To determine the velocity that the electron has inside the Wien Filter I used energy conservation

$$E_i = E_f \tag{1}$$

$$\frac{1}{2}m\left(v_0 + \Delta v\right)^2 = -Eq\Delta y + \frac{1}{2}m\left(v_0 + \Delta v + \Delta v_{wf}\right)^2 \tag{2}$$

$$= -\frac{4Em\Delta v}{B} + \frac{1}{2}m\left(v_0 + \Delta v + \Delta v_{wf}\right)^2 \tag{3}$$

We have chosen E and B such that

$$E = -\frac{d_{tot}}{d_w}\frac{Bv_0}{4} \tag{4}$$

We combine 3 and 4 to get

$$\frac{1}{2}m\left(v_0 + \Delta v\right)^2 = \frac{d_{tot}}{d_w}m\Delta v v_0 + \frac{1}{2}m(v_0 + \Delta v + \Delta v_{wf})^2 \tag{5}$$

$$\left(v_0 + \Delta v\right)^2 = 2\frac{d_{tot}}{d_w}\Delta v v_0 + \left(v_0 + \Delta v + \Delta v_{wf}\right)^2 \tag{6}$$

$$-2\frac{d_{tot}}{d_w}\Delta v v_0 + \left(v_0 + \Delta v\right)^2 = \left(v_0 + \Delta v + \Delta v_{wf}\right)^2 \tag{7}$$

$$\sqrt{-2\frac{d_{tot}}{d_w}\Delta v v_0 + \left(v_0 + \Delta v\right)^2} = v_0 + \Delta v + \Delta v_{wf} \tag{8}$$

$$\sqrt{\left(v_0 + \Delta v\right)^2 - 2\frac{d_{tot}}{d_w}\Delta v v_0} - v_0 - \Delta v = \Delta v_{wf} \tag{9}$$

From Martin's analysis, the equation for Δv_{wf} is

$$\Delta v_{wf} = \frac{4E\Delta v}{Bv_0} \tag{10}$$

using (4) as above gives

$$\Delta v_{wf} = -\frac{d_{tot}}{d_w}\Delta v \tag{11}$$

Both (9) and (11) depend on the ratio $\frac{d_{tot}}{d_w}$ and Δv. Since (11) is the ideal case, this gives us a pulse width of 0. By beforming a first order approximation on (9) gave back equation (11). To determine the error, I will compare a second order approximation to the first order.

The second order approximation of (9) is:

$$\Delta v_{wf} = \sqrt{(v_0 + \Delta v)^2 + 2\frac{d_{tot}}{d_w}\Delta v v_0} - v_0 - \Delta v \tag{12}$$

$$= (v_0 + \Delta v)\sqrt{1 + 2\frac{d_{tot}}{d_w}\frac{\Delta v v_0}{(v_0 + \Delta v)^2}} - v_0 - \Delta v \tag{13}$$

$$= (v_0 + \Delta v)\left[\sqrt{1 + 2\frac{d_{tot}}{d_w}\frac{\Delta v v_0}{(v_0 + \Delta v)^2}} - 1\right] \tag{14}$$

$$= (v_0 + \Delta v)\left[1 + \frac{d_{tot}}{d_w}\frac{\Delta v v_0}{(v_0 + \Delta v)^2} - \frac{1}{2}\left(\frac{d_{tot}}{d_w}\frac{\Delta v v_0}{(v_0 + \Delta v)^2}\right)^2 - 1\right] \tag{15}$$

$$= (v_0 + \Delta v)\left[\frac{d_{tot}}{d_w}\frac{\Delta v v_0}{(v_0 + \Delta v)^2} - \frac{1}{2}\left(\frac{d_{tot}}{d_w}\right)^2\frac{\Delta v^2 v_0^2}{(v_0 + \Delta v)^4}\right] \tag{16}$$

$$= \left[\frac{d_{tot}}{d_w}\frac{\Delta v v_0 (v_0 + \Delta v)}{(v_0 + \Delta v)^2} - \frac{1}{2}\left(\frac{d_{tot}}{d_w}\right)^2\frac{\Delta v^2 v_0^2 (v_0 + \Delta v)}{(v_0 + \Delta v)^4}\right] \tag{17}$$

$$= \left[\frac{d_{tot}}{d_w}\frac{\Delta v v_0}{v_0 + \Delta v} - \frac{1}{2}\left(\frac{d_{tot}}{d_w}\right)^2\frac{\Delta v^2 v_0^2}{(v_0 + \Delta v)^3}\right] \tag{18}$$

We can expand $(v_0 + \Delta v)^3$ to:

$$v_0^3 + 3v_0^2\Delta v + 3v_0\Delta v^2 + \cancelto{0}{\Delta v^3} \approx v_0^3 + 3v_0^2\Delta v + 3v_0\Delta v^2 \tag{19}$$

Substituing back into (18)

$$\Delta v_{wf} = \left[\frac{d_{tot}}{d_w}\frac{\Delta v v_0}{v_0 + \Delta v} - \frac{1}{2}\left(\frac{d_{tot}}{d_w}\right)^2\frac{\Delta v^2 v_0^2}{v_0^3 + 3v_0^2\Delta v + 3v_0\Delta v^2}\right] \tag{20}$$

$$= \left[\frac{d_{tot}}{d_w}\frac{\Delta v}{1 + \frac{\Delta v}{v_0}} - \frac{1}{2}\left(\frac{d_{tot}}{d_w}\right)^2\frac{\Delta v^2 v_0^2}{v_0^3\left(1 + 3\frac{\Delta v}{v_0} + 3\frac{\Delta v^2}{v_0^2}\right)}\right] \tag{21}$$

$$= \left[\frac{d_{tot}}{d_w}\frac{\Delta v}{1 + \frac{\Delta v}{v_0}} - \frac{1}{2}\left(\frac{d_{tot}}{d_w}\right)^2\frac{\Delta v\frac{\Delta v}{v_0}}{1 + 3\frac{\Delta v}{v_0} + 3\frac{\Delta v^2}{v_0^2}}\right] \tag{22}$$

I performed another second order approximation on this function and which resulted with

$$\Delta v_{wf} = -\frac{d_{tot}}{d_w}\Delta v - \frac{1}{2}\frac{\Delta v^2}{v_0}\frac{d_{tot}}{d_w}\left(\frac{d_{tot}}{d_w} - 2\right) \tag{23}$$

The difference in the ideal velocity change and the actual is Δv_{err}

$$\Delta v_{err} = -\frac{1}{2}\frac{\Delta v^2}{v_0}\frac{d_{tot}}{d_w}\left(\frac{d_{tot}}{d_w} - 2\right) \tag{24}$$

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The time of flight difference is:

$$\Delta t_{err} = d_w \frac{\Delta v_{err}}{v_0^2} \tag{25}$$

$$\Delta t_{err} = d_w \frac{-\frac{1}{2}\frac{\Delta v^2}{v_0}\frac{d_{tot}}{d_w}\left(\frac{d_{tot}}{d_w} - 2\right)}{v_0^2} \tag{26}$$

$$\Delta t_{err} = \frac{1}{2}\frac{\Delta v^2}{v_0^3}d_{tot}\left(2 - \frac{d_{tot}}{d_w}\right) \tag{27}$$

I've realized that $\left(2 - \frac{d_{tot}}{d_w}\right)$ doesn't make sense so I'll go back to (20)