To determine the velocity that the electron has inside the Wien Filter I used energy conservation

$$E_i = E_f \tag{1}$$

$$\frac{1}{2}m(v_0 + \Delta v)^2 = -Eq\Delta y + \frac{1}{2}m(v_0 + \Delta v + \Delta v_{wf})^2$$
(2)

$$= -\frac{4Em\Delta v}{B} + \frac{1}{2}m\left(v_0 + \Delta v + \Delta v_{wf}\right)^2 \tag{3}$$

We have chosen E and B such that

$$E = -\frac{d_{tot}}{d_w} \frac{Bv_0}{4} \tag{4}$$

We combine 3 and 4 to get

$$\frac{1}{2}m(v_0 + \Delta v)^2 = \frac{d_{tot}}{d_w}m\Delta v v_0 + \frac{1}{2}m(v_0 + \Delta v + \Delta v_{wf})^2$$
 (5)

$$(v_0 + \Delta v)^2 = 2\frac{d_{tot}}{d_w} \Delta v v_0 + (v_0 + \Delta v + \Delta v_{wf})^2$$
(6)

$$-2\frac{d_{tot}}{d_{vv}}\Delta v v_0 + (v_0 + \Delta v)^2 = (v_0 + \Delta v + \Delta v_{wf})^2$$
(7)

$$\sqrt{-2\frac{d_{tot}}{d_w}\Delta v v_0 + (v_0 + \Delta v)^2} = v_0 + \Delta v + \Delta v_{wf}$$
(8)

$$\sqrt{\left(v_0 + \Delta v\right)^2 - 2\frac{d_{tot}}{d_{vv}}\Delta v v_0 - v_0 - \Delta v} = \Delta v_{wf} \tag{9}$$

From Martin's analysis, the equation for Δv_{wf} is

$$\Delta v_{wf} = \frac{4E\Delta v}{Bv_0} \tag{10}$$

using (4) as above gives

$$\Delta v_{wf} = -\frac{d_{tot}}{d_{w}} \Delta v \tag{11}$$

Both (9) and (11) depend on the ratio $\frac{d_{tot}}{d_w}$ and Δv . Since (11) is the ideal case, this gives us a pulse width of 0. By beforming a first order approximation on (9) gave back equation (11). To determine the error, I will compare a second order approximation to the first order.

The second order approximation of (9) is:

$$\Delta v_{wf} = \sqrt{\left(v_0 + \Delta v\right)^2 + 2\frac{d_{tot}}{d_w}\Delta v v_0} - v_0 - \Delta v \tag{12}$$

$$= (v_0 + \Delta v) \sqrt{1 + 2\frac{d_{tot}}{d_w} \frac{\Delta v v_0}{(v_0 + \Delta v)^2}} - v_0 - \Delta v$$
(13)

$$= (v_0 + \Delta v) \left[\sqrt{1 + 2 \frac{d_{tot}}{d_w} \frac{\Delta v v_0}{(v_0 + \Delta v)^2}} - 1 \right]$$
 (14)

$$= (v_0 + \Delta v) \left[1 + \frac{d_{tot}}{d_w} \frac{\Delta v v_0}{(v_0 + \Delta v)^2} - \frac{1}{2} \left(\frac{d_{tot}}{d_w} \frac{\Delta v v_0}{(v_0 + \Delta v)^2} \right)^2 - 1 \right]$$
 (15)

$$= (v_0 + \Delta v) \left[\frac{d_{tot}}{d_w} \frac{\Delta v v_0}{(v_0 + \Delta v)^2} - \frac{1}{2} \left(\frac{d_{tot}}{d_w} \right)^2 \frac{\Delta v^2 v_0^2}{(v_0 + \Delta v)^4} \right]$$
(16)

$$= \left[\frac{d_{tot}}{d_w} \frac{\Delta v v_0 (v_0 + \Delta v)}{(v_0 + \Delta v)^2} - \frac{1}{2} \left(\frac{d_{tot}}{d_w} \right)^2 \frac{\Delta v^2 v_0^2 (v_0 + \Delta v)}{(v_0 + \Delta v)^4} \right]$$
(17)

$$= \left[\frac{d_{tot}}{d_w} \frac{\Delta v v_0}{v_0 + \Delta v} - \frac{1}{2} \left(\frac{d_{tot}}{d_w} \right)^2 \frac{\Delta v^2 v_0^2}{(v_0 + \Delta v)^3} \right]$$
 (18)

We can expand $(v_0 + \Delta v)^3$ to:

$$v_0^3 + 3v_0^2 \Delta v + 3v_0 \Delta v^2 + \Delta v^3 \approx v_0^3 + 3v_0^2 \Delta v + 3v_0 \Delta v^2$$
(19)

Substituing back into (18)

$$\Delta v_{wf} = \left[\frac{d_{tot}}{d_w} \frac{\Delta v v_0}{v_0 + \Delta v} - \frac{1}{2} \left(\frac{d_{tot}}{d_w} \right)^2 \frac{\Delta v^2 v_0^2}{v_0^3 + 3v_0^2 \Delta v + 3v_0 \Delta v^2} \right]$$
(20)

$$= \left[\frac{d_{tot}}{d_w} \frac{\Delta v}{1 + \frac{\Delta v}{v_0}} - \frac{1}{2} \left(\frac{d_{tot}}{d_w} \right)^2 \frac{\Delta v^2 v_0^2}{v_0^3 \left(1 + 3\frac{\Delta v}{v_0} + 3\frac{\Delta v^2}{v_0^2} \right)} \right]$$
(21)

$$= \left[\frac{d_{tot}}{d_w} \frac{\Delta v}{1 + \frac{\Delta v}{v_0}} - \frac{1}{2} \left(\frac{d_{tot}}{d_w} \right)^2 \frac{\Delta v \frac{\Delta v}{v_0}}{1 + 3 \frac{\Delta v}{v_0} + 3 \frac{\Delta v^2}{v_0^2}} \right]$$
(22)

I performed another second order approximation on this function and which resulted with

$$\Delta v_{wf} = -\frac{d_{tot}}{d_w} \Delta v - \frac{1}{2} \frac{\Delta v^2}{v_0} \frac{d_{tot}}{d_w} \left(\frac{d_{tot}}{d_w} - 2 \right)$$
(23)

The difference in the ideal velocity change and the actual is Δv_{err}

$$\Delta v_{err} = -\frac{1}{2} \frac{\Delta v^2}{v_0} \frac{d_{tot}}{d_w} \left(\frac{d_{tot}}{d_w} - 2 \right) \tag{24}$$

The time of flight difference is:

$$\Delta t_{err} = d_w \frac{\Delta v_{err}}{v_0^2} \tag{25}$$

$$\Delta t_{err} = d_w \frac{-\frac{1}{2} \frac{\Delta v^2}{v_0} \frac{d_{tot}}{d_w} \left(\frac{d_{tot}}{d_w} - 2\right)}{v_0^2}$$

$$\tag{26}$$

$$\Delta t_{err} = \frac{1}{2} \frac{\Delta v^2}{v_0^3} d_{tot} \left(2 - \frac{d_{tot}}{d_w} \right) \tag{27}$$

I've realized that $\left(2-\frac{d_{tot}}{d_w}\right)$ doesn't make sense so I'll go back to (20)