

$$\begin{aligned}
m\dot{v}_x &= e[v_y B_z(y)] \\
m\dot{v}_y &= e[-v_x B_z(y) + E]
\end{aligned}$$

$$\begin{aligned}
B_z(y) &= B_0 + \beta y \\
\dot{v}_x &= 0
\end{aligned}$$

$$\dot{v}_y = \frac{e}{m}[-v_x(B_0 + \beta y) + E]$$

$$\begin{aligned}
v_x &= \sqrt{(v + dv)^2 \cos\left(\frac{d\theta}{2}\right)^2 - \frac{eBvd_{\text{tot}}d_{\text{w1}} \sin\left(\frac{d\theta}{2}\right)}{2md_{\text{w}}}} \\
&\approx v + dv - \frac{e}{m} \frac{B}{8} \frac{d_{\text{tot}}d_{\text{w1}}}{d_{\text{w}}} d\theta \\
\alpha &= \frac{e}{m} \frac{B}{8} \frac{d_{\text{tot}}d_{\text{w1}}}{d_{\text{w}}} \\
v_x &= v + dv - \alpha d\theta \\
\dot{v}_y &= \frac{e}{m}[-(v + (dv - \alpha d\theta))(B_0 + \beta y) + E] \\
\dot{v}_y &= \frac{e}{m}[-(v\cancel{B_0} + (dv - \alpha d\theta)B_0 + v\beta y + \cancel{(dv - \alpha d\theta)\beta y}) + E] \\
\dot{v}_y &= -\frac{e}{m}[(dv - \alpha d\theta)B_0 + v\beta y]
\end{aligned}$$

$$\begin{aligned}
y''[t] &= -\frac{e}{m} (v\beta y[t] + (dv - \alpha d\theta)B) \\
y'[0] &= (v + dv) \sin \frac{d\theta}{2} \\
y[0] &= d_{\text{w1}} \sin \frac{d\theta}{2} + \frac{mdv}{eB} \\
x''[t] &= 0 \\
x[0] &= 0 \\
x'[0] &= v_{\text{xwf}}
\end{aligned}$$