical properties of metal.

d) What way do physicists have to solve this problem? Explain it. Name both terms used for this method.

e) How could we the physicists' solution method in heuristics?

5. Bugs

a) What is sizewisely remarkable about the method in this paper?

b) What destinquishes random graphs from non-random graphs?

c) Given two or graphs, how could you determine which is more random?